Gas Dynamics and Propulsion
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Lecture - 11
Fanno Flow

In the last class we looked at flow with friction through a pipe.

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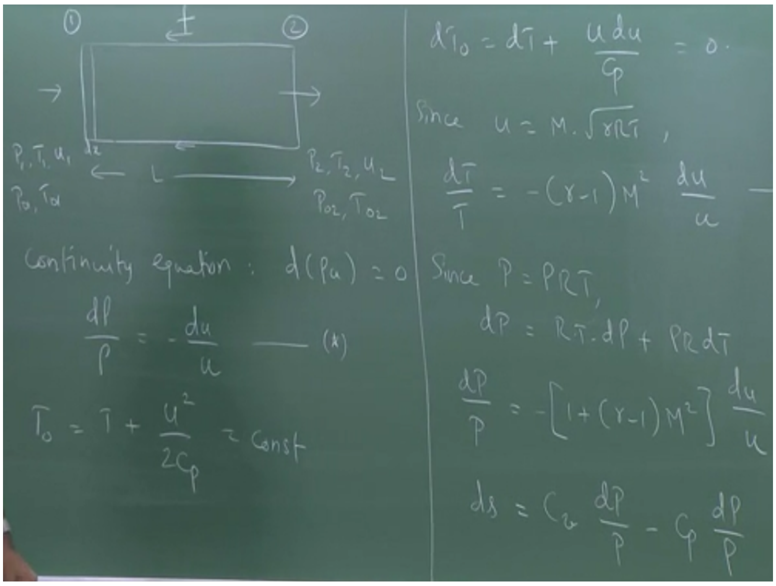

So this was the scenario that we were looking at. So there was a pipe of, pipe or duct, or length L and the flow comes in at state 1 and flow leaves at state 2. And we said that there was a shear stress on the walls of the duct or pipe and the friction factor was known. So given the flow of properties at the state 1, the objective now is to find out what happens at state 2, or what are the state points or property values at state 2. So we are looking at P1, T1 and U1 being given at state 1 and consequently we know P01 and T01.

So the objective is to determine P2, T2, U2 and P02, T02, that is what we are trying to find out for a given duct length and frictional factor f which as we said earlier we assumed to be a constant. That is the objective of the exercise. Before we do that, just like what we did earlier starting from the inlet stage we are going to look at a small length of the duct or point, let us say length = dx.

And then we will try to determine how the properties change when we go from state 1 through an incremental distance dx. So we do that and then once we know this state we can then look at another small section dx and then we can keep building until we go from inlet to the outlet. So same is what we did earlier. So in order to do that we start with the differential form of the governing equation.

And if you remember the differential form of the continuity equation, looks like this, d of rho u = 0, so if you expand this rewrite this, this look like this d rho/rho = -du/u that is a differential form of the continuity equation and the stagnation, definition stagnation temperature T0 = T+u square/2 Cp and since there is no heat addition in this particular case T0 is a constant.

There is no heat addition or work addition, so T0 is a constant, so let us write like this, T0 is a constant and if I take a differential of this expression dT0 = dT+U du/Cp and this is = 0. And if I use the fact that U can be written as M * square root of gamma RT, this expression can be rewritten like this, dT/T = -gamma-1*M square*dU/U. The next equation that we are looking at is the equation of state since P = rho RT.

I can take a differential of this expression and write it like this, dP = RT*d rho+rho R*dT. And if I substitute for dT from here and if I substitute for d rho from here into this equation I can write this eventually like this dP/P =-1+ gamma – 1*M square*dU/U. So we will denote these equations with the star, so these are the changes in quantities or properties that we are looking at. So this we will denote with a *, this will we will denote with a star.

The next question that we are going to write is the entropy equation. So this I can write like this, dS = Cv dP/P – Cp d rho/rho, we wrote this equation down in the beginning itself. So dS can be written as Cv dP/P-Cp d rho/rho and if you substitute for dP/P from here and you substitute for d rho over rho from this equation, then we get the following.

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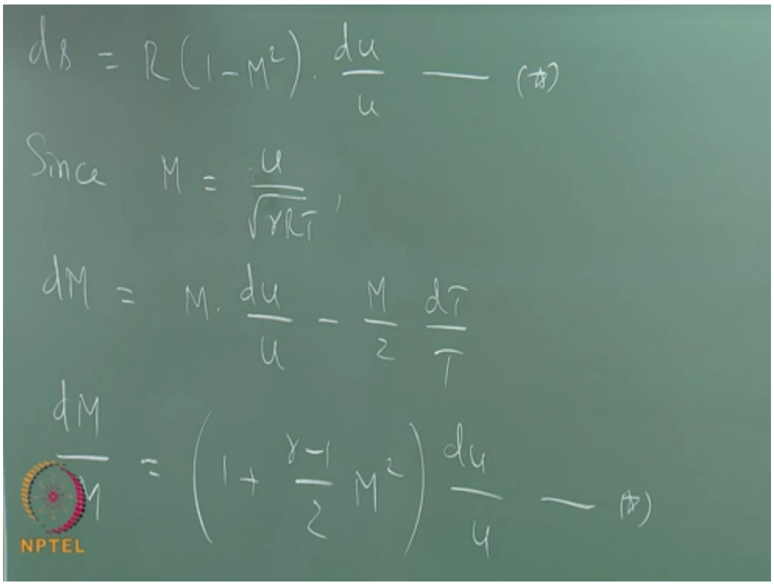

We can write dS = R*1 – M square*dU/U. Now from the definition of Mach number, since M = U/square root of gamma RT, so take the differential of this equation, I can write this as dM = M*dU/U-M/2*dT/T and once again if I write this in the form dM/M which is more convenient for us, I can write this as, if I divided by M on both sides, I can write this as dM/M = 1+gamma – 1/2*M square times dU/U.

So this completes the list of property changes that we were looking for. So this gives us differential changes in density temperature, pressure, entropy and Mach number. Although this form of this expressions are useful like we did earlier, remember when talked about flow with heat addition, we wanted to relate incremental changes in properties to incremental heat addition.

That was what we were looking at because we were looking at flow with heat addition. So we relate it, we ask the question how do the properties change when I add an amount of it delta q or I change the stagnation temperature by dT0. That was what we did eventually, we wrote everything in terms of dT0 over T0. Similarly, here the objective is, there is a fictional force over small length of the pipe, how do the properties change because of that.

These equations as they are written tell me what happens if the velocity changes were small amount, how do the other properties change. What I want to know is how do the properties change if I move by say a distance dx, that is what we eventually need. So if I move by a distance dx I know that whether the flow is subsonic or supersonic as a result of moving through a distance dx in the duct or the pipe, the entropy will always increase.

So the entropy always points in the direction of the flow in this case. So it will be more convenient if I rewrite all these incremental changes in terms of dS rather than dU. The way it is written, each one of this equation tells me what happens if dU changes by certain amount. What I want is how do these properties change if I move an incremental distance dx. Now incremental distance dx results in an increase in entropy by dS, irrespective of whether it is subsonic or supersonic.

So it is more advantageous to rewrite this in terms of dS. So what we do is to use this equation and eliminate dU/U from all the equations and write everything in terms of dS. That is a very easy thing to do and that is what we are going to do now. So we will rewrite everything in terms of dS and let us see what that looks like.

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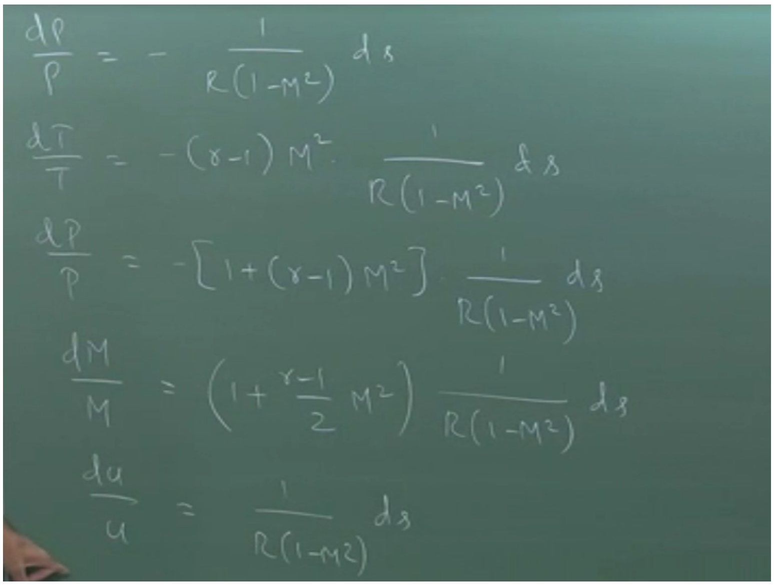

So I can easily do the algebra and write d rho over rho = -1/R*1-M square dS, dT/T can be written like this, is =-gamma-1*M square*1/R times 1-M square dS. Similarly, dP/P can be written like this, is = -1+gamma-1*M square*1/R*1-M square dS. dM/M can be written similarly, 1+gamma-1/2*M square*1/R*1-M square TS and dU/U=1/R*1-M square dS.

So now the equations are in the form which I want them. So if I move a small distance dx which means there is an increase in entropy by an amount dx then I know how the property changes are going to be. Now let us tabulate this property changes and then see what happens. So we are going to tabulate this property changes in the same manner as we did earlier.

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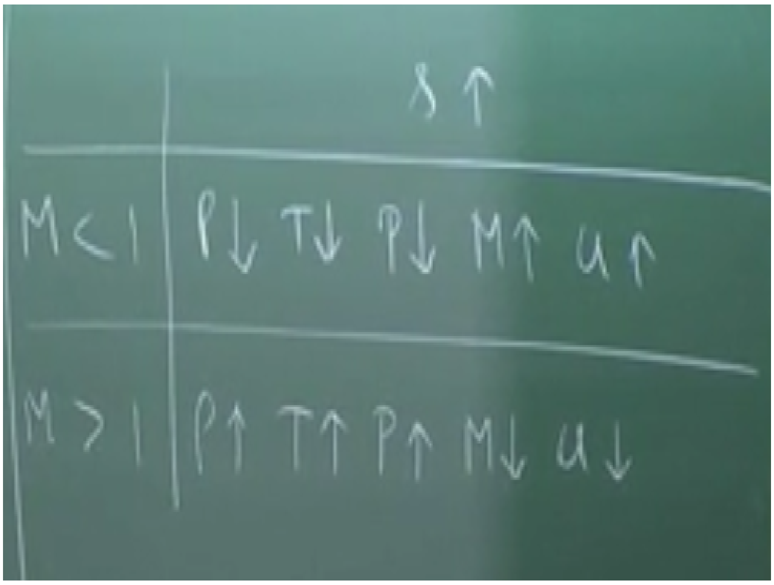

So let us look at 2 situations one corresponding to subsonic case and one corresponding to the supersonic case. Remember in both cases as I travel along the duct or the pipe the entropy is going to increase. So that means dS is positive. There is no situation where dS can be negative, so it is enough if I consider dS being positive. So if dS is positive, this tells me that if the flow is subsonic, this quantity is positive, this is positive, so density d rho is positive.

So density, did we do this correctly? dS is positive, this is positive, but there is a negative sign in the front, right? So density decreases in this case, so rho decreases, next one dS positive, 1-M square positive for subsonic Mach number. No problem with this term. There is as negative sign, so that means temperature decreases. If you look at pressure same thing, dS is positive, this is also positive. The term within the square bracket is also positive.

There is a negative sign here that means P decreases. Mach number positive, this is positive, this term is also positive, so Mach number actually increases in this case as a result of friction in a subsonic flow Mach number increases and if you look at this term here, dS is positive, this term is also positive, so U increase. So dU is positive, so U increases for the subsonic case.

Now let us look at the supersonic case, dS is positive, this term is negative for a supersonic case M is >, so that cancels with this negative term so the density for a supersonic case actually d rho becomes positive, so density increases. So the same thing here, dS is positive.

This is negative and that negative sign cancels this negative sign, so that the temperature is going to increase in ths case, so T also increases. What about P?

This is positive, this is negative, this entire thing is positive. So this negative sign, this negative sign with the result that P increases in a supersonic flow as a result of friction. Mach number dS positive, this term is negative, this term is positive, so Mach number decreases as a result of friction. And velocity dS positive, 1-M square negative so that dU also is negative. So U decreases as a result of friction.

Remember the idea that we started out with is the following. Starting from the initial state we want to locate all the subsequent state so that we can go from the inlet to the exit, that was the objective we started out with. So this kind of tells me how things are headed. So if I am going to illustrated this on a TS diagram, if I start with a subsonic inlet, then the temperature decreases, entropy increases.

That means state point is going to lie to the right and below of the state we have now and we keep proceeding like this. It becomes easier like I did before I write down the equation for the Fanno curve itself. Let us look at the Fanno curve and then we will proceed from there. So if I look at this equation for a TS diagram this equation gives me dT dS. So that is the equation which connects all the states that I am looking for.

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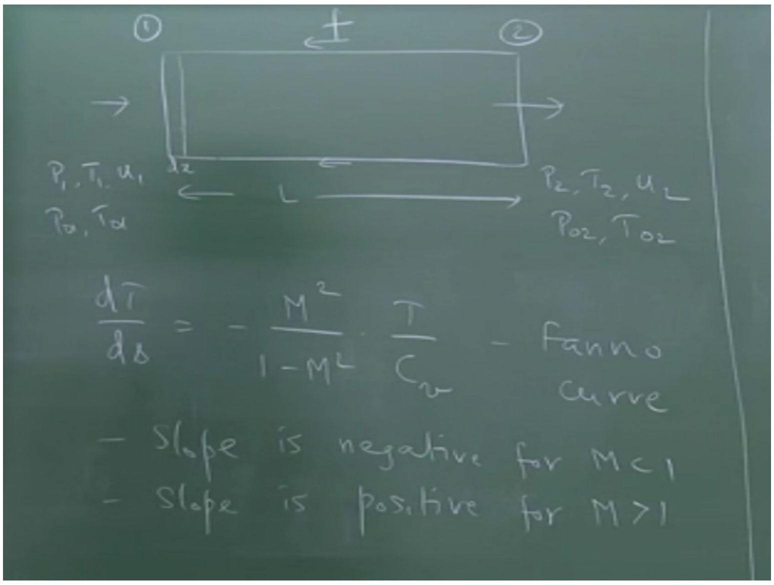

So this gives me dT dS, so I can write this as following. So dT dS =-M square/1-M square*T/Cv. So this is the equation that connects all the subsequent states for every incremental movement. I know how the states are connected. So this curve is the locus of all the possible downstream states starting from the inlet state 1. So the inlet state 1 and then we have next state, next state and so on.

So all those states are connected by this particular curve. This is called the Fanno curve. Just like the Rayleigh curve this is the Fanno curve on a TS diagram. And let us write down some inferences before we transfer this information to the diagram. Some of the important inferences are, when M is < 1 this term is positive, this is also positive, there is a negative sign here.

So slope of the Fanno curve is negative for the subsonic Mach numbers, right. Negative for any subsonic state. So that means on a TS diagram the subsonic part of the curve will look this way, slope is negative. Now slope is positive for supersonic states.

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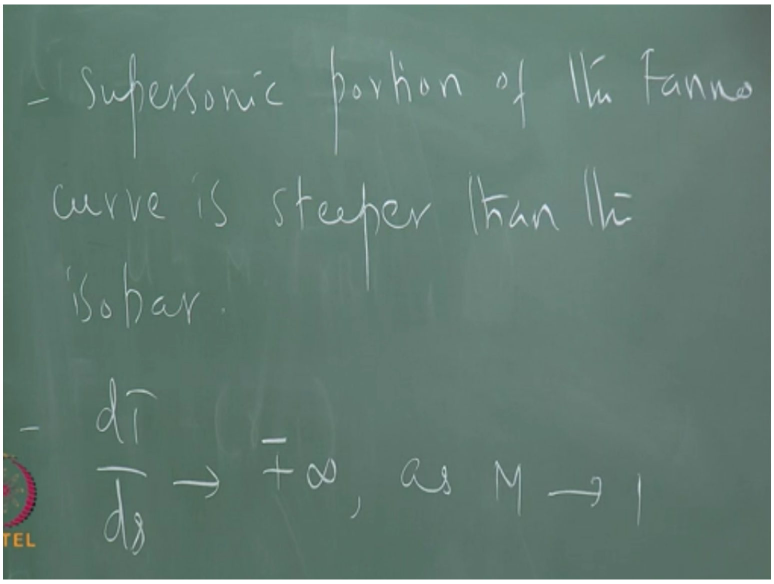

And if I compare this equation with the expression that we wrote down earlier for the slope of a isobar in a TS diagram you can easily see that the supersonic part of the Fanno curve is steeper than the isobar and as M goes to one, notice that on the subsonic portion the slope goes to-infinity and as M goes to 1 from the supersonic side, the slope goes to + infinity. So dT dS tends to - + infinity as M goes to 1.

So with this we can now roughly sketch what the Fanno curve will look like on a TS diagram and let us take a look at that.

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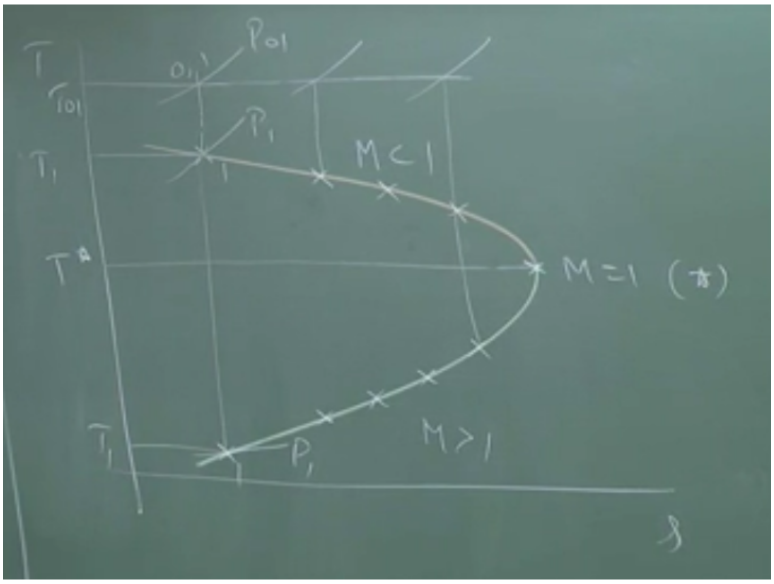

Next initially start with the subsonic state, let us say that this is my state one which is a subsonic state. Okay, so this is T1 and this is xi p1 and this is xi p1 and let us say this is my stagnation state this is T01 now starting from this state this is my inlet state, starting from that state if I try to sketch the locus of all the possible downstream states I realize that since M1 is subsonic the slope is going to be negative.

And the slope will keep increasing until it becomes-infinity. So I can roughly draw this curve like this, right, as I move along the pipe corresponding to different lengths of the pipe, let us say if the pipe length is certain value here, then my downstream state may lie let us say somewhere here. That will be my downstream state. For a longer pipe, my downstream state will lie further down.

Then I say keep increasing the length, the state keeps moving further and further downstream along this curve, until I reach the point when M = 1 this is the sonic state. And remember in this case unlike the Rayleigh flow case the stagnation temperature remains constant. Which means that T* also remains constant. So my T* is going to be constant throughout based on this value of T01 and stagnation temperature remains constant.

So I can extend this line, for anyone of this downstream states this will be the stagnation temperature and this will be the stagnation pressure. And once again you can see that due to the increase in entropy because we are moving to the right there is a loss of stagnation pressure. We will show this mathematically but you can see it graphically from this diagram so far.

Once again you can see that starting from an initial state if I make the pipe long enough of length equal to let us say L*, then I can reach the sonic state. So this is for a certain length, increase the length I go to state 2, I increase it further I go to this state, if I make it sufficiently long let us say equal to a critical value L* then I reach the sonic state. Now what would happen if I increase the length beyond that?

Again it is a question that we will answer afterwards. In the same manner this takes care of the subsonic portion of the Fanno curve, in the same manner I can draw the supersonic portion of the Fanno curve and that would look something like this. The curve is not symmetric about M = 1, okay? But for illustrative purposes it is alright. So if this is my initial state 1, where the Mach number > 1 let us say this is my T1, initial T1 and remember, the supersonic portion of the Fanno curve is steeper than the isobar.

So which means that the isobar itself would look like this. So this is T1 and starting from this for different lengths of the pipe I get different downstream states until I make it long enough so that the final state is the sonic state. So this is the supersonic portion of the Fanno curve and once again you can see that the stagnation temperature remains the same, so that the stagnation pressure continues to decrease as we move along the curve, okay? Let us quickly take a look at the curve.

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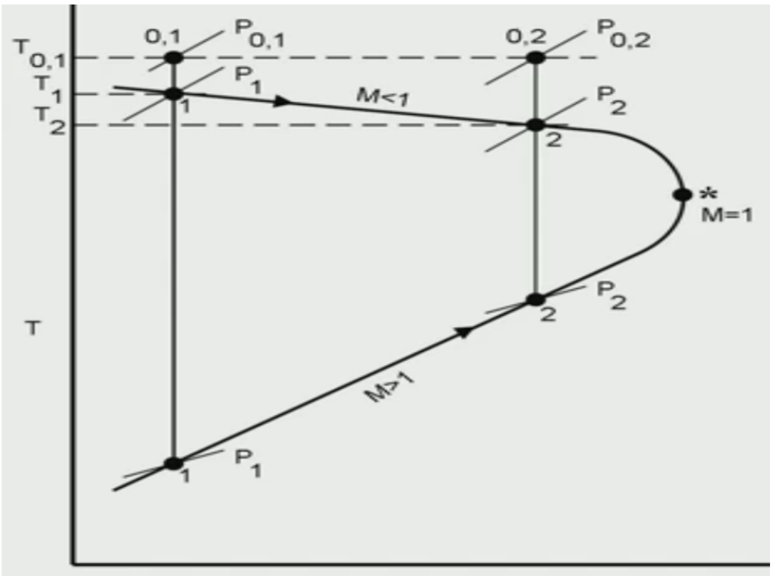

We have a nice picture of the curve here. So if you look at this curve, you will see that the information that we just talked about is included here, so starting from this is the subsonic portion of the Fanno curve. This is the supersonic portion of the Fanno curve. So starting from a subsonic state 1 notice that the subsonic portion of the Fanno curve is less steep than the isobar.

This is why the isobars are like this. So as I move along the subsonic portion of the Fanno curve, we notice that the pressure decreases when I go from 1 to 2. There is also a reduction in stagnation pressure when I go from state 01 to state 02. And eventually if I make the pipe or duct long enough I can reach the sonic state as shown here. And if I start from a supersonic Mach number which is 1 here.

Again you notice that the supersonic portion of the Fanno curve is steeper than the isobar. So as I move along the supersonic portion of the curve, the pressure increases as I go from here to here and then I keep going like this until if I make it long enough, I can reach the sonic state in this pipe.

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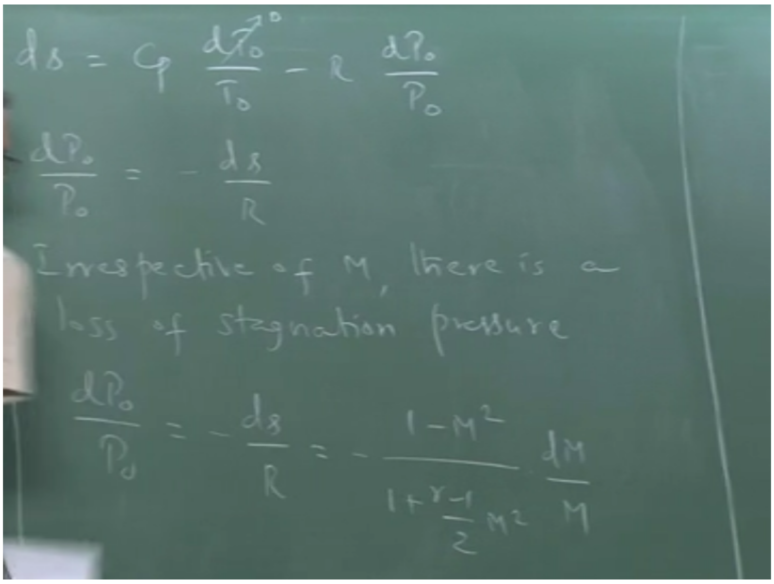

Let us now derive an expression for a change in stagnation pressure as we move along the pipe. That is an extremely important quantity and if you remember earlier we had derived the, we have shown the following equation in one of the earlier chapters, $dS = dT_0/T_0 - R*dT_0/T_0$ and in the case of Fanno flow, dT_0 is 0, so I can set this term to 0, so that $dT_0/T_0 = -dS/R$.

So I can see that irrespective of the Mach number there is always a loss of stagnation pressure dS is always positive as I move along the duct, so irrespective of whether the flow is subsonic or supersonic there is a loss of stagnation pressure. And if I write dS in terms of Mach number I can eventually write this equation like this, dP0 over P0 = -dS/R and that is = – 1-M square/1+gamma-1/2*M square*dM/M.

So this is also in the same form as the other equation, so once I know the Mach number I can calculate the changes in stagnation pressure using this equation. Now we said that it is possible to start from any initial state either subsonic or supersonic and if I make the pipe long enough I can reach the sonic state. So in the earlier case with Fanno flow we saw that when you add heat.

If you add sufficient amount of heat then we can start from any state subsonic or supersonic and reach the sonic state, we call that thermal choking because the choking was coming because of heat addition. Choking refers to the Mach number becoming = 1 at some point. The cause of that is heat addition in the previous situation, so we called it thermal choking.

So this type of choking is called friction choking because the choking is happening due to friction and let us see the consequences of friction choking.

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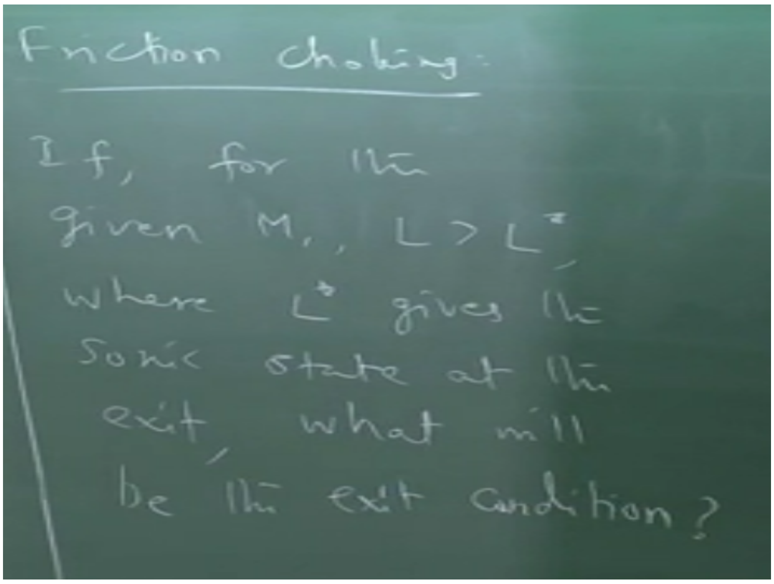

So the question that we wish to answer is if for the given Mach number L is > L* corresponding to, L* is the length of the pipe which should give me sonic condition at the exit. If L is more than L* what would the flow conditions in the pipe be, right? So the L* gives the sonic state at the exit, what will be the exit condition? Supersonic or subsonic if the length of the pipe is > L* what would be the flow condition at the exit?

So we will consider 2 cases when the inlet Mach number is subsonic and another case when the inlet Mach number is supersonic. So if M1 is < 1 and L is > L* corresponding to M1, in this case the flow is subsonic and if the length is more than the L* corresponding to M1 it turns out that the answer to this question is same as what we had before, there is an adjustment of the inlet static condition and the mass flow rate reduces.

So in this case if L is > L* corresponding to M1 there is a reduction, there is a change in the inlet static state and a reduction in the mass flow rate. So we move from one Fanno curve corresponding to a certain value of mass flow rate to a Fanno curve which has a lesser mass flow rate. So if I show this on a TS diagram it looks like this. Let us say that this is my initial state 1.

Let us say this is T01, and this corresponds to a certain constant value of M dot/A and we want a reduction in the mass flow rate without the change in the stagnation conditions. So remember we said even in the earlier case that static conditions can be changed because it is a subsonic flow but the stagnation conditions musts remain the same at the state 1 because stagnation condition at state 1 maybe changed only due to heat or work addition of (()) (33:35) state 1 which we are not doing now.

So stagnation condition at 1 remain the same and the static conditions change so that we climb on to a different Fanno curve which looks like this and now this becomes my new state 1 prime. So this is the sonic state T*, so you can see what has happened. So this curve, if you look at this curve, you can see that the L* corresponding to this inlet condition approximately is this.

So since the actual L is more than L*, I can get on to a Fanno curve where the L* is more. So state 1 prime is such that the given length of the duct = L*. So that is what happened.

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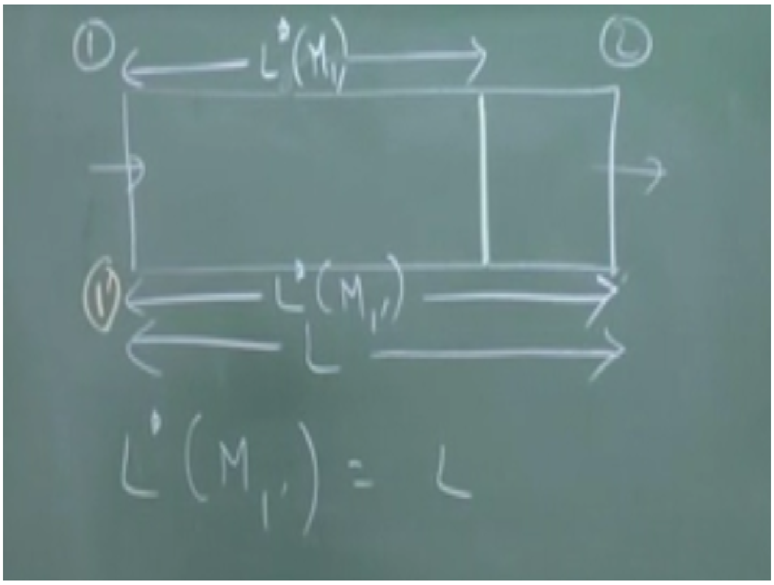

So pictorially if you see we had a duct or a pipe where we had state 1, state 2 and this was L. Now corresponding to inlet state 1, this L was more than L*. So in other words the L*, or let us say, something like this, so this is, I am going to denote this as L1* or I can also denote this as L* corresponding to M1. So the given L is more than L corresponding to M1, correct? L* is the length of the duct that would take the flow from the inlet state to sonic state after length L*.

Since this L is more than L* the flow adjust itself so that we get a new state at the inlet which we denote as 1 prime. For 1 prime, this would be L would be = L* of M1 prime, right? So L* of M1 prime = L. So the important concept here is the reduction in the mass flow rate and the loss of stagnation pressure. So if you remember, we started this chapter by saying that compressors and storage tanks were located far away from buildings where you have the equipment.

So if the equipment is designed for let us say a certain stagnation pressure and a certain mass flow rate, you want a certain mass flow rate in the equipment when the storage tanks and forms were located further away and if you do not design the pipes on the ducts properly and if their friction choking in between your equipment and your storage tank then we can see that you are not going to get the mass flow rate that you want.

So if the distance between your equipment and your storage tank is more than L* then you notice that your mass flow rate reduces and you cannot get the kind of mass flow rate you want, number one + there is also a loss of stagnation pressure. So if I want a stagnation pressure of 10 bar for my equipment, I need to know what the stagnation pressure in the storage tank should be so that after all these losses I get 10 bar here.

So I want 10 bar here and let us say a mass flow rate of 2 kg per second I need to know this to determine what my reservoir condition should be. And if the reservoir is located let us say 500 meters from an equipment, then I really need to figure out what the reservoir conditions have to be for the given piping, okay? So in that case you can intuitively see because this is M dot/A and if I want to minimize the stagnation pressure loss and if I want to avoid friction choking what do I do?

I use pipes of larger cross sectional area, larger diameter or larger cross sectional area until I reach very close to the compressible flow equipment itself. There is also a cost fact involved, if you make the pipes larger they cost more per unit length than smaller pipes. All this says is that I cannot use a 1-inch diameter pipe from tank which is located 500 m away to my equipment.

So I need to use a bigger pipe and pipes have increasingly smaller diameters until I reach the equipment itself. That is why understanding this effect is very important in practical situation. We will also do numerical examples that explain this idea. So this is for the case when the inlet Mach number is subsonic.

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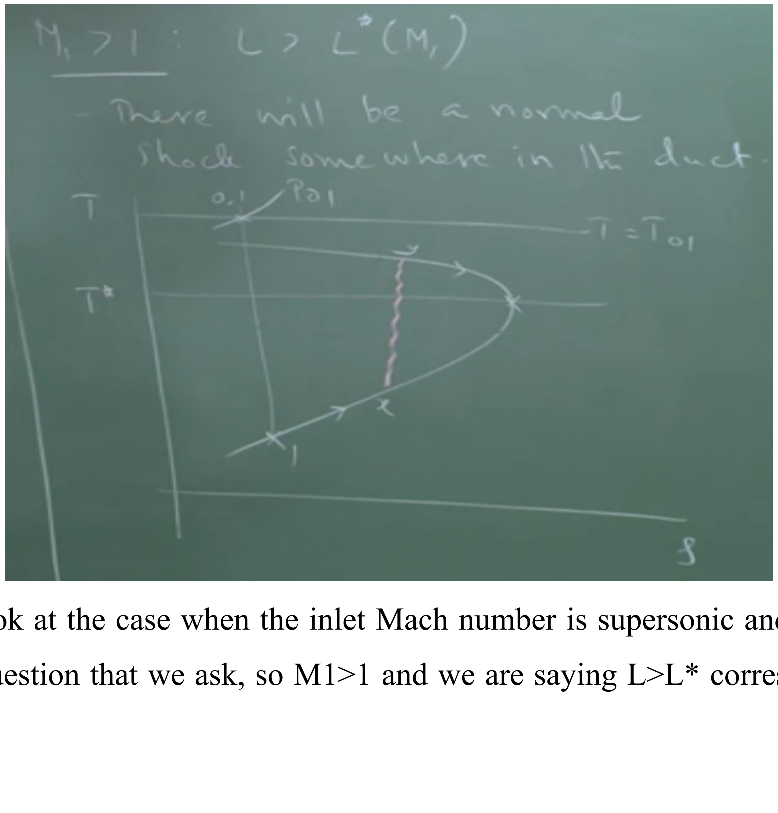

Now we will look at the case when the inlet Mach number is supersonic and once again the answer to the question that we ask, so M1>1 and we are saying L>L* corresponding to M1.

So what would happen to the downstream state. The answer to this question is also the same as before, there will be a normal shock somewhere in the pipe. So in other words, if I show this on a TS diagram, let us say this is my initial state, this is T*, let us say this is T = T01.

So this is my stagnation state. It is P01. So there is going to be a normal shock that sits somewhere along the duct So let us say that the normal shock is here, so we sketch the normal shock like this. So let us call this state just before the normal choke as x and the state just after the normal shock as y. So we go from state 1 up to state x and then there is a normal shock.

We then jump to the subsonic branch of the Fanno curve and then we go along the subsonic branch of the Fanno curve. Now the concept that you need to remember is the following. L* corresponding to M1 is this, that is L* corresponding to M1. What is L* corresponding to x? L* corresponding to x is also the same quantity because they are both on the same branch, right? L* corresponding to x is also equal to this. So what basically I am saying is the following.

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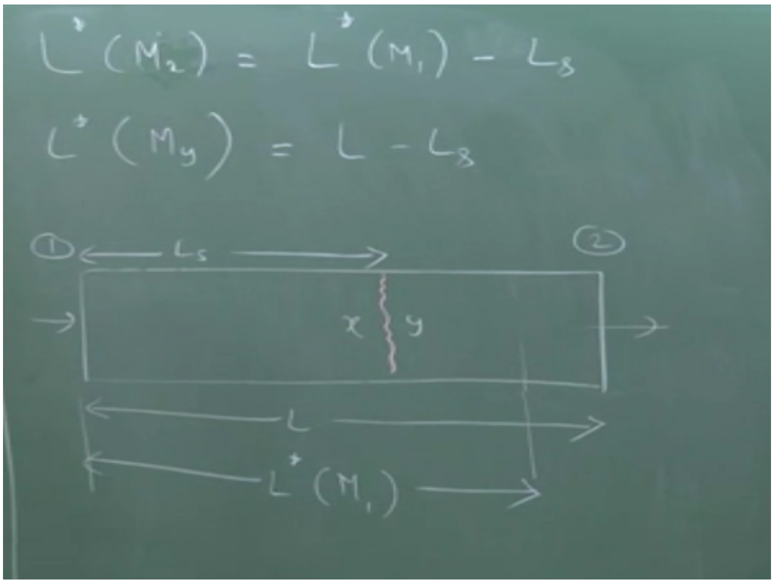

L* of x state x or corresponding to Mach number x is equal to the length of the pipe. L* corresponding to M1-the length of the pipe and travel the length L along the pipe, so L* corresponding to this Mach number is going to be the L* for this-this length of the pipe. Now I go here, notice that L* corresponding to My is going to be different. So that is how we accommodate the length L which is > L* corresponding to this.

Now I have a L* corresponding to My also. So the remaining length will be such that L* corresponding to this, the remaining length of the pipe comfortably that is what we are seeing here. Do you understand that? Is that clear. So what we are saying is L* corresponding to My is =, what is that equal to? The remaining length of the pipe. So that is the remaining length of the pipe, L-Ls where Ls is the location of the shock.

So let us show this pictorially state 1, state 2, total length of the pipe or duct is L and we have a shock which sits xi over there, Xi that the shock occurs at a distance Ls, okay? So this is state point x and this is state point y. So the shock occurs at such a location which is such that the L*corresponding to My is = the total length of the duct-this Ls, okay? So if you want to determine the location of the shock that has to be done iteratively using these ideas.

Both these constraints must be satisfied. The state x should be such that L* corresponding to Ms = Ls corresponding to M1-this and this is also be satisfied. Both these conditions must be satisfied this normal shock can position itself appropriately. Is that clear or is there any doubts in this case? There is one change that we need to make here. Remember L* corresponding to M1 let me just make this small change here, L* corresponding to M1 maybe something like this.

This is L* corresponding to M1. Remember L* corresponding to M1<length of the pipe, so this is L* corresponding to M1. This should also be, please make this following change, so this should be, so L* corresponding to this Mx should be this-this. Remember, this I am travelling a length Ls along the pipe. So L* corresponding to Ms should be L* corresponding to state 1-Ls, correct, is that clear?

Let me pull down this diagram also, so this distance this is say Ls, this is L* corresponding to M1-Ls, and that is = L* corresponding to Mx, okay? And this distance is L* corresponding to My and that is nothing but L-Ls. Okay, we will do a numerical example to illustrate this, but you understand these ideas clearly. Yes, please go ahead. **"Professor-student conversation starts"** Exit conditions, always Mach 1 are there? **"Professor-student conversation ends"**

If nothing is specified then we assume exit conditions to be sonic state, in some problems or in some applications the exit pressure maybe known. In those cases, we have to obtain the solution, the normal shock will locate itself, so that after the flow goes through this remaining length of the pipe, let us say the static pressure here will be equal to the static pressure that is required by the device, or that is set up by the device.

So the exit condition was specified then the normal shock will locate itself to match those exit conditions, where not given then we assume the exit states to be sonic state and proceed with the analysis, alright? For example, if you have a supersonic flow that comes in and I maintain the pressure at this end to be higher than what the supersonic flow would give, then there will be a normal shock and the shock will locate itself so that it matches the exit pressure, okay?

Is this concept clear to everybody? This is very important because these are the 2 constraints which we will use to solve this problem. So what we basically would do is, we assume the shock to occur at some locations, at some distance, so we assume a value for Ls, once I do that, I can then calculate L* and then I can solve the problem iteratively and then it locates itself properly, we will do that next in the next class.