Conduction and Radiation

Prof. C. Balaji

Department of Mechanical Engineering

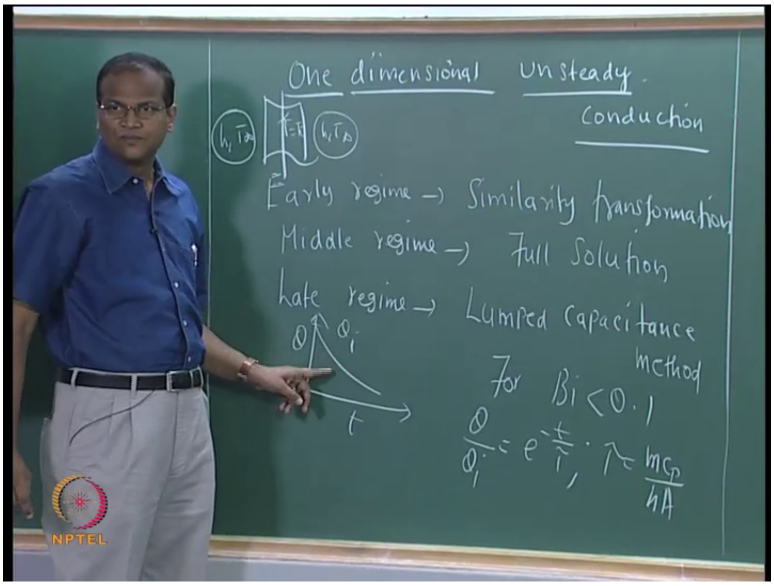
Indian Institute of Technology, Madras

Lecture No. # 40

Transient Conduction contd

So, anyway it is topical that we are looking at transient conduction where things change with time as with any other situation in life so in fact this is beautiful. Yesterday we saw a 1 dimensional unsteady conduction you can have 3 regimes, the early regime only certain portion of the solid is affected by the thermal disturbance at the boundary.

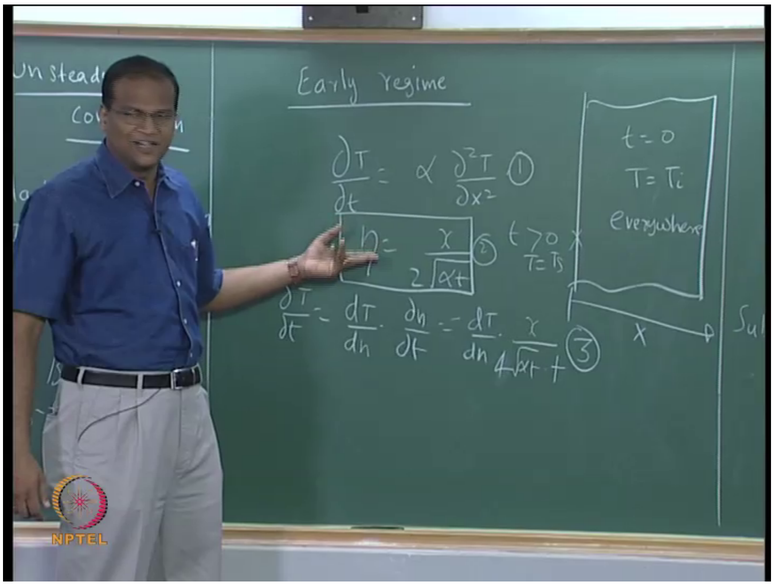
(Refer Slide Time: 00:33)



So, we took something like this. So, this is t is equal to T i throughout and then we suddenly cooled the sides, heat treatment of a ball bearing, heat treatment of a so heat treatment of a helical coil spring or a carrot spring of your truck or helical coil spring of your car or whatever, if you do so that if you can thermal conductivity is very high and if the biot number is much smaller compared to 1 you can use the lump capacitance system that is 1 thing. If you cannot do that but, you are still in the early portion of the cooling then the centre will not know that there is a disturbance, so the disturbance propagates from the end.

In the early regime we can do something called the similarity transformation. The whole idea of a similarity transformation is to convert the partial differential equation into an ordinary differential equation and ODE is easier to solve. In middle region you have to use the full solution and in the late regime you can use what is called the lump capacitance system where more or less the system is reaching the temperature of the surrounding so you can lump the whole solid to be at 1 temperature and so yesterday we saw the exponential temperature distribution is even if you plot lot of systems in real life follow this last 10 days if you plot the attendance in this core sources time it will follow it theta with eta is e to the power of minus e by tau, that is law of nature that is exponentially decaying, exponentially decays with time so that is the solution for the lump capacitance system.

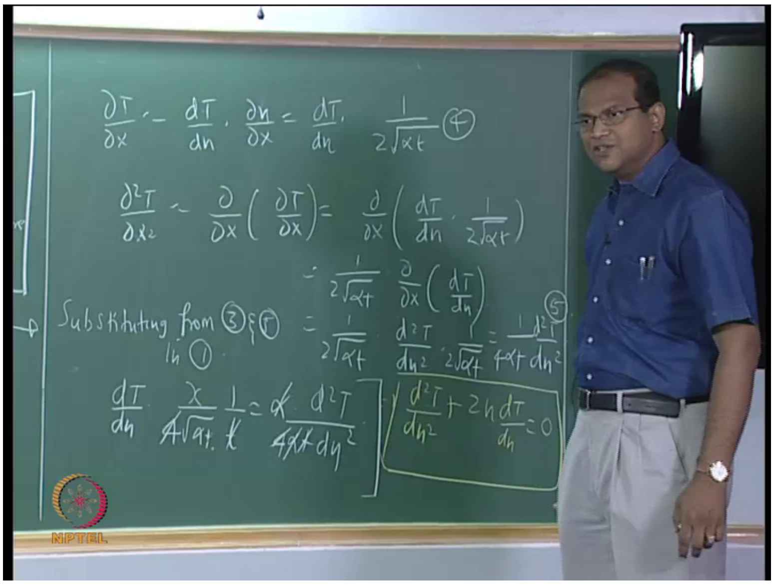
(Refer Slide Time: 02:14)



Now, in yesterday's class I looked at this we looked at this early regime where we introduced a similarity variable called eta, eta is x by 2 root alpha t temperature is a function of both x and t therefore, if you introduce x by 2 root alpha t it is an intelligent choice of 2 variables combined in such a way that when you work out when you get the when you try to use the similarity transformation automatically the PDE is converted into an ODE.

Yesterday, some people had a doubt where there was no problem with this d of temperature by d of time is dT by d eta because temperature is a function of eta only into dow eta by dow T, this is like this now we got dT by dx is same dT by d eta into dow eta by dow x.

(Refer Slide Time: 02:50)

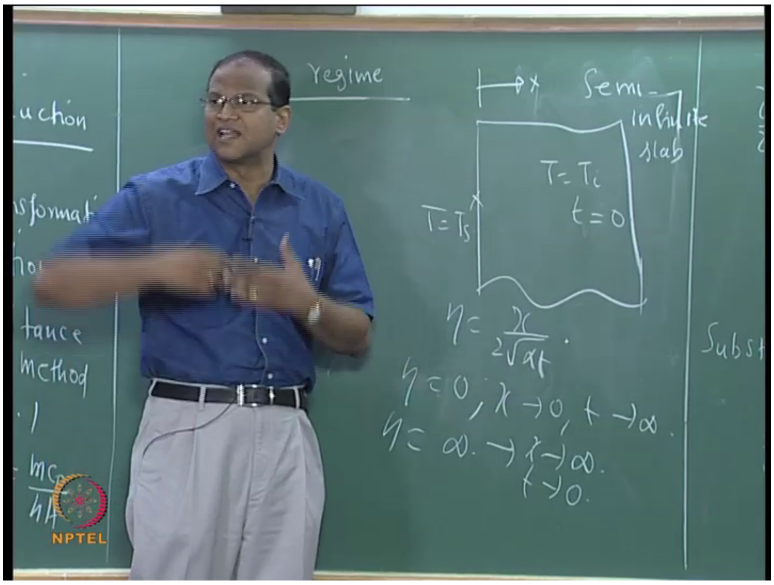


So, dow eta by dow X is 1 by 2 root alpha t that is given here but, when you are going to dow square T by dow X square so first you do dou by dow X of this. Once you do dow by dow X of this, this 1 by 2 root alpha t has got no business with X so it called be pulled out so you just pull out 1 by 2 root alpha t then you have got dow by dow X of this, dow by dow X of this is d by d eta of this and dow eta by dow x. Once you do that then this alpha all this get cancelled then I again have that x by root alpha t, x by 2 root alpha t is eta therefore, x by root alpha t will be 2 eta.

So, d square T by d eta square plus 2 eta dT by eta equal to 0. So, this is a similarity where this is the ODE which you obtained by using this similarity transformation. The beauty of this is as follows. You can assume that dT let dT by d eta be equal to p, then this will become dT by d eta plus 2 eta p equal to 0. So, it will just become a first order equation and apply the boundary conditions and you can solve it and you will get what is called the error function.

So, it is a very simple way of solving a transient. So, you have simplified the original partial differential equation because we exploited the concept of limited thermal disturbance in the initial portion of the cooling or heating. The whole of the solid is not affected. Now, if you look at the boundary conditions.

(Refer Slide Time: 04:37)



Let us take a solid like this x is like this, so it is T i T equal to T i t less than or equal to 0 some people do not like less than equal to 0. So, this T equal to T s so eta equal to 0 and eta tending to infinity, eta tending to 0 means what? That means eta tending to infinity is x tending to infinity and t.

See the see the beauty of this. Please, look at the board eta tending to infinity means somewhere further assume that this is a this is called a semi infinite slab that is why it is called a semi infinite slab. At a given, at a given time the penetration depth delta is much much smaller compared to the l that is why it is called semi infinite in extent, this delta the penetration thickness may be 5 mm, the slab may be 30 centimetres long that means 29.5 centimetres of the slab do not does not know that there is a disturbance at the first 5 millimetres.

Now, eta tending to infinity what is the story of eta tending to infinity? Is x tending to infinity, x tending to infinity do you know the disturbance or not at the x tending to infinity, we do not know the disturbance therefore, what is the boundary condition for temperature.

((T equals to T i)).

T equal to.

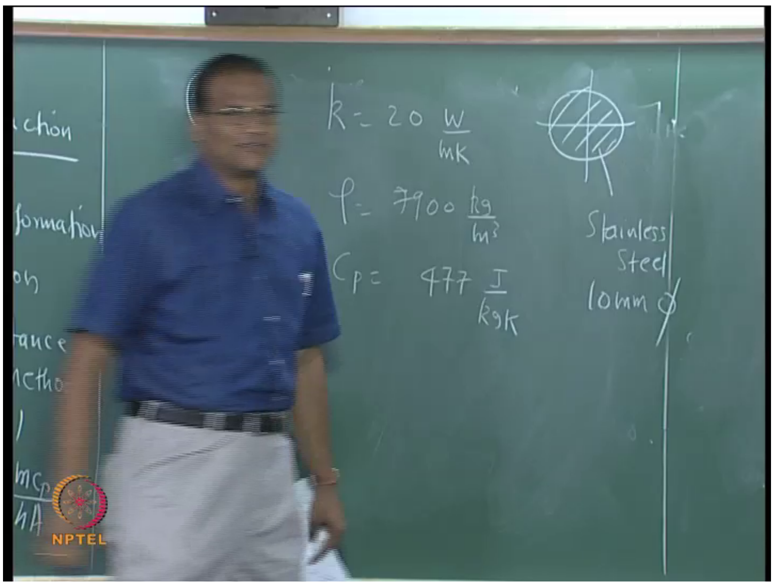
T i.

T i T equal to T i is the boundary condition for eta tending to the infinity but, the beauty is T equal to T i is also the initial condition that corresponds to t equal to 0 everywhere therefore, by an intelligent choice of eta you are able to absorb 1 boundary condition and 1 initial condition that is why it is beautifully reduces the partial differential equation into an ordinary differential because the original equation, the original equation has to support 3 conditions, 2 boundary conditions on x and 1 initial condition on time because your dT by dT and dow square T by dow x square. Now, it has become second order in eta.

Therefore, you cannot it cannot support 3 conditions but, physically those 3 conditions are real for the problem therefore, these eta's absorb 2 of these conditions into 1 condition on eta equal to infinity are you getting the point. So, there is no degeneracy, there is no loss of information because we tried to use this procedure. So, the funda clear now, we will solve a problem. in the lump capacitance method problem number 48 very good stainless steel balls 10 millimetre in diameter are annealed a n n e a l e d are annealed by heating to 1200 Kelvin stainless steel balls 10 millimetres in diameter are annealed by heating to 1200 Kelvin and then slow cooling to 375 Kelvin in air, then slow cooling to 375 Kelvin in air, I am using telegraphic line h equal to 25 watts per metre square Kelvin.

If somebody is a purist you will say a heat transfer coefficient at the interface you can write all that h equal to 25 watts per metre square per Kelvin and T infinity equal to 40 degree centigrade.

(Refer Slide Time: 09:56)

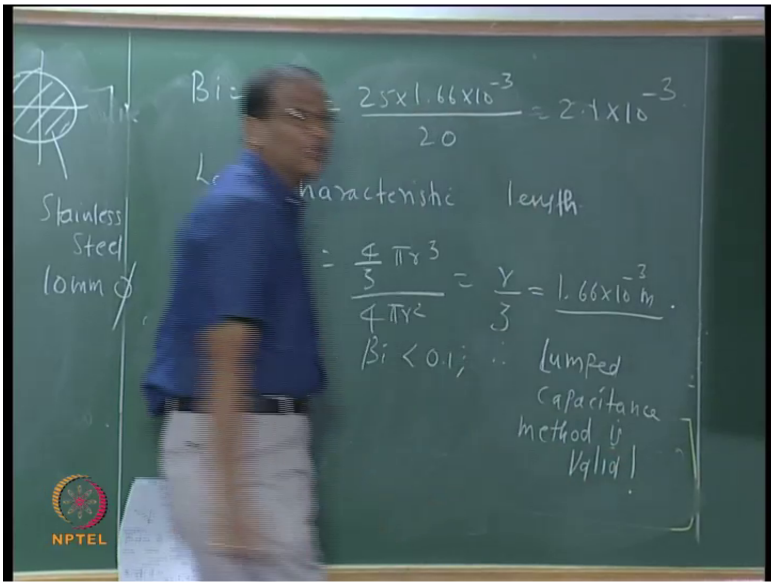


Properties of stainless steel are thermal conductivity 20 watt per metre per Kelvin, density 7900 kilo gram per metre cube, C p 477 joules per kilo gram per Kelvin. What is a question? Now, what is the question?

((Time required)).

What is the time required for the cooling? This is the question which can be answered only by the heat transfer engineer. What is the time required for cooling? 10 millimetres phi 10 millimetres in diameter the first step in technology is biot number first check whether biot number is alright. If characters characteristic length is not

(Refer Slide Time: 11:11)



so apparent, take the ratio of volume to area 4 by 3 pi r cube. Similarly, you can work out for slab cylinder or even an arbitrarily, an arbitrary shape volume. So, r by 3, how much is it?

((one)).

1.66 correct. Now, biot number. So, the biot number is ridiculously small for a ball bearing and all that blindly you can start assuming it is a lump capacitance, good thermal conductivity very small size, good heat transfer coefficient I mean it should not be too good then h u is in the numerator reasonable heat transfer coefficient so what is it biot number.

((Two point))

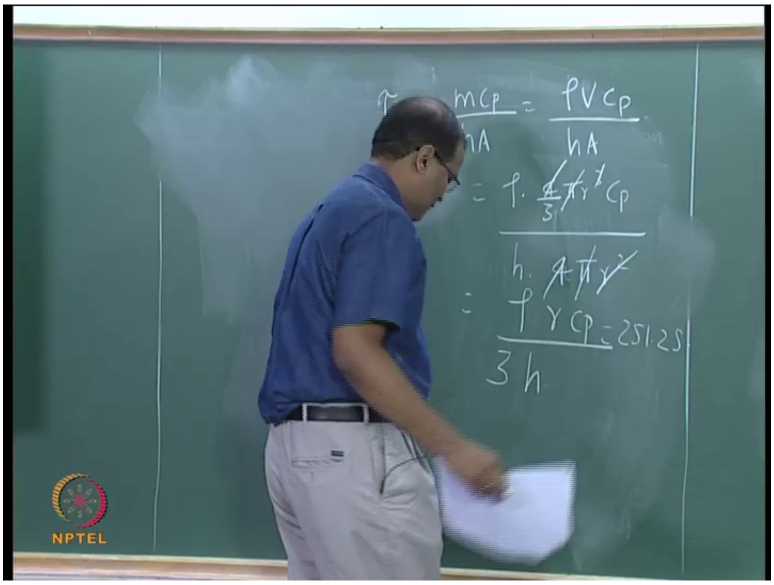
Two point.

(( )).

((I mean I'll put 2.1 into 10 (( )) minus 3))

So, I will write like this biot number less than 0.1 therefore, therefore, the lumped capacitance method is valid.

(Refer Slide Time: 13:37)



So, time constant. Umesh finish.

(( )).

This is that is all is it correct. I made some mistake what is that.

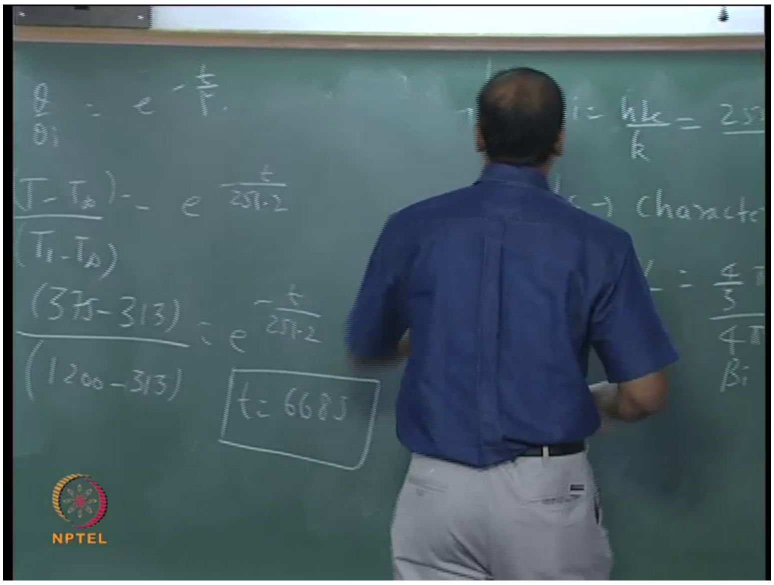
((3 h)).

How much is it.

((251.1.))

251 point what Deepak.

(Refer Slide Time: 14:51)



So, theta by theta i theta, it must get cooled to 375, 313 correct 1200 minus so take l n both sides on the calculator multiply by 251 and you will get some decent time 700 seconds, 800 seconds how much is it?

((6685.))

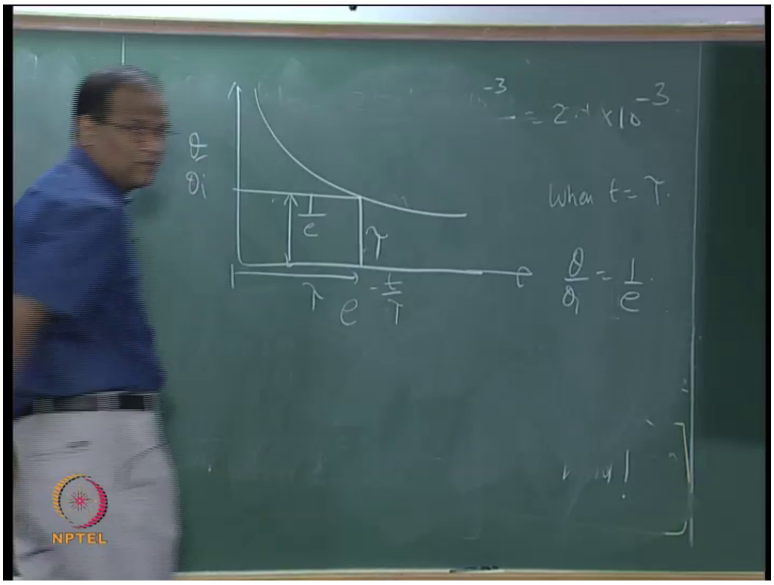
((668)).

So, after about 11 minutes or so we can pull out this stainless steel ball bearing it will be it will come to this temperature, its good. Suppose, there is a radiative heat transfer even then the lump capacitance system can be used but, you have to do a equivalent radiative heat transfer coefficient. I have told you the linear radiation t to the power of minus t to the power of 4 minus t infinity to the power of 4 is T square minus T infinity square into T square plus T infinity square T square minus T infinity square is T minus T infinity into T plus T infinity combine all the terms and call it as h r radiative heat transfer coefficient into T minus T infinity and add it.

Suppose, I give you a problem in which combine convection radiation is there so total heat transfer coefficient must respect the biot number less than 0.1. Anyway, we leaving the mathematics apart what is this time constant now.

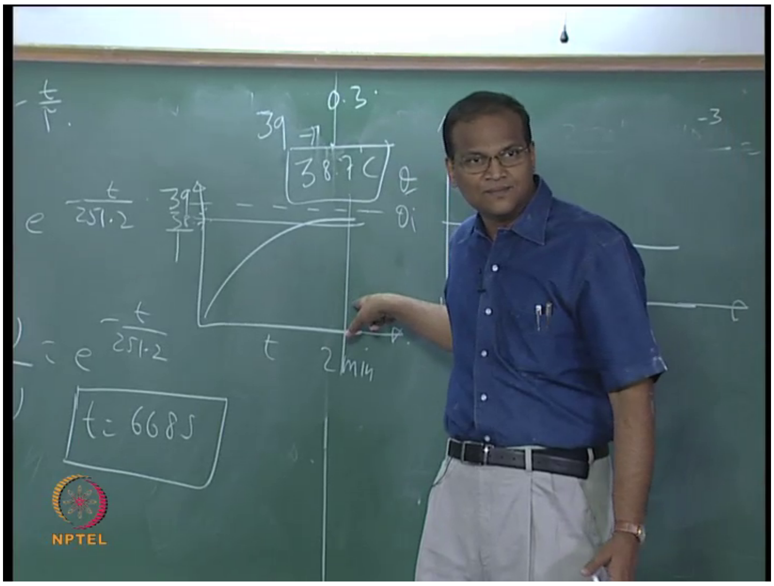
Time required to (( )).

(Refer Slide Time: 16:53)



Time required to for the temperature to drop to 66 percent of initial whatever correct. So, theta by when t is equal to tau 1 by e correct, did I make it did I mess it up or it is 1 by e.

(Refer Slide Time: 17:44)



Therefore, I may similarly, let us take t and suppose the doctor wants to get the fever temperature accurate to let us say 0.22 is for him, 0.05, 0.05, 0.1 is too fine a number they are not engineers they cannot handle such numbers, what is a good accuracy for a doctor?

((Less than 0.5))

Less than 0.5 why I am doing this is if it is less than 0.5 let us say somebody has a 0.5 centigrade or Fahrenheit.

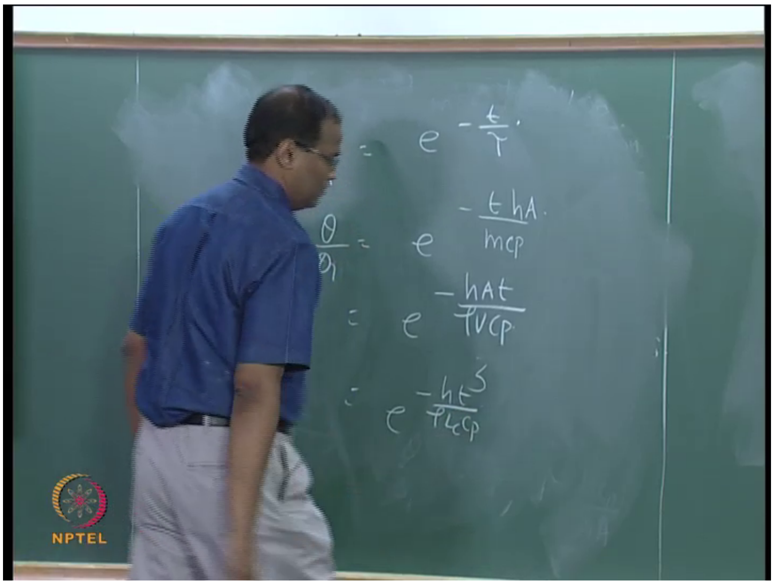
Fahrenheit.

Fahrenheit so we have to give some respect to doctor,so give 0.5 Fahrenheit that is about 0.3 centigrade so let us say that somebody has a fever of 40 is too much 104 let us say 39 is a decent fever so he wants accuracies about 0.3 so if it reaches about 38.7 degree centigrade it is okay for him. He will live with that error.

So, if the if it is like this so if the actual temperature is 39. So, he wants to know at what time it will reach 38.7, that will be related to the time constant invariably the doctor pulls out at the end of 2 minutes. So, correct at the end of 2 minutes it that is directly related you can find out the time constant of this mercury in glass thermometer and find out.

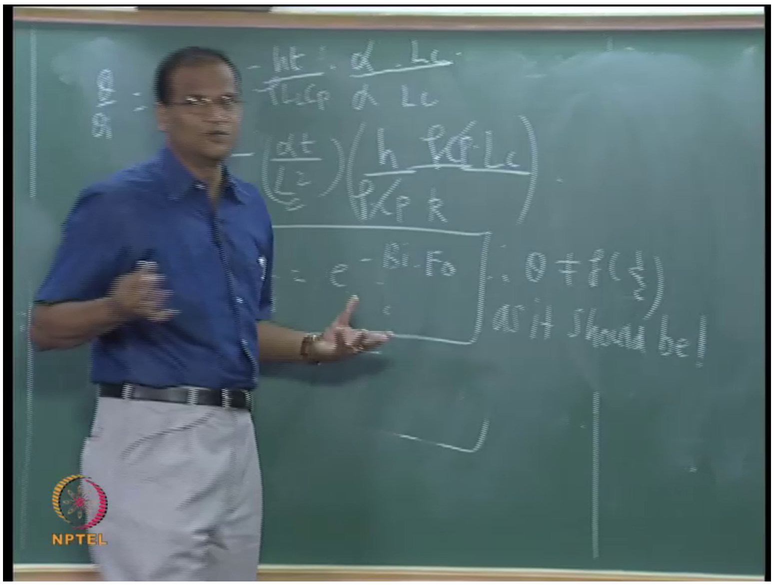
Now a days you got electronic this thing which it will which gives a beep and but, if you take you can fake a fever by going to different he is having a coffee and then going to the or you can take a Pepsi and doctor he gets scared hypothermia so or in Europe what they do is they insert the thermometer into the ear and ear temperature or they will measure the forehead temperature because the core temperature inside is not actually known, these are all diagnostic of overall temperature is diagnostic of something happening inside or you have to use a infrared thermograph or something ear ear thermometer is a slowly getting popular here also, they are expensive.

(Refer Slide Time: 20:21)



So, this is the idea of how much time you want. So, the time is related to the accuracy. Now, what is this theta by theta i? Is that what is alpha Fourier number.

(Refer Slide Time: 20:57)



((Alpha (( )) by l (( )))).

What is it? What is the story now? Alpha, what happened to the.

(( )).

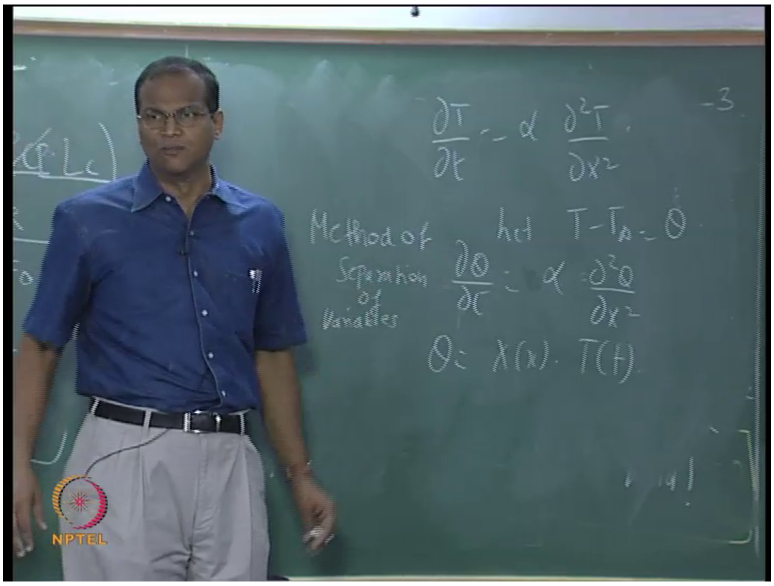
Not biot number, then where is the k oh there is a k there. Now, what else is there h rho c p alpha is denominator k by rho c p some L is there somewhere, what is h L C by k? what is this?

(( )).

So, theta by theta i is e to the power of minus biot number into Fourier number, I am saved, that is the way it should be if you recall when we started the non dimensionalization I told you that theta is a function of xi biot number and Fourier number. xi gives this spatial variation of temperature, biot number gives the conduction convection coupling and Fourier number gives the variation of temperature with respect to time. Now, it is a lump capacitance system therefore, the spatial variation should go therefore, the dependence of theta on xi has vanished because it is lump therefore, as it should be.

See, if you are working on a project or heat transfer problem on research or something, so we just take paper and pencil and keep on working before going to the computer so much is possible, you can extract lot of information without actually solving the equation by non dimensioalization, scaling analysis and that is how you get the physics.

(Refer Slide Time: 23:48)

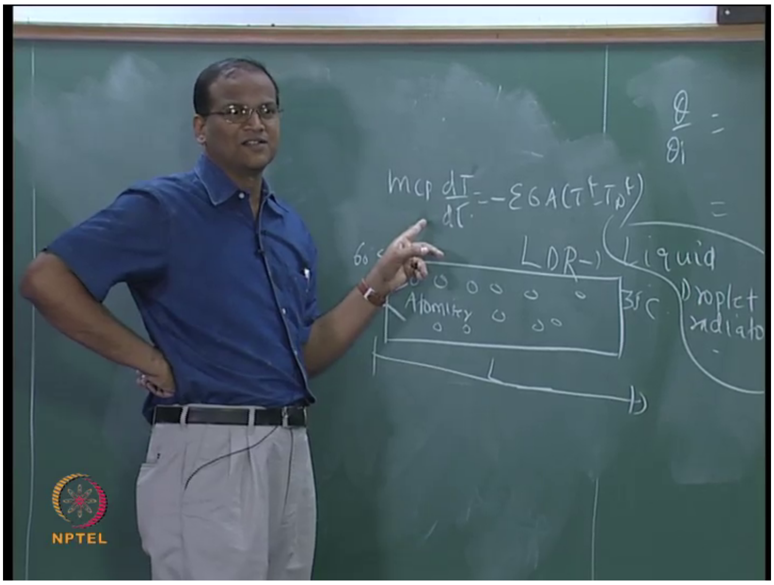


Now, let us go to the last part and look at the I did not tell you the full solution the full solution will be, what I basically do is I assume that theta is a product of 2 solutions X of x into T of t where x is a function only of x and t is a function only of t. Now, I take d theta by dT d theta by dX d square theta by dX square and then I will arrive at terms on the left side and right side, left side 1 left side will be a function only of time, right side will be function only of x. If both have to be equal both of them have to be individually equal to some constant I call it as lambda square. Then I am essentially breaking down the PDE into 2 ODE's. I solve this. Thisis called the superposition principle. I combine the solutions and using the boundary conditions I get a b c d.

So if you do that what you are essentially doing is called the method of separation of variables. I should not put method of separation of variables here because the separation of variables has not come here. In the next 2 steps it will come when you put d theta by dX and d theta by d. So, this is the full solution which is applicable for the middle regime as well as for the early regime and the late regime. However, for the early and late regime this complicated solution is not required we can use the lump capacitance for the late regime and use the similarity variable for the early regime.

And these have been solved and these are available and these charts are also available. Standard heat transfer books and the so called data books will give you the will give the various charts for theta as a function of xi biot number and Fourier number. So, you have to calculate biot number first then Fourier number and find out x by l and for cylinder sphere and slab charts are available in several text books. You can use it or you can program in mat lab and get the solution right away, you can use mat lab solution and use the PDE solver.

(Refer Slide Time: 26:21)

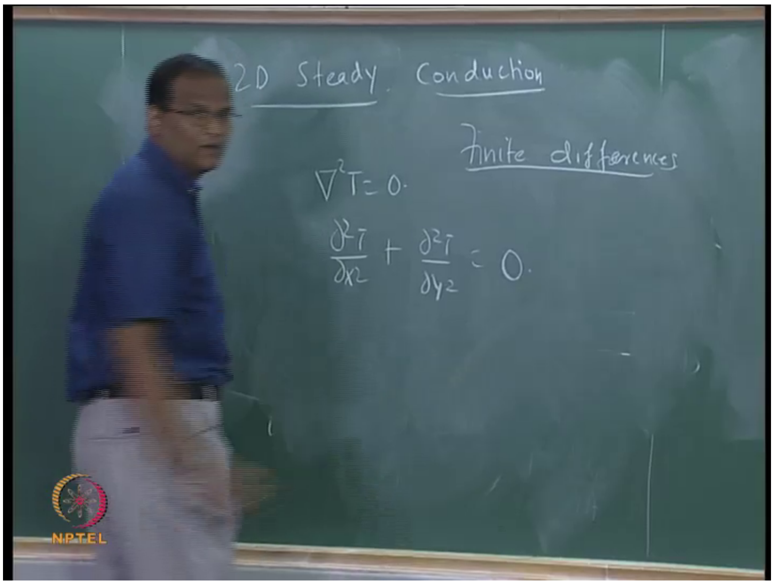


Now, the last part is we look at the Laplace equation. So, if I have a lump capacitance with heat generation so it will be that is I have a dipper heater, immersion heater initially it will rise then the temperature becomes so high that the loses will be equal to Q then this will vanish then Q will be equal to h A into T minus T infinity, you will get the steady state temperature. Then if you switch off or you can have situation where so 1 good example would be basically I am cooling some electronics in a spacecraft and then I am using a fluid, an organic fluid to cool it, then the fluid gets hot that hot fluid has to be cooled again so that it can continue the cooling process.

Therefore I can have a glass, I can have a device which is made of glass like this then I can have a I can have an atomizer, a pump which just sprays this particles, oil particles. The oil particles move with a velocity. Now, each of this can be considered to be spatially isothermal and then if you know all the properties and now if the ambient is just out outer space at 3 Kelvin, the cooling will be accomplished so the design will be what should be the length. If I know what is a velocity with which I push this drops out what should be the length of this device such that by the time it comes out so if it enters at 60 I want it back at 35 degree centigrade.

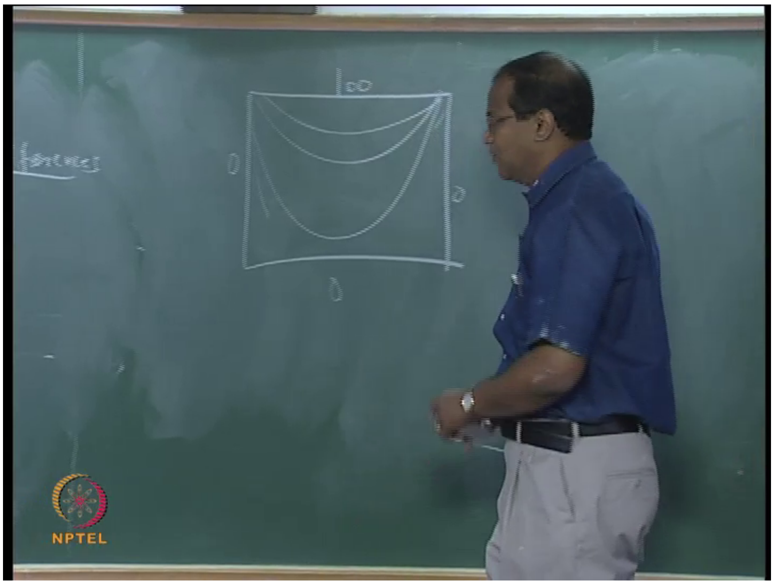
So, this is called LDR a liquid droplet radiator. So, the even the lump capacitance system looks so simple but, it has got so many applications. Vinay you dint get it? So, the governing equation will be this, there will be lot of fun because dT should be taken to this side, then you have to use a method of partial fractions, tan inverse of lon of all these will come. I can just give you a 15 minute course on finite differences, finite differences can be taught for 40 hours.

(Refer Slide Time: 29:01)



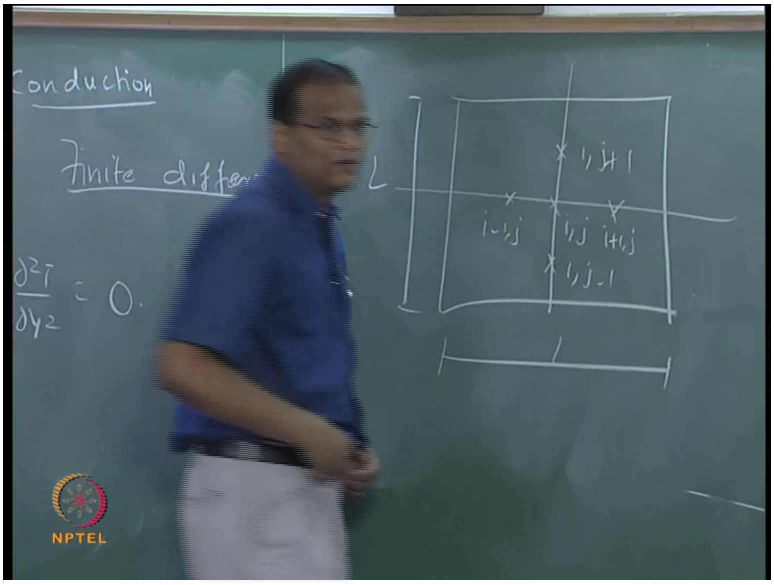
So, let us consider a simple Laplace equation which is del square T equal to 0, this can be derived from the general equation. So, if you had the heat generation term it is called the Poisson equation.

(Refer Slide Time: 30:04)



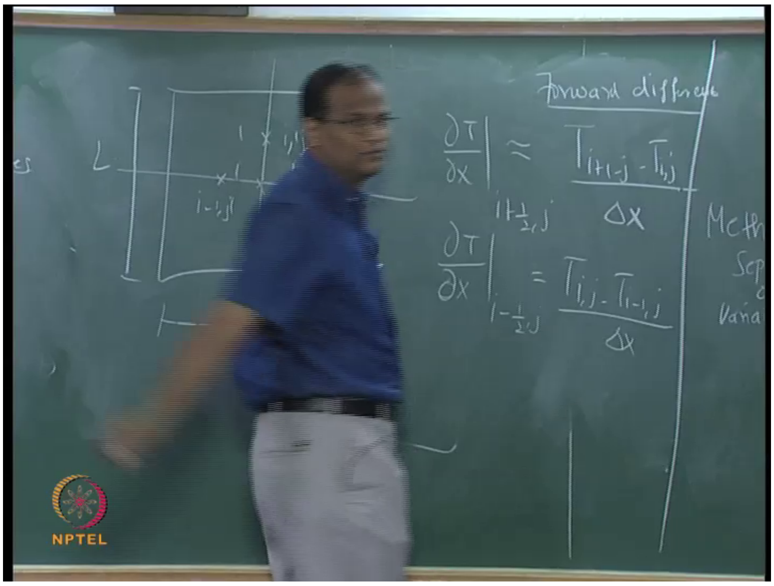
Let us consider a situation you remember when I started discussing the Fourier's law this figure I used to discuss heat flux lines and isotherm. This is the 100 degree isotherm this is the 0 degree isotherm, all the other isotherms will be like this. Is it okay.

(Refer Slide Time: 30:35)



Now, this can be solved using the method of separation of variables and how do you solve it using finite difference. Let us take a square slab L, so what I do is I take a particular node called i comma j then I take 1 more node i plus 1 j, I take here i minus 1 j, I take here i j plus 1, I take here i j minus 1.

(Refer Slide Time: 31:15)

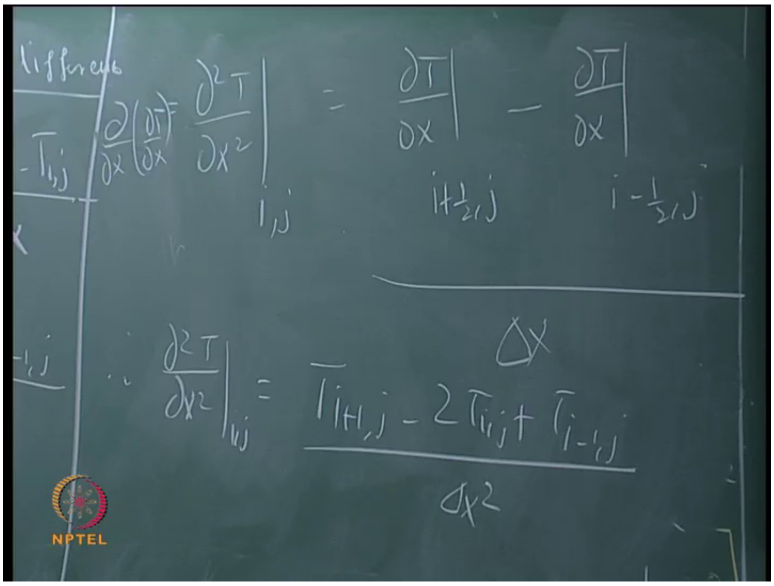


Now, what I do is I can calculate dT by dT by dX at i plus half comma j that is here in the middle, the temperature gradient here using a tailor series approximation retaining only the linear terms will be t of the node ahead it minus the T of node behind it divided by the delta

X. That is a finite difference approximation so I am replacing it replacing dow T del T and replacing dow X by del X.

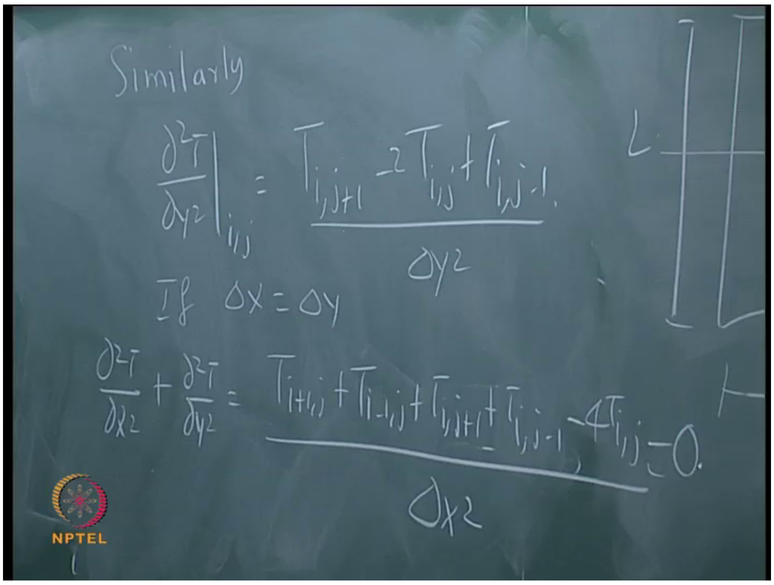
So, this is just be assuming that all the delta X's are same otherwise I have to put x i plus 1 minus x i similarly, dow dT by I can take the derivative here that will be T, so this will be T i of j minus so what I am essentially doing is called the forward difference. Is it forward difference. I am doing central difference here I am doing central.

(Refer Slide Time: 32:45)



Now I will say what is d square T by dX square at i j is nothing but, dT by dX at i plus half j minus dT by dX at i minus half j divided by delta X, very good because this is nothing by this is nothing but, dow by dow X of dT by dX. What is this? Use that therefore, I got a central difference approximation for the second derivative.

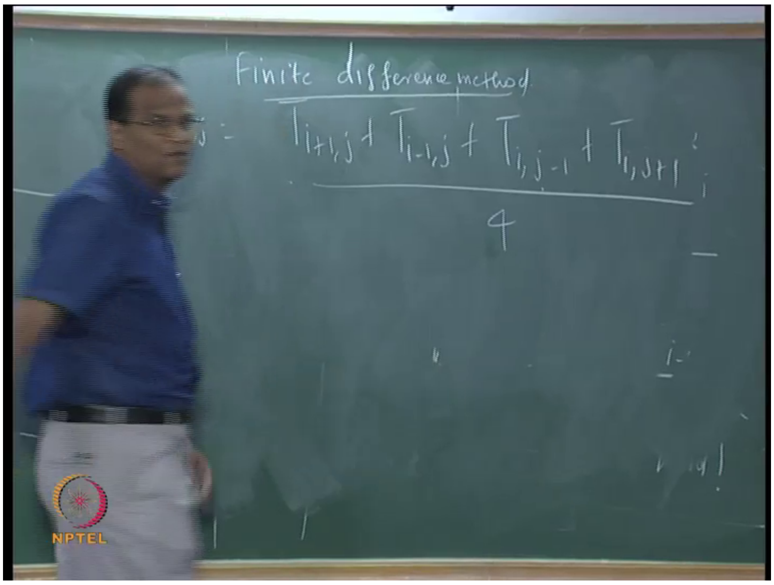
(Refer Slide Time: 34:04)



Similarly, T i j plus 1. You can make delta x different from delta Y. In the same place case will be delta X equal to delta Y. Why are we doing all this? We want to get an expression for T i j in terms of its neighbours. What is a fun in doing this? This i can vary from 2 to n minus 1, the j can vary from 2 to m minus 1, if it is a non-uniform grid, then each of the T i j's is related to its 4 neighbouring nodes, then when you do this at i for i equal to 1 and i equal to n for j equal to 1 and j equal to m the boundary conditions are given to the problem the 4 boundary conditions are given.

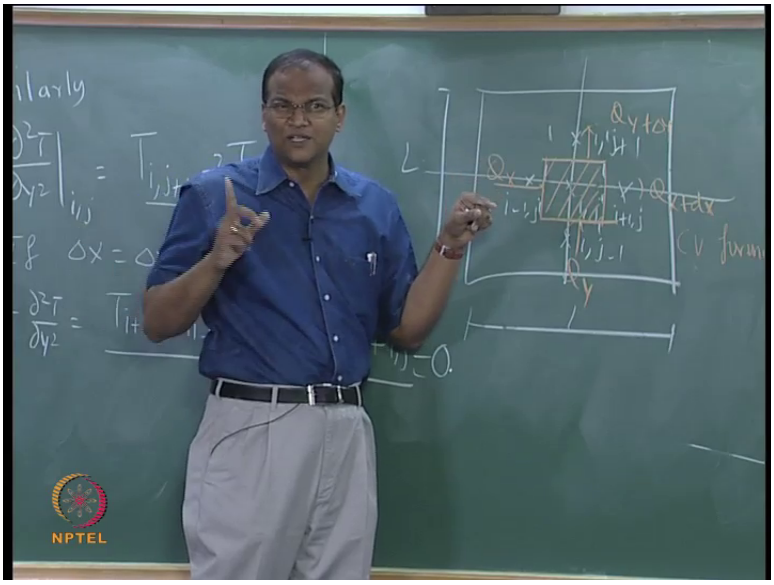
So, the boundary conditions will start feeding and we can solve this system of simultaneous equations for T i j's either using matrix inversion or Gauss Jodan elimination or Gaussian elimination or Guass Seidel method, all the c f d software work on essentially this. Finally, you get a system of simultaneous equations for a particular node T i j, instead of T i j you can have u i j, u velocity v i j rho i j for density concentration and all that however it would not be the diffusion equation, apart from diffusion you will have the conviction or the advection terms u dow u by dow x plus v dow u by dow y and so on.

(Refer Slide Time: 36:38)



This is essentially numerical heat transfer therefore, T i j equal to T i plus 1 j plus T i minus 1 j plus T i j minus 1 plus T i j plus 1 divided by 4. So, this is called finite difference method. Please, look at the board I can derive the same equation in a slightly different fashion.

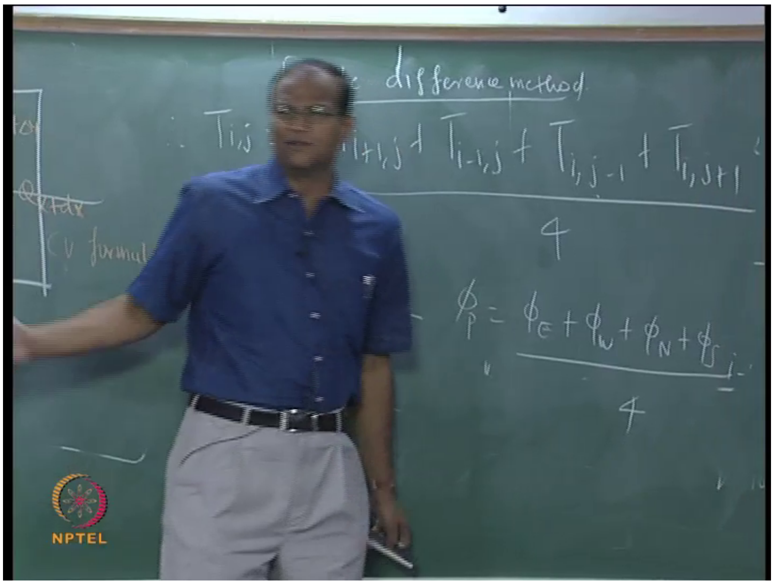
(Refer Slide Time: 37:24)



I can derive the same equation in a slightly different please look at the board, I can take half so my control volume is delta x by 2 into delta y by 2. I can find out what is that Q which is entering here, what is the Q which is leaving here, what is the Q which is entering here, what is the Q.

The net will be k into t of this minus t of this divided by del X square. I can write I can replace the Fourier's law by just using finite difference. I can do an energy balance so this is called a control volume formulation, if you take delta X equal to delta Y and you will, it will result in the same, it will result the same equation you can do a flux balance, the net flux which is coming will be equal to 0. Actually, when you say that the net flux is equal to 0 you are actually integrating the governing equation once. So, essentially what I am using is the integral formulation of the governing equation. So, if the finite difference uses the differential formulation the finite volume and the finite element method will integrate the governing, integrate means what we are across a volume we are finding what is a story.

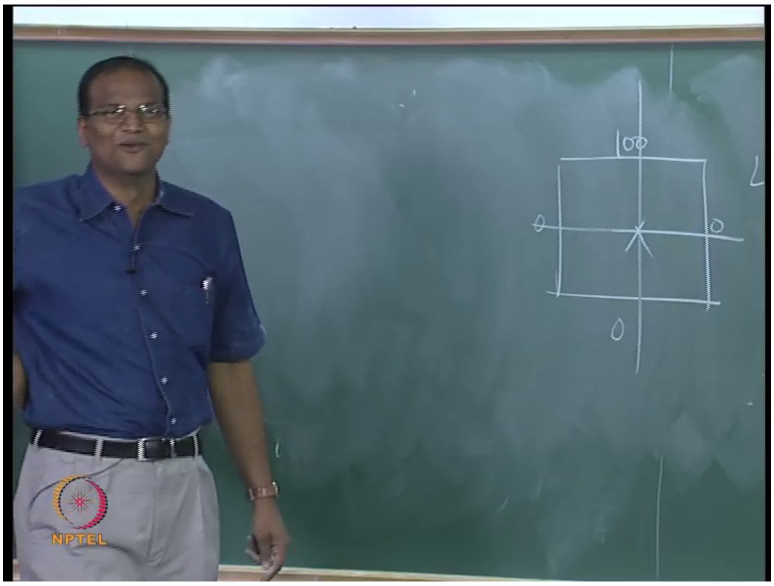
(Refer Slide Time: 38:52)



Anyway let us not get into that we will get buried deeply. If I have 1 more hour I can derive it. I will derive it for the Poisson equation but, now there is no time so the basic story is the nodal value for a diffusion equation is east plus west plus north plus south divided by 4, that divided by 4 comes because you get 25 percent weightage to all this otherwise in our when you write your CDF software it is always like this, always in life there are 4 people, they always talk about 4 people. If 4 people are happy you can do anything, I mean such type of stories are also there.

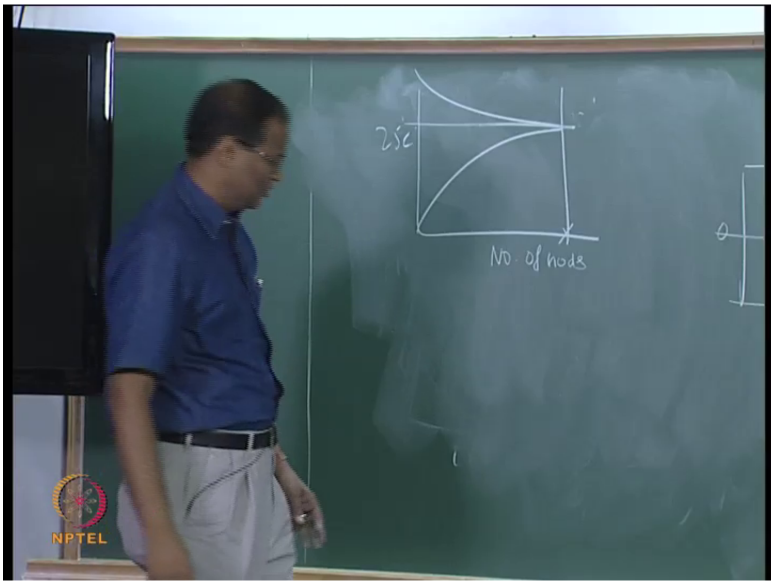
Eventually also, some 4 people are ego head. So, here also there are 4 neighbours. Here also, here also there are 4 neighbours. So, the value of the function at a point is a weighted average of its neighbours. If all the neighbours are at equal distance we give 25 percent weightage to each of them, if they are at different distance it will be w east into value at east plus W west into value at west, the sum of all the W's will be equal to 1 that is the weighted which is the weighted mean. So, this is essentially the diffusion you can put the convection advection also.

(Refer Slide Time: 40:28)



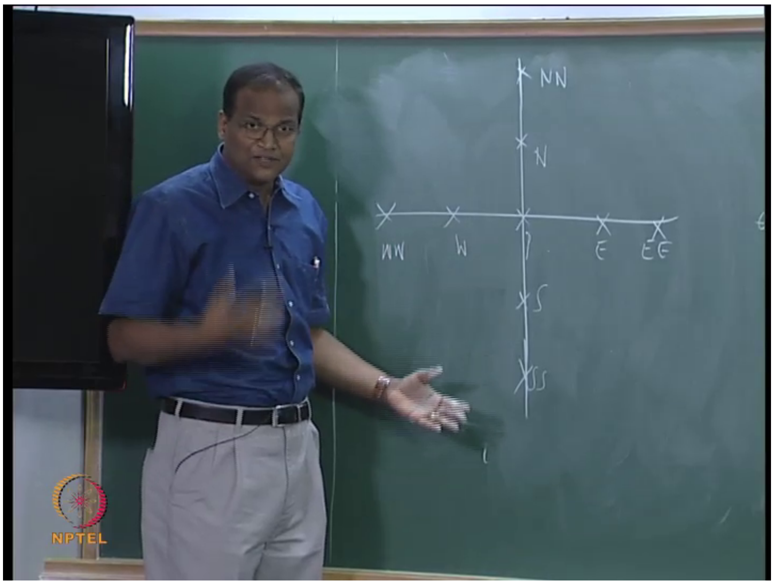
Now, what will a mathematician do. This everybody knows what is this. So, what he will do is, so what will be a quick answer to this problem 100 0 0 0 if it is a very crude method what will be the centre temperature according to the algorithm? 25. So, this is the first program whoever wants to do b d b with be this is will be the first program I ask them to do. Solve it using finite difference and prove that the centre temperature is 25, you have change the number of grids.

(Refer Slide Time: 40:51)



So, the centre temperature will approach either like this or it will approach like this. This will be number of nodes, when it has reached somewhere then you say it is grid independent. These are all part of a numerical study for any numericals.

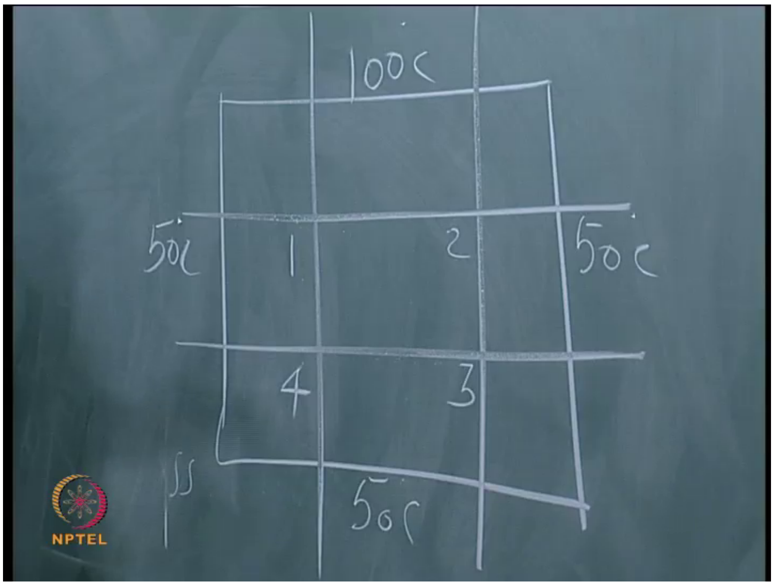
(Refer Slide Time: 41:11)



So, what I have done essentially is called a what is called a 5 point stencil. In computational heat transfer or CFD they will say it is a 5 point stencil that means the value of the node is dependent on 4 neighbours, that node itself plus 4 others it is called a 5 point stencil. You can have a higher order scheme I do not know you can call it east east, you include these fellows also what stencil is it 9 point stencil they have 13 point stencil, 21 point stencil then they have stencil which is equal to the number of grids. I have seen I have reviewed papers for high class because and they will prove that compared to 2 point stencil they are able get with much less grid they are able to get accuracy blah, blah, blah all that.

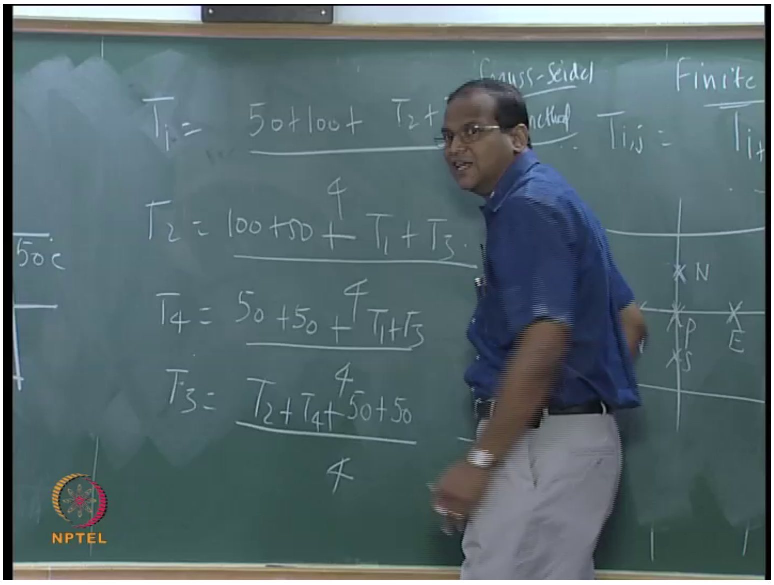
So, are you getting the point. When will you get the 9 point stencil in the tailor series you stop with linear then you go to the second order terms, you go to the third order term then we will get higher order stencil, delta x need not be equal to delta y then the average will not be the it would not be 25 percent, 25 percent, 25 percent. Now, if you have once you write for p you can similarly, write for E, you can write for N, you can write for this and then what you do is basically.

(Refer Slide Time: 42:53)



Suppose, I give you a problem. I give you the grid pattern suppose in the exam I ask you a question like this. There is a square slab 10 centimeters in 10 centimeters long and 10 centimeters wide and 1 unit depth in the direction perpendicular to the plane of the board. So, various boundary conditions are given, there is no heat generation constant thermal conductivity steady sate. Then using any iterative technique using a central difference approximation get the temperatures T 1, T 2, T 3, T 4 to a reasonable accuracy. Problem will go like this, how do we solve this problem? I will write the finite difference for T 1 first, T 1 is west east plus west plus north plus south.

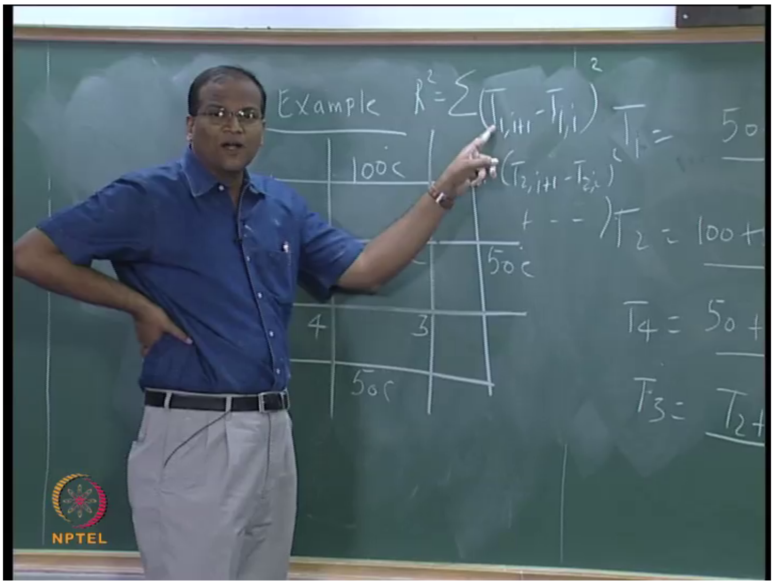
(Refer Slide Time: 43:52)



So, 50 plus 100 plus T 2 plus T 4 by 4, T 2 equal to 100 plus 50 plus T 1 plus T 3 by 4, T 4 equal to 50 plus 50 plus what T 1 plus T 3 by 4, T 3 equal to T 3 equal to T 2 plus T 4 plus 50. How do you start the operations? Let T 1 equal to T 2 equal to T 3 equal to T 4 is equal to 70 degree centigrade, first iteration. Once it is 70 I will come to first T 1 70 plus 70 plus 100 plus 50 plus by 4 I already have T 1.

Now, watch carefully watch carefully. Now, T 2, T 3, T 4 are 70 but, T 1 is calculated so when I go to T 2 I use the latest update of T 1 so that if I do that that is called the Gauss Seidel method. It has to satisfy diagonal dominance and all that fortunately it automatically satisfies you do not have to worry about this here. Then you will get T 1 and T 2 get the latest updates and at the end of the first iteration compare to your original 60 degrees assumption find out how much T 1, T 2, T 3, T 4 have changed.

(Refer Slide Time: 45:44)



So, you can put a criterion as sigma is called a nom, R squared nom, you can put a nom that is T 1 previous iteration minus old iteration square plus T 2 old previous this iteration minus whole square whole square and then this R square must reach a reasonable value, 10 to the power of minus 4, 10 to the power of minus 5.

Very easily we can implement it on the computer, if it is on the computer why use only 4 point I can use 400 point but, whether 400 points are required or not if you are trying to establish that then you are trying to establish what is called grid dependence or grid independence, the solution is independent of the grid pattern, the solution has converged but the solution has may also converge to a wrong answer therefore, apart from grid dependence you have to validate. In this case you know 100 temperature 25 is a validation.

For certain other problems when you are in the wilderness then you have to take recourse to literature, if somebody has already has an analytical solution or somebody has an experimental result then you validate. That cannot be a research problem because already somebody has done. Now, you say that he did only this much. Now, I am considering some more and then you proceed with your work. We can keep on going like this we can write down the finite difference algorithm for the Poisson equation and when you start writing the finite difference for cylindrical and spherical coordinates this will become very very interesting.