Gas Dynamics and Propulsion

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Lecture - 12
Fanno Flow

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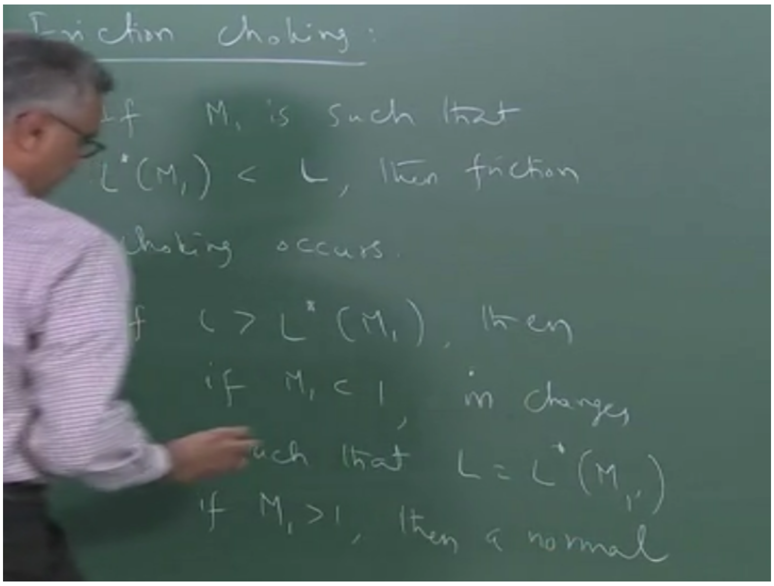

In the previous class we looked at friction choking and its consequences. So in the subsonic and the supersonic case, if the Mach number M1 is such that the L* corresponding to that M1 is < the length of the duct L, then friction choking occurs and we said that if friction choking occurs depending upon the length of the duct in comparison to L*, in the subsonic case, the mass flow rate is adjusted.

So if for example L is > L* corresponding to the given inlet Mach number. Then if the Mach number M1 is < 1, M dot changes. So we saw that in the subsonic case, the mass flow rate through the duct changes to accommodate the new length. So M dot changes such that the L* corresponding to the new one for L = L* corresponding to the new operating condition.

And if the Mach number initial, the inlet Mach number is supersonic, if M1>1, then a normal shock occurs in the duct.

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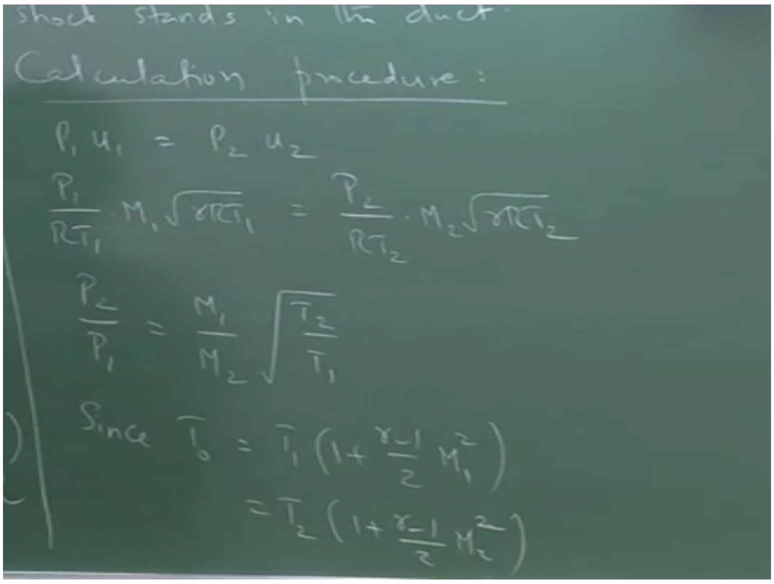

So the location of the normal shock has to be determines iteratively. We will work out numerical example that illustrates how this iterative process work. We will look at both the cases through worked examples. So now we take a look at calculation procedure. How do we do practical calculations using the theory that we have developed so far. So calculation procedure here is the same as what we did for Fanno flow.

So if you remember from our governing equations we have written down rho 1 U1 = rho 2 U2 and if I substitute for rho 1 in terms of P1, so I can write this as P1/RT1 and for U1 I can substitute in terms of Mach number and the speed of sound, so this is M1*square root of gamma RT1 that is = P2/RT2 times M2*square root of gamma RT2. And if I rearrange this, I get P2/P1 = M1/M2*square root of T2/T1.

Now in this particular flow there is no heat addition, so stagnation temperature remains constant. So we can write this as since T0 = T1*1+gamma-1/2*M1 square and this is also = T2*1+gamma-1/2*M2 square.

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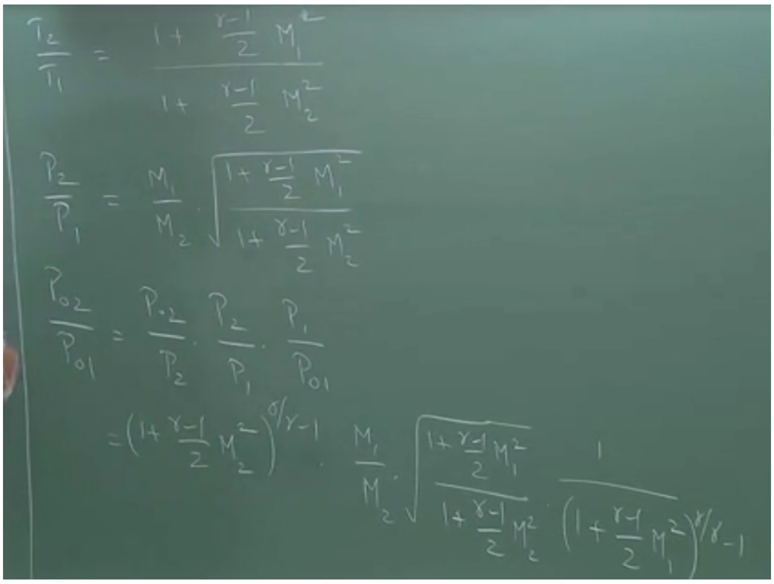

So I can write T2/T1 as follows. T2/T1 = 1+gamma-1/2*M1 square divided by 1+gamma-1/2*M2 square. And I can actually substitute for T2/T1 from here into this equation. So I still have T2/T1 in the right hand side. So if you substitute from there, I can get P2/P1 to be = M1/M2*square root of 1+gamma-1/2*M1 square/1+gamma-1/2*M2 square. And I can write P02/P01 in the same way as we did earlier.

So P02/P01, remember this is a flow with friction which is an irreversibility, so the entropy increases and there is a loss of stagnation pressure as a result of the increase of entropy. So P02/P01 can be written as P02/P2*P2/P1*P1/P01. And you know that P02/P2 is nothing but 1+gamma-1/2*M2 square raise to the power gamma/gamma-1 and P2/P1 is given from here. So this can be written as M1/M2*square root of 1+gamma-1/2*M1 square/1+gamma-1/2*M2 square.

And P01/P1 is going to be 1/1+gamma-1/2*M1 square raise to the power gamma-1. So now we have all the quantities that we are looking for. M1 is known, so if you look at these four expressions, M1 is known, that is the Mach number at the inlet. If I know M2 then all the quantities that I need are known. So we calculate M2 from the momentum equation. So we will do something like this. So we will start with the differential form of the momentum equation.

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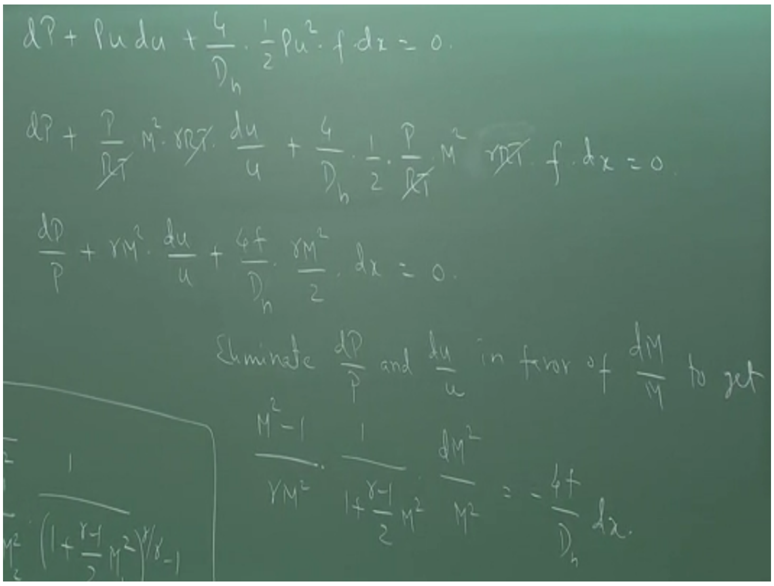

So we can write the momentum equation in differential form like this. If you take a small control volume and write the differential form of the momentum equation, it looks like this. F is the friction factor, remember f is the Darcy friction factor. Please remember that. And if I make use of the equation of state and the definition of Mach number I can write this as dP+P/RT*M square*gamma RT*dU/U.

Notice that I have multiplied and divided this term by U. So I get a U square here and a dU/U. So U square can be written as M square*gamma RT. And I have done the same thing, here 4/DH*1/2*P/RT* again U square is M square*square root of gamma RT times f.dx = 0. Oh, I am sorry, this is M square*gamma RT. The square root disappears, so there is no square root here.

So we can easily see that the RT cancels out in both these terms and the P can be brought to the denominator. So I end up with an equation that looks like this, dP/P plus gamma M square* dU/U plus 4f/Dh*gamma M square/2 times dx = 0. And we derive relationships earlier for dP/P and dU/U in terms of dM/M. So if you use those relationships, so eliminate dP/P and dU/U in favor of dM/M so we can get relationship which looks like this.

So, I am going to leave the algebra to you. So I will just write down the final expression. It is M square-1/gamma M square*1/1+gamma-1/2* M square* dM square/M square = -4f/Dh*dx. So this equation that we have written down, finally gives me an expression involving duct length and change of Mach number.

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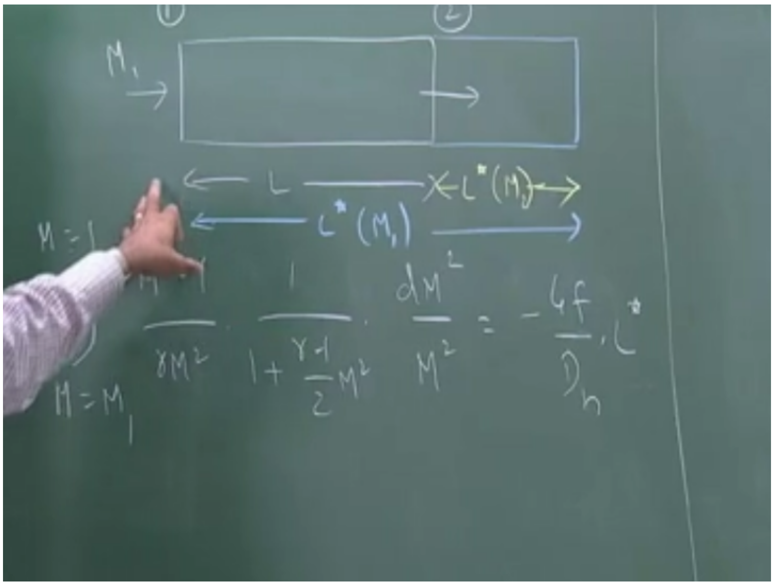

So I have two ways in which I can proceed now, just like we discussed earlier if I have a duct of length let us say L, and this is state 1, this is state 2. M1 is known here, I can integrate this equation, left hand side from M1 to some M2 which is not known and the right hand side can be integrated from 0 to L. So that will give me L. So right hand side when integrated from 0 to L will give me L.

So I will have a highly non linear equation involving M2 from the left hand side. I can solve that and get M2. Now this means I have to solve a non linear equation every time. So I need to have very complex tables. The other problem is such non linear equations also have multiple solutions. So we avoid this by using the same trick that we used before. So what I am going to do is I am going to take a certain value of M1 and integrate from that M1 to M = 1.

So the right hand side if I do that will give me L* corresponding to that Mach number, right? So let me write this down and then we will see. So the left hand side, I am going to integrate from sum M = M1 to M = 1. So this is M square – 1/gamma M square*1/1+dM square/M square = -4f/Dh times, in this case this becomes the L* corresponding to L1. Notice that we are assuming that the friction factor f is constant along the length of the duct.

In fact we are not accounting for variations in the friction factor usually for this problems you will notice that for the kind of Reynolds number that we are operating at, friction factors will not vary significantly. So this is a very good engineering assumption. So this is L* corresponding to M1. Notice that now I am not solving any equation. I am just obtaining this integral this can actually be evaluated in close form.

So what I do is, I take M1 to be point 1, point 2, point 3, point 4, so I tabulate this for all values of subsonic Mach numbers. I do the same thing for supersonic Mach numbers also, right? I can start from 1.1, 1.2, so I tabulate this and then I don't have to solve the equation. So if I know M1, I calculate L* corresponding to that M1. So L* corresponding to that M1 maybe something like this, right?

So this maybe L* corresponding to M1. But because we are operating on the same branch of the Fanno curve, this is nothing but L* corresponding to M2. So with the known value of M1, I calculate L* from this tabulation, I know L* corresponding to M1. I subtract the length of the duct; I get L* corresponding to M2. So I go to the table again and see what is the value of M2 which corresponds to this value of L*.

That is the basis of the calculation procedure. Same is what we did for Rayleigh flow also, okay? So if this is tabulated, we don't have to solve any equations at all. It is a very effective way of doing calculation and that is what we are going to do. We are going to use this table to do these calculations. Any questions before we do that? **"Professor - student conversation starts"** (()) (15:48).

The friction coefficient depends upon, primarily upon or only upon the Reynolds number, the friction stress depends up on the hydraulic diameter. But the friction factor depends only the Reynolds number. And you will notice that for the kind of speeds with which we are working, the Reynolds number based on hydraulic diameter will be so high that you are essentially operating in a region where the friction factors are constant.

So in fact you are far to the right on the Moody's chart where you notice that the friction factor is independent of Reynolds number, that is the kind of region that we are operating in. So that is a very good engineering approximation to use. **"Professor - student conversation ends".** So let us do a worked example. Next we will do two of the examples in this module, one for a subsonic entry Mach number, another one for a supersonic Mach number.

And in both cases, we will look at cases, we will look at situations when L is < L* and L is also > L*. So the first example that we are going to do reads like this. Air enters 3 cm diameter pipe with a stagnation pressure and temperature of 100 K pascal and 300K and a velocity of 100 m/s/. Compute A, the mass flow rate, B, the maximum pipe length for this mass flow rate and C mass flow rate for a pipelength of 14.5 m. Take f to be 0.02 for the calculation.

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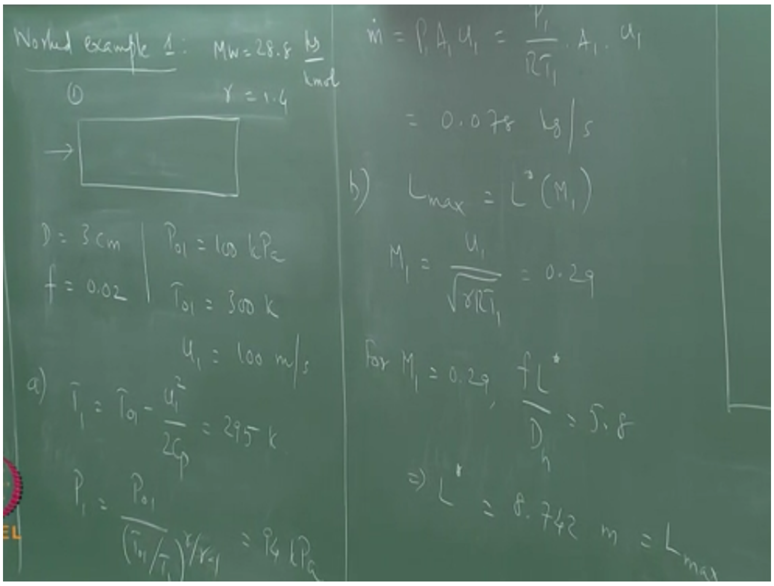

So let us write down the information here. So this is, let us call the worked example 1. So are given a pipe. The diameter of the pipe is given to be 3 cm. Friction factor f is given to be 0.02, remember this is Darcy friction factor and at entry to the duct, so this is state 1, we have been given, P01 = 100, T01 = 300 kelvin and U1 is given to be 100 m/s. So we are asked to calculate first the mass flow rate, M dot through the pipe.

So to calculate mass flow rate we need to know the static condition, stagnation conditions are given here. So let us start with the static conditions. So T1 is going to be T01 minus U1 square/2Cp and if you substitute the numbers that are given here for T01 and U1 and you will also say that molecular weight, we will use molecular weight for air to be 28.8 kg per kmol and gamma for air to be 1.4.

So if you substitute these numbers we will get static temperature to be 295 kelvin. And this static pressure P1 is nothing but P01 divided by T01 divided by T1 raised to the power gamma-1/gamma, I am sorry, gamma/gamma-1. And so the static pressure comes out to be 94 kilo pascal. So the mass flow rate, M dot is rho 1 A1*U1 and rho 1 is nothing but P1/RT1 times A1*U1.

So all the quantities are known in this. A1 is nothing but pi d square/4 where d is the diameter of the duct. So if you plug in the numbers, you get the mass flow rate to be 0.078 kg per second for part A. Now part B asks us to determine the maximum length of the pipe that I can have for this mass flow rate. So the maximum length is going to be = L* corresponding to the inlet Mach number, so let us look at part B.

So part B for the mass flow rate to remain the same, the maximum length that I can have is L* corresponding to M1. And M1 we can calculate like this, M1 = U1/square root of gamma RT1 and this comes out to be roughly 0.29. So we have to use the tables for this.

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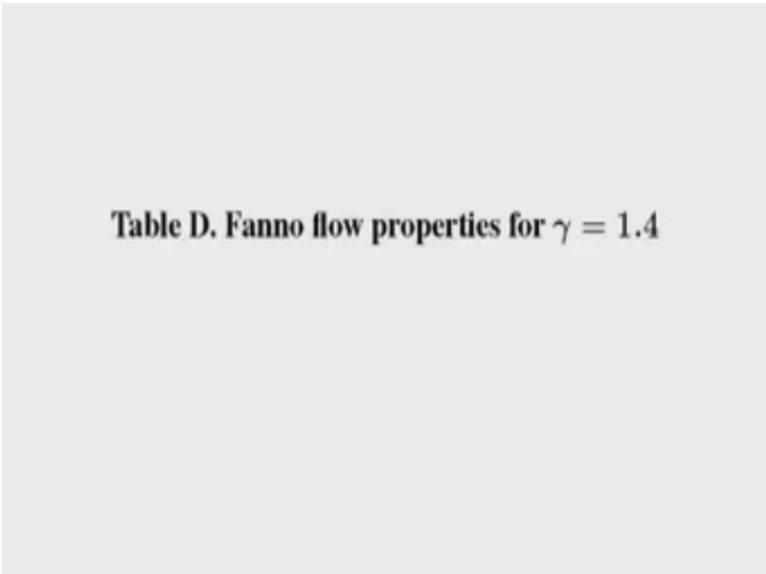

So let us look at the tables. So we are now using table B which corresponds to Fanno flow and you will notice that the table lists data like this Mach number.

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M	$\frac{P}{P^*}$	$\frac{T}{T^*}$	$\frac{\rho}{\rho^*}$	$\frac{P_0}{P_0^*}$	$\frac{fL^*}{D_h}$
0.01	1.09543E+02	1.19998E+00	9.12880E+01	5.78738E+01	7.13440E+03
0.02	5.47701E+01	1.19990E+00	4.56454E+01	2.89421E+01	1.77845E+03
0.03	3.65116E+01	1.19978E+00	3.04318E+01	1.93005E+01	7.87081E+02
0.04	2.73817E+01	1.19962E+00	2.28254E+01	1.44815E+01	4.40352E+02
0.05	2.19034E+01	1.19940E+00	1.82620E+01	1.15914E+01	2.80020E+02
0.06	1.82508E+01	1.19914E+00	1.52200E+01	9.66591E+00	1.93031E+02
0.07	1.56416E+01	1.19883E+00	1.30474E+01	8.29153E+00	1.40655E+02
0.08	1.36843E+01	1.19847E+00	1.14182E+01	7.26161E+00	1.06718E+02
0.09	1.21618E+01	1.19806E+00	1.01512E+01	6.46134E+00	8.34961E+01
0.10	1.09435E+01	1.19760E+00	9.13783E+00	5.82183E+00	6.69216E+01
0.11	9.94656E+00	1.19710E+00	8.30886E+00	5.29923E+00	5.46879E+01
0.12	9.11559E+00	1.19655E+00	7.61820E+00	4.86432E+00	4.54080E+01
0.13	8.41230E+00	1.19596E+00	7.03394E+00	4.49686E+00	3.82070E+01
0.14	7.80932E+00	1.19531E+00	6.53327E+00	4.18240E+00	3.25113E+01
0.15	7.28659E+00	1.19462E+00	6.09948E+00	3.91034E+00	2.79320E+01
0.16	6.82907E+00	1.19389E+00	5.72003E+00	3.67274E+00	2.41978E+01
0.17	6.42525E+00	1.19310E+00	5.38533E+00	3.46351E+00	2.11152E+01
0.18	6.06618E+00	1.19227E+00	5.08791E+00	3.27793E+00	1.85427E+01
0.19	5.74480E+00	1.19140E+00	4.82190E+00	3.11226E+00	1.63752E+01
0.20	5.45545E+00	1.19048E+00	4.58258E+00	2.96352E+00	1.45333E+01
0.21	5.19355E+00	1.18951E+00	4.36613E+00	2.82929E+00	1.29560E+01
0.22	4.95537E+00	1.18850E+00	4.16945E+00	2.70760E+00	1.15961E+01
0.23	4.73781E+00	1.18744E+00	3.98994E+00	2.59681E+00	1.04161E+01
0.24	4.53829E+00	1.18633E+00	3.82548E+00	2.49556E+00	9.38648E+00
0.25	4.35465E+00	1.18519E+00	3.67423E+00	2.40271E+00	8.48341E+00
0.26	4.18505E+00	1.18399E+00	3.53470E+00	2.31729E+00	7.68757E+00
0.27	4.02795E+00	1.18276E+00	3.40556E+00	2.23847E+00	6.98317E+00
0.28	3.88199E+00	1.18147E+00	3.28571E+00	2.16555E+00	6.35721E+00
0.29	3.74602E+00	1.18015E+00	3.17419E+00	2.09793E+00	5.79891E+00

For each Mach number you get all the quantities that you want, P/P*, T/T*, rho/rho *, P0/P0* and FL*/Dh. So now our Mach number is 0.29, so let us see, so corresponding to 0.29 which is over here, fL*/Dh is 5.79891, right? Let us write this down. So corresponding to so for M1 = 0.29, fL*/Dh comes out to be approximately 5.8. Let us use 5.8 for this. So from which I can calculate my L* corresponding to this smart number to be about 8.742 meters.

“Professor - student conversation starts” Sir when do we use whole fL square by Df? I have consistently said throughout that we are using the Darcy friction factor. Now, in the aerospace community they use a friction factor called manning friction factor which is actually off from this number by a factor of 4. The Darcy friction factor is the most widely used friction factor in connection with flow through pipes and ducts and so on.

So that is what we are using, that is why you don’t see the 4f there. So the manning friction factor for the same case instead of f being 0.02, the manning friction factor for this case would have been 0.005. So the 4 times that is equal to this. But Darcy friction factor is the most widely used, all the correlation gives only this which is why we are using this, okay? **“Professor - student conversation ends”**.

Please note that when you are using the Darcy friction factor we should not use tables that have the last column as 4 fL*/Dh, if you see 4 fL*/Dh in the last column, notice that the last column in the table that we are using is fL* over Dh, the last column is 4 fL* over Dh, that would usually mean that the f that is being used that is the manning friction factor and not the Darcy friction factor.

So this tells me that the maximum length that I can have for this mass flow rate is 8.742 meters. So if I use a 3 cm diameter pipe, so I have my experimental set up for some other device set up over here and my compressed air tank is let us say further down, if the pipe length that I am using is more than 8 meters then I will not get this mass flow rate, it has to be below 8.7 meters for me to get the same mass flow rate that I am looking for, okay?

Part C of the question says, what would be the mass flow rate if the length of the pipe is 14.5 meters more than this maximum value that is part C.

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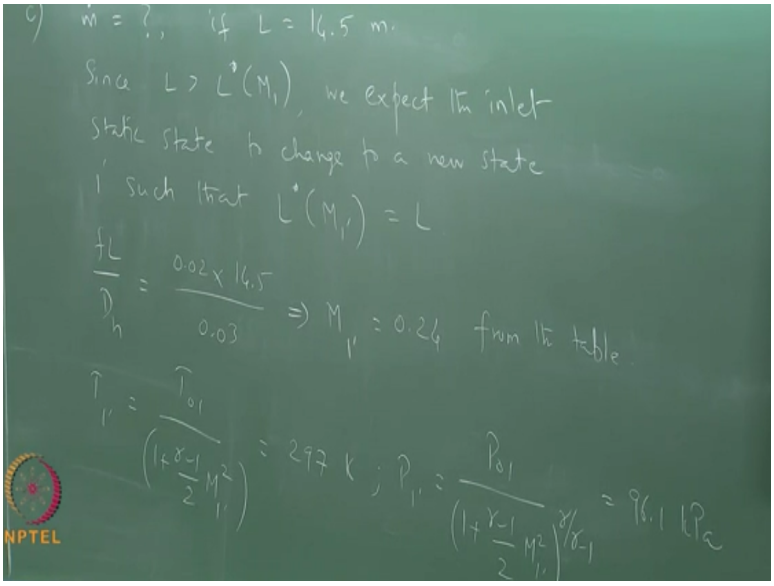

So part C m dot = 1 if L = 14.5 meters. So now since L is > L*, is > L* corresponding to M1 which is 0.29, we expect the mass flow rate to decrease and the inlet conditions to be different. Since L>L*, we expect being at static state to change to a new state which we are going to call s let us say 1 prime such that L* corresponding to M1 prime = L.

And again we have not been given any information about the exit pressure. If exit pressure is specified, then we need to determine an inlet static state which is such that the pressure at the exit of the duct matches the exit ambient pressure. Since nothing has been given, we will assume that it comes out at the sonic state, okay? So from the given length of 14.5, then let us calculate fL/Dh.

For this value comes out to be 0.02 times 14.5 centimeters divided by this is a pipe, so you know that hydraulic diameter of the pipe is same as the diameter of the pipe. So that is 14.5

meters, so this is 0.03 meters. So if I go to the tables with this value, so this corresponds to M1 prime which is = 0.24 from the table. So the inlet Mach number changes from 0.29 to 0.24.

But the stagnation conditions remain the same, so T1 prime = T01 divided by 1+gamma-1/2*M1 prime square and T01 remains the same, so this comes out to be 297 kelvin. And P1 prime can be calculated in the same manner (()) (29:16) P01/1+gamma-1/2*M1 prime square raise to the power gamma/gamma-1 and this comes out to be 96.1 kilopascal.

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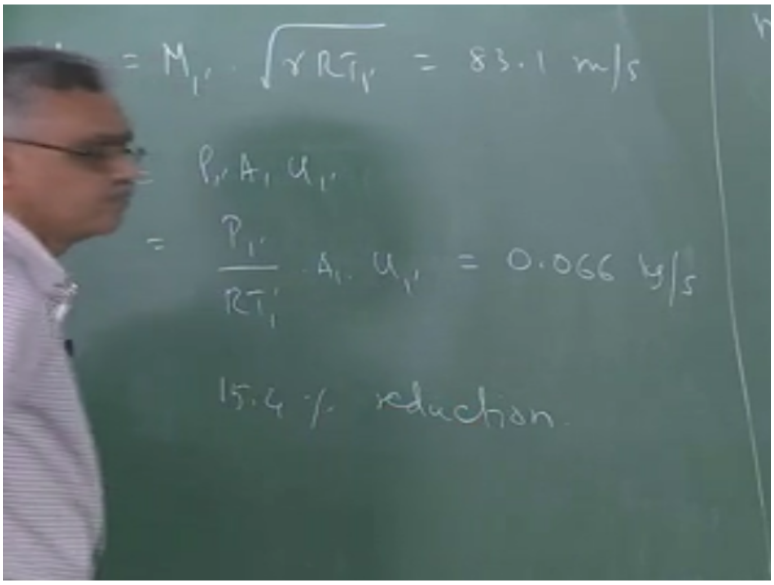

Now the velocity U1 prime is nothing but M1 prime*square root of gamma R T1 prime and if you substitute the numbers the velocity comes out to be 83.1 meter per second, slightly different from what we had before, earlier we had 100 m/s, now we have 83 meter per second. So the new M dot is called the M dot prime, is once again rho 1 prime A1 U1 prime and that is nothing but P1 prime divided by R T1 prime*A1*U1 prime.

And this works out to 0.066 kilogram per second which is 15.4% reduction. So if you design compressible equipment expecting certain inlet conditions and if your pipe is let us say longer than, if you don't know the design and you just simply use a very long pipe then the mass flow rate as you can see is going to be substantially different from what you are hoping to get. Okay, that is the reason why it is very important to study Fanno flow.

The effective of friction can be quite significant and the change in inlet condition is also quite significant. Notice that at the exit state now, which you are using a 15-meter-long pipe the exit side Mach number is one. Whereas you might have designed the equipment for some other inlet Mach number, the equipment which is connected to the end of the pipe, you may be expecting some Mach number and some mass flow rate whereas the conditions are totally changed now.

So if you want to use a longer pipeline then what you need to do to avoid friction choking is to increase the diameter of the pipe, is the larger diameter and then design the pipes properly so that you get the conditions that you are looking for, okay? So that is the big example where we looked at the inlet state, subsonic inlet state, the next example that we are going to do will use a supersonic inlet state and will see how the changes takes place.

The next worked example reads as follows, air enters 5 cm x 5 cm square duct at 300 kelvin, 100 kilopascal and a velocity of 905 meter per second. If the duct length is 2 meters find the flow properties at the exit, take f to be = 0.02. So here we are given a square duct, so the cross section of the duct is square. 5 cm x 5 cm, f is given to be 0.02.

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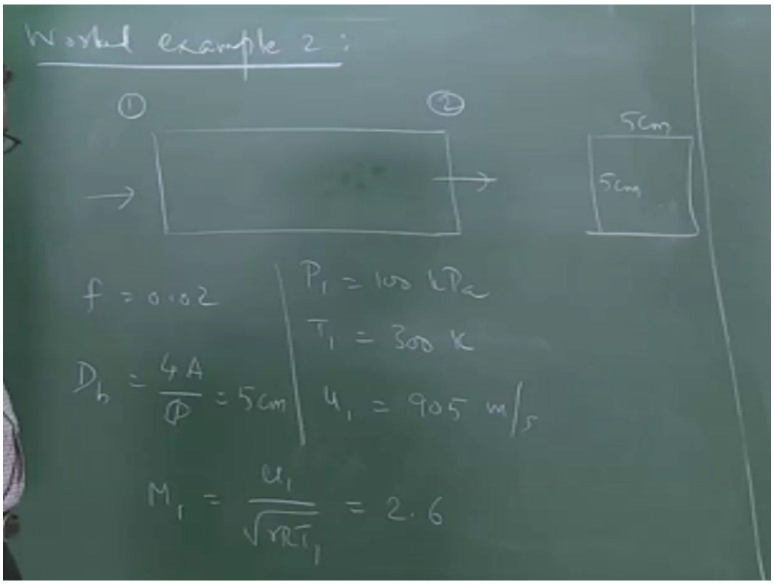

The inlet state we are given the let us assume these to be static conditions since nothing is said, so we take P1 to be 100 Kpa and T1 to be 300 kelvin and U1 is given to be 905 meter per second and the hydraulic diameter the cross section is given, so the hydraulic diameter, so you remember is 4 times the area divided by the perimeter. So 4 times the cross sectional area divided by the perimeter.

For this case gives me the hydraulic diameter to be 5 centimeters again, coincidentally, okay? Now you may wonder how this can possibly happen in real life. Why would I want to have a flow that is entering at velocity of 905 meter per second into a duct. This can very easily happen if for example you are doing let us say supersonic flow experiments in a wind tunnel, right, so we may have a nozzle which is connected to this.

This maybe your test section where you are going to keep some objects and other things that you want to test let us say in a supersonic stream. So in such cases you will definitely get this type of flow situation. So it is very important that you study this type of flow, so they do occur in real life. So let us start this, since the static conditions are already given we can calculate the initial Mach number M1 as U1/square root of gamma R T1 and this I get to be 2.6.

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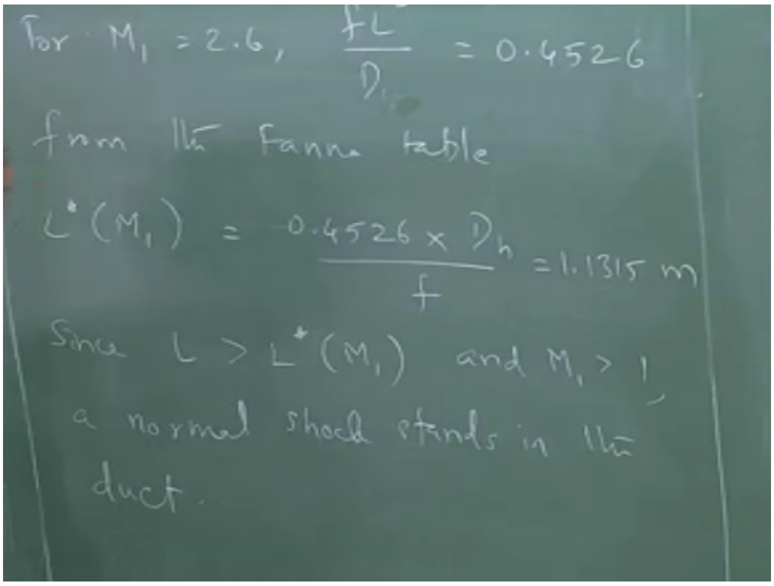

So for M1 = 2.6, f L*/Dh comes out from the table as, I am sorry, f L*/D from the table it can get to be 0.4526 from the Fanno table. So this gives me L*, corresponding to the Mach number M1 to be 0.4526*Dh/f which comes out to be 1.1315 meter. And the problem statement stated that the length of the duct was 2 meters. Okay? Questions? **"Professor - student conversation starts"**

Sir that thing, using (()) (38:48) is 1.3 likely. Please look in the supersonic (()) (38:58) 2.6 Mach number over (()) (39:02)1.4., so 1.3/3. No, what I have is correct. That maybe 4 FL* but we are not using that, we have to use a table with is FL*. Using Darcy friction factor, not Fanno friction factor. Okay, 1.1315 meters, that is what we have used, okay? So notice that what we have done here is we have gone to the Fanno table corresponding to 2.6 fL* comes out to be 0.4526, that is what we have used, okay?

"Professor - student conversation ends". So since the given length L is > the L* corresponding to M1 and M1 is > 1, the inlet is supersonic, there is going to be a stance in the duct. So we have to locate where the normal shock is and then determine the exit properties. We discussed this scenario in the previous class. Let us recap this scenario before we proceed with our calculations, okay?

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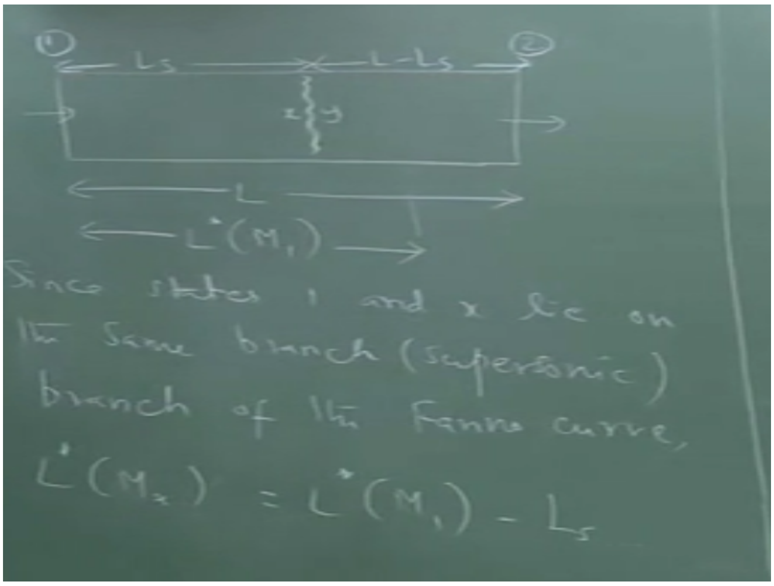

So we have a duct, so this is state 1, this is state 2, this is L, okay, now let's say this is L* corresponding to M1, right, remember for this example L was 2, but L* corresponding to M1 was < L. So let us say this is L* corresponding to M1. So normal shock stands somewhere in the duct like this. Let us say that the state 1 before the normal shock let us call that y, okay?

That is further state that the stock stands let us say at a distance LS from the entrance of the duct, okay? Now state point 1 and state point x lie on the same branch of the Fanno curve, correct? So since states 1 and x lie on the same branch and that is the supersonic branch of the Fanno curve L*corresponding to Mx is going to be L* corresponding to M1 minus Ls, okay? So that is one relationship that we have.

Now if no information about the exit is given, then we assume the exit states to be the sonic state.

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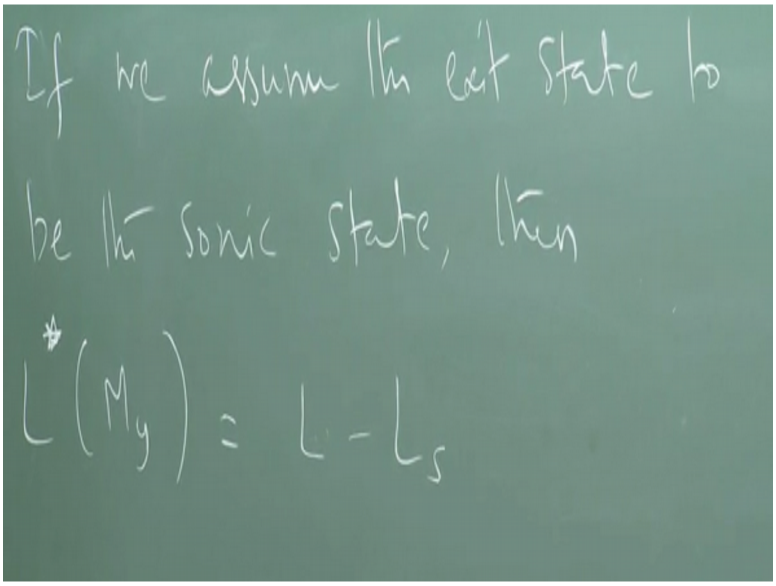

If you assume, then notice that L* corresponding to My is nothing but total length minus Ls, that is the remaining length, right, so this is L-Ls and that has become = L* now, so if you assume the exit state to be sonic state then L* corresponding to Ly is L-Ls. So the calculation has to proceed iteratively now. We assume the shop to be at some location. That is Ls is not, so then corresponding to M1 we calculate L*.

Then since we have assumed the value for Ls, I can get L* of Mx. Corresponding to L* of Mx from the supersonic part of the Fanno table I calculate Mx. With the value of Mx, I go to normal shock table to get my My. With the value of My, I go to the subsonic part of the Fanno table to calculate my L* corresponding to My. I checked to see whether this value is equal to this value.

If they are not equal, then I adjust the shock location, I use a different value for Ls and repeat the same thing. That is the procedure that we are going to use. So we will write a table and then do these calculations in the next class.