Gas Dynamics and Propulsion
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Lecture - 15
Quasi One Dimensional Flows

In the previous class, we derived the area Mach number relationship for flow through a varying area passage.

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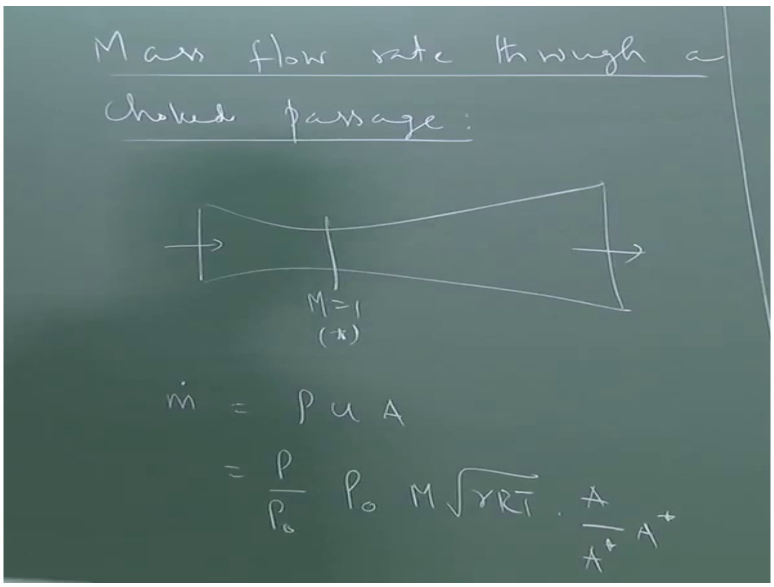

In the next module, we are going to look at mass flow rate through a choked passage. So here we are considering a passage in which the flow is choked at some location, so that means the flow has reached the sonicstate at some location in the middle. For example, this can be convergent divergent nozzle which looks like this, and we can say that the flow is choked at this location, remember we showed earlier through the area velocity relationship that in such a passage, If the flow chokes it must do so at the point where the area of cross section is minimum okay.

So for example in this case we may reach the sonicstate here, and once the sonicstate is rich we wish to find out what is the mass flow rate through this, if the sonicstate is not reached then there are other ways of finding out mass flow rate, but for most applications we are interested in the mass flow rate when the passage is choked, because the sonicstate is reached at the throat, the flow through the nozzle is said to be chocked.

So the flow through the nozzle is said to be choked, and we wish to derive an expression for the mass flow rate through the nozzle or the passage. So I start by writing my m dot, m dot is nothing but at any section m dot is rho times u times area at the section, and if I write this in terms of known quantities for example I can multiply and divide by rho 0 and write this as rho/rho 0 times rho 0.

And the u I can write as Mach number times square root of gamma R T speed of sound, and the A I can write like this A/A star times A star since the flow is choked I can write this as A/the cross-sectional area that the flow choke times A star right, so A star is the location where the Mach number reaches 1. Now I can take this static temperature and write that as T/T0 times T0.

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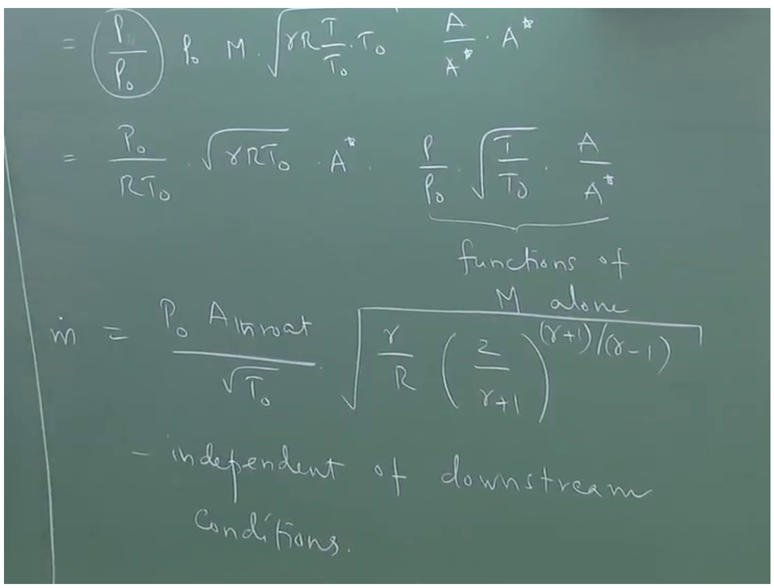

If I expand this equation further I can write this as I am sorry rho/rho 0 times rho 0 times M times square root of gamma R T/T0 times T0 all within the square root times A/A star times A star. Notice that rho/rho 0 is known to us as a function of Mach number right, we know the relationship T/T0 is also known to us as a function of Mach number, A/A star is also known to us as a function of Mach number because that is what we derived in the previous class area Mach number relationship.

So if you rearrange this expression I can write the rho 0 as P0/R T0, so let us go ahead and do that P0/R T 0 times square root of gamma R T0 times A star times rho/rho 0 times square root of T/T0 times A/A star right, these 3 quantities are functions of Mach number alone. So if I go ahead and substitute these functions and then simplify I end up with the expression for m dot which looks like this, so m dot=P0.

Remember we said using our area velocity relationship that if the flow chokes it must do so at a point of minimum cross-sectional area right, so I can replace this A star with A throat and so I replace the area A star with A throat, so P0 times A throat/square root of T0 times we have another quantity here which we write like this gamma/R times 2/gamma+1 to the power gamma+1/gamma-1, so this is expression for mass flow rate through a choked passage okay.

The expression that we have derived here is an extremely important expression in gas dynamics, this appears in many, many applications. In fact, this controls the operation of many devices which handle compressible flow okay, the probably one of the most important or most remarkable think about this expression is what? The most important or most startling aspect of this expression is that no exit quantities or exert pressure is involved in this right.

This depends only on the upstream stagnation pressure, throat area and the quantity inside this depends only on the nature of the gas right that is the most remarkable think about this. So what this tells you is that once the flow is choked we cannot control the flow by adjusting the downstream condition, so once the flow is choked I can adjust conditions or mass flow rate only by adjusting these quantities.

For example, I can change the throat area, I can change my stagnation pressure, or I can change my stagnation temperature, but not any of the downstream quantities that is the reason why we said that the sonicstate is very important state. Remember we achieved a sonicstate here, so what this tells me is once I achieve the sonicstate here I cannot control the mass flow rate by changing things downstream, the flow will be completely oblivious to whatever changes that are taking place here.

If I want to change the mass flow rate once it is changed, I need to change the stagnation pressure here or the stagnation temperature, or I can also change the throat area okay. So let us write that down, so this is independent to put it in a slightly different way, if for example I am trying to control the flow by adjusting conditions downstream, I can do that up to a point I can continue to do that up to a point the point being attainment of the sonicstate.

As long as I do not attain the sonicstate I can control the mass flow rate by adjusting conditions downstream, but once I attain the sonicstate at the throat I can no longer control the flow by changing conditions downstream that is what this says. In other words, this is the maximum mass flow rate that can be achieved by controlling the downstream state, this is not the maximum mass flow rate, this is the maximum mass flow rate that can be achieved by controlling downstream.

"Professor - student conversation starts" yes, Sir by changing downstream area we can change mass flow rate? That does not appear here right, I mean if you change this area then the flow is not choked right, then you will see exit quantities appearing here, if the flow is not choked for the same situation if the flow is not choked then we need to specify the downstream static pressure also, and we can control the downstream static pressure to get the mass flow rate that is also possible correct **"Professor - student conversation ends."**

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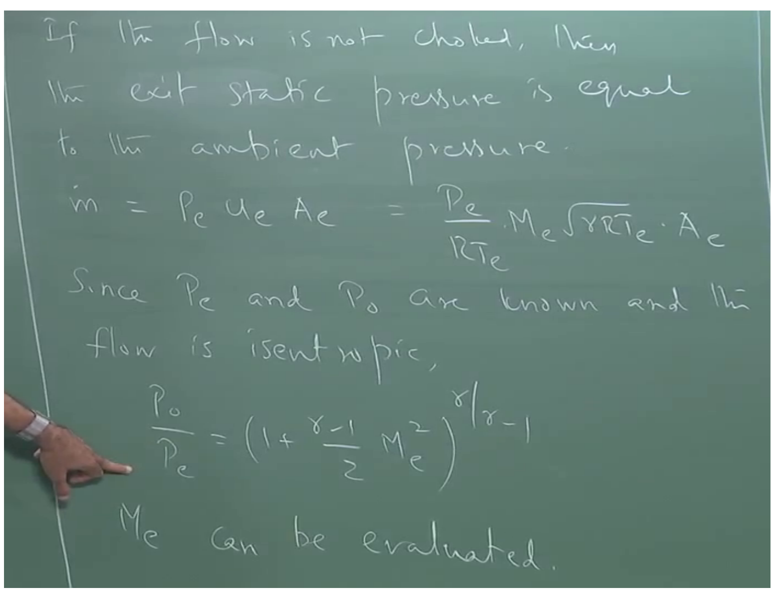

If for example, you look at that situation, so if the flow is not choked then the downstream static pressure now I can write this as exit static pressure. We are going to discuss these concepts next but we can discuss them now that is not a problem, the exit static pressure is equal to the ambient pressure right. Then mass flow rate through the nozzle in this case can be written as m dot=rho e times ue times Ae where the subscript e refers to exit conditions right.

We can rewrite this in the same manner as we did earlier, I can write this as Pe/R Te and ue itself can be written as Me times square root of gamma R Te and I can leave Ae as it is exit area is known okay, since the exit pressure is known, stagnation pressure is also known and the flow is isentropic right. Since Pe and P0 are known and the flow is isentropic, this means I can get my exit Mach number by using the isentropic relationship right.

So this means that P0/Pe=1+gamma-1/2 times Me square raise to the power gamma/gamma-1 right, P0 is known, Pe is known, so I can calculate Me, once I know Me exit static temperature can also be calculated right T0 is known right. So for example so from this I can calculate Me can be evaluated once Me is known other things can be evaluated.

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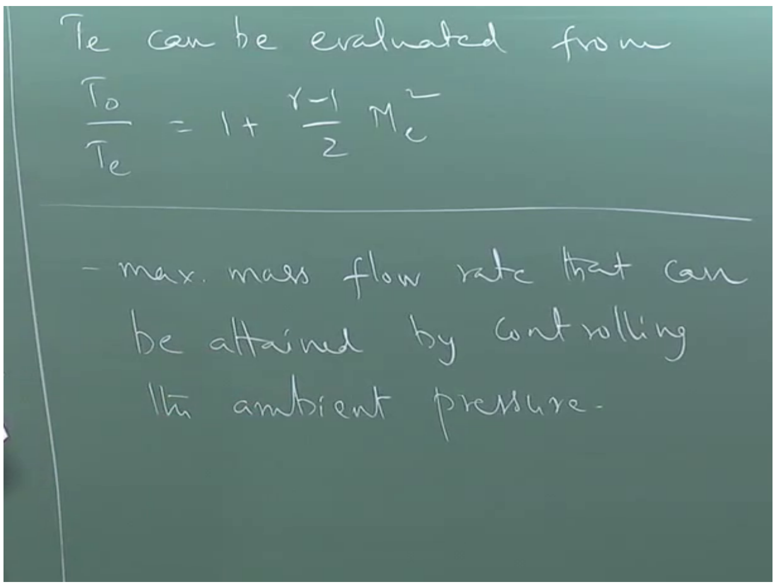

So Te can be evaluated from this relationship T0/Te=1+gamma-1/2 times Me square, T0 is known, so Te can be evaluated. And once I know that, notice that I can now evaluate the mass flow rate through the nozzle, this is if the flow is not choked. Once the flow is choked the exit pressure is no longer sensed by the flow, which is why? We go to this type of situation okay, so once it is choked any further change in mass flow rate has to be achieved only by changing these quantities okay.

So the exit pressure appears in all these equation, notice that the exit pressure is the key here, because we use the exit pressure to get Me, once I get Me all the other quantities can be evaluated. Once the exit pressure disappears then I no longer have control over the flow from the downstream side right. So let us if you come back to this expression, now what we are going to say now for this expression is that this m dot is the maximum mass flow rate that can be attained by controlling the ambient pressure.

This is probably one of the most poorly understood concepts in Quasi one dimensional flow, normally it is said that this is the maximum mass flow rate that we can have in the nozzle that is not true, this is the maximum mass flow rate you can have if you are trying to adjust the flow from downstream. Obviously, if you look at this expression you can easily see, if you look at this expression you can easily see that even after if it is choked.

Even after the flow is shocked if I increase my P0 I can get more mass flow rate, or if I decrease my T0 I will get more mass flow rate, so the maximum mass flow rate that can go through a nozzle what is the limit on that? Unlimited there is no limit that the mass flow rate that a nozzle can pass, if I am changing P0 and T0. If I am changing Pe then this is the maximum that I can achieve that is a very important point.

So this is the maximum mass flow rate that can be attained by controlling downstream pressure. If I am controlling stagnation pressure and temperature I can have any mass flow rate that I want, but some other adjustments will take place, which is what we are going to discuss next okay. And you also remember from our discussion of normal shocks, flow with heat addition and flow with friction right.

If there is a loss of stagnation pressure upstream or the nozzle then the mass flow rate that can go through the nozzle is also reduced right, if there is a pipe long pipe then there is a loss of stagnation pressure which means m dot become less than what it can be. In fact, if there is a heat addition before the nozzle that is even worse because heat addition, what is heat addition do? It increases T0 and it decreases P0.

So you can see that this increases which reduces the mass flow rate this reduces which also reduces the mass flow rate even more right, so heat addition add of the nozzle is actually very I will not say detrimental effect causes the mass flow rate that can go through the nozzle tremendously. This kind of application is important, as we will see later when we talk about after burners, in after burners you heat before it goes through the nozzle.

So this is an extremely important point for after burner design, the reduction in mass flow rate is much higher if you add heat before the nozzle, because of the double effect, this increases and this also decreases okay. We will actually have an opportunity to revisit this expression for mass flow rate many times during our discussion as we go along, this is an extremely important discussion, because what this tells me is that you know that this is the mass flow rate that is going through.

And in some cases when you have multiple throats this expression will determine what flow solution are possible downstream, for instance we said that you know the nozzle is insensitive to downstream conditions as they keep changing downstream conditions, this kind of thing mass flow keeps coming through, at some point the imbalance maybe so much that you can have very strong changes taking place in the flow field okay, that also is something that we will discuss as we go along okay.

This is an extremely important expression which you must remember, if not the functional form if not the exact form you must definitely remember the functional form P0 A throat/square root of T0 okay alright.

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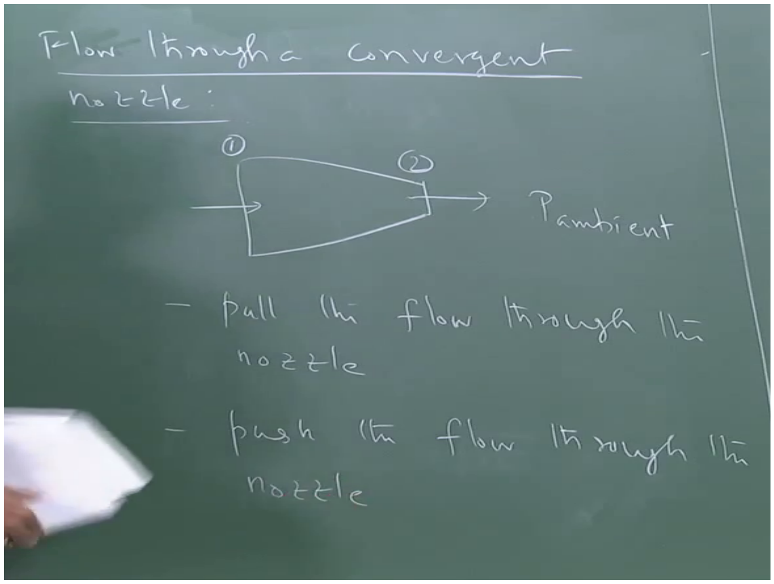

Now we are going to look at flow through a convergent nozzle, so we have let us say a convergent nozzle like this, let us label this inlet as 1, this outlet as 2, and let us say that the ambient we say that the ambient pressure is P ambient, and the stagnation conditions are known. When you have a nozzle like this, we can establish a flow through the nozzle in 2 different ways, one way is to pull the flow through a nozzle.

So how do we pull the flow through nozzle? We keep changing the ambient pressure to pull the flow through the nozzle. The other way is to push the flow through nozzle, we do this by changing the stagnation conditions or the stagnation pressure keep increasing the stagnation pressure that will push more and more flow through the nozzle. Changing the ambient pressure will pull the flow through the nozzle okay.

These are the two different ways in which we can established the flow, let us see what happens as the flow goes through in each one of these cases, what is the change of state for example along the length of the nozzle as to do this okay that is what we are going to see, and we are going to actually illustrate this on a TS diagram.

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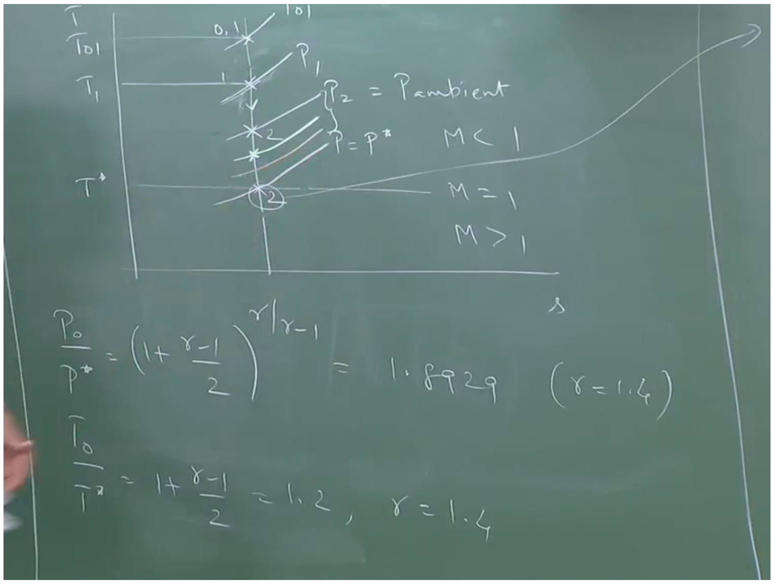

Let us start with the first one, so we start by pulling the flow through the nozzle, let us say that the inlet state is over here, let me do like this so this is 1, let us say this is T1, and this is P1, and let us say that this is my T01, and this is my P01, and this is my stagnation state 0, 1. So this is my inlet state, and I have shown this stagnation state also, now the ambient pressure in this case we are going to change the ambient pressure right.

So corresponding to this T01 I can evaluate a T star right, so let us evaluate a T star, let us say that this is my T star and you know that T star corresponds to M=1, anything below T star we know that it is a supersonic state and anything above you know is a substance state. And of course once I T star I also know my P=P star because this is the sonicstate right that is the sonicstate. So now what I do is let us say that I lower the ambient pressure a little bit <P1 okay.

So the ambient pressure here is just a little bit <P1, so the flow we establish a flow and when it comes out, it comes out with a certain velocity, and remember this is a converging passage Mach number is subsonic. So we know from the area velocity relationship that the velocity increases probably not to sonic value for this value of ambient pressure but the pressure decreases velocity increases temperature also decreases, so we probably will end up something like this.

So this is maybe our, so this value of ambient pressure our state 2 is going to be over there okay, so where we have lower the ambient pressure a little bit from the P1 right, so the flow happens like this, so you can see that the same stagnation condition u2 is going to be >u1 right, because this distance what is this distance? u2 square/2 CP right that is this distance, so u2 has increased pressure has decreased temperature is also decreased.

So it has gone through an expansion process isentropic expansion process from state 1 to state 2. Now if I lower the ambient pressure a little bit more right, I can get more flow to go through the nozzle, so let us say that I lower the ambient pressure something like this right, now my state 2 will come here, the flow accelerates some more exit velocity increases some more. So as I keep lowering the ambient pressure if I do this some more, then I get my ambient pressure to come to this value right P2 once again =P ambient, as I do this P2=P ambient in all this cases correct.

Now if I finally lower my P ambient to a value=P star right, we know the value for P star right, we know the isentropic relationship so P0/P star =1+gamma-/2 we said M=1, so this raised to the power gamma/gamma-1, and for gamma=1.4 this comes out to be 1.8929. So if I know my stagnation pressure and I lower my ambient pressure to a value equal to this, then the exit velocity reaches the speed of sound, and exit state becomes the sonicstate.

So the flow expands all the way from here to this state, so this becomes the exist state at this point, and the pressure =P star, the temperature is also=T star, remember T star can also be evaluated in the same way, so T star=1+gamma-1/2 right, so this is nothing but gamma+1/2 and this=1.2 for gamma=1.4. So at this point the exit pressure is still equal to ambient pressure and the ambient pressure is itself =P star corresponding to that value of stagnation pressure okay.

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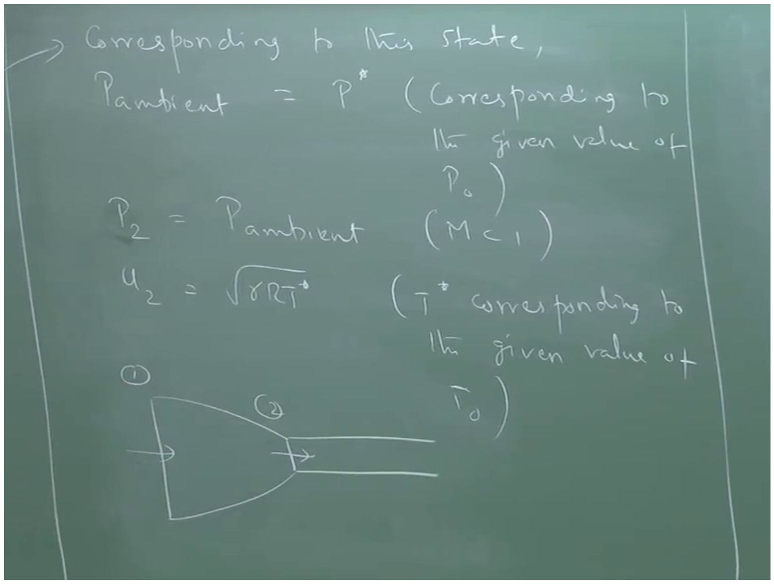

So when we attain this state, so corresponding to this state let me write it here so corresponding to this state, P ambient becomes=P star corresponding to the given value of P0 right, P0 remains constant throughout, we are changing the ambient pressure right, we are pulling the flow through the nozzle, so P0 remains constant. For the given value of P0 I calculate P star from this expression and when the ambient pressure reaches value equal to that I attain this state okay.

That is the most important point and the velocity at the exit let me finish this, so P ambient reaches this value, and P2 which is the exit pressure recharge is always=P ambient right. So as long as we are here on the subsonic person, the exit pressure is always equal to the ambient pressure okay, so this is always true when M<1 or just at the instant when you reach M=1, when the flow is subsonic the exit pressure is always equal to the ambient pressure.

And the exit velocity u2= gamma R T star, where the T star corresponds to given value of T0, so this is a critical state when we just reached the sonicstate from above okay. So in this case if you look at the flow that comes out of the nozzle, let us sketch this separately if I look at the flow that comes out of the nozzle, so this is the convergent nozzle state 1, state 2, so jet comes out of the nozzle and the pressure at the exit P2=the ambient pressure that means the jet comes out like this.

The diameter of the jet as it comes out=the exit diameter when the pressures are matched okay. Now we look at the situation when we lower the ambient pressure even more okay, when I lower the ambient pressure even more what will happen to this flow? The flow in the nozzle if I lower the ambient pressure some more right, let me sketch that separately here.

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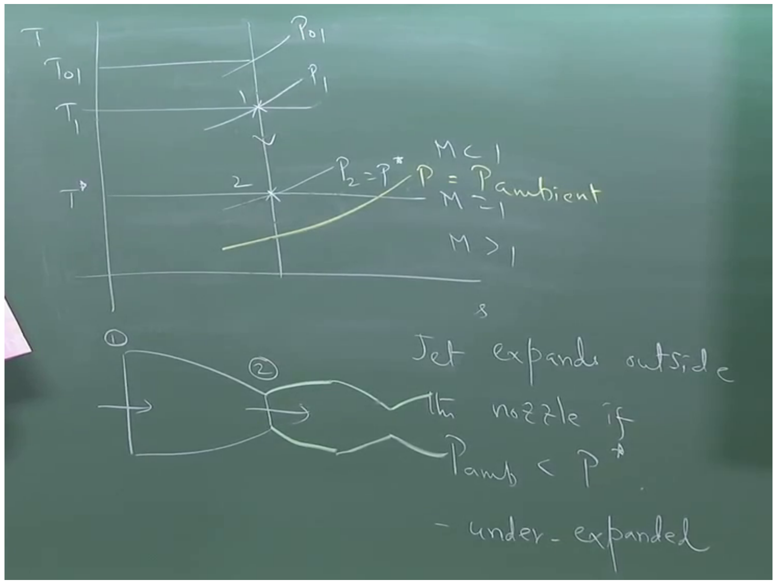

So we have attained that state right, so let us do it like this P1, T1, P01, T01, this is state 1, this is T star, this is my state 2, P2=P star. Now I have lowered my ambient pressure to a value even below this, for example let us say that this is my ambient pressure now, so the flow has expanded from state 1 to state 2 will it now expand to this state or will it remain the same? It will remain the same there is no scope for it to expand to this value, because then what would happen here if it expands to this state, what would be the Mach number? Supersonic.

So if the Mach number is supersonic here that means it must have gone through the sonicstate at some other cross-section right, so we are saying that the nozzle looks like this, so if the flow is supersonic at the exit right this is the exit state now, if it expands to this pressure this is the exit state. So if the Mach number is supersonic here that makes the sonicstate should have been accomplished somewhere here right this is < 1.

So M=1 should have come somewhere here which is not possible, because M=1 can occur only at the throat where dx= 0, so that means it is not possible for this flow to go further down, it will continue to stay here but the ambient pressure will continue to get lower, which is why we said that once the nozzle chokes there can be no change in the mass flow rate by controlling the flow from downstream.

However, one thing that happens is because when the jet comes out P2 is now>P ambient, P2>P ambient that means that jet is under expanded it is coming out at a pressure which is >ambient which means it must expand to match the ambient pressure that is why it is called under expanded right. So the jet expands outside the nozzle, if P ambient<P star okay so in this case the jet is said to be under expanded.

So what we mean by that expands outside? What is it mean when I say jet expands? What happens to the here we said that the diameter is the same as the exit diameter when the pressures are match. Now here when I say jet expands I literally mean expands, so the jet actually swells right, so the expansion process takes place like this, so jet actually swells when it comes out, but it cannot continue to do this you know the jet will expand and then it will shrink and then it will expand again.

So it will keep going like this until it finally equilibrates with the atmosphere or the ambient condition then the jet diameter will become like this, we will actually as we go along in course we will try to evaluate these angles that the jet will make, and the places where it contracts, we will look at the detailed structure of this jet as we go along, but this is what happens when you have a situation where the ambient pressure<P star.

Notice that you know we are talking about aircraft engines and thrust is what we are ultimately interested in, so when this jet expands against the atmosphere, that is not useful right, remember what we derived the expression for thrust. We derived 2 expressions, one was i2-i1, the other one was what? Integral PDA on the surfaces. So only when the jet or the fluid expands against the surface that results in useful thrust.

If it expands against the atmosphere that is not useful for me right that is lost right that is not of any use, so how will I make this thrust useful? What can I do? By putting a divergent section here that is what we will talk about next, if I put a divergent section here then the fluid can expand against that and produce useful thrust okay. So this is the sequence of events when we actually pull the flow through the nozzle.

So this is what we are doing when we pull the flow through nozzle, see any further reduction in pressure here we argued, we can argue this in many different ways any further reduction in pressure it is still not possible because of the conditions that we said from the area velocity relationship. The other physical way of stating this is if I reduce the pressure some more the flow at the exit of the nozzle is already travelling with the speed of sound.

Now reduction in pressure is trying to that information is trying to travel upstream, but this is already travelling at speed of sound this pressure signal is also travelling at the speed of sound with result that nothing gets passed this M=1 location, which is why the flow upstream of the nozzle is completely insensitive to any changed pressure conditions downstream that is a physical way of arguing this okay.

Of course keep in mind that as long as the pressure disturbances travel with speed of sound this is true, what if I make the pressure disturbances travelled with velocity greater than the speed of sound, I close the valves suddenly then the speed of propagation is actually much higher can be supersonic, if it is supersonic then definitely it can go upstream okay, as long as I do this gradually this all I can do.

But unsteady compressible flows are ready really not within the purview of this course okay, I am just telling you that it is possible but that is not something that we will look at okay alright. So what we are going to do next is look at the sequence of events that happens when we push the flow through a nozzle keeping the ambient pressure constant, remember this kind of scenario actually happens in real life you may wonder when can the ambient pressure change right.

The ambient pressure is always constant, so why are we talking about this, think about an aircraft engine you know that is what we are going to do next right, an aircraft engine takes off from sea level then it claims to an altitude of 35000 feet, definitely the ambient pressure changes right from sea level to 35000 feet. So this is not something which is contrived this is very real this happens all the times.

If you think about rocket is even worse, it takes off from sea level and goes up to vacuum almost vacuum in the outer space, so this is something that happens all the times so this is very very important, this is very real and very important. The next scenario that we are going to look at is pushing the flow through the nozzle by increasing the stagnation pressure for example okay.

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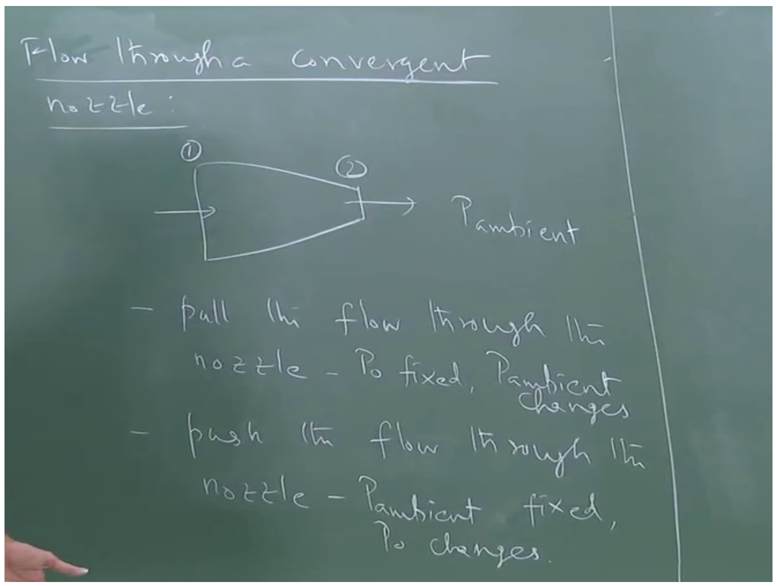

So we are going to look at the next set of sequence of events, when we push the flow through the nozzle, the difference between these 2 is that here P0 is fixed, P ambient changes right. Here, P ambient is fixed, P0 changes okay. We do not really look at situations where T0 changes you know that is tantamount to reducing T0 that is not something that we want to do, increasing P0 is not a problem right? when we run a compressor, we can get higher and higher P0, so that is what we are going to look at P0 changes or P0 increases P ambient remains fixed okay.

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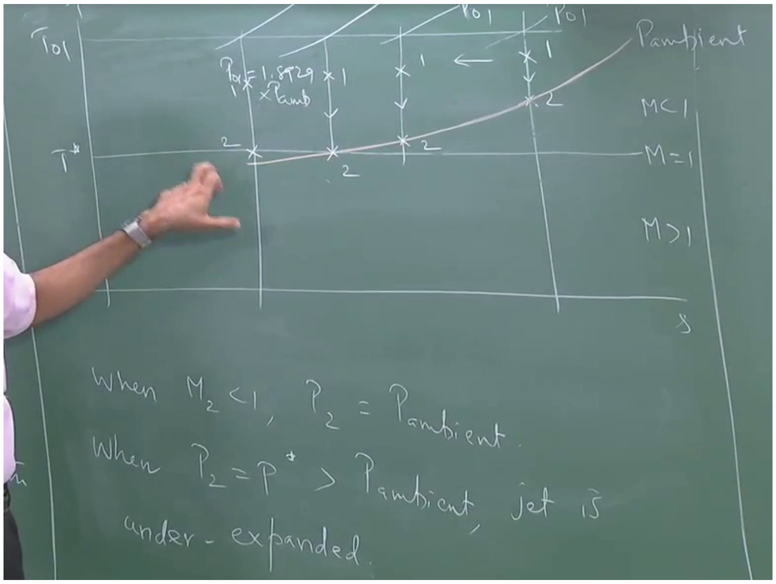

So let us look at the sequence of events on a TS diagram, so same way we have state 1 here and state 2 here, let us say that this is my T, and this is my T01, we will assume that T0 remains the same throughout there are no changes in T01. So this is my T star corresponding to M=1, this is M<1, this M>1, so let us say this is my state 1. For some value of P0 I am establishing flow, so the flow expands so this is my expansion from state 1 to state 2.

Remember the ambient pressure is fixed right, so when the Mach number is subsonic the exit pressure is always equal to the ambient pressure correct, when the-Mach number< 1 when M2<1 P2=P ambient right. So I can draw the isobar corresponding to P ambient like this, this is my isobar corresponding to P=P ambient, so you can see that the exit pressure P2=P ambient in this case. Now I increased the stagnation pressure right.

So what happens to the inlet state? Where do I show it? I increase the stagnation pressure, remember isobars diverge from each other, so when I increase the stagnation pressure I need to move to the left, so I need to move to the left so I operate at different value of inlet stagnation pressure P01, and so my inlet state maybe something like this, and my outlet state something like this, so I may have expansion like this.

So I have increased the stagnation pressure, so I am actually going this way but the exist state continuous to lie on the P ambient isobar, if I increase my stagnation pressure again to a value which we wrote down earlier P0/P star, so if I increase the ambient pressure to a value which is 1.8929 times the ambient pressure right. So if I increase my, so this value of P01=1.8929 times P ambient, so if I increase my P01 to that value then my state 1 is probably somewhere here, state 2 is here.

So we have attained the sonicstate at exit, M becomes=1 okay, so for example this P01 can be say 1.1 times P ambient, this is let us say 1.5 times P ambient, this is now 1.8929 times P ambient that is the critical case right, exist state M=1 right, the mass flow rate keeps increasing like this. Now what would happen if I increase the stagnation pressure to a value which is let us say even higher than this right?

So we are coming along in this direction, there is nothing that stops me from doing this I can increase the stagnation pressure here what would the state line look like in this case? What would the process line look like in this case? State 1 will continue to be here, what about state 2? Where will state to be? State 2 will still be over here this will be 2 and this will be my state 1 right, there will be no change state 2 will continue to be in the sonicstate.

But what happens to the pressure at step 2? Pressure is now higher compared to the ambient. In the previous case also we said that because we were lowering the ambient pressure we said that exit pressure>ambient pressure, so the jet was under expanded. In this case because we are increasing P0, the exit pressure is once again >ambient pressure and jet is under expanded. When one quantity so we take P2 and P ambient.

When we say P2 is >ambient that can be a result of P ambient decreasing or it can be result of P ambient remaining the same and P2 increasing right. So for example if the ambient pressure=let us say 100 kilopascal right, this could be something like 110 kilopascal, 120, 189.29, this could be let us say 200 kilopascal. So 200 kilopascal exit pressure will be >100, so the jet in this case will become under expanded right.

So when P2 which =P star becomes >P ambient jet is under expanded same as before, the only difference between this case and the other case is that the mass flow rate continuous to increase.

In the previous case once I attain this type of situation the process curve did not change it remain the same right.

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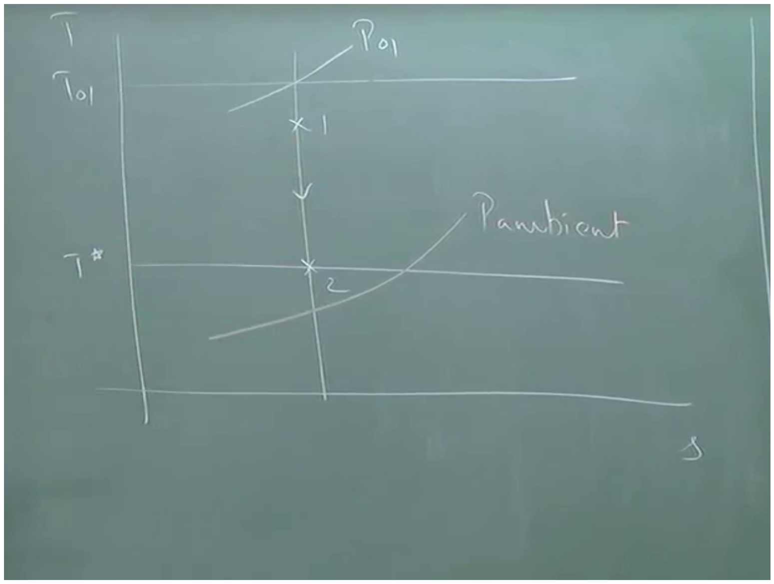

So if I draw the other curve for comparison, if you recall the previous case look like this, so this was the process curve when we attained the sonicstate at the exit, and if I lowered the P ambient, the P ambient curve looked like this right, so this was P ambient. So if I keep lowering the P ambient, the process curve does not change at all which means the mass flow rate remains the same. In contrast here as you can see the process curve keeps changing exit state is sonicstate.

But the process curve keeps changing, which is why the mass flow rate in this case reaches a maximum and remains there, whereas in this case the mass flow rate will continue to increase as I keep increasing this, this curve will keep sliding to the left and the mass flow rate will continue to increase, that is the biggest difference between this and that. Here, after attain the critical state the process curve remains the same I cannot change it anymore by lowering the ambient pressure.

Whereas here I can change the process curve by increasing P0 continuously at will, so mass flow rate can also be changed at will as I in keep increasing P0 that is the difference between these 2 okay. Any questions? What we will do in the next class is go through the same type of argument for a convergent divergent nozzle okay, what happens? What are the sequence of events when we pull a flow through a convergent divergent nozzle?