Gas Dynamics and Propulsion
Dr. Babu Viswanathan
Department of Mechanical Engineering
Indian Institute of Technology – Madras

Lecture - 14
Quasi One Dimensional Flows

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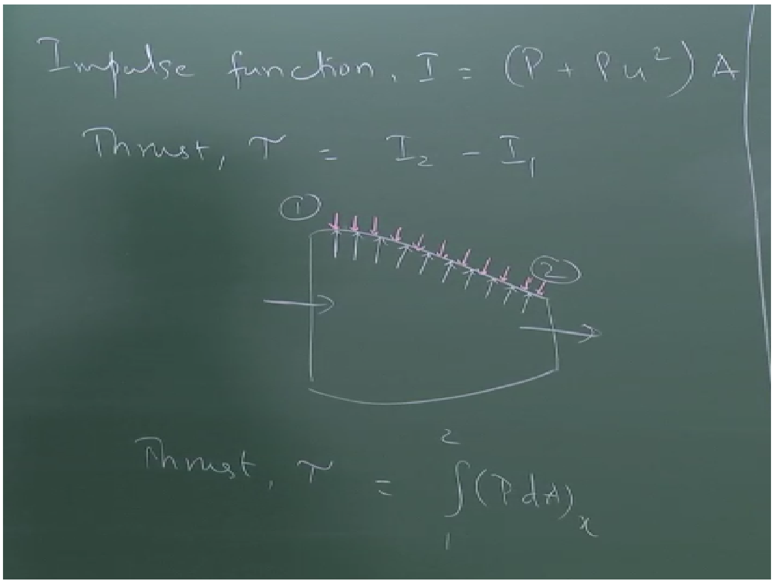

In the last class, we showed that the impulse function I, which was defined as P + rho u square*the cross sectional area on the stream tube at that location, can be used to calculate the thrust that is produced, thrust or drag that we get from different components, and the thrust could be evaluated as I2 - I1, where the state point 2 refers to the exit of the device, and state point 1 refers to the inlet of the device.

So if you look at convergence nozzle for example, let us say this is state 1 and this is state 2, then the net force acting on the nozzle can be evaluated by - 1 from I2. Now when we do this, in this particular case, we do not know whether the nozzle is going to produce thrust or whether the force is going to be in the opposite direction, we can determine this for example by using the fact that thrust is also =.

We showed yesterday, that thrust T should also be = integral 1 to 2 PdA with the x component being calculated for evolving the thrust. So when you do this if you look at this kind of a device as the fluid flows through, notice that the pressure forces are acting normal to the surface. So the pressure forces would act like this on both surfaces, remember this is actually the pressure force exerted on the walls.

So that is, it is very important remember that, and the pressure decreases as we go along the length of nozzle, because the flow undergoes an expansion process. So if you take the x component of the pressure force, on the top and the bottom side, you will then realize that in this case the net force that is exited on the nozzle is in the positive x direction, which means this is actually a drag force and not a thrust force.

So even if you evaluate I2 - I1, you should get the same thing. This is what we showed yesterday, when we showed the breakup of the thrust produced by different components of the air craft engine. Positive thrust was generated in the intake, compressor, diffuser and the combustor, and the thrust that was produced in the turbine and nozzle was actually negative, meaning that indicating that this was a drag force.

However, if you take an entire air craft engine and evaluate I at the exit of the engine - I at the inlet of the engine you will still produce positive thrust, otherwise the air craft will not fly. If you take the nozzle alone, and look at the forces, net force exerted here, then this is the way it will be. So it is important to realize that, however we need the nozzle, because we want to convert the enthalpy of the gas to kinetic energy. So that is still required.

So we cannot just because the force is acting on the nozzle in this direction, we cannot dispense with the nozzle, nozzle is still required. But the calculation of the thrust on net force acting on a component must be done as impulse function at the exit - impulse function at the outlet, or if you know the pressure it should be the integral of the PdA with the appropriate component being evaluated.

Solid rocket is having only convergence nozzle, then it will not be able to produce thrust? It would not be able to produce thrust. The solid rocket will not have a convergence nozzle part alone, because the stagnation pressure in a solid rocket motor, in a rocket engine stagnation pressure is very high. So a convergence nozzle will not be enough to actually, Diwali rocket and all that small stuff, toy rocket, if they are having only convergence nozzle.

They are designed slightly differently with enough surface to get an upward force from the rockets, if you look inside you will see that you know they have enough things to propel them upwards. We will take a closer look and at this expression and evaluate this for an air craft engine when we start discussing air craft engine. That is the next module of the course. We will rewrite this expression for an air craft engine where some simplifications can be made.

But what you must remember is that the pressure force, this is the internal pressure force that acts on the device. There is also an ambient pressure force which is exerted on the outside. So the net force is the difference between the 2, so if we show the ambient force like this, the ambient force is a constant value. So the ambient pressure force acts like this. So the net pressure force on the surface of the nozzle is this pressure - this pressure*by the area.

So its advantages to actually evaluate this pressure with reference to the ambient pressure at that location. So that I need not worry about this and just take this - this*by the area and that is an idea that we will use later on when we derive the expression for the thrust of the air craft engine. Measuring the static pressure with reference to the local ambient pressure and no the sea level ambient pressure.

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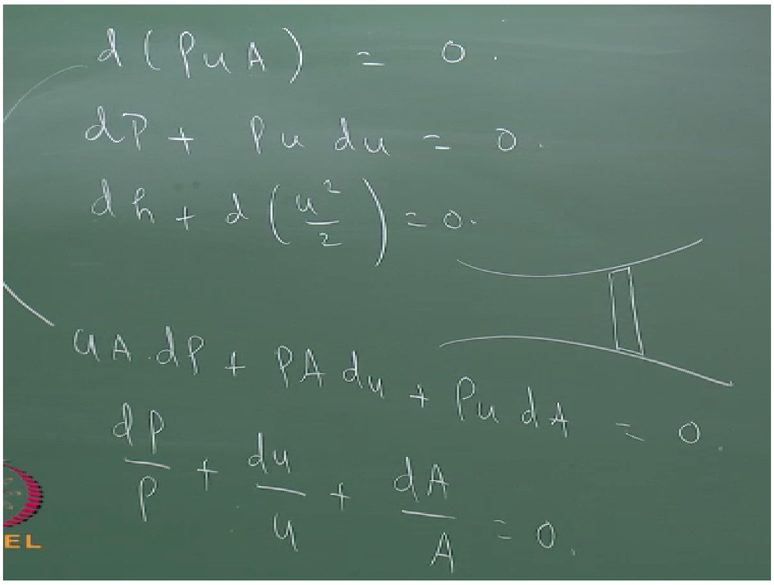

The next topic that we are going to take out is area velocity relationship, which is a very important relation when dealing with flow through varying area passages. So here we try to determine the relationship between changes in velocity and changes in cross sectional area. So what happens when the cross sectional area decreases, or what happens when the cross sectional area increases, that is what we are going to try to figure out.

So we start with the differential form of the continuity equation which can be written like this, and the differential form of the momentum equation can also be written like this dP +rho U dU = 0. Notice that the differential form of the continuity equation does not contain a term which is the integral of the pressure force on the walls, because that is a very small quantity. So if you think about a nozzle like this actually, this my differential control valve over which I am trying to, for which I am trying to write the momentum equation.

So I have pressure force which is acting in this direction, and P + dP*dA A+dA which is acting in this direction, that is a change of momentum. Notice that this is very small, so the integral of the pressure force on this wall will be 0, because I am integrating over an incremental distance, I am trying to evaluate an integral over an incremental distance which means it is = to 0.

That is why the integral term is not present in the differential form of the governing equation, whereas if I apply it between inlet and outlet, so the control volume goes from inlet to outlet, then I need to account for the net pressure force that acts on the surface. That is a very important distinction to keep in mind, and the differential form of the energy equation looks like this, so if I take this equation and differentiate things through.

Then I get the following, uA*d rho+rho A*du+rho u*dA = 0, and if I divide through by rho uA, then I can get this to be d rho/rho+du/u+dA/A = 0. Now we are looking for an area velocity relationship, which means a relationship must contain only changes in area, changes in velocity and area and velocity. So that means I would like to eliminate d rho/rho in favour of du/u, that is what we are going to do next.

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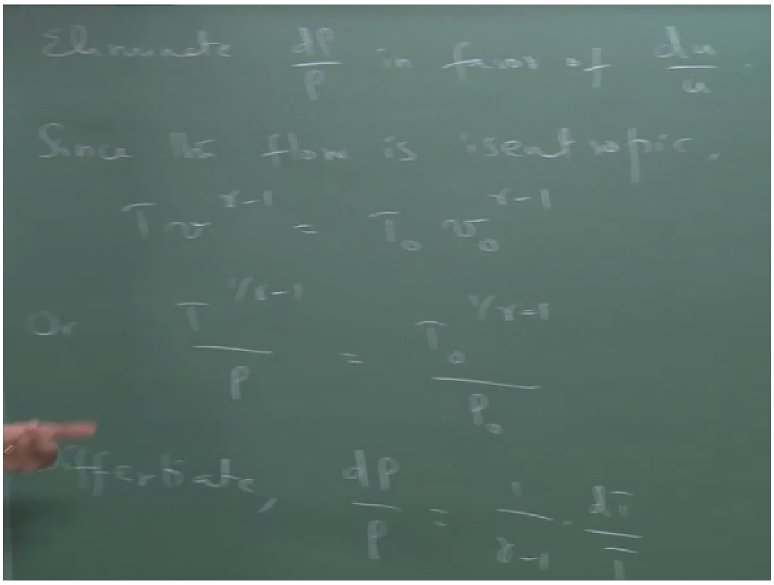

So we will eliminate, since we have the assumed the flow to be isentropic, I can write from my entropy equation, which says entropy is constant. I can write this as Tv to the power gamma-1 = T0v0 to the power gamma - 1 where T0 and v0 are the stagnation quantities or if I rewrite this in terms of density, I can actually write this as T to the power 1/gamma-1/rho=T0 to the power 1/gamma-1/rho 0, and if I differentiate this expression I get d rho/rho= 1/gamma-1*dT/T.

So we wanted to write d rho/rho in terms of du/u, we have managed to write d rho/rho in terms dT/T. Now we have to relate dT/T to du/u, changes in static temperature have to be related to changes in velocity, how do we do that? We use a definition of the stagnation temperature and the fact that stagnation temperature is a constant.

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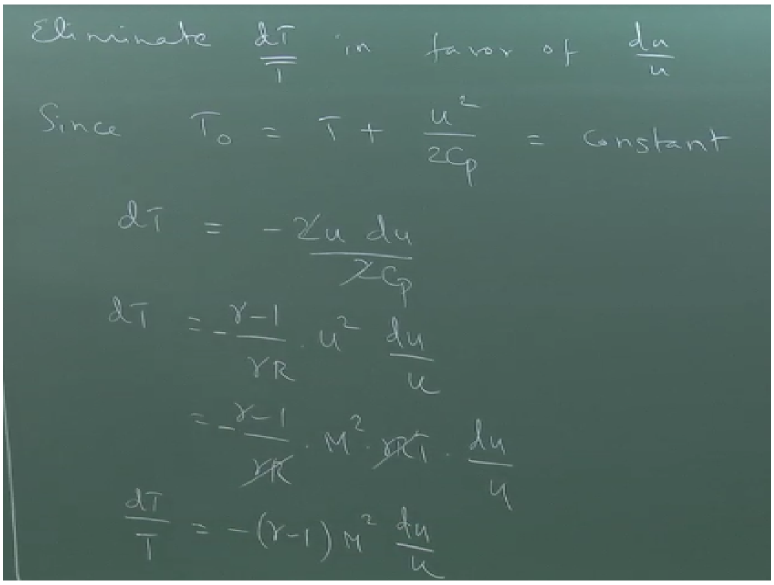

So we can write, so we eliminate dT/T in favour of du/u, since the stagnation temperature is T0=T+u square/2CP, and that is a constant, because flow is isentropic there is no heat addition or work addition, so this is constant. So if I take the differential on both sides I get dT=-2udu/2CP or I can also write this as dT=CP is gamma R/gamma-1, so I can write this as gamma-1/gamma R and if I*and / by a u, then I get this to be u square*du/u.

And if I use the fact that u is M*square root of gamma RT, I can write this as gamma-1/gamma R*u square then becomes m square*gamma RT du/u, so the gamma R cancels out and I end up with an expression where I have dT/T =, I am sorry I missed out a negative sign here, so there should be a negative sign here and there should be a negative sign here. So dT/T = - gamma-1*M square*du/u.

So I finally have, what I was looking for, I wanted to eliminate d rho/rho in favour of du/u. So here I express d rho/rho in terms of dT/T and then I now eliminate dT/T in terms of du/u, so I finally end up with an expression where I can say d rho/rho.

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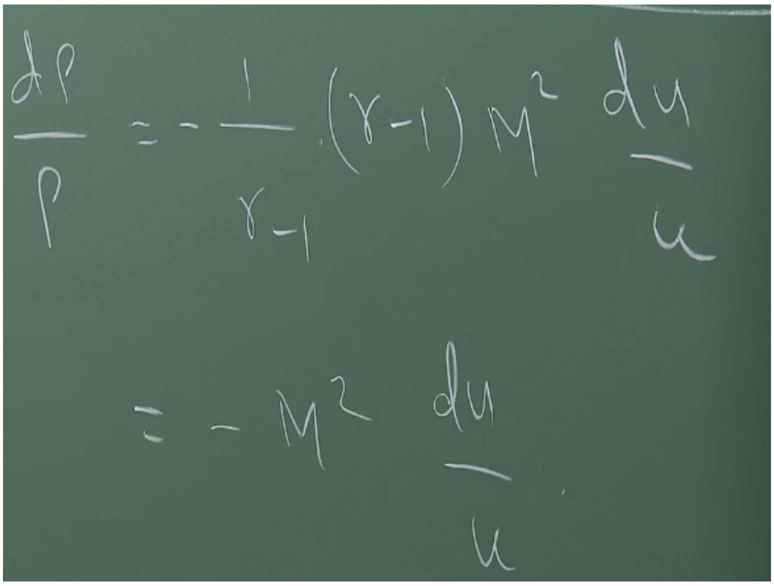

So therefore d rho/rho is = 1/gamma - 1*dT/T, but dT/T itself = gamma-1*M square* du/u, so I am going to write this as gamma-1*M square*du/u, so we can say that d rho/rho = -M square*du/u. So if I substitute the expression for d rho/rho, if you remember d rho/rho is written in terms of du/u here, and if I substitute this into my differential form of the continuity equation here, I finally get a relationship which involves only dA and du, which is what I am looking for in my area velocity relationship.

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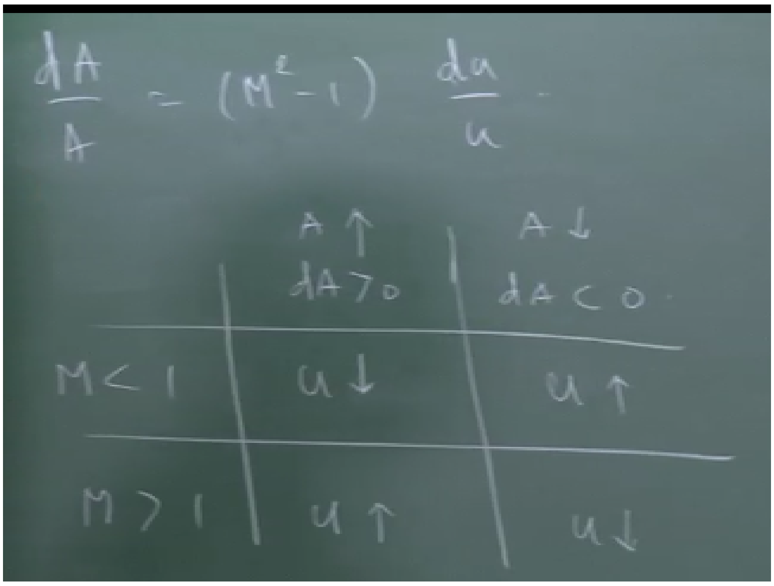

So let us go ahead and do that, so if I do this I end up with the following expression dA/A = M square-1*du/u. So this is the so called area velocity relationship, where changes in velocity are related to changes in the cross sectional area of the passage. Now with this, we can get an idea of what changes in cross sectional area of passage does to velocity, does it increase or decrease the velocity that will depend up on whether the flow is subsonic or supersonic because the Mach number also involved in this expression.

Let us take a look at the next. So we are going to actually do the same thing that we did earlier we will tabulate the changes as we did earlier, if you remember for Rayleigh flow and Fanno flow, we did the same thing, M<1, M>1 and there are 2 possibilities dA>0 which means that A increases, and dA<0 which means that A decreases. So if the cross sectional area increases, dA>0 for a subsonic flow.

For a subsonic flow this term is negative, when dA is positive then du becomes negative. So a subsonic flow in a diverging passage decelerates. So u is negative, or if I write this symbolically as always done we can write it like this, u decreases. Similarly, if dA is negative this term is also negative for a subsonic flow so du becomes positive, so u increases in a converging passage for a subsonic flow.

I can show easily that u for a supersonic flow u increases in a diverging passage and u decreases in a converging passage. So this tells me that subsonic flow decelerates in a diverging passage and accelerates in a converging passage, and supersonic accelerates in a diverging passage and decelerates in a converging passage. We can also interpret this slightly differently by using earlier 2 equations, where we have written down d rho/rho in terms of du/u and the differential form of the continuity equation.

The argument goes like this, let us say that the, let us say that we are looking at a subsonic flow. At 1 point let us says that the velocity increases slightly and let us say du is positive, that means d rho/rho is negative and since M is also <1 the magnitude of the quantity is also <du/u, that is negative the magnitude is also <du/u. So now if I use that information in my continuity equation for mass conservation, this sum must always be = 0.

So this has increased slightly because I said du was positive, now this has become negative but < this. That means for this entire equation left hand side to sum to 0 what should dA be? dA should be negative right, so it is as if this is 10, this is -8, so this has to be -2 to make the whole thing go to 0, which means that dA has to be negative. So if du is positive and the flow is subsonic, then dA has to be negative in order to satisfy mass conservation.

On the other hand, let us say the flow is supersonic and the velocity increases slightly du is positive, then d rho/rho is actually negative and >du/u. So when I look at mass conservation now, what happens is, this was we said was 10, now instead of this being -8 this has become let us say -12, then this would have to be +2 to make the whole thing go to 0, so that means dA has to be positive.

So the change in behaviour, when the flow is subsonic to when the flow is supersonic comes becomes of the Mach number dependence of the change in density. What his tells me is that for the same change in velocity, for the same change in velocity, for the same value of du the change in density is less in the case of subsonic flow and more in the case of supersonic flow.

And if you remember in our very first lecture when we drove lines of v = constant on a TS diagram, we said that the lines are spaced closely for a lower temperature and spaced farther apart for a higher temperature. Now that is what is causing this effect, so the same change in du results in larger change in density if the flow is supersonic and smaller change in density if the flow is subsonic.

And that is what is causing the flow to behave differently exactly opposite, depending upon whether it is subsonic or supersonic. So change in density for a given change in velocity is a much more when the flow is moving at supersonic speed and much less when it is moving at subsonic speeds. So that is an extremely important conclusion that comes out of this equation. Now notice that there are actually 3 possibilities in this equation for dA.

We have looked at dA positive, dA negative, but we have not looked at dA=0, that is also a possibility which we might entertain, and that brings us to the next topic which is.

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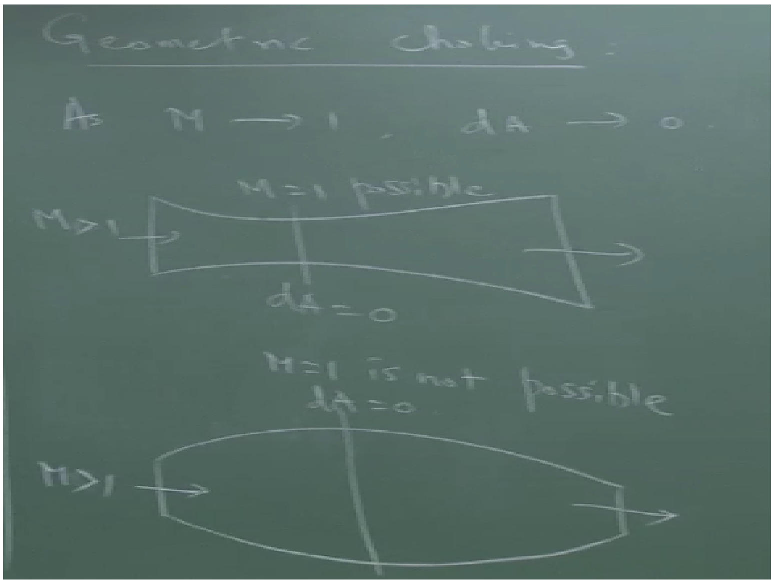

We have looked at thermal chocking where enough heat addition could cause the Mach number could go to 1 either from a subsonic Mach number or a supersonic Mach number with enough heat addition we can make the Mach number go to 1 or the flow attains a sonic state. We saw with friction also that if you make the pipe long enough we can attain the sonic state in the pipe if you make it long enough, now that was friction chocking.

Now we are looking at geometric chocking that with the geometry being designed in a certain way we can attain the sonic state once again in this flow also, that is what we are going to discuss next. Now from this relationship as M goes to 1, we can see that dA goes to 0 very simple. So this means that what does this mean, does this mean that wherever dA is 0, we attain M=1 not so fast, we need to look at different possibilities to see what kind of solutions are allowed.

Let us look at 2 scenarios where dA=0, then we will see M can become =1 in both this cases. The first scenario is a passage that looks like this, so at this location dA=0 and we look at another scenario where the passage is inverted inside out, inside out of the same thing, so the passage looks something like this, and you see that here dA is 0, so now whether M=1 can be attained in these 2 situations.

Let us say that we start with the first case. Let us initially say that M is < 1, so the flow enters the passage with the subsonic Mach number, and then what will happen to the Mach number, as it goes through the converging passage, the Mach number increases, the flow accelerates the subsonic flow, so it is possible for me to accelerate to M=1 here, and then here onwards the flow will decelerate perhaps or accelerates depending up on the back pressure conditions it will decelerates or accelerate.

But it is possible for me to have M=1 here, do you understand that. So it is possible to achieve if it is subsonic, what if the flow is supersonic. If I say that the incoming flow is supersonic, let us say M>1 if it is supersonic flow, it enters a converging passage what happens to the flow, the flow decelerates, that means M decreases. So it is possible for me to decelerate from the supersonic Mach number to M=1.

So both cases irrespective of whether it is subsonic or supersonic it is possible to achieve M=1 at this location where dA=0. Now if you look at this case let us say the flow enters with a subsonic Mach number. Let me just add an arrow here, to show the direction of flow, so let us say that the flow enters with the subsonic Mach number and then the subsonic flow, what happens to the subsonic flow in a diverging passage, the flow decelerates.

So there is no way I can actually attain M=1 at dA=0 because the Mach number is decreasing, so attaining M=1 at this dA is not possible in this case. Similarly, if the flow enters with the supersonic Mach number then the flow accelerates in the diverging passage so I cannot attain M=1, in the supersonic case also. So which means that in this case although dA is 0, M=1 is not possible, so dA may be 0, but M=1 is not possible.

So what this tries to tell me is that if M=1 occurs, then it can occur it is not sufficient if you have dA=0, it also says that it must be a point of minimum cross sectional area, dA can be 0 for minimum as well as maximum cross sectional area. So what this tells means if M=1 occurs, then it must occur at a point of minimum cross sectional area and such a location point of minimum cross sectional area is usually called a throat.

Now this relationship, the other aspects to this relationship which we will see next. So we have looked at this situation as M goes to 1, dA goes to 0, that is okay. Now what about the converse, if you look at this equation we have seen that as M goes to 1 dA has to go to 0. The converse is if dA goes to 0 what can we say about M. The left hand side of this equation if it tends to 0, what can we say about the right hand side does M have to go to 1.

That is not necessary, because the right hand side has 2 terms, so if the left hand side goes to 0, either this can go to 0 or it is possible that this can goes to 0, in which case M can be anything, it need not go to 1. So the converse is not always true, if M=1 occurs, it must occur at the throat, but just because you have a throat does not mean that M should always be 1 there, that is a very important point.

So as dA goes to 0, we can then write either M tends to 1 or du can go to 0, which one we actually see in practice depends upon the conditions that you are maintaining in the flow situation, the downstream pressure the upstream stagnation pressure, stagnation condition and so on. It depends upon the actual situation, so depending upon the situation either M will tend to 1 or du will go to 0, if du goes to 0, then M can have any value.

So we can sum up these 2 as follows if M goes to 1, it can go to 1 only at a throat, but just because there is a throat, M does not have to be 1 there. Remember this is an extremely important point in gas dynamics here, M=1, but M need not always be 1 at the throat. So a good example is given right here, we showed that M cannot be = 1 in this case, but the flow can come in with some Mach number.

If it is supersonic it will accelerate reach a higher Mach number, for example it can enter a 2, may be reach 2.5 and then decelerate here and exit at 2 or 1.8 or whatever it wants, it is perfectly okay. Similarly, in this case also it can enter at some Mach number, let us say supersonic, decelerate so it starts from 2. Let us say it decelerates to 1.5 and then can again accelerate and exit at Mach number, let us say 3, that is allowed.

It can come at Mach number 0.4, accelerate to Mach number 0.8 and leave with Mach number maybe 0.3 or 0.5 or 0.6 whatever. So M need not always be 1 here, whether you get M=1 here or not depends upon the actual flow condition, the upstream stagnation conditions and the pressure that you are trying to maintain downstream will depend on many things, because it is possible to attain the sonic state here.

If the sonic state is attained, we call this geometric chocking. But the important point is we need not actually see a sonic state here, just like what we did with Raleigh flow and Fanno flow, the sonic state need not occur, but we can extend the situation and assume that a sonic state occurs and then relate flow properties to the sonic state, because the sonic state is a very convenient state since M is also known there, M is always 1 at the sonic state.

So I can always imagine situation where M becomes = 1. For example, in this flow situation M does not become = 1 here, then I can conceive a flow situation just like what I did for Raleigh flow and Fanno flow, I can extend this for example in a do something like this, where a sonic state, I ensure that for the conditions the sonic state is reached here.

Then I can relate flow properties at any 1 of this section to the flow property at the sonic throat just like we did for Raleigh flow and Fanno flow and then proceed with our calculation, which is what we are going to do next. We are going to derive something called an area-Mach number relationship, which will allow us to relate the Mach number at any cross sectional area to the cross sectional area at a sonic state.

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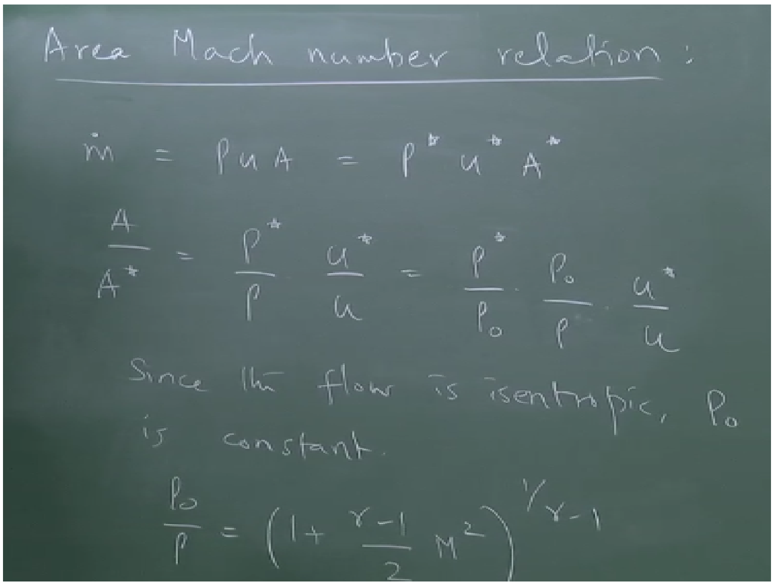

We are now looking at area-Mach number relationship. So the previous 1 was area-velocity relation, now we are looking at area-Mach number relation, because as you know once I know the Mach number at a station, all the other quantities, all the other flow properties can be evaluated. So we can derive a relationship involving this, then we are okay. So let us assume that a sonic state occurs somewhere in our flow.

We are looking at a flow with passage whose area of cross section varies. So the M dot at any section can be written rho ua and assuming that a sonic state exists like I showed in the previous diagram, I can also write this as rho*u**a*. So if I rearrange this expression, I can write this as a/a* = rho */rho*u*/u and I can simplify this as or rewrite this as rho*/rho not, where rho not is the stagnation density*rho not/rho **u*/u.

This is rho */rho, there is no star here. Since the flow is isentropic, rho not is constant. So that the stagnation density at the point where the sonic state occurs is the same as the stagnation density at any other point in the flow. Otherwise, I will not be able to use this. For example, Raleigh flow or Fanno flow, this will also keep changing, because either T not keep changing or P not keep changing. So this will also keep changing.

The stagnation density where the sonic state occurs will be different from the stagnation density at any other state. So I will not be able to make use of this in the other two cases, but here because the flow is isentropic, rho not is constant, I can write it like this. If you recall from our first module rho not/rho can be written as 1+gamma -1/2*M square raised to the power 1/gamma-1.

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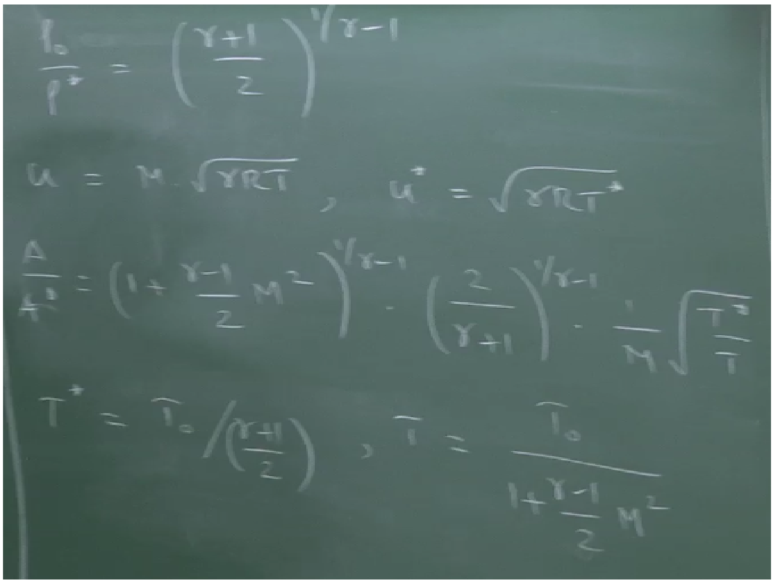

If I substitute M=1 in this relationship, then I get rho not/rho*as. The idea is to write, remember we are looking for area-Mach number relationship. The area is on the left hand side. The idea is to write everything on the right hand side in terms of Mach number and Mach number only. So now I have this in terms of gamma, I have this in terms of Mach number. I need to rewrite this in terms of Mach number. That is what we are going to do next.

We will write u as being M* square root of gamma RT and u*, if you remember u* is the speed of sound u*=square root of gamma r T*. So substitute all these into this expression for A/A*. I can write this as A/A*=1+gamma – 1/2*M square raised to the power 1/gamma – 1*2/gamma+1 raised to the power 1/gamma-1*1/M*square root of T*/T. So we are making progress.

The only term that is left is this T*/T, but I can write T* and T in terms of t not and finally do what I have always been wanting to do. If you remember T*=T not/gamma+1/2 and T=T not/1+gamma -1/2*M square. So substitute everything into this, you end up with an expression that looks like this.

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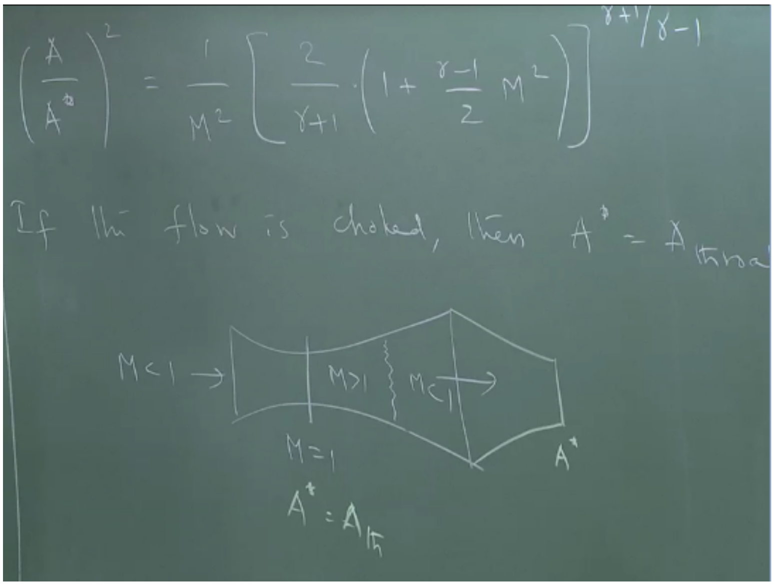

This is the so called area-Mach number relationship where we have the area term on the left hand side and the Mach number term on the right hand side. Here A* is the area or cross section where the Mach number reaches 1 and as I said earlier, A* need not be = A throat. If the flow is choked, then A* is indeed = A throat, otherwise we have to determine A* in some other manner.

This kind of situation may arise later on when we look at, for example flow through a convergent divergent nozzle, we will look at examples involving this. So let us say that we have flow through a convergent divergent nozzle. This is the throat. It is quite possible that we have a situation like this, subsonic Mach number at entry. M becomes = 1 at the throat and there is a normal shock that stands somewhere here.

So M accelerates to a supersonic speed and then it becomes subsonic after this. In such as a situation, A* for this part of the flow will be = A throat. A* for this part of the flow = A throat, but A* for this part of the flow field is not = A throat. It has to be something else. So that means we are actually imagining extending this section in such a way that, for example the A throat or A* for this second case has to be something like this.

Whereas this is A*=A throat for this case. This need not be the same as this. In fact, this will not be the same as this. That is what I meant when I said A* need not be = A throat. Different parts of the flow field can have different A*, although there is only a single throat, different parts can have different A*. Now this equation for a given value of A/A*, notice that for a given value of A/A*, this equation can give you two solutions, one corresponding to a subsonic flow and another corresponding to a supersonic flow.

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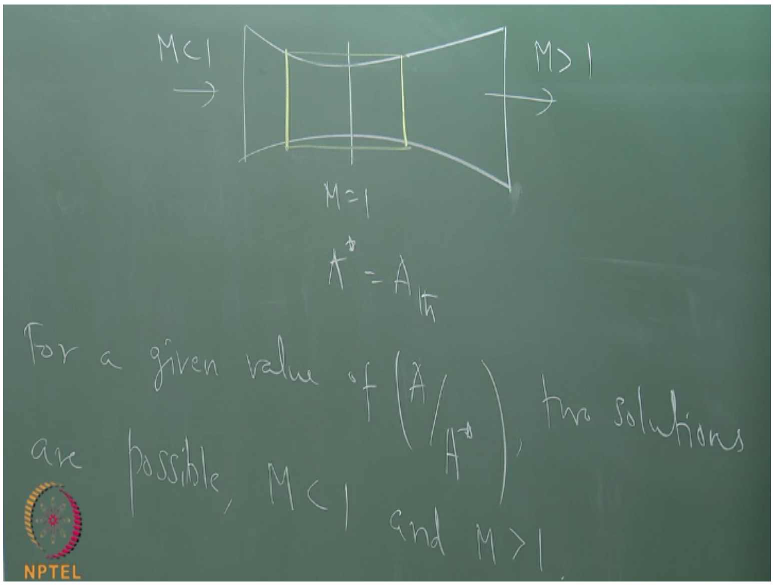

That is also clear if you look at the same flow situation. So for example, if you look at a flow situation like this, where M=1 and if for example the flow enters at a subsonic Mach number and exits with a supersonic Mach number, then the A* in this case = A throat. However, if I consider 2 sections like this, this cross section and this cross section, notice that A/A* for both these cross sections are the same.

But here the flow is subsonic, here the flow is supersonic. That is what I meant when I said for given value of A/A^*, I can get 2 solutions, 1 corresponding to the subsonic branch and another one corresponding to the supersonic branch. Let us write this down formally. So $M<1$ is 1 solution and $M>1$ is the other solution. Once again, what we try to do is, we do not try to solve this equation for different values of M.

We use the same strategy as what we did earlier. We tabulate this function for different values of M. So I take $M=0.1$, 0.2, 0.3 all the way up to $M=5$ and I substitute the value here, I calculate A/A^*. So I make a table. So whenever I have a given value of A/A^*, I go into the table and figure out my M depending upon whether I am looking for the subsonic solution or the supersonic solutions.

So this function is tabulated and we will use this table to do our calculations rather than solving this equation, we have written. What we will do in the next class is to look at an expression for the mass flow rate through choke nozzle because that is one of the most important expressions that we will encounter in gas dynamics that has far reaching implications on many, many equipment handling compressible flow, especially air craft engines. That is what we will do in the next class.