Gas Dynamics and Propulsion

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Lecture - 10
Rayleigh Flow/Fanno Flow

Okay in the previous class, we completed our discussion of flow with heat addition. We looked at situations where the head added was < q star, which was the amount of heat required to take the inlet state to a sonic state for both supersonic and subsonic cases. What we are going to do next is to work out an illustrated example to see how the actual calculations can be carried out. That is what we are going to do next.

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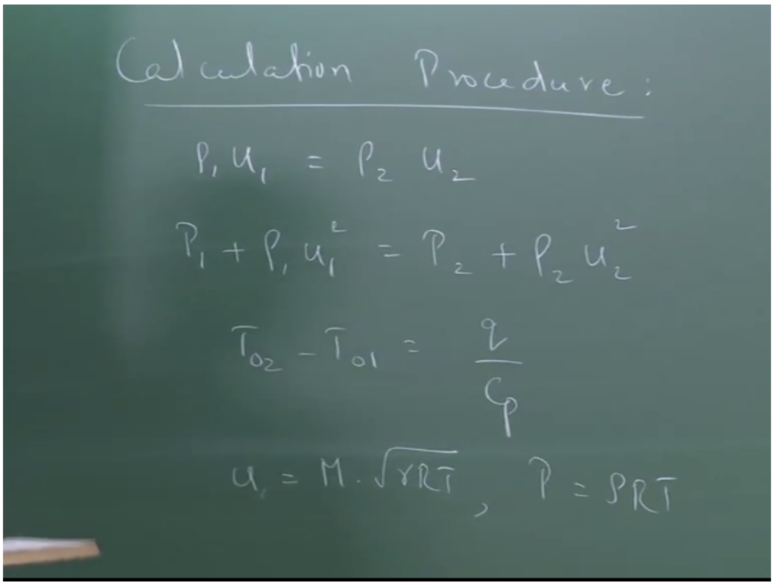

Before we do that let us device a calculation procedure that will allow things to be done in a very systematic and easy manner that is what we are going to do. So for this purpose we start with the original governing equations that we looked at. So the states between inlet and outlet are related like this rho1 U1=rho2 U2 and if you remember P1+rho1 U1 square=P2+rho2 U2 square and the energy equation itself could be written like this.

T02-T01=q/Cp and in addition to this we also have the definition of Mach number U1 or U=the Mach number at any point time square root of gamma RT is a definition and we also have the equation of state, which says P=rho RT. At any point, using these relationships we can actually write down the following equations, which connect the inlet and the outlet state in terms of Mach numbers alone okay.

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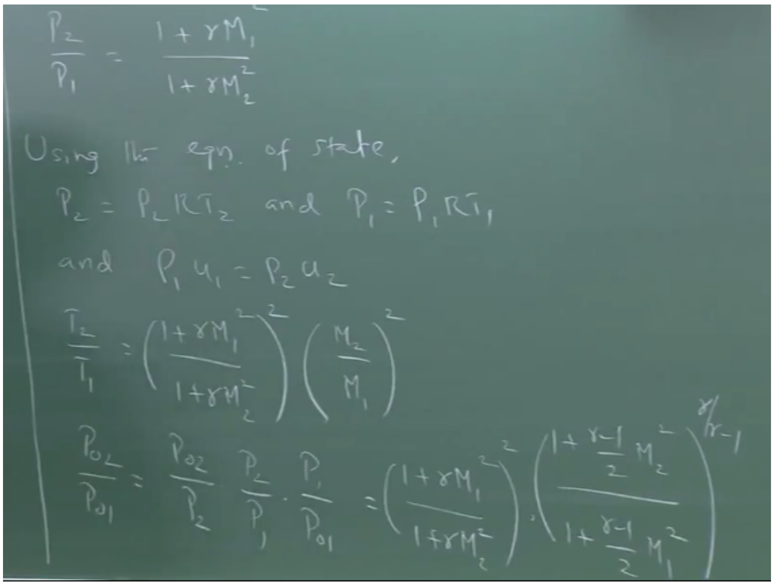

So we can write for example P2/P1=1+gamma M1 square/1+gamma M2 square that is one relation and we can also write down since using the equation of state P2=rho2 RT2 and P1=rho1 RT1 and in addition we can actually use the fact that rho1 U1=rho2 U2 and we can combine these 3 equations and write T2/T1 as 1+gamma M1 square/1+gamma M2 square the whole squared times M2/M1 whole square.

So we can write T2/T1 also in terms of M1 and M2 so this is very similar to what we did earlier in the normal shock relations right. I know M1, I know T1, I know P1, if I know M2 then I can evaluate all the downstream states using this relationship and further to this we can also write for example the ratio of stagnation quantities P02/P01 can be written as P02/P2 times P2/P1 times P1/P01.

So this we can simplify and write in terms of this expressions as follows 1+gamma M1 square/1+gamma M2 square the whole square times 1+gamma-1/2 times M2 square/1+gamma-1/2 times M1 square the whole thing raised to the power gamma/gamma-1.

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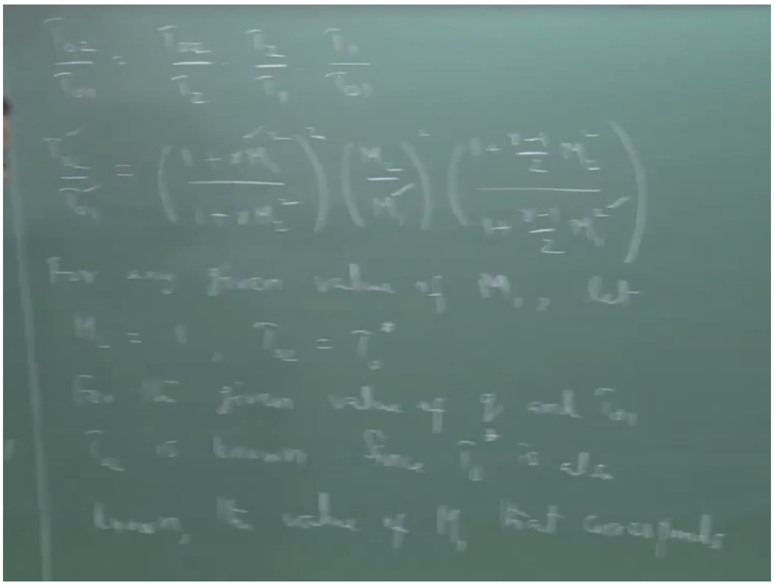

And in the same way we can write T02/T01 like this is = T02/T2 times T2/T1 times T1/T01 and once again I can write it like this 1+gamma M1 square/1+gamma M2 square whole square times M2/M1 square times 1+gamma-1/2 times M2 square/1+gamma-1/2 times M1 square. So now if I look at these expressions notice that T02/T01 is known to me because we have specified q for the problem.

Remember we are looking at flow with heat addition in a duct, so inlet state is specified. We have also specified the amount of heat that is being added right so q is known to me, which means that q is known to me so T01 is known to me so that means T02 is known, which means that I can actually calculate M2 from this. So in this expression so if I write it like this right T02/T01 right.

So if you look at this expression notice that q is specified in the problem, T01 is also known so I can calculate T02 from this equation. So once I calculate T02 from this equation I can evaluate the left hand side and M1 of course is completely known so we know M1. So this is the equation that I can solve for M2. It is a very complicated equation, but hopefully it will have multiple solutions also.

But hopefully it is something that we can solve so once I obtain M2 then I can calculate all the other quantities like P02/P01 and T2/T1, P2/P1 and so on. All the other things can be calculated once I know M2 and I know how to calculate M2 also from this. So this is a viable solution procedure, but made difficult by the fact that this equation is a highly complicated equation.

It will have multiple solutions and it may be very difficult to solve. A much easier way is to use a tabulated form of this equation. So what we do is the following. Let us say that we take M2 to be = 1 right so for a given value of M1, for any given value of M1 let M2 be = 1. So if I let M2 to be = 1 then this equation is simplified T02 then becomes=T0 star right.

So then there is nothing for me to solve. There is no equation to solve because I have let M2=1 I can directly evaluate T0 star, I am not solving for T0 star I am evaluating T0 star from this. I know T0 star from this, I can calculate all the other quantities also setting M2=1. Once I do this for a given value of q, but we have not supplied q star, we have only supplied q.

What this gives me is the final state if I had supplied q star correct, but the given value is only q so what I do is the following. Now I know my T02 for the given value of q. So what I do is the following. I do this I evaluate T0 star for different values of M1 and I tabulate the result. So I do this for M1=0.1, 0.2, 0.3 and so on and for each value I tabulate T0 star corresponding to that value.

Now I know q, I know T01 which means I know T02 right. Now I know T0 star also so what I do is I know do a reverse look up where I see for this value of T0 over T0 star what is the value for M from here okay? Do you understand that procedure? So it uses a tabulated procedure so we do not actually solve any equation at all from this okay. That is basically what we are doing.

So from this table we calculate T0 star right, I know T02, so I know T02/T0 star because the table tabulates values of T02/T0 star or T0/T0 star. So once I know this I can look up the value of M, which corresponds to that value of T0 now I have my M2 right and then I can proceed with the calculation. Is that procedure clear to you? Right so that is what we are going to illustrate.

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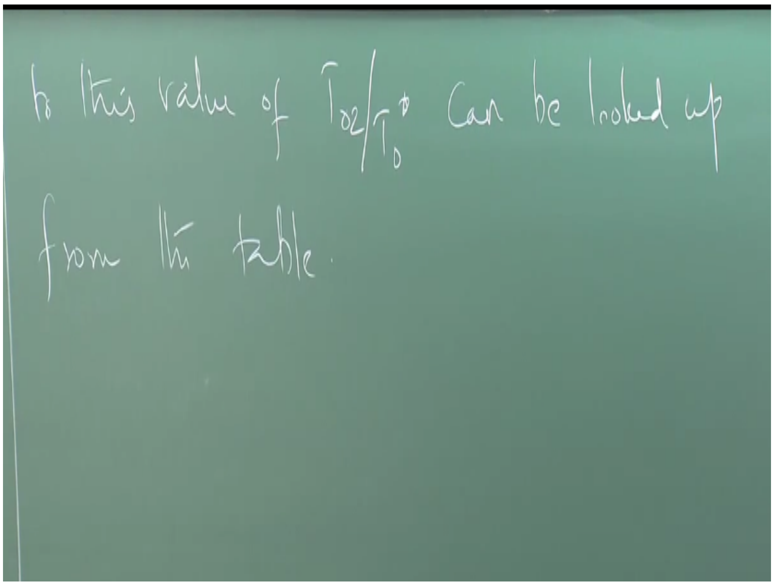

So T02 is known since T0 star is also known, the value of M that corresponds to the value of M1 for example that corresponds to this value of T0/T0 star or T02/T0 star can be looked up from the table. So that is the calculation procedure for any given value of q and we are going to now demonstrate this with the worked example.

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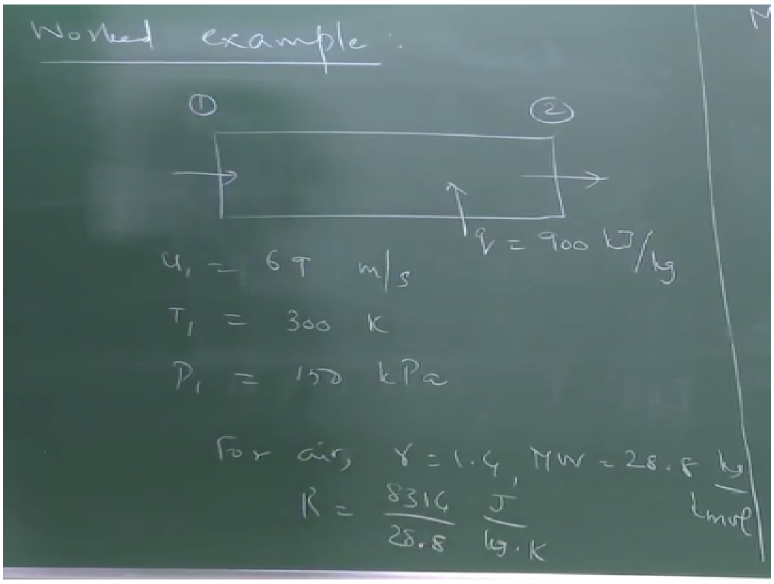

So the worked example states the following, air enters a combustion chamber at 69 meter per second, 300 kelvin and 150 kilopascal where 900 kilojoule per kilogram of heat is added. Determine a, the mass flow rate per unit that area; b, exit properties and c, inlet Mach number if the heat added is 1825 kilojoule per kilogram. So let us sketch the flow situation that we are looking at.

So we have a duct so initially it is given that U1=69 meter per second, T1=300 kelvin and P1=150 kPa. Heat is now added here to the amount of 900 kilojoule per kilogram. So we are asked to determine the exit properties, which should mean M2, P2, T2 and the stagnation quantities that is what we are asked to determine and we will assume the following. For air, we take gamma to be 1.4 and the molecular weight of air to be 28.8 kilogram per kilo mole okay.

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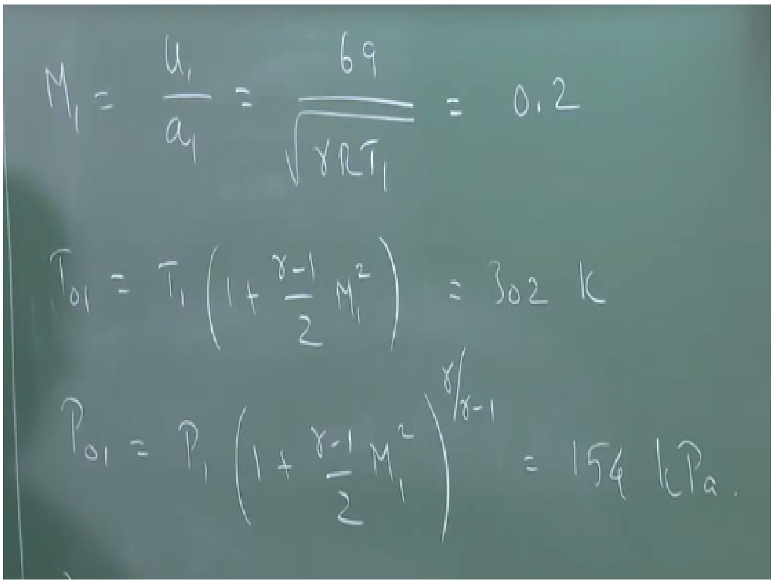

So we first determine the Mach number M1 so M1=U1/A1 so that is nothing but 69/square root of gamma RT1 and if you substitute the values remember R=this R is the particular gas constant so this is = 8314/28.8 and this is in units of joule per kg kelvin. So if you plug in these values for gamma R and T1, we get M1 to be = 0.2 and T01 can be calculated, T01=T1 times 1+gamma-1/2 times M1 square.

And if you substitute the numbers, you get this to be 302 kelvin and P01=P1 times 1+gamma-1/2 times M1 square raised to the power gamma/gamma-1 and this comes out to be 154 kilopascal.

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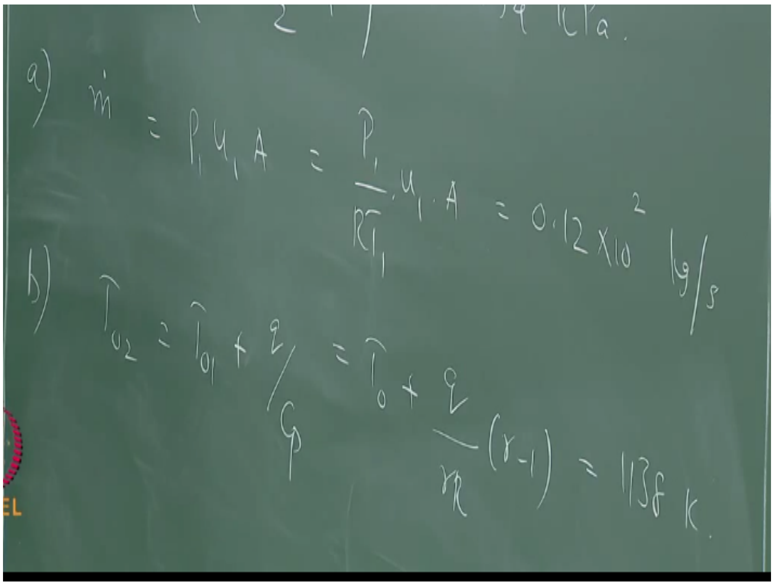

Now we are first asked to calculate in a, the mass flow rate per unit that cross-sectional area. So m dot is going to be rho1 U1 times A and A is given to be 1-meter square, rho1 is nothing but P1/RT1. So if you substitute the values, you get this to be 0.12 times 10 to the 2 kilogram per second as the mass flow rate through the duct for these conditions.

“Professor - student conversation starts.” a, I have taken to be 1-meter square, so I am writing this as kg per second. If you write a mass flow rate in terms of kg per meter square per second that is extremely confusing, so with the understanding that A is 1 meter per second we will write it as kg per second. **“Professor - student conversation ends.”** Part b, we are asked to calculate the exit properties for heat addition, which is = 900 kilojoule per kilogram.

And we are going to use the tables for this, but before we do that let us calculate T02 first so T02 if you remember=T01+q/Cp and this is nothing but T01+q, Cp is nothing but gamma R/gamma-1. So if you substitute the numbers you get T02 to be 1138 kelvin. Now M1 is given to be 0.2, so we use the table to determine T0 star corresponding to M1 right, T0 star/T0 that is what we are going to calculate.

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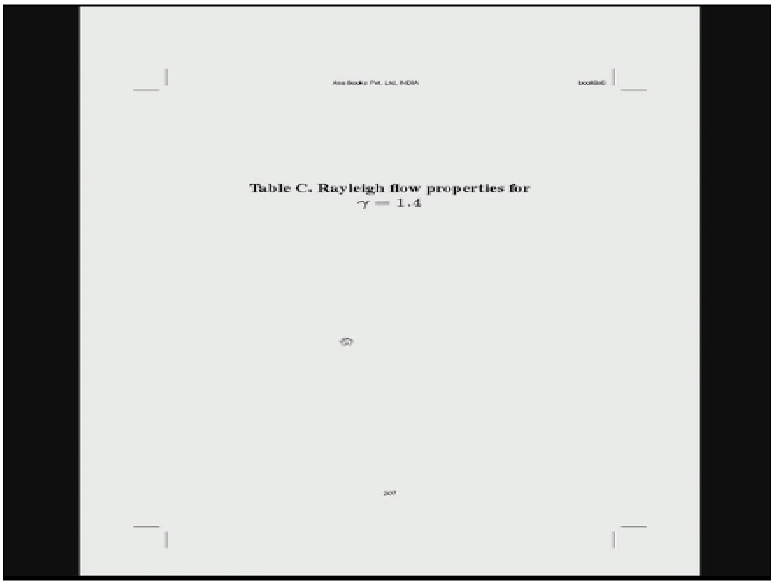

So this is the table that we are going to use Table C, which gives Rayleigh flow properties for a fixed value of gamma.

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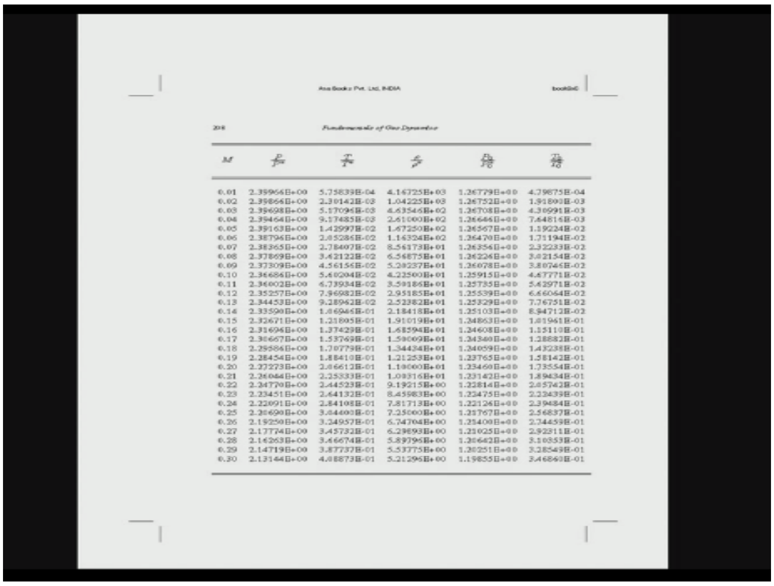
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M	$\frac{p}{p^*}$	$\frac{T}{T^*}$	$\frac{\rho}{\rho^*}$	$\frac{p_0}{p_0^*}$	$\frac{T_0}{T_0^*}$
0.01	2.39966E+00	5.75839E-04	4.16725E+03	1.26779E+00	4.79875E-04
0.02	2.39866E+00	2.30142E-03	1.04225E+03	1.26752E+00	1.91800E-03
0.03	2.39698E+00	5.17096E-03	4.63546E+02	1.26708E+00	4.30991E-03
0.04	2.39464E+00	9.17485E-03	2.61000E+02	1.26646E+00	7.64816E-03
0.05	2.39163E+00	1.42997E-02	1.67250E+02	1.26567E+00	1.19224E-02
0.06	2.38796E+00	2.05286E-02	1.16324E+02	1.26470E+00	1.71194E-02
0.07	2.38365E+00	2.78407E-02	8.56173E+01	1.26356E+00	2.32233E-02
0.08	2.37869E+00	3.62122E-02	6.56875E+01	1.26226E+00	3.02154E-02
0.09	2.37309E+00	4.56156E-02	5.20237E+01	1.26078E+00	3.80746E-02
0.10	2.36686E+00	5.60204E-02	4.22500E+01	1.25915E+00	4.67771E-02
0.11	2.36002E+00	6.73934E-02	3.50186E+01	1.25735E+00	5.62971E-02
0.12	2.35257E+00	7.96982E-02	2.95185E+01	1.25539E+00	6.66064E-02
0.13	2.34453E+00	9.28962E-02	2.52382E+01	1.25329E+00	7.76751E-02
0.14	2.33590E+00	1.06946E-01	2.18418E+01	1.25103E+00	8.94712E-02
0.15	2.32671E+00	1.21805E-01	1.91019E+01	1.24863E+00	1.01961E-01
0.16	2.31696E+00	1.37429E-01	1.68594E+01	1.24608E+00	1.15110E-01
0.17	2.30667E+00	1.53769E-01	1.50009E+01	1.24340E+00	1.28882E-01
0.18	2.29586E+00	1.70779E-01	1.34434E+01	1.24059E+00	1.43238E-01
0.19	2.28454E+00	1.88410E-01	1.21253E+01	1.23765E+00	1.58142E-01
0.20	2.27273E+00	2.06612E-01	1.10000E+01	1.23460E+00	1.73554E-01
0.21	2.26044E+00	2.25333E-01	1.00316E+01	1.23142E+00	1.89434E-01
0.22	2.24770E+00	2.44523E-01	9.19215E+00	1.22814E+00	2.05742E-01
0.23	2.23451E+00	2.64132E-01	8.45983E+00	1.22475E+00	2.22439E-01
0.24	2.22091E+00	2.84108E-01	7.81713E+00	1.22126E+00	2.39484E-01
0.25	2.20690E+00	3.04400E-01	7.25000E+00	1.21767E+00	2.56837E-01
0.26	2.19250E+00	3.24957E-01	6.74704E+00	1.21400E+00	2.74459E-01
0.27	2.17774E+00	3.45732E-01	6.29893E+00	1.21025E+00	2.92311E-01
0.28	2.16263E+00	3.66674E-01	5.89796E+00	1.20642E+00	3.10353E-01
0.29	2.14719E+00	3.87737E-01	5.53775E+00	1.20251E+00	3.28549E-01
0.30	2.13144E+00	4.08873E-01	5.21296E+00	1.19855E+00	3.46860E-01

And we do for M1=0.2, we can see from here that M1=0.2 comes right here so all the values are given. So we go to the last column and we pick up T0/T0 star from there, other quantities can also have picked up from here okay. So we take the values from here T0/T0 star.

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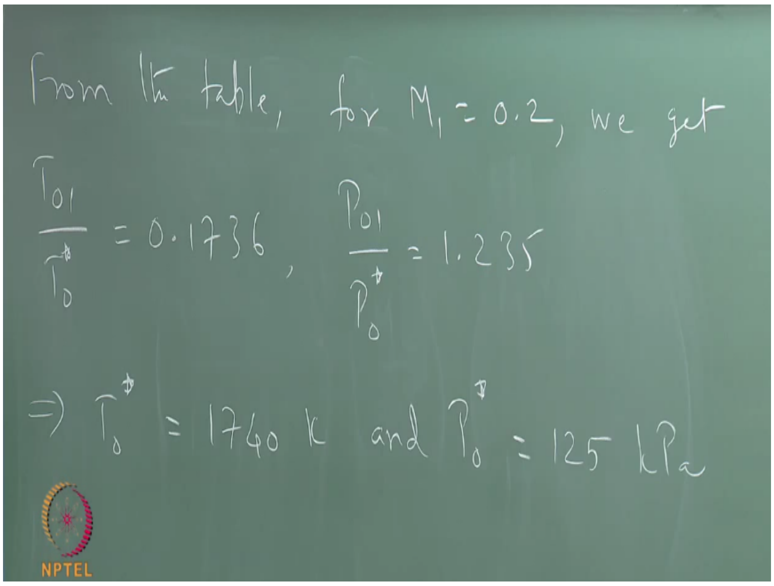

So let us write it down from the table for M1=0.2 we get T01/T0 star to be approximately 0.1736 and P01/P0 star to be 1.235. So this allows me to calculate my T0 star to be 1740 kelvin and my P0 star to be 125 kilopascal okay.

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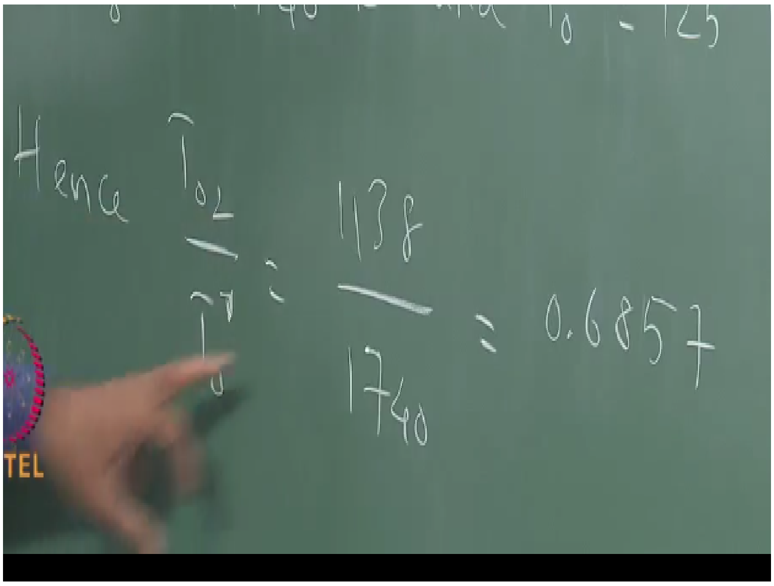

As we stated earlier, now T02 is known, T02/T0 star can be calculated to be 1138/1740, which comes out to be 0.6857. So we now go to the tables corresponding to this value of T0/T0 star we try to retrieve what the Mach number is going to be. That is the calculation procedure okay. So we actually do not solve any equation, we simply do a table look up which is very, very convenient to do.

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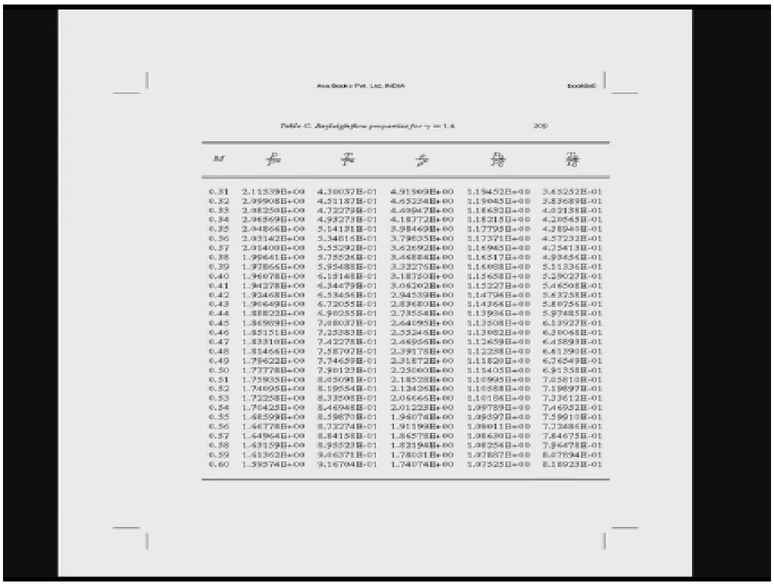

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Table C. Rayleigh flow properties for $\gamma = 1.4$ 209

M	$\frac{p}{p^*}$	$\frac{T}{T^*}$	$\frac{\rho}{\rho^*}$	$\frac{p_0}{p_0^*}$	$\frac{T_0}{T_0^*}$
0.31	2.11539E+00	4.30037E-01	4.91909E+00	1.19452E+00	3.65252E-01
0.32	2.09908E+00	4.51187E-01	4.65234E+00	1.19045E+00	3.83689E-01
0.33	2.08250E+00	4.72279E-01	4.40947E+00	1.18632E+00	4.02138E-01
0.34	2.06569E+00	4.93273E-01	4.18772E+00	1.18215E+00	4.20565E-01
0.35	2.04866E+00	5.14131E-01	3.98469E+00	1.17795E+00	4.38940E-01
0.36	2.03142E+00	5.34816E-01	3.79835E+00	1.17371E+00	4.57232E-01
0.37	2.01400E+00	5.55292E-01	3.62692E+00	1.16945E+00	4.75413E-01
0.38	1.99641E+00	5.75526E-01	3.46884E+00	1.16517E+00	4.93456E-01
0.39	1.97866E+00	5.95488E-01	3.32276E+00	1.16088E+00	5.11336E-01
0.40	1.96078E+00	6.15148E-01	3.18750E+00	1.15658E+00	5.29027E-01
0.41	1.94278E+00	6.34479E-01	3.06202E+00	1.15227E+00	5.46508E-01
0.42	1.92468E+00	6.53456E-01	2.94539E+00	1.14796E+00	5.63758E-01
0.43	1.90649E+00	6.72055E-01	2.83680E+00	1.14366E+00	5.80756E-01
0.44	1.88822E+00	6.90255E-01	2.73554E+00	1.13936E+00	5.97485E-01
0.45	1.86989E+00	7.08037E-01	2.64095E+00	1.13508E+00	6.13927E-01
0.46	1.85151E+00	7.25383E-01	2.55246E+00	1.13082E+00	6.30068E-01
0.47	1.83310E+00	7.42278E-01	2.46956E+00	1.12659E+00	6.45893E-01
0.48	1.81466E+00	7.58707E-01	2.39178E+00	1.12238E+00	6.61390E-01
0.49	1.79622E+00	7.74659E-01	2.31872E+00	1.11820E+00	6.76549E-01
0.50	1.77778E+00	7.90123E-01	2.25000E+00	1.11405E+00	6.91358E-01
0.51	1.75935E+00	8.05091E-01	2.18528E+00	1.10995E+00	7.05810E-01
0.52	1.74095E+00	8.19554E-01	2.12426E+00	1.10588E+00	7.19897E-01
0.53	1.72258E+00	8.33508E-01	2.06666E+00	1.10186E+00	7.33612E-01
0.54	1.70425E+00	8.46948E-01	2.01223E+00	1.09789E+00	7.46952E-01
0.55	1.68599E+00	8.59870E-01	1.96074E+00	1.09397E+00	7.59910E-01
0.56	1.66778E+00	8.72274E-01	1.91199E+00	1.09011E+00	7.72486E-01
0.57	1.64964E+00	8.84158E-01	1.86578E+00	1.08630E+00	7.84675E-01
0.58	1.63159E+00	8.95523E-01	1.82194E+00	1.08256E+00	7.96478E-01
0.59	1.61362E+00	9.06371E-01	1.78031E+00	1.07887E+00	8.07894E-01
0.60	1.59574E+00	9.16704E-01	1.74074E+00	1.07525E+00	8.18923E-01

So we go to the tables and for this value of T0/T0 star if you go down the table you can see that approximately this is 0.6857 so we get approximately 6857 comes somewhere in between these 2 right so we take the Mach number to be between 0.49 and 0.5.

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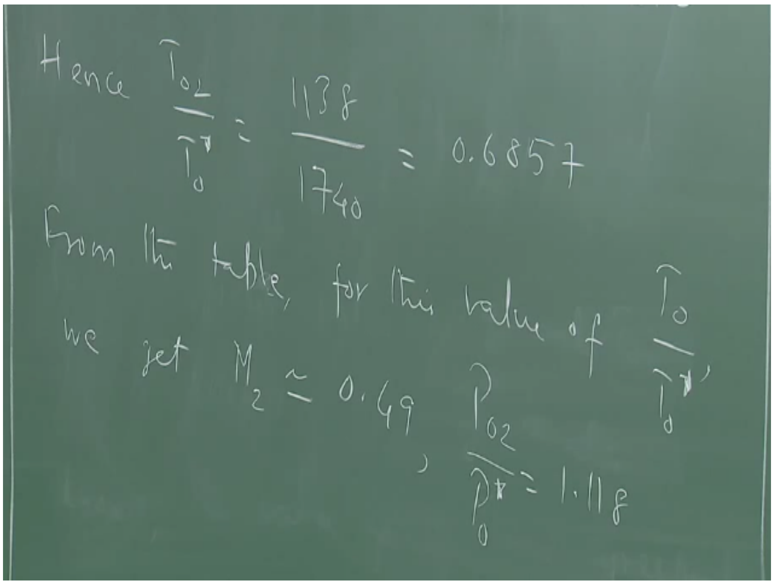

So from the table for this value of T0/T0 star we get M2 to be we will approximate it as 0.49, it actually falls in between 0.49 and 0.5 and I can also retrieve P02/P0 star, I can retrieve this also in one go to be 1.118. P0 star is already known to me from here right, P0 star is 125 kilopascal, so I can now evaluate P02 from this.

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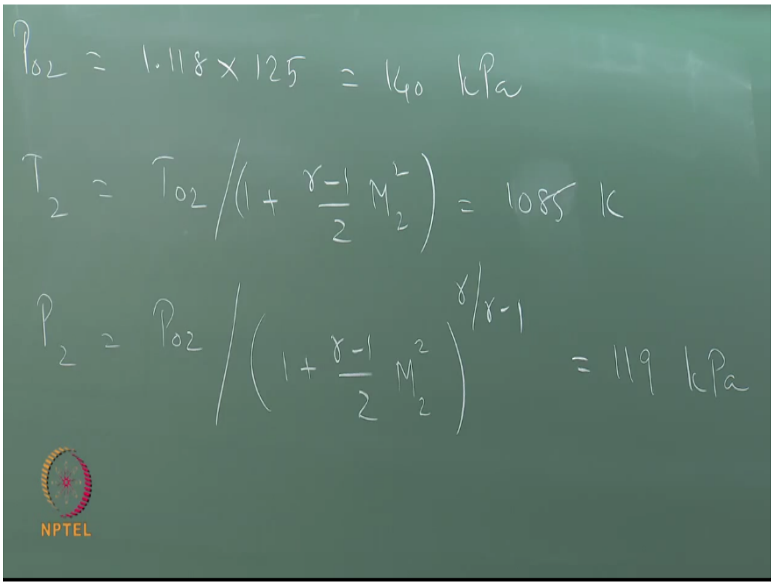

So P02=1.118 times 125 which is nothing but 140 kilopascal. So you should remember that our P01 was calculated to be 154 kilopascal was P01 and notice that P02 is 140 kilopascal, which means there is a loss of stagnation pressure due to the heat addition that is consistent with what we expect from these calculations. So we are asked to calculate T2 the static state also.

So T2 I can calculate this way, T2=T02/1+gamma-1/2 times M2 square and if I substitute the numbers, I get T2 to be 1085 kelvin and P2 we calculate in a similar manner. P2=P02/1+gamma-1/2 times M2 square raised to the power gamma/gamma-1. So this comes out to be 119 kilopascal. Remember this is a subsonic flow. The Mach number is subsonic at the inlet.

So we expect heat addition, we expect the static pressure to decrease as a result of heat addition and that is what we are seeing here also. “Professor - student conversation starts.” T2 is 1085 yes “Professor - student conversation ends.” So the reduction in static pressure as a result of heat addition is also consistent with our expectations.

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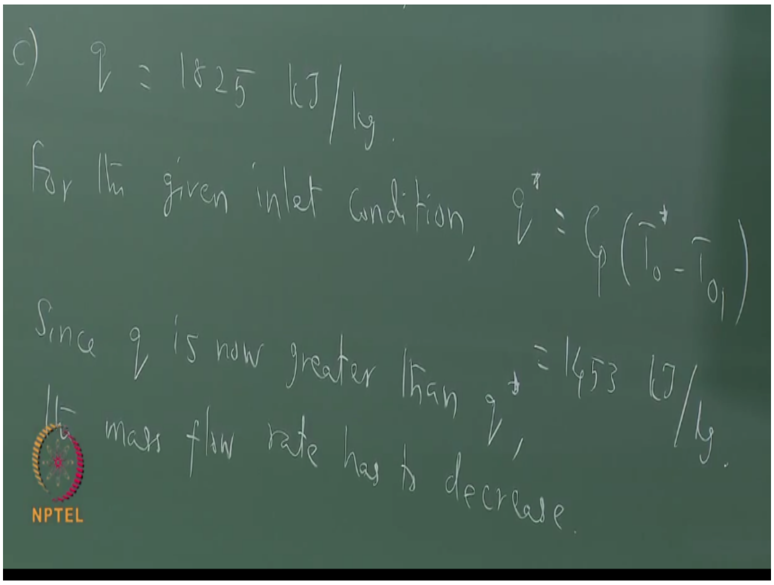

Now part c, the heat instead of 900 kilojoule per kilogram the given heat addition is 1825 kilojoule per kilogram is the amount of heat that we are going to add in the same duct, so we are asked to determine the change inlet conditions for this much addition of heat okay. So we need to first determine what q star is? We need to see whether this is < q star or not. If it is < q star, then the inlet conditions will not change.

There is no problem. If this is more than q star, then there will be a change in the inlet conditions usually a reduction in mass flow rate to accommodate this heat release okay. So first we need to calculate q star right. So for the given inlet condition, q star can be calculated as Cp times T0 star-T01. I know T0 star, so I can calculate q star from this without any difficulty and this I get to be 1453 kilojoule per kilogram.

Since the amount of heat to be added is more than q star > q star, the mass flow rate has to decrease. So we will calculate the new inlet static condition to accommodate this value of heat.So the principle is that the mass flow rate has to be such that the amount of heat that I have to add which is 1825 kilojoule per kilogram will be = the q star corresponding to the new inlet state right. So that is what we are going to calculate.

What is the new inlet state for which q star=1825 kilojoule per kilogram? We are assuming the exit state to be sonic state right.

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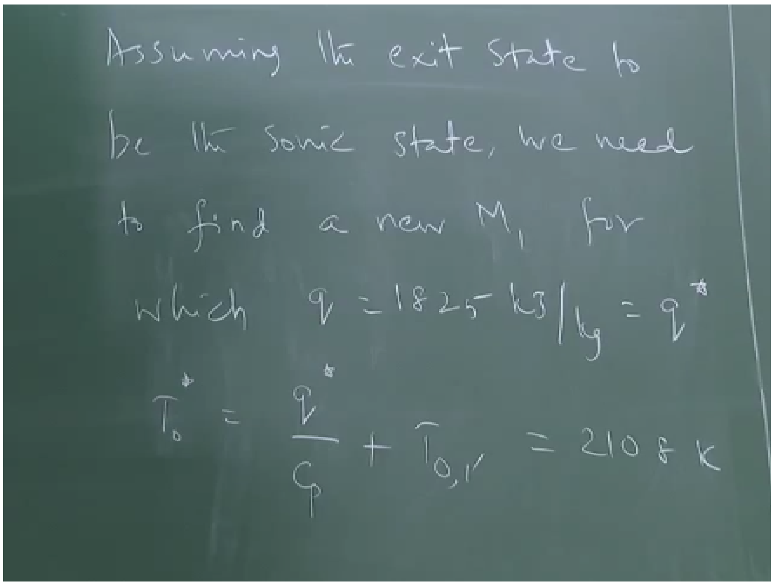

So assuming to be the sonic state we need to find a new M1 for which this q which is 1825 kilojoule per kilogram=q star right. That is what we are going to do next because no information is given about the exit state, we really cannot do anything. We simply assume it to be the sonic state. So this means for the new inlet condition T0 star=q star/Cp+T01. Let us denote the new star with the prime.

Because it is a new star, we cannot use 1 anymore, we use prime, but remember the stagnation temperature for the new state is also the same, the stagnation conditions do not change. Only the static conditions change, we argued that before. So only the static condition is going to change. So if you substitute the numbers, T01 prime=T01 right, so if you substitute the numbers you get this to be 2108 kelvin.

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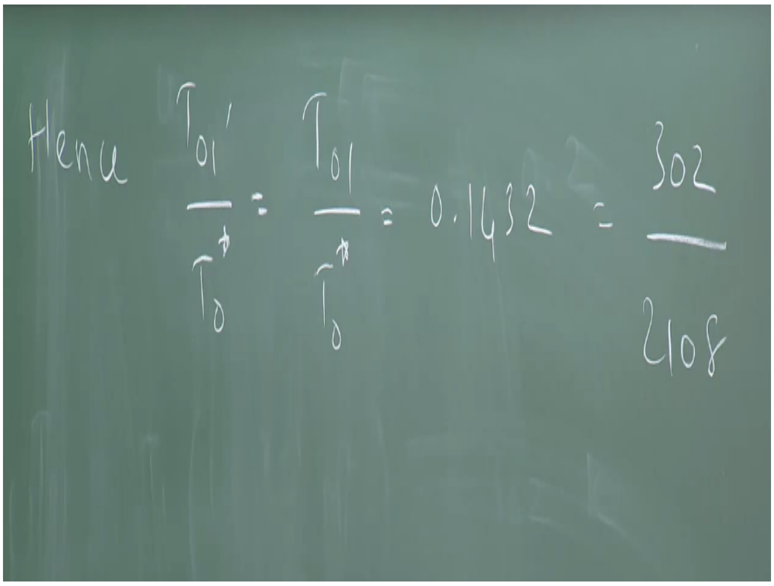

Hence, T01 prime/T0 star=remember T01 prime=T01 so T0 star and this is = 0.1432. So basically what we have done is we have substituted T01 was 302 kelvin/the new T0 star is 2108. So that is what we have done here to get 1432. So with the value of T0/T0 star=1432, we go to the tables to find out what value of Mach number this corresponds to okay.

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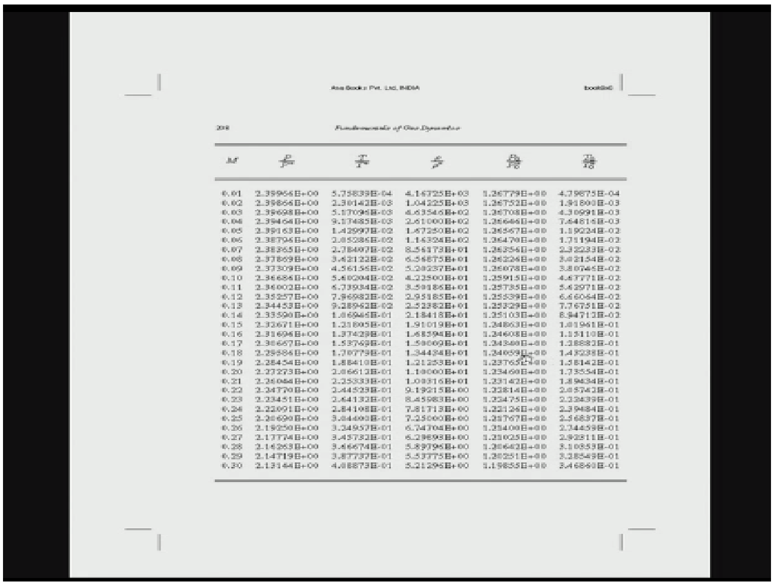

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M	$\frac{p}{p^*}$	$\frac{T}{T^*}$	$\frac{\rho}{\rho^*}$	$\frac{p_0}{p_0^*}$	$\frac{T_0}{T_0^*}$
0.01	2.39966E+00	5.75839E-04	4.16725E+03	1.26779E+00	4.79875E-04
0.02	2.39866E+00	2.30142E-03	1.04225E+03	1.26752E+00	1.91800E-03
0.03	2.39698E+00	5.17095E-03	4.63546E+02	1.26708E+00	4.30991E-03
0.04	2.39464E+00	9.17485E-03	2.61000E+02	1.26646E+00	7.64816E-03
0.05	2.39163E+00	1.42997E-02	1.67250E+02	1.26567E+00	1.19224E-02
0.06	2.38796E+00	2.05285E-02	1.16324E+02	1.26470E+00	1.71194E-02
0.07	2.38365E+00	2.78407E-02	8.56173E+01	1.26356E+00	2.32233E-02
0.08	2.37869E+00	3.62122E-02	6.56875E+01	1.26226E+00	3.02154E-02
0.09	2.37309E+00	4.56156E-02	5.20237E+01	1.26078E+00	3.80746E-02
0.10	2.36686E+00	5.60204E-02	4.22500E+01	1.25915E+00	4.67771E-02
0.11	2.36002E+00	6.73934E-02	3.50186E+01	1.25735E+00	5.62971E-02
0.12	2.35257E+00	7.96982E-02	2.95185E+01	1.25539E+00	6.66064E-02
0.13	2.34453E+00	9.28962E-02	2.52382E+01	1.25329E+00	7.76751E-02
0.14	2.33590E+00	1.06946E-01	2.18418E+01	1.25103E+00	8.94713E-02
0.15	2.32671E+00	1.21805E-01	1.91019E+01	1.24863E+00	1.01961E-01
0.16	2.31696E+00	1.37429E-01	1.68594E+01	1.24608E+00	1.15110E-01
0.17	2.30667E+00	1.53769E-01	1.50009E+01	1.24340E+00	1.28882E-01
0.18	2.29584E+00	1.70779E-01	1.34424E+01	1.24059E+00	1.43238E-01
0.19	2.28454E+00	1.88410E-01	1.21253E+01	1.23765E+00	1.58142E-01
0.20	2.27273E+00	2.06612E-01	1.10000E+01	1.23460E+00	1.73554E-01
0.21	2.26044E+00	2.25333E-01	1.00316E+01	1.23142E+00	1.89434E-01
0.22	2.24770E+00	2.44529E-01	9.19215E+00	1.22814E+00	2.05742E-01
0.23	2.23451E+00	2.64132E-01	8.45983E+00	1.22475E+00	2.22439E-01
0.24	2.22091E+00	2.84108E-01	7.81713E+00	1.22126E+00	2.39484E-01
0.25	2.20690E+00	3.04400E-01	7.25000E+00	1.21767E+00	2.56837E-01
0.26	2.19250E+00	3.24957E-01	6.74704E+00	1.21400E+00	2.74459E-01
0.27	2.17774E+00	3.45732E-01	6.29893E+00	1.21025E+00	2.92311E-01
0.28	2.16263E+00	3.66674E-01	5.89796E+00	1.20642E+00	3.10353E-01
0.29	2.14719E+00	3.87737E-01	5.53775E+00	1.20251E+00	3.28549E-01
0.30	2.13144E+00	4.08873E-01	5.21296E+00	1.19855E+00	3.46860E-01

So we go to the table, T0/T0 star=0.1432. So T0/T0 star=0.1432 so you can see that approximately I am getting M1 to be 0.18 correct I am getting M1 to be approximately 0.18.

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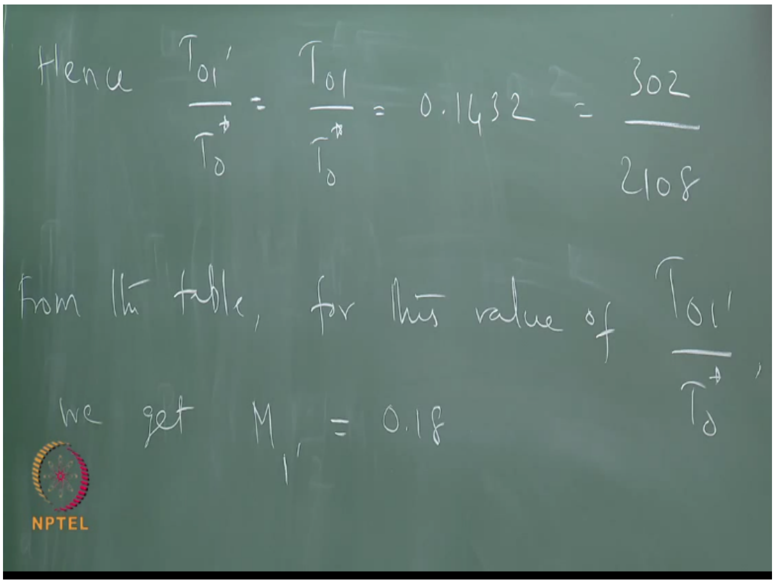

So from the tables, for this value of T01 prime/T0 star, we get M1 prime, which is the new Mach number at the inlet to the duct to be 0.18 okay. From this, I can calculate all the other quantities, I know T01 prime, I know P01 prime. I can calculate T1 prime and P1 prime from this without any problem okay. Mass flow rate can also be evaluated to see how much lesser it is going to be okay. Any questions or doubts? So this brings us to a close of this chapter on heat addition and Rayleigh flow.

The next chapter is going to be flow with friction or Fanno flow.

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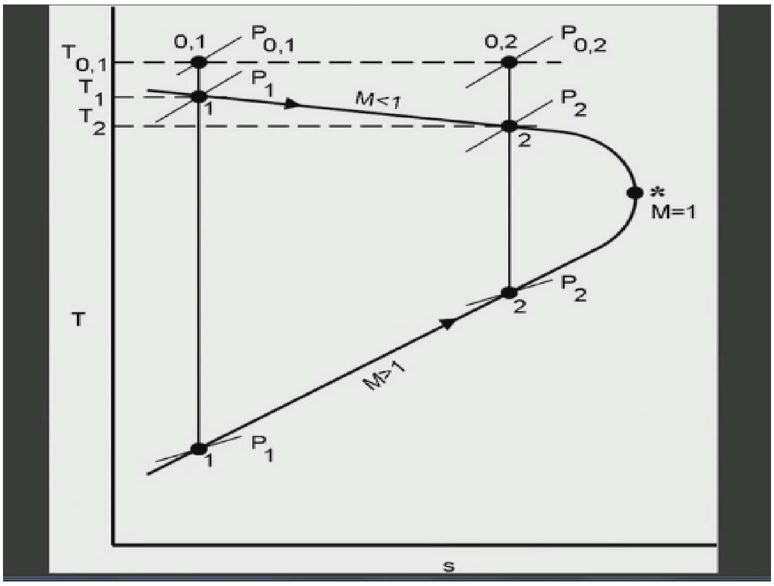

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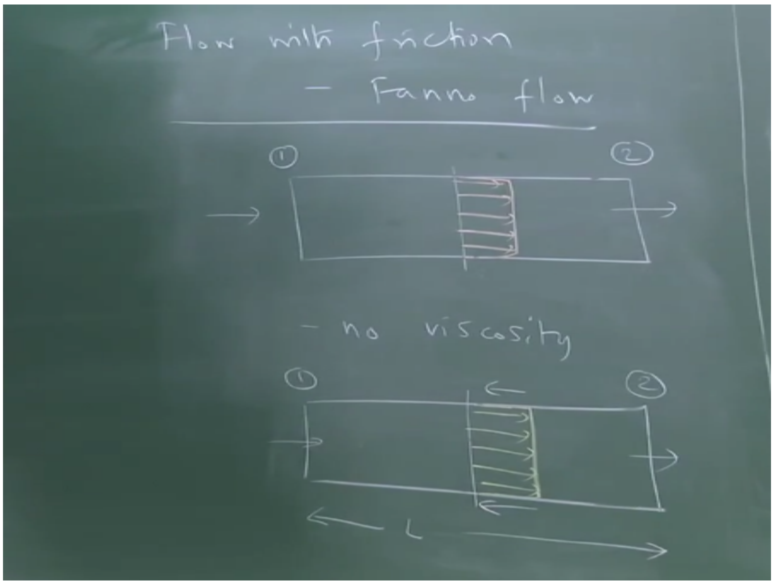

Now flow with friction occurs in many real life applications especially compressible flow with friction. The kind of applications that we are looking at in the context of this particular chapter would be real life situations where for example you have a compressor that provides compressed air, which is located at say outside a building. Normally, compressors provide air which is stored in large compressed air tanks.

Huge compressed air tanks will store this air maybe at a pressure of 10 bar, 15 bar something like that. Then the air is taken from the tank to the equipment, which uses the air. The equipment itself will be located quite far away from the storage tank. So that maybe an experimental set up or some other device, which uses the compressed air. So these 2 are connected using pipes.

When you do this when you connect high pressure air reservoir to an equipment using a pipe, you need to really understand flow with friction because many times you would have designed the equipment for a certain stagnation pressure and certain mass flow rate, but unless you design the piping properly, you will not get that stagnation pressure or the mass flow rate.

You may get the stagnation pressure, but you may not get the mass flow rate. So in order to design how to actually connect experimental set ups to these types of reservoirs so that we get the proper inlet conditions that you want, you need to understand flow in pipes and ducts with friction where frictional effect is present and that is the main reason why we are studying this.

In fact, there are many places for example in the western countries where the compressor and the storage tanks will be located several kilometers away, they call that a compressor form. There are lot so compressors which supply huge amount of air, which is stored in tanks and then it is taken through pipes several kilometers long to experimental set ups or wind tunnels and so on.

So we really need to understand Fanno flow so that we can design the pipes properly and also ensure that the equipment gets the air at the conditions that you desire okay. So that is the reason why we are looking at this particular application. So to simplify the scenario is almost the same as what we were looking at earlier. So we have a duct or a pipe, constant cross-sectional area.

And air enters let say inlet state is 1, air enters at inlet state 1 and air exits at state 2. Now you must remember that the fluid that we are dealing with is a calorically perfect fluid. There are not viscous effects. Our momentum equations talked only about pressure and momentum. There are no terms related to viscosity. That is a very simple assumption which allows us to get nice solutions.

And we will still continue to do that okay. So we will still assume that there is no viscosity and that the fluid is calorically perfect that is still okay. So what we are going to do is we actually take a look at the real life situation. So when you have flow through a pipe or a duct like this, the speeds are usually quite high. Remember we are looking at compressible flow, subsonic, supersonic Mach numbers.

So the speeds are quite high that the frictional effects, effects due to viscosity are confined to very thin regions near the walls of the pipe. Remember the wall of the pipe is stationary right, fluid is moving with the certain velocity so the wall exerts a frictional effect and that is felt only in very thin layers near the surface. So if I were to draw a velocity profile at this section, the velocity profile may look something like this.

For the kind of flows that we are looking at the velocity profile would look something like this. This is the velocity profile, so notice that the effect of friction is confined to a very thin layer near the wall of the pipe. So what we are going to do is we are going to treat this flow as if these layers are not present, but instead we would impose whatever shear stress this imposes we would simply impose that.

In the previous case, we said we are going to add so much heat. In the present case, we are going to say we are going to exert so much frictional force from the wall okay. So that the effect of this thin layers can be simulated without having the thin layers present at all. If you include the thin layers that means the fluid is viscous. So we will not include the thin layers but instead of doing this what we are going to do is at the same section we will assume the velocity profile to look something like this.

Let me draw the new situation. So the kind of problem that we are going to solve we are looking at the same duct and in the same section we are going to do the following. We will assume the velocity profile to be like this, but now the drag force that the wall exerts on the fluid will be simulated by specifying a drag force on the wall. So this is an externally imposed drag force that we are applying on the wall okay.

How we actually do this is not of any concern to us. As long as I know what drag force the wall exerts from the fluid I can exert the same drag force here right. Such a flow is called the

Fanno flow and this is what we are going to study. For a given inlet condition, pipe length and frictional force, I want to know what the exit state is, static pressure, stagnations pressure, static temperature, stagnation temperature and so on okay.

That is what we are going to do. So notice that we are still saying that the fluid there is no viscosity and notice that there is no boundary layer right. So this thin layer is called the boundary layer. Notice that in the Fanno flow that we are looking at there is no boundary layer okay.

The velocity profile is flat, but the effect of friction exerted by the walls is modeled by applying an external drag force on the fluid okay, which is felt by the entire fluid not just near the wall. That is the model that we are looking at, such a flow is called the Fanno flow okay.

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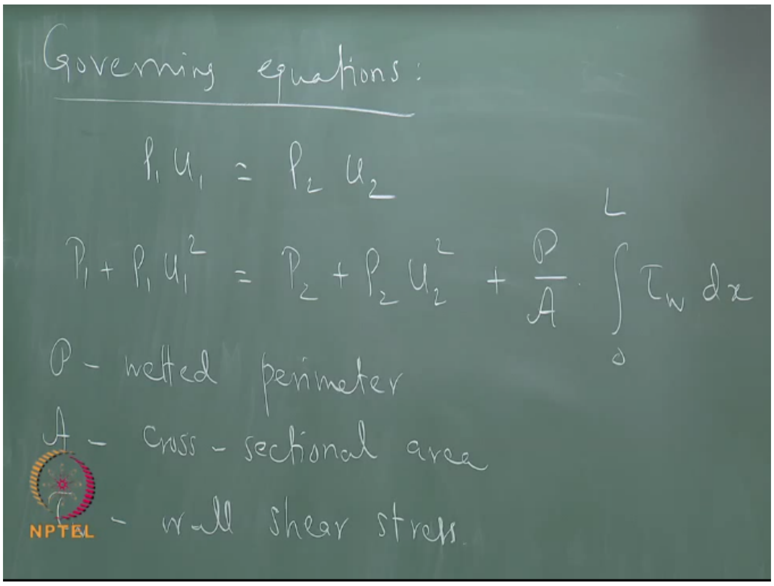

Now the governing equation for the flow look like this, rho1 U1=rho2 U2 is a constant cross-section so there is no change and the momentum equation remember now the momentum equation has to account for the shear force on the wall right. So we account for that like this, so we write P1+rho1 U1 square=P2+rho2 U2 square+P/A integral 0 to l tau w dx, so we assume the pipe to be of length l so let me sketch that also here.

So we assume the pipe to be of length l and tau wall is the wall shear stress that is exerted on the pipe surface. So let us write down all these quantities. So P is the wetted perimeter, A is the cross sectional area, and tau wall this is the externally applied wall shear stress. Remember we are applying this stress externally. In the real case, this would automatically come out because of the viscosity of the fluid, but now we are modeling it and exerting this externally.

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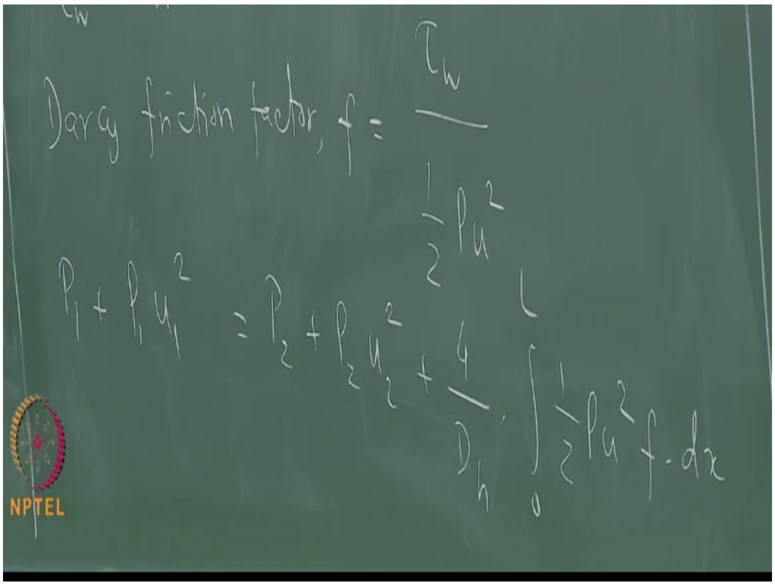

Now in basic fluid mechanics, we define something called Darcy friction factor. Basically, this Darcy friction factor is nothing but a dimensionless shear stress okay. It is a dimensionless shear stress so we take shear stress, which has units of newton per meter square right. So the non-dimensionless is this way.

So if I now use this relationship in this equation, I can write this equation as P1+rho1 U1 square=P2+rho2 U2 square+4/Dh times integral 0 to l 1/2 rho U square times f times dx.

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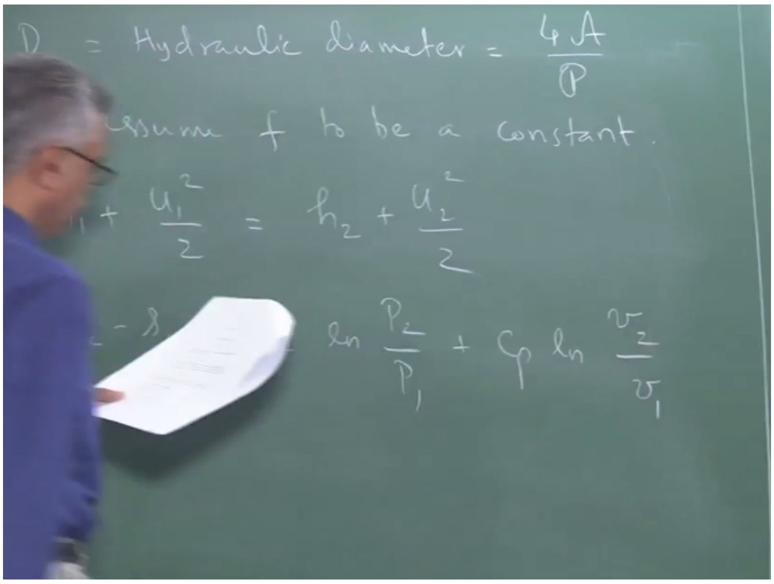

Where this quantity Dh is known as the hydraulic diameter of the duct and is defined as 4 times the cross-sectional area/the wetter perimeter okay. Now from the definition of f you can see that U varies from one point to another right. So it appears that f is also going to vary along the length of the pipe, so U varies from inlet to outlet, it appears that f can also vary from inlet to outlet.

But in reality if you look at Moody's chart, you will notice that for the kind of velocities and Reynolds numbers that we are talking about, f is practically a constant so we need not worry about f varying from inlet to outlet. So we usually assume f to be a constant and that is an extremely good engineering approximation okay. So that is the momentum equation. Let us write down the energy equation.

Energy equation is h1+U1 square/2=h2+U2 square/2, there is no heat addition or work addition so energy equation is the basic form without the q or any work addition terms and one more equation, which is entropy right, s2-s1=Cv natural log P2/P1+Cp natural log V2/V1 and what do we expect, s2-s1 to be no heat addition right, but there is friction force, which means there is a reversibility so we expect s2 to be > s1 okay.

So what we will do in the next class is adopt the strategy same as what we did earlier. Starting from the inlet, we will illustrate the subsequent states on a TS diagram, connect them to look at the process curve and then we will proceed further just like what we did before in the next class.