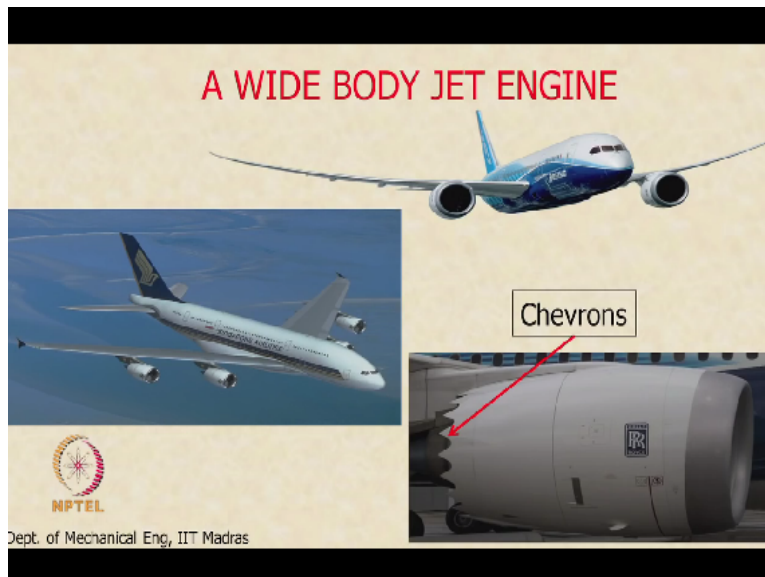


Gas Dynamics and Propulsion
Dr. Babu Viswanathan
Department of Mechanical Engineering
Indian Institute of Technology - Madras

Lecture – 01
Introduction

Good morning. I will start our lecture today with brief introduction to the material that you are going to cover in the course, okay. We are going to primarily talk about aircraft engines in this course and what kind of engines are we talking about?

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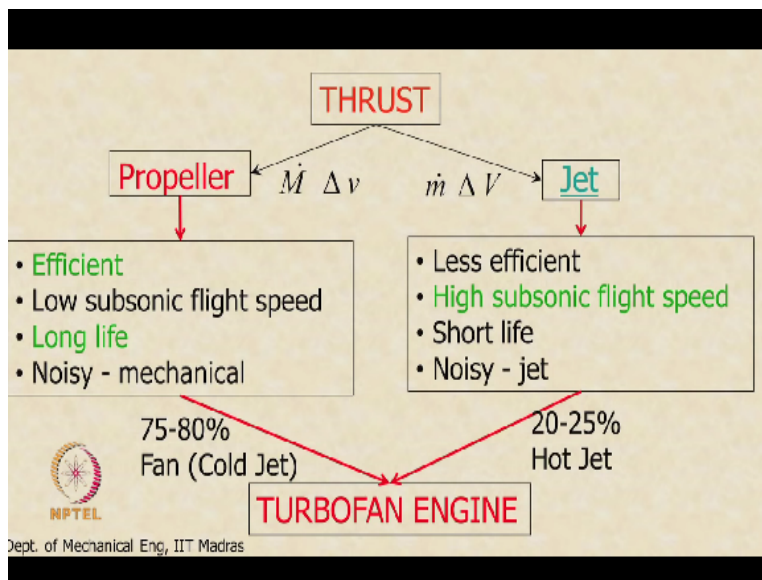


These are the modern aircraft engines that are being used as of today. What you see here is the Airbus A380 super jumbo. We can see the twin deck, the bottom deck and the top deck and this is powered by the latest of the Rolls-Royce engines and what you see here is the Boeing 787 Dreamliner which is also powered by the latest Rolls-Royce and the GE engines. These are the latest and the greatest in aircraft propulsion technology.

And the courses are primarily going to focus on the nature of these engines, the various components in the engine, how these engines operate and how do we do thermodynamic calculations relating to the engine, for example what are the different state points as the air goes through the engine? what kind of thrust does the engine produce? how much fuel does it consume per unit thrust that it produces?

These are very important performance metrics and what we are planning to do in the course is to eventually be able to calculate these kinds of quantities from the engine and also get a detailed idea about how these engines were fabricated? what are the design aspects of the various components of the engine and so on and that is the primary objective of the course that is what we are going to cover. Now any proportioned device, the basic idea in any proportion device that we are going to talk about is to generate thrust so that the vehicle can be propelled forward.

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So what we are looking at is generation of thrust. Now historically, thrust is generated by 2 means. First one is especially in the context of aircraft propulsion, before World War II or maybe even during World War II, propellers were primarily used for generating thrust, okay. So what did the propeller does as this term indicates is the propeller takes a large amount of air and imparts a small velocity change to it, okay.

That is why I have shown the v in small letter and the M in capital letter to indicate that M is very large. So a propeller typically moves a large mass of air through a small velocity change, okay. So the product of the 2 from Newton's second law then produces the thrust and the net rate of change of momentum that you are imparting to the air is then exerted on the propeller itself and the airframe as thrust. So that is Newton's third law.

So the force is calculated using Newton's second law and the reaction force is what the aircraft feels. So you are exerting this kind of force on the air mass and so the air mass exerts an equal and opposite force on the airframe which is what is generated as thrust. So that is typically what a propeller does. This is the basic idea and it has its own advantages and disadvantages. A propeller actually is extremely efficient in terms of fuel economy and in terms of operational speed and certain other parameters.

So it is extremely efficient which I have indicated in green but the problem is it is restricted to low subsonic flight speeds. So for example, you may not be able to fly at speeds more than Mach 0.5 or 0.6. So it is restricted to low subsonic flight speed because as you increase the flight speed, we need to make the propeller larger and larger and eventually we get into problems with the tip speeds of the propeller exceeding the speed of sound.

Then you have other complications and the efficiency drops off. So the propeller is efficient only so long as you fly at low subsonic fly speed, the flight speeds; otherwise, you lose the efficiency. So because it is very efficient, it has a long life. There are 2 reasons for the long life, probably the most important reason is that the propeller handles only cold air, not hot air. So it takes in a certain amount of air, imparts a velocity change and then sends it out.

So it takes in cold air and so it is never exposed to high temperatures which is why it has a long life but as probably all of you know, the propeller is extremely noisy from a mechanical point of view. You know it makes a lot of noise, even modern turboprop engines actually make a lot of mechanical noise when you fly in one of this, okay. So this is one means of generating thrust. Now the biggest limitation is the low subsonic flight speeds.

For commercial aviation industry to profit, we need to be able to move people or fly aircraft at high subsonic Mach numbers that is probably the most efficient way. So this was the biggest obstacle which had to be overcome and this was overcome by introducing what is called a jet propulsion. So jet propulsion made it possible to overcome this limitation on the flight speed. Now in contrast to propeller, jet propulsion system moves or imparts a large velocity change to a small mass of air.

That is why I have shown the V in capital letter and the M in small letter. So you can see the contrast between the 2. So it imparts important a large velocity change to a smaller mass of air. So that by we again produce thrust. We can produce large amounts of thrust and importantly, the amount of thrust that we produce, actually can improve the flight speeds to high subsonic flight speeds.

So turbo jet engines for example which utilises technology or capable of flying at Mach 0.9 or 0.95 very easily but it is less efficient and we will see the reason for this as we go along but generally this means of generating thrust is less efficient than what the propeller does and consequently, the engine also has short life but now the noise problem really does not go away. A popular is very noisy from a mechanical perspective but a jet engine is very noisy from an air acoustic perspective because we have a very high-speed air.

Remember we are important a large velocity change to the air. So because we have very high-speed air which is coming out of the engine and mixing with the ambient, that mixing noise actually is a very high noise, very low noise comparable to, probably comparable to the kind of noise that a propeller engine makes. The origin of the noise is different between the 2 but probably the intensity of the noise is more or less the same between the 2, okay.

In the first case, this noise is felt both probably by the passengers who are sitting inside the aircraft and in the second case, this noise is felt by people who are living next to the airports, that is the only difference between the 2, okay. Now the short life in this is due to the operating environment of the jet engine because we are trying to impart a very high velocity to the jet, we need to actually heat the jet.

We will see that in a minute and because the engine has to handle or is exposed to high temperature, the life of the engine goes down because the propeller handles only cold air and is not exposed to high-temperature, it has a long life. In contrast, this has a short life. Now the less efficient part of this actually gave the impetus to develop more efficient engines but with still this types of flight speeds.

So if you remember, the low subsonic flight speed was what costs the industry to move to a different form of propulsion so that we can overcome that obstacle. In this case, the obstacle is the lesser efficiency. Today for commercial aviation to be profitable, we need to have extremely high fuel efficiencies and that drove the next generation of engines to be developed. What is the next generation of engines?

This was what I showed you in the previous slide that is the turbofan engine which is the modern aircraft engine that is being used in the industry today, that is what you also saw in the previous picture. Now the turbofan engine actually combines the advantages of the propeller engines. In that the longer life and the efficiency comes from the propeller mode. So it generates about 75% to 80% of the thrust using a fan which is very similar to a propeller, very similar but not the same.

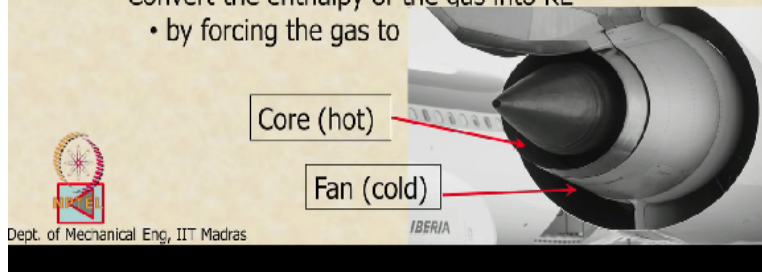
This is very similar to a propeller. So 80% of the thrust is generated using cold air which will definitely increase the life of the engine because it is not exposed to high temperatures. This definitely increases the life of the engine. Again remember up to 80% of the thrust is generated using the fan and about 20% to 25% of the thrust is generated using a hot jet. So it retains the advantages of both and hence it has an extremely good fuel efficiency and it is used very widely today in the industry. So as we go along, what we are going to focus on in this course is only the jet propulsion.

So we will look at engines that use jets to propel themselves. Of course one aspect that we will discuss later is, how do you actually drive these engines is an aspect that we will look at later on, okay. Now when I say jet propulsion, what are we talking about. Remember the requirement is for us to generate high-speed jets, so we want to impart, we want to take in air and impart a large velocity change to it, right. How do we do that? So let us take a quick look and that will explain why?

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How can a gas be accelerated to high velocities?

- Increase the specific enthalpy of the gas
 - by increasing the pressure
 - by increasing the temperature
- Convert the enthalpy of the gas into KE
 - by forcing the gas to



So the question is, how do we accelerate a gas to high velocities. We accelerate the gas to high velocity is by increasing its enthalpy, right. So we increase the specific enthalpy of the gas. Once we increase the specific enthalpy of the gas, right, we can do the following. Once we increase the specific enthalpy of the gas, we can convert the enthalpy of the gas into kinetic energy by using a nozzle, right.

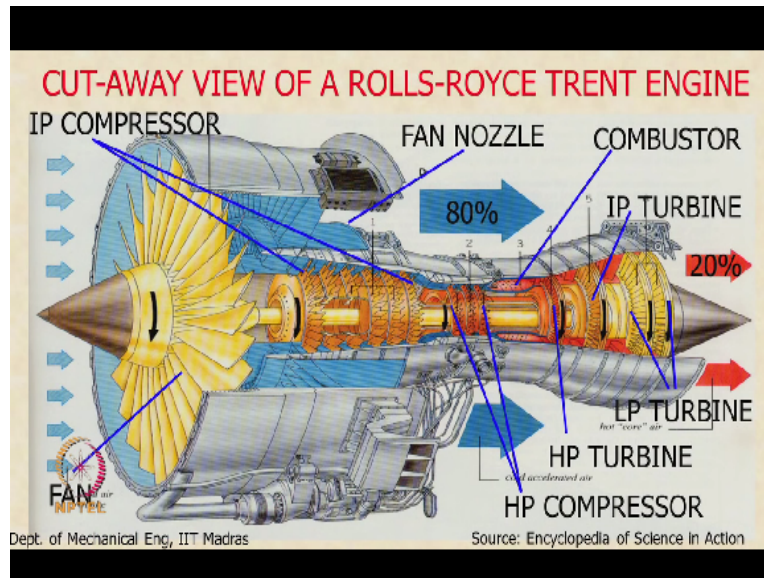
That is what a nozzle does. So we increase the specific enthalpy of the gas, then we convert enthalpy into kinetic energy in a nozzle. So that is what basically an aircraft engine does. So here you are looking at a turbofan engine. You see the core nozzle which handles hot gases and you see the fan nozzle which handles the cold air alone. So the fan nozzle produces about 75% to 80% of the thrust usually and this nozzle produces about 15% to 20% of the overall thrust, okay.

So this is how we accelerate a gas to high velocity. Now how do we increase the specific enthalpy of the gas. We increase the specific enthalpy by increasing the pressure, right. Remember enthalpy is nothing but $U + PV$ where U is the internal energy, PV is the product term, right. So we increase the specific enthalpy by increasing the pressure and by increasing the internal energy. How do we increase the internal energy of a gas? By increasing its temperature, right.

So this is what the engine must do. So any engine that utilises jet propulsion, must have a way of

taking the air, increasing the pressure and increasing the temperature and then sending it out through a nozzle so that the enthalpy can be converted into kinetic energy, right. So that is how this engine operates. How do we increase the pressure? How do we increase the temperature? We will take a quick look but we will go into the details of some of these design choices later on.

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So here is a look at a modern turbofan engine from Rolls-Royce. This is a Rolls-Royce Trent engine. So you see the fan in the front and then you see 3 rows of compressors, fan itself is a compressor, then you have an intermediate pressure compressor, you see these axial compressors with the blades on them, right and then you see this high-pressure compressor which again has blades but a smaller cross-sectional area as the gas is compressed and then, so this increases the pressure.

Remember we said that we wanted to increase the pressure, we wanted to increase the temperature. So we have increased the pressure now. We then take this to the combustion chamber which is located over here. So that is the combustor as we can see. So the air goes into the combustion. We inject a fuel, typically aviation kerosene or aviation turbine fuel. We inject the fuel; we burn the fuel.

The calorific value of the fuel is converted into internal energy of the gas. So the internal energy of the gas goes up. So now we have done, increased the pressure. We have increased the

temperature. We then take it to the turbine. We do not take it directly to the nozzle because we need to do work to compress the air, right. We cannot increase the pressure without doing work. So we need to do work to increase the pressure.

So the work that is required to drive the compressor comes from this turbine. Again you see an intermediate pressure turbine and a low-pressure turbine which matches with the high-pressure turbine which matches with each one of this. So the high-pressure turbine drives the high-pressure compressor. The intermediate pressure turbine drives the intermediate pressure compressor and the low-pressure turbine drives the fan. So each one is a matched set.

So the work required to run these things, comes from the turbine. So we have increased the enthalpy of the air here so much that we can extract the work that is required to drive this, then the remaining enthalpy, the air is then taken to the nozzle where the enthalpy is converted to kinetic energy which then produces thrust, okay. The turbine is present mainly to generate the work that is necessary to drive the compressor, okay.

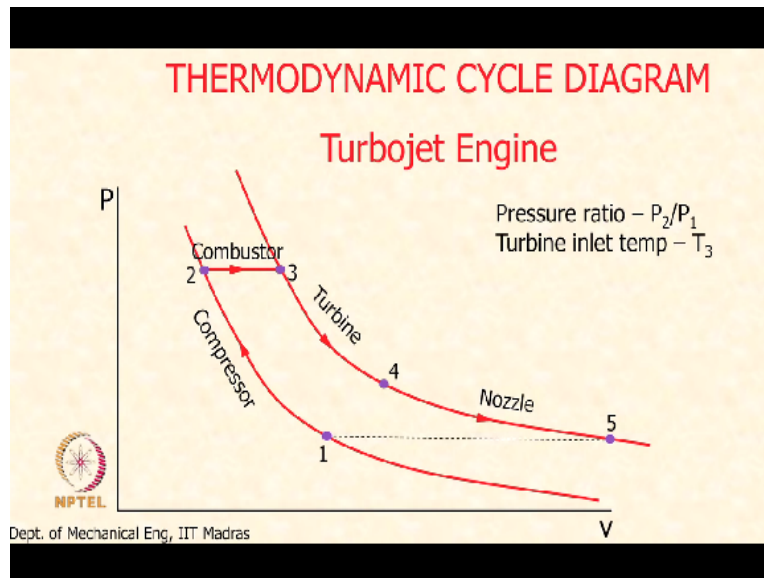
So this is how the modern aircraft engine operates and you can also see additional things, we can see the fan nozzle and the core engine nozzle is present over here, okay. Notice that the fan generates thrust by increasing the pressure of the air and then forcing it to go through a nozzle and in that respect, it is different from a propeller. The propeller does not have a nozzle. Propeller generates the thrust directly.

It imparts a velocity change and generates the thrust directly that is the difference between a fan and a propeller. The fan increases the pressure of the air then the air is forced to go through the nozzle where the enthalpy is converted to kinetic energy in both cases, okay. So this is the operation of an engine and as we go along in the course, we will take a detailed look at the operation of each one of these components, okay.

The fan, the compressors, combustor, nozzle, the shafts and the spools, we will take a detailed look at the operation of all these things, how they are put together, what is operating condition, what kind of materials need to be used for this and so on, all the details. We will look at material

details, design issues, thermodynamics of this component and then we will follow it up with calculation to eventually calculate the thrust that the engine will produce, okay. So now you see a picture of the engine here. Now let us try to put some numbers, associate some numbers with this engine.

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Let us go through the thermodynamic cycle before we look at the numbers. So if you take a simple turbojet engine and if you draw the process that the air undergoes on, a PV diagram. We can see that we take in air at the ambient condition which is the state 1 and then we compress the air along process 1-2 which ideally will be an isentropic process, right. So we compress it along 1-2 and once we do the compression, we have to add heat or we raise the temperature of the air.

So we actually add heat in the combustor which is a constant pressure process from 2 to 3 and then once that is done, we then extract part of the enthalpy to generate the work for the compressor. So that is an expansion process in the turbine from 3 to 4. So 3 to 4 expansion is required to produce the work that is necessary to drive the compressor and then the remaining enthalpy is extracted in the nozzle, that is process 4 to 5.

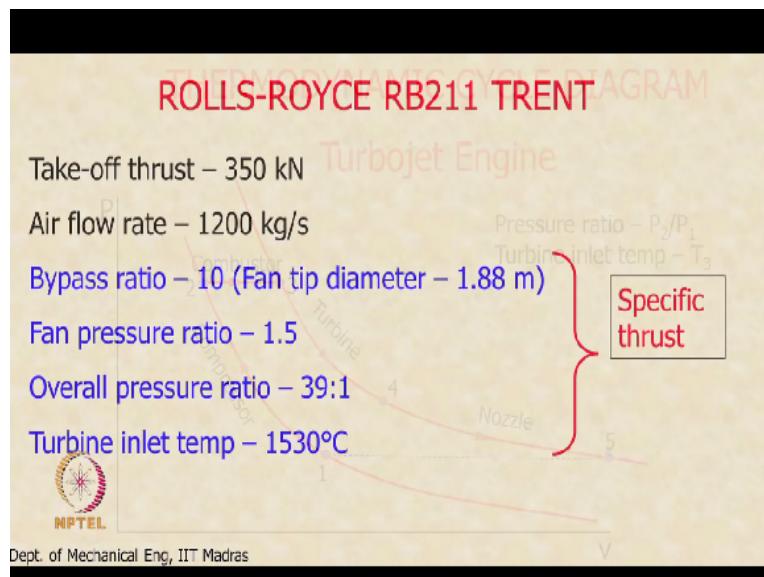
So the thrust is generated in 4 to 5. Notice that the working fluid does not actually undergo cyclic process here because we do not come back to state 1. So we take in air at state 1, it executes these processes and then it leaves at state 5, right. We do not take it back to state 1. Important

parameters for a turbojet engine or the pressure ratios, P_2/P_1 , these are the parameters that can be changed.

These are the only adjustable parameters in the cycle, P_2/P_1 and the turbine inlet temperature T_3 . These are the quantities we can adjust and depending upon the values for these quantities, we then get different values of thrust and different sizes for the engine and so on, okay. So these are the 2 important operating parameters for the engine. Once you fix this, everything else is fixed, right. Now what kind of values are we talking about for the pressure ratio, right.

So remember we said that, you know, we needed to increase the enthalpy of the air and we did that by increasing the pressure and by increasing the temperature. So you can see that these are the 2 parameters which represent that, P_2/P_1 is the increase in pressure that we give and turbine inlet temperature is the increase in temperature that we are providing, right. So what kind of numbers are we talking about. For the modern aircraft engine, what numbers should these things have.

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So these are some numbers for Rolls-Royce RB211 engine that I showed you. It produces the takeoff thrust of about 350 kilonewtons, okay. It takes in when the engine is at ground level or static level or sea level. The engine takes in about 1200 kilograms per second of air, that is 1.2 tons of air per second that the engine sucks in and it is a huge engine and here, we are talking

about bypass ratio here.

So this means the bypass ratio of 10 means 10 times as much air goes through the fan as goes through the core engine. In fact, for every kilogram of air that goes through the core engine, 10 kilograms goes through the fan, right. So the total amount of air that goes through the engine is $10+1=11$ kg per second. So for every 1 kg through the core engine, 10 kg goes through the fan, that is what bypass ratio means and we also get an idea about the physical size of the engine.

This engine is about the fan tip diameter is about 1.88 meters. So the overall engine diameter will be about 2 meters or so, okay. The fan pressure ratio is 1.5. I told you that the fan is different from the propeller. In that, it increases the pressure of the air and then converts that pressure into or converts that enthalpy into thrust by using a nozzle. So you can see that the pressure rises very modest, it is only 1.5 but if you look at this number, 1200 kilogram per second.

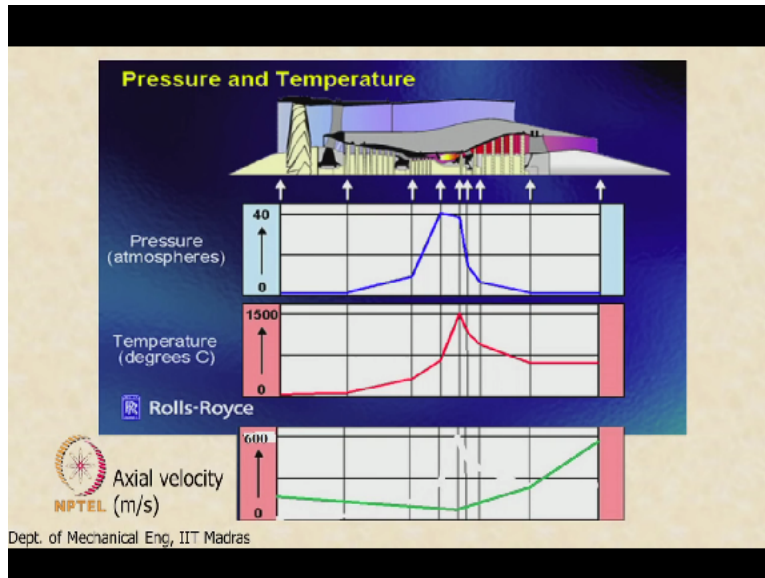
And you look at the bypass ratio, you realize that nearly 1000 kg per second of air is going through the fan and the pressure of that much air is being increased by 1.5. So now if you start thinking about $M \cdot \Delta V$ for this air, you say that M is very large, right and this pressure wise will only give rise to a small velocity change. So you can see that M is very large, ΔV is very small for the fan. It is very small to the propeller from that perspective, okay.

Overall pressure ratio, we talked about P_2/P_1 being the parameter. So for the turbojet engine, overall pressure ratio for modern aircraft engines are typically of the order of about 35 to 40, okay. So the air is compressed by a factor of 35 to 40 in these engines. The turbine inlet temperature is typically or in the ranges of about 1500 degree Celsius for this engine. What is challenging and interesting about this is that this temperature of 1500 degree Celsius, if you recall from our earlier discussion, we showed the engine, right.

So the air goes into the combustor where we raise the temperature to about 1500 degrees Celsius, then it comes to high-pressure turbine, right. This temperature is about 200 to 300 degrees Celsius above the melting point of the blade, metal. Again, still the blade does not melt, okay. That is what is challenging and interesting about this technology that the engine can operate at

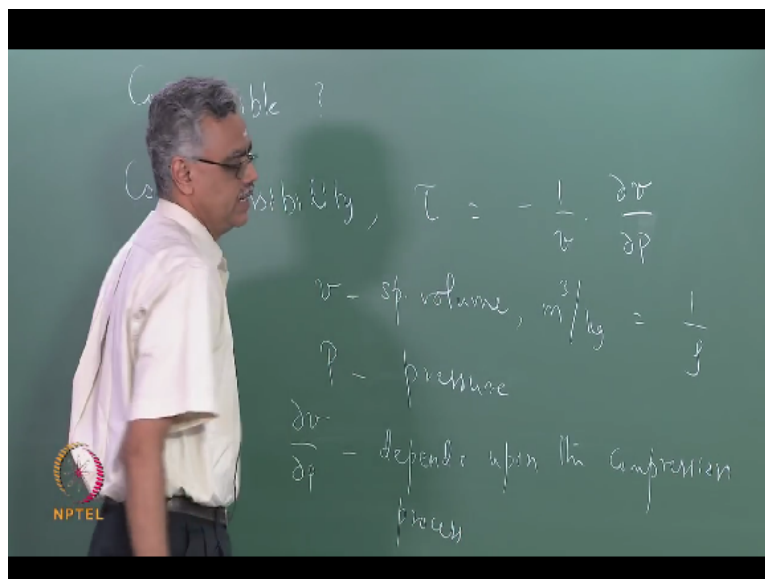
temperatures which are about 200 to 300 degree Celsius above the melting point of the blade metal and still continue to operate without any problems, okay.

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Now before we go in to the slide, we need to take a look at, we want to do a component by component analysis of the engine and the first thing that we need to decide is the nature of the thermodynamics and the flow in each one of the components. So basically we are looking at fan, compressor, combustor, turbine and nozzle. So we want to have an idea about the nature of the flow in thermodynamics in each one of this component, okay.

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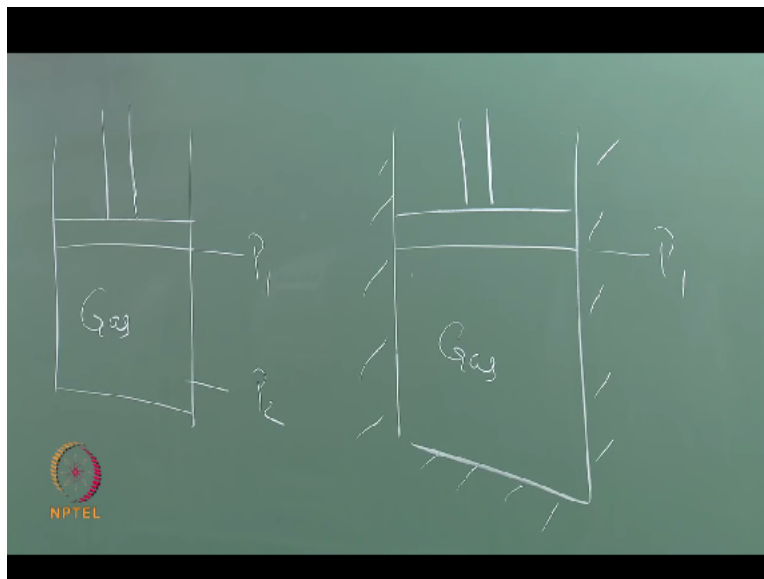
So to do that, the first question that we want to answer in this situation is, is the flow

compressible. Do we need to do compressible fluid dynamics or do we assume that the flow is incompressible and then we use the simplified equation. That is the first question we must ask, right. So to answer this question, 2 concepts are important. First one is the so-called compressibility.

Compressibility is usually given a symbol β and it is written as, here v is the specific volume in meter cube per kilogram and it is equal to the reciprocal of the density ρ and p is the pressure. So this is how compressibility is defined. The negative sign ensures that when since this quantity is negative, the negative sign here ensures that compressibility is a positive number, okay. Of course, the value itself, the exact value for partial v , partial p will depend upon whether, so that depends upon the compression process.

In other words, if I take a fluid at a certain pressure and specific volume, then I make it undergo an isothermal compression process, the change in the specific volume will be different for a given change in pressure but if we make it undergo an adiabatic compression process, definitely the value at the end of the process will be different, right.

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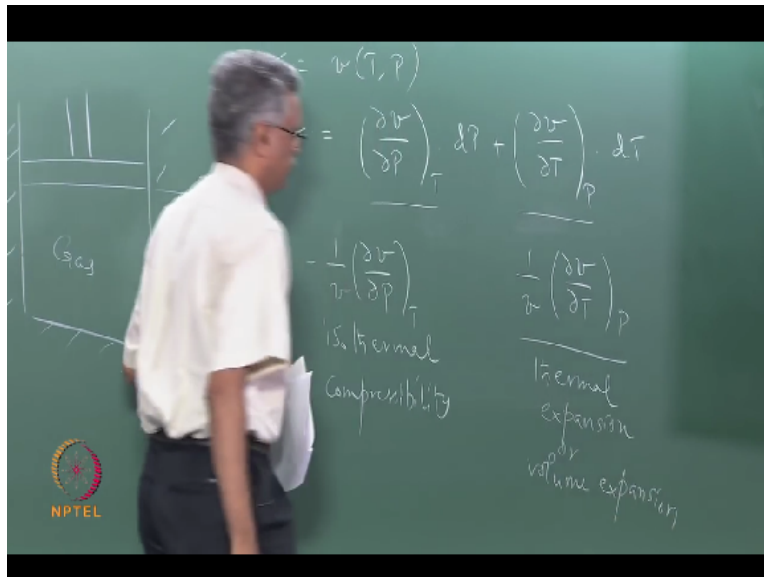
So if for example I take a certain amount of gas and I compresses this gas certain initial pressure to a certain final pressure, let us say this is P_1 to let us say P_2 and I compress this gas if I allow the compression process to be isothermal, that is the gas remains at constant temperature, so it

loses heat to the surroundings, then I end up with a different specific volume. If I insulate the cylinder, so this is isothermal.

If I insulate the cylinder, starting from certain P1, then when I compress it from P1 to P2, then I can achieve that with a slightly different change in specific volume. I will not be able to compress it all the way down here. I will probably be able to compress it only up to there, partial v, partial P depends upon the process whether we are using isothermal process or adiabatic process or some other process, right.

So this is dependent upon the process. So the compressibility is also defined, we can have an isothermal compressibility, we can have an isotropic compressibility which ensures that the values are different for different processes, that is what we are talking about. Now this is actually very restricted definition.

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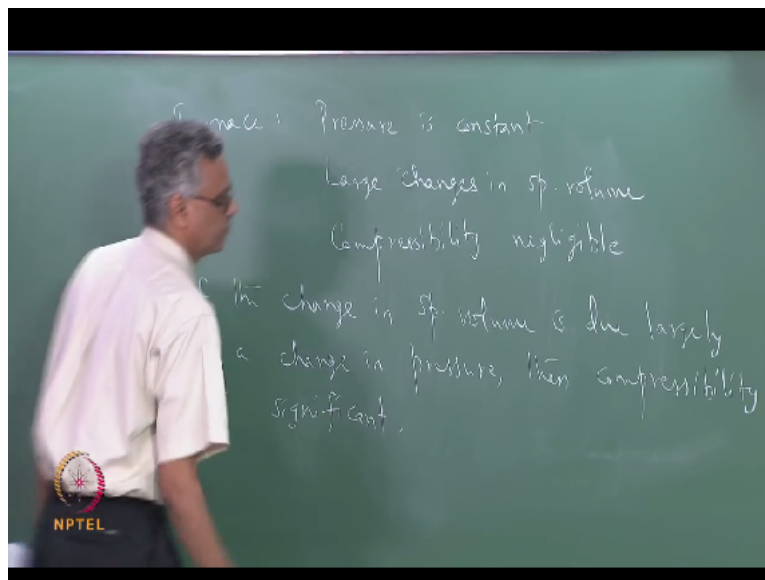


In fact, we can write the specific volume v as a function of temperature and pressure. You know that in thermodynamics, any variable depends upon 2 independent properties. So I can write specific volume as function of temperature and pressure. So any change in specific volume, I can use chain rule, okay. So use chain rule or differentiation to write the total derivative, so what this tries to tell me is there a change in specific volume can be due to 2 things, number one due to a change in pressure or due to change in temperature.

You all know that when I heat a gas, its density decreases or spacing volume increases. So that is what this term, the first 2, what is a change in specific volume due to just a change in temperature, right. This term talks about change in specific volume due to change in pressure. So this term is what relates to compressibility, right. So this term, in fact, I can expand this and write this as, if I write this as $-1/v$ partial v partial P, right. This is isothermal compressibility.

This is isothermal compressibility and this quantity, this quantity is called or volume expansion. That is the coefficient of volume expansion. So you know that when I heat a gas, its density decreases and that is nothing but volume expansion and this is compressibility. So when we talk about this equipment, what we need to do is, how much of the change in specific volume in each one of this device is due to change in pressure, okay.

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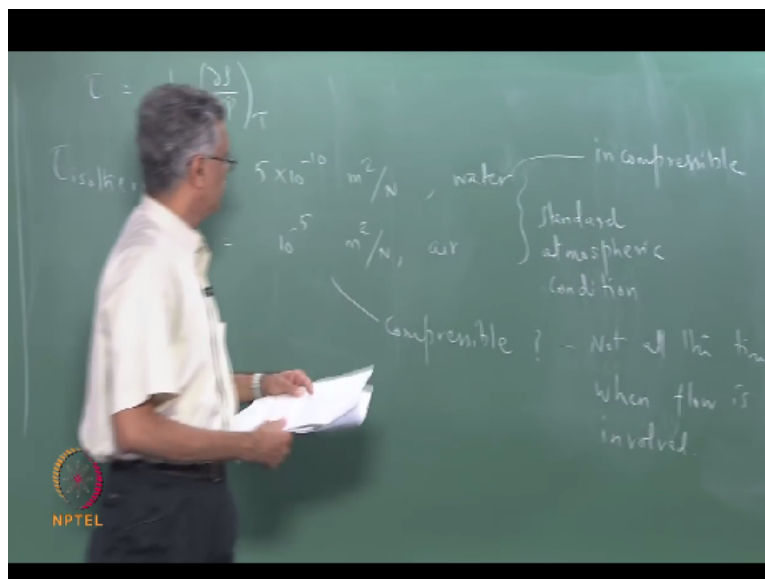
Now if I look at these 2 terms, right, if I think about let us say large furnace or combustion chamber, so if you look at a large furnace or combustion chamber, you know that the change in temperature is very large, right. In the furnace, the change in temperature is very large and so the change in specific volume is also going to be quite large. So the change in spacing volume due to the change in temperature is very large but the question is, is the flow inside a furnace, compressible or not, is the compressibility effect important inside the furnace or not, right.

What about the pressure inside the furnace or combustion chamber? Does it also change or does it remain more or less constant? So the furnace can actually operate at atmospheric pressure but it can have large changes in temperature and specially volume. So pressure but it has large changes in... so which means that the second term is large but the first term is going to be very small in the case of a furnace, right. So that means in this case, compressibility effects are negligible.

So the important point is that you can have large changes in specific volume but if the change in specific volume is largely due to a change in pressure then we say the compressibility is important; otherwise, compressibility effect is not importance. So what this equation tells me is if the change in specific volume is due largely to a change in... then compressibility is significant. So when we look at these devices and then try to decide whether compressibility effect is significant or not, we must keep this in mind, okay.

Now there is also another factor which is important in these cases and that is what we are going to look at next.

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So if I rewrite the equation for compressibility in terms of density, I can write tau as $1/\rho$ partial rho partial p. This is in terms of density and for example the isothermal compressibility of water comes out to be 5×10^{-10} meter square per Newton for water and it comes out to be

approximately 10^{-5} meter square per Newton for air under standard atmospheric conditions.

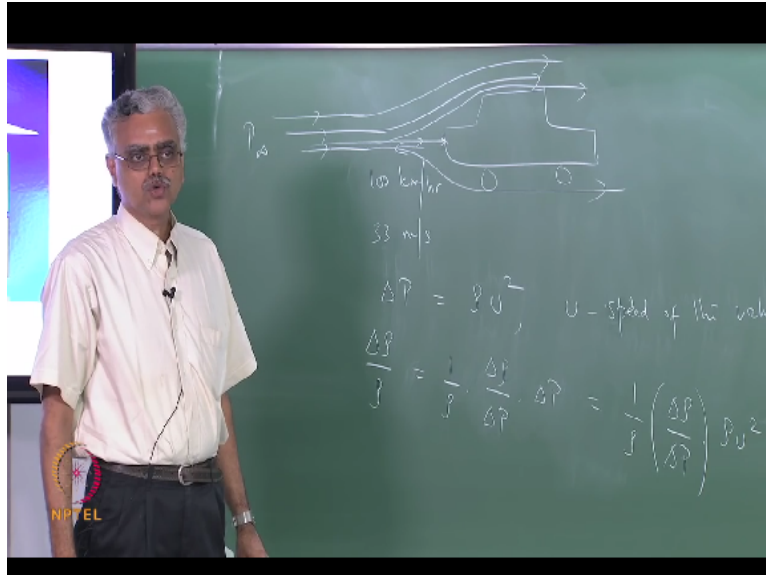
So comparing these 2 numbers, we have said tau isothermal, by saying tau isothermal what we are saying is, this is going to be $T=\text{constant}$, right. It is going to be isothermal process. So if you calculate this quantity for water and air, you see that for air, the number is 5 orders of magnitude more than that of water. So it is clear that water can be essentially treated as incompressible, might.

So if your working fluid is water then we can essentially treated as incompressible and if it is air, then do we say that it is compressible all the time that is the question? Do we say that compressibility effects are significant all the time, just based on this alone, the compressibility alone, can we say it? The answer is, if flow is involved, then this may not be true all the time. Whether a particular flow exhibits compressible effects or not, depends upon the natural of the flow and that is what we have to take a look at next, okay.

So not all the time, right. When flow is involved. Notice that this definition of compressibility uses only thermodynamic variables, right, density, temperature and pressure. It does not involve anything from flow. So if you want to look at flow effects then we need to bring in some quantity that is related to the flow and that is what we are going to do next.

This is the useful criterion in this, there is no doubt about it, right. When the flow effects are absent but when flow effects are present, when flow is taking place, we need to have a slightly different method by which we can assess the compressibility of the flow, okay and what are we going to do for that. Let us take an example.

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So let us say that you are moving in an automobile. So this is moving at a speed of let us say 100 kilometers per hour, right which works out to roughly 33, about 33 meter per second roughly, right. So the air flows around the automobile, right. So the air flows around the automobile and we want to find out whether compressibility effects are significant in this flow, right. So now we have one idea about the velocity and the speed is actually, it seems quite high.

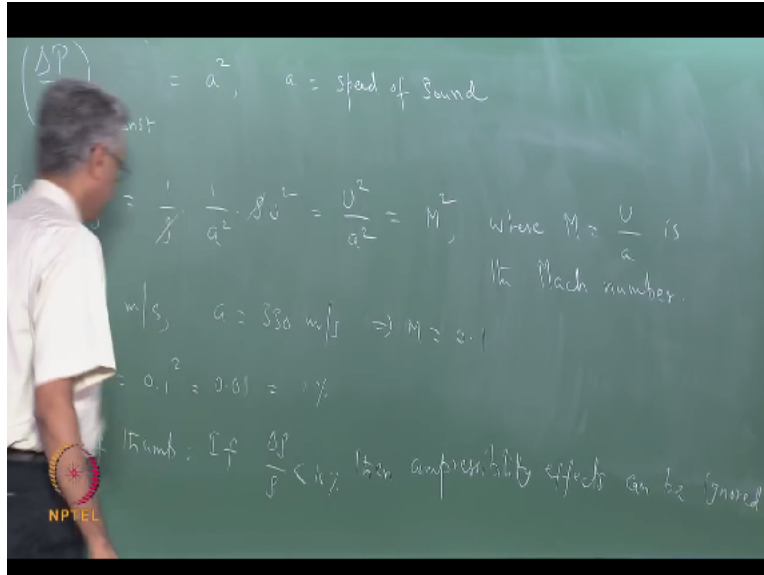
So we are trying to figure out whether compressibility effects are significant in this type of flow. In order to do this, what we are going to do is the following. We are going to bring in an estimate of changes in density and changes in pressure to evaluate this quantity and we will relate that to the flow velocity, okay. So any change in, the maximum change in pressure. So the fluid as it approaches, it is at about a free stream pressure of let us say P_∞ , right when the fluid is coming here far away from the vehicle, the fluid is at this pressure of P_∞ .

As it goes around, there is one streamline which is this stagnation streamline. So where the flow is brought to a halt completely, right. So from P_∞ , the fluid is brought to a rest. Now if we use Bernoulli's equation, then we know that the increase in pressure in this streamline is the maximum and that scale is roughly as the density of the fluid times the velocity of the vehicle, right. So U is the speed of the... or free depending upon your frame of reference.

Now if I look at change in density, for example change in density over unit density, I can write

this quantity as follows, right, I multiply and divide by delta P and I can write this quantity like this. So if I substitute for delta P, I get the following. Now most of you know from your high school physics that this quantity if you evaluate it for an isotropic process is the square of the speed of sound.

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So we know from high school physics that delta rho delta P/delta rho that is equal to constant which is an isotropic process, is nothing but the square of the speed of sound. So a is the speed of sound, right. So if I introduce that here, I get... So I cancel out these 2 quantities, I end up with the characteristic P/a square, the speed of sound which is nothing but the square of the Mach number. So where M is the Mach number.

So now we can see that the fractional change in density is now related to a flow quantity, right. Use the flow speed characteristic speed in the flow is the speed of sound. So now we actually can answer the question that we asked earlier, this compressibility effect significant in this flow, that the speed is 33 meter per second. So if you substitute 33 meter per second into this, so U=33 meter per second, right and a is approximately 330 meter per second for this case.

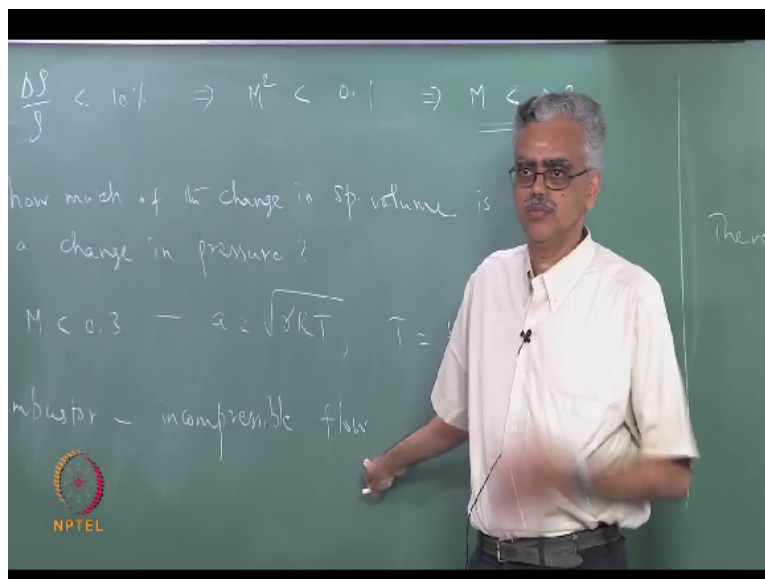
So which means that you Mach number for this case comes out to be 0.1, right. That means fractional change in density is only about, right, so fractional change in density over rho is 0.1 square so that is 0.01 which is nothing but 1% of the mean density. So the maximum change in

density is only about 1% of the mean density which means it is very very small. So for all intents and purposes, we can ignore compressibility effects in this particular flow field and treat it as incompressible.

This criterion is advantageous because it relates the changes in density, remember we are talking about change in specific volume, right. For compressibility, we talked about change in specific volume. So this relates the same quantity, change in density to a flow quantity, it says this is proportional to M square. General rule of thumb is if the change in density, as long as the change in density is about 10% of the mean density, then compressibility effects can be ignored.

In this case, it is 1%, it is very clear and it is very small. In general, this is a rule of thumb. This is not based on any actual physics, right. This is rule of thumb if $\Delta \rho / \rho < 10\%$, then incompressibility effects can be ignored, right. Let us see what this means in terms of Mach number.

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So if I say that $\Delta \rho / \rho < 10\%$. This implies that $M^2 < 0.1$, right which then says that approximately M should be less than 0.3 approximately. So if the Mach number is less than 0.3, then in general compressible effects in the flow can be ignored. So that is where, this is where these arguments come from that $M < 0.3$ because if I square this, I get something like 0.09 which is close to 0.1.

So approximately if $M < 0.3$, then compressibility effects in the flow can be ignored. The reason why speed of sound is important in these types of situation is that any disturbance that is generated in the flow field travels with the speed of sound. So when the actual flow speed is comparable to the speed of sound then that means the compressibility effects are very significant, right. If the actual speed is much less compared to the speed of sound, then compressibility effects are small, right.

So we will use both the criterion, namely, so we said 2 criteria for assessing compressibility. One, how much of the change in specific volume or density, both are the same, right, is due to the change in pressure. We said that that is very important and number 2, Mach number < 0.3 . So these are the 2 criteria that we can use to evaluate whether the particular component, in the particular component, compressibility effects are significant or not, okay.

Let us take a look at each one of these components. So here, we are sketching the variation of the pressure along the length of the engine. So you see that the pressure ratio is about 40 in this case. So you can see the pressure increasing slightly in the fan, drastically in the compressor and then pressure remains more or less constant in the combustor, falls rapidly in the turbine as the fluid expands, even generates the work required for the compressor, then continues to drop as it goes through the other turbine and then the nozzle.

You see a similar trend for temperature. Temperature is very small, it increases as the gas is compressed and it increases very drastically in the combustor as we add fuel and then burn the fuel and then it begins to drop as the fluid undergoes an expansion process, right. Now the axial velocity, axial velocity along engine is shown in this figure. So we are starting at about, you know, 150 meter per second, velocity more or less remains constant as it goes through the compressor and then drops a little bit in the combustor, then begins to increase in the turbine and then increases drastically in the nozzle, right.

So this is how these 3 quantities are changing. Now we want to apply these criteria to see which of the component is handling incompressible flow, which of the component is handling

compressible flow. If the majority is incompressible flow, then we need not worry about compressibility effects, right, that is what we are looking at. Now right off the back here we can see for example, if you look at the combustor, you can see that the pressure in the combustor is more or less constant, right.

It is the large change in temperature which means there is going to be a change in density as a result of this but the pressure is remaining constant. So which means if the combustor, the flow in the combustor is essentially incompressible. We need not even look at the velocity, right. We see that the pressure is remaining constant with heat addition, right. So that means the combustor essentially handles only incompressible flow. So that we can identify very easily.

The flow itself may be moving at high speed but the temperatures are very high. So Mach numbers are also low, that also tells us indirectly that we do not have a problem with the combustor. You can see that the velocities are here. Now the speed of sound, if you remember, speed of sound for an ideal gas varies as square root of γRT , right. So the speed of sound when you apply this rule, this means that a is nothing but square root of γRT , where T is the temperature.

We will derive this relationship later on but for now, it is sufficient if you know this, right. So this varies with the temperature and as the temperatures are the highest in the combustor, speed of sound is also the highest. So when I take this velocity and divide it by the speed of sound, I can easily see that the Mach number is going to be very low in the combustor. So both this condition, the fact that the pressure is constant with heat addition and the fact that M is going to be very small, tells me that the flow in the combustor is essentially incompressible.

Now in contrast when we look at ramjet or a scramjet engine later on, you will see that when you add heat to flow where compressibility effects are important, the pressure will not remain constant, the pressure will also increase. Not only will the temperature increase in a ramjet or scramjet engine with addition of heat, the pressure will also increase. So that tells you that the change in specific volume is also due to a significant change in pressure which means when we go to a ramjet or a scramjet combustor, we need to account for compressibility effects in the

combustor.

Whereas in a gas turbine combustor, because addition of heat does not increase the pressure significantly, we know that compressibility effects are negligible. In fact, when we are talking about heat addition to a compressible flow, we will actually demonstrate this relationship mathematically, right, that heat addition to an incompressible flow does not cause an increase in pressure, whereas heat addition to a compressible flow causes an increase in pressure.

We will prove that mathematically but we can see that here from this relationship now. Now what about the other components in this figure, okay. We can see that the velocity here is small. So if I take the speed of sound and try to calculate the Mach number, it may seem that the Mach number is actually not very large in these spots also. However, this criterion is trying to tell me that the pressure is changing continuously, right.

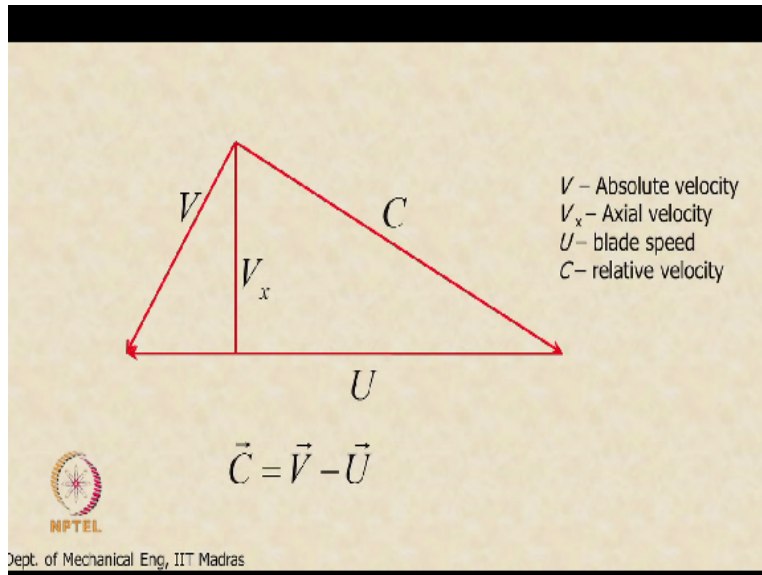
So any change in specific volume is going to be due to both change in pressure and change in temperature. So when I look at this too, the first criterion, I am getting a feeling that compressibility effects are significant, right but when I look at the axial velocity alone, this is about you know may be 150 meter per second. It is going down here. So the temperature is increasing.

So the Mach number is actually decreasing. It may be high here, right where the speed of sound may be 300 meter per second or 330, so Mach number can be 0.5 here but as the heat more or less keeps going down slightly, the temperature keeps going out, speed of sound increases and Mach number goes down. So it tells me that may be in this compressor, compressibility effect may not large, that is what this one is trying to tell me.

But the problem is when you are dealing with the turbo machines, please remember that the blades are spinning. So axial velocity is not the only velocity that is present in the flow. The blade is also spinning, right. So we need to take that into account. In a combustor when we did this, there is only axial velocity, there is no spinning blade. In the case of a turbo machine, because we have a spinning blade, we have to be very careful about how we evaluate the Mach

number.

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Although the axial velocity is small and the Mach number based on the axial velocity may be small. The relative velocity is very high and the Mach number based on the relative velocity is quite high. So when I interpret this particular diagram, I need to be careful about this. We will finish this in the next class and then continue from there.