Conduction and Radiation

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Lecture No. # 35
Conduction – 1D, heat generation

In the last class we looked at the simple problem of conduction in a one dimensional plane wall without heat generation. But, we saw an interesting boundary conditions, we saw the case of interesting boundary conditions where you have got two fluids at different temperatures T infinity one and T infinity two on the two sides, which give rise to two different convective heat transfer coefficient h one and h two.

And, though the temperature profile was still linear, the algebra was quite involved. It took quite some time for us to figure out the solution. Under the special case of h one and h two turning to infinity, then you get the simple case; where q is equal to k into delta t by l. And, the temperature distribution is simply that linear temperature distribution which T 1 and T 2 specified at the ends.

Towards the end of yesterday's class, we looked at the problem of the same plane wall with a variable thermal conductivity. The variable thermal conductivity can have two variants. The k can be a function of x and the k can be a function of T. When k is a function of x, we figured out a generic formulation to get the solution. And, we wrote it in terms of k of x d x, right, in yesterday's class. So, if I tell you, if somebody tells you what the k of x is, that is, if it is written as a plus p x and a and b are specified to you either from experiments or from theory or from theory, you will be in a position to calculate the temperature distribution and the heat transfer rate for the case of variable thermal conductivity.

Right. So, please remember, we are able to work out analytical solutions because it is simple enough and it is only one dimensional and so on. The moment it becomes two dimensional unsteady or three dimensional unsteady and so on, then you have to solve it numerically. Either you have to write your own code or you can take resource to software.

So, now we look at the case of variable thermal conductivity where k is a function of T. So, temperature is what you are seeking. That is the solution to the problem. That is a variable, dependent variable in the problem is temperature and the independent variable is x. Unfortunately the k which is the property, thermal conductivity itself depends on T.

Many times k is a function of temperature. Therefore, variable thermal conductivity is not just an academic exercise. It is an important practical problem. So, we will look at a… not such a simple, not a very elementary case, but a reasonable linear model for thermal conductivity. So, it goes as k is equal to k naught into one plus alpha T. right. So, if you have got, if you have done experiments, you got various values of k measured for various values of temperature, you can do least square regression and get the values of alpha and k naught if you have a model like this. right.

As you can see, as the temperature increases one plus alpha T increases. Therefore, k is; it is an increasing model for thermal conductivity. Right. As temperature increases, k increases. Right. Now, we will have to get the temperature distribution and the heat transfer rate for a problem in which k exhibits this behavior. We will work out the generic solution and stop in between, we will put we will put some values for all these alpha, k naught, the thickness of the wall and all that. And, get a hang of the whole thing by solving a numerical example, before going to the case of one dimensional plane wall with heat generation. So, I want to take up two things today. One dimensional plane wall variable thermal conductivity; one dimensional plane wall constant heat generation rate, which mimics what happens in a nuclear fuel rod.

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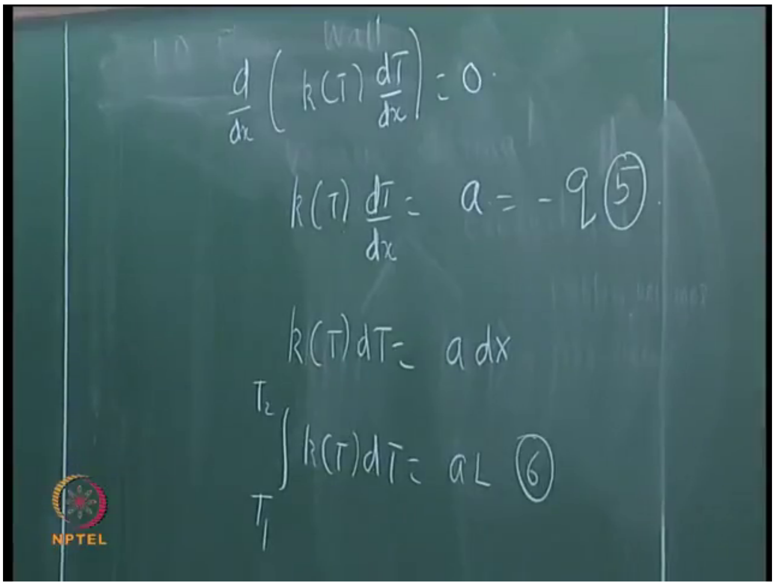

Let us look at this equation. So, we have… sometimes I use q v, sometimes I use q triple time. So, you can be consistent. So, whichever you prefer, you use. Basically, it is watts per meter cube. Now, I told you it is steady. So, this term gets knocked off. I also told you q v equal to zero. So, the two terms gets knocked off. Unfortunately, I cannot take k out of the differential because k is a, k is the function of temperature.

Therefore, B c's boundary condition; we are understanding that T 1 is greater than T 2. Right. It can also be the other way. So, please remember if T 1 equal to T 2, in this case there is no heat transfer. It is not the case with radiation. They will continue to emit radiation because of provost law, but net radiation will be zero. right. So, temperature difference is responsible for heat transfer generally. Why I put generally because it is not applicable for radioactive heat transfer. Ok.

So, now we will have to solve this. I can use the linear model for thermal conductivity k. Before using the model, I can just start off with the generic formulation. So, d by d x. what is k d t by d x? k minus of … correct. I would like to use q itself and q v for volumetric heat generation rate. Suppose you do not like it, put the double prime so that, you are sure that it is watts per meter square.

Let us keep it like this. Is it correct? a is a constant integral d x zero d l, is that okay? People who came late, we are doing this problem. That is, the thermal conductivity is varying with temperature. That is the problem we have solved. I am proceeding from six. So, what can I say about this?

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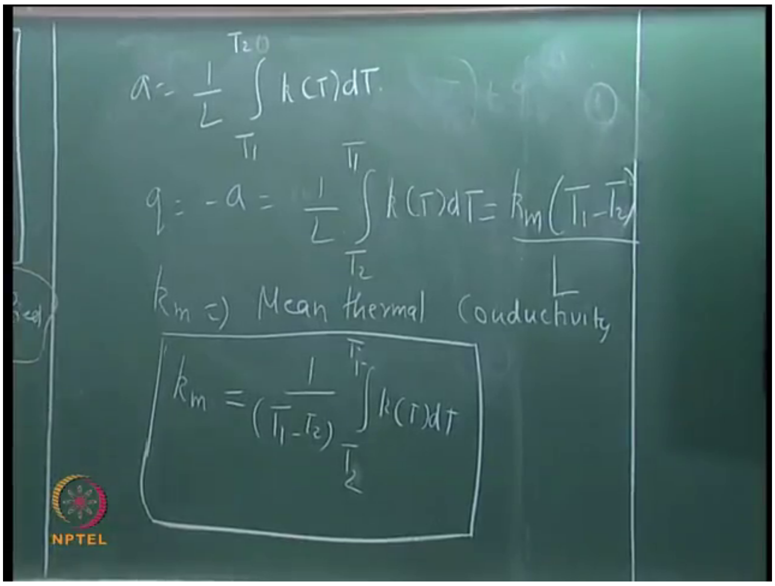

a is equal to… q is equal to minus a. I just changed T 1 to T 2. No, it is straight forward. If I know the functional form of k of T, it is possible to insert it into the integral, integrate this and substitute the limits T 2 and T 1 and we are home. But, that is only one part of the story. If somebody says what is the temperature in between at the middle of the, in the, at the middle of the slab or twenty five percent from the left side, seventy five percent from the right side or left side, we are we are not done yet. But, if somebody wants heat transfer rate, we have a, we have an answer to that.

Now, can we say that, this also equal to some mean thermal conductivity k m multiplied by the temperature difference divided by the width of the slab. I am proposing because it may be very useful from an engineering point of view to define something like a mean thermal conductivity, right. Where… Did you guys do this in the basic heat transfer course? three one seven you have not done. people from outside you have done ok

So, now we get an expression for k m. What is k m? The first step in solving such a problem would be to get the expression for k of T. Put it into the integral, evaluate the mean thermal conductivity. And, mean thermal conductivity into delta T divided by L will directly give the q. First part of the problem is over. The second part involves getting the temperature distribution. It is slightly more involved. Is this clear now? Now, let us assign some value. Otherwise, it is getting very dry.

So, we will assign some values and try to solve this problem. Problem number Forty. New notebook Vikram? Forty two. Problem number forty two. Good. Problem number forty two. Consider the one dimensional; consider the one dimensional slab given in the figure. Consider the one dimensional slab given in the figure.

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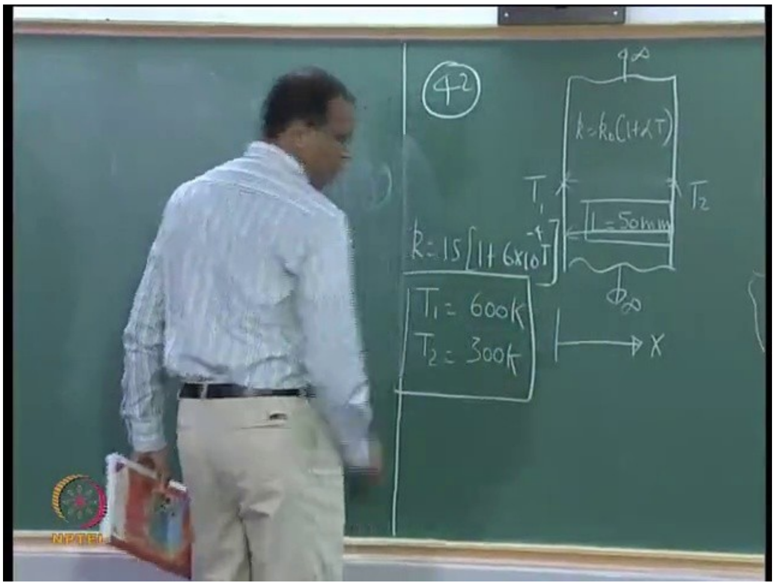

So, k is 15 into 1 plus 6 into ten to the power of minus four T. Consider the one dimensional slab given in the figure. The thermal conductivity of the slab material varies as k equals 15 into 1 plus 6, then the minus four into T. Consider the one dimensional plane wall given in the figure. The thermal conductivity of the slab material varies as k equal to 15 into 1 plus alpha into T, where alpha is 6 into ten to the minus four or 10 to the power of minus four.

The slab thickness L is 50 millimeter, the slab thickness L equal to 50 millimeters. The left side temperature T 1 is 600 kelvin; the right side temperature T 2 is 300 kelvin. So, left side temperature is 600 kelvin; right side temperature is 300 kelvin, q v equal to zero and steady state exists in the slab. q v equal to zero and steady state exists in the slab. q v is equal to zero and steady state exists in the slab. For these conditions for these conditions, determine the heat flux across the wall. For these conditions, determine the heat flux heat flux across the wall and the temperature at the mid plane and determine the heat flux across the wall and the temperature at the mid plane and the temperature at the mid plane ok. We will we will start solving the first part.

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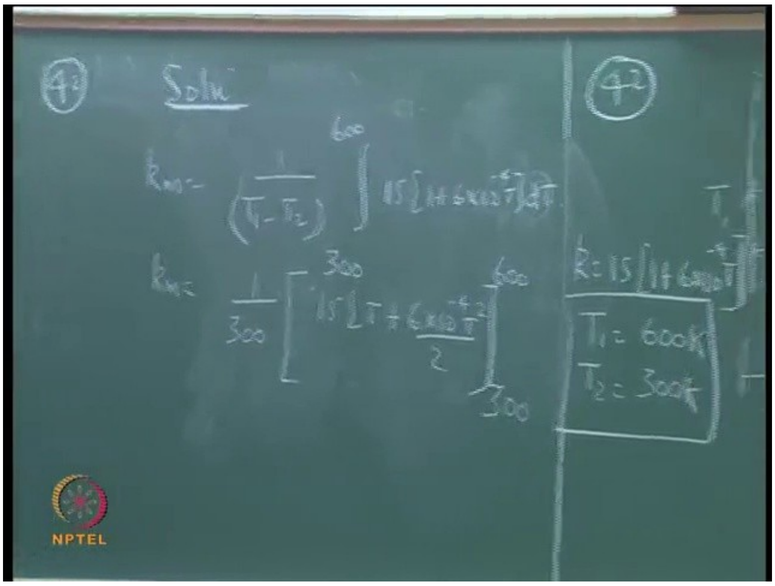

So, first get the mean thermal conductivity. Correct. Yes. Now, tell me the value of k m.

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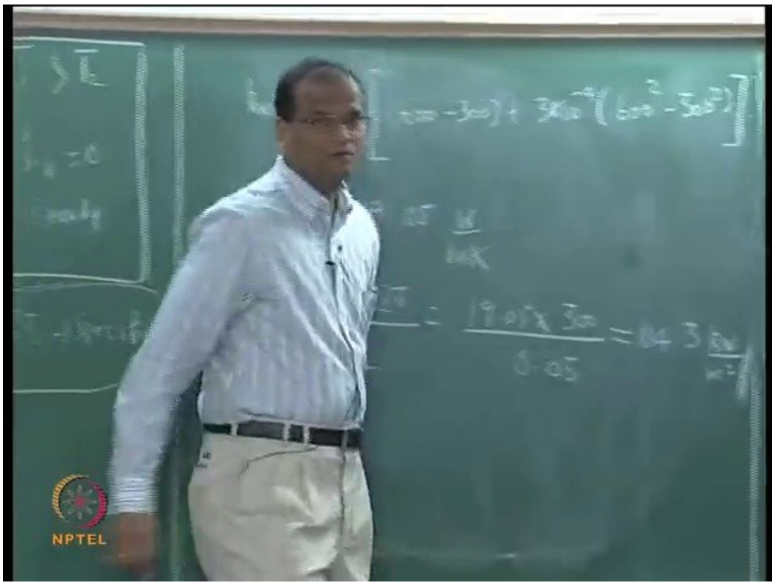

So, k m; what is it Deepak? 19.05. 19.5. 19.5. Yeah. 19.05. 19.05 what? Watt per meters per kelvin. Right. Now, we can substitute in the expression q. The heat flux is a product of mean thermal conductivity into delta T by L. … quite something, some kilowatts, 114.3 kilowatt, kilowatt per meter square. Correct. 114… So, it is a reasonable thermal conductivity corresponding to stainless steel, terrific temperature difference of 300 kelvin. So, we expect a reasonable heat flux of 114 kilo watt per meter square. All right. But, this is not the full story.

The second part is more involved. What is the temperature at the mid plane? That is, at x equal to 2.5 centimeter or x equal to 0.025 meter. So, that is going to be a little more involved. Let us see how it. So, this is okay, up to this. Rohit is it clear? kaustav.

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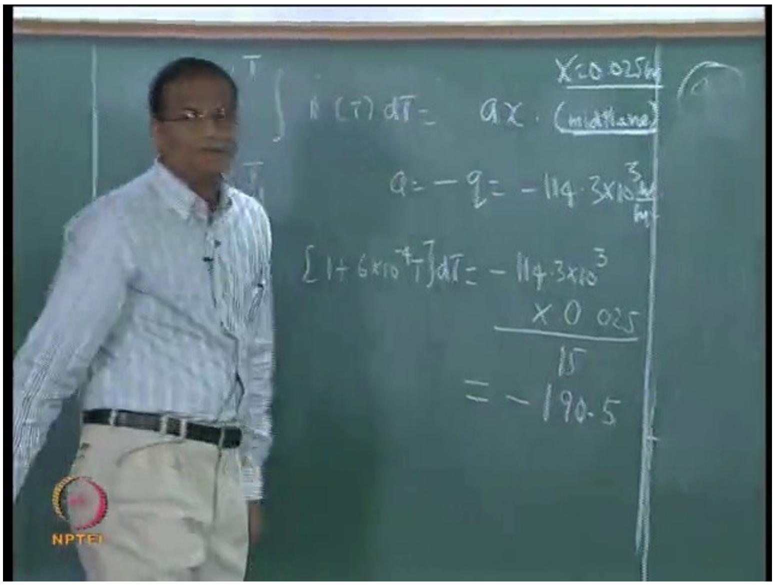

Now, the second part of the story is difficult. … So, we just now evaluated a is a constant, heat flux is a constant; one dimensional plane wall, no heat generation steady state, whatever is coming from the left side has to go on to the right side. Where else can it go?

So, therefore… what is the a x we want to do? x equal to, x equal to 0.025 mid plane. Yeah. Just get me the right side. Shall, we take this fellow also here? What do you say? Yes. Tell me what is this? ketan right side? Watch out kilo watt. That is all, everything taken care of minus 190.5. Very good. If we do not get the final answer, we will catch Vikram. 190.5. Ok.

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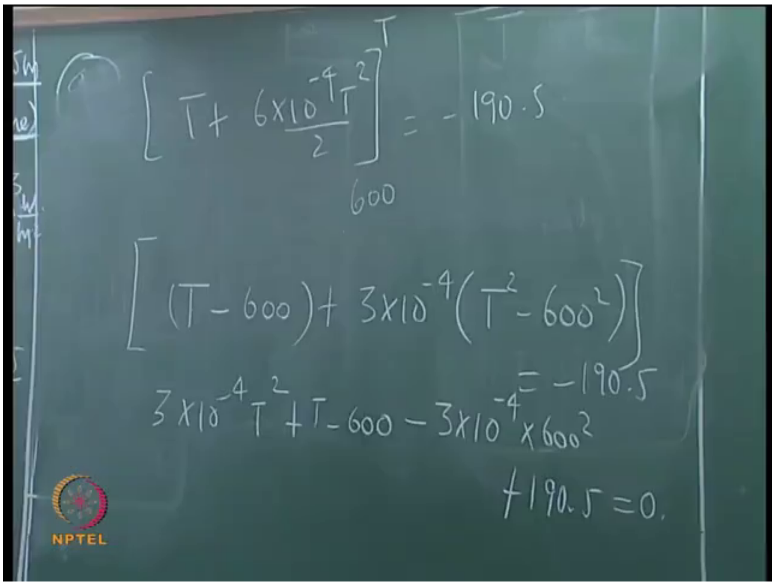

Now, left side. What about the left side? What is T 1? … What is happening now to the temperature distribution? You have to solve a quadratic. See, you have to a very simple linear temperature distribution. Now, linear temperature distribution, constant thermal conductivity; linear thermal conductivity, quadratic temperature; quadratic thermal conductivity, cubic temperature; so, it will keep on increasing, difficulty level will escalate. Now, I hope to get a reasonable temperature. I do not know. So, let us see.

So, this is 3 into ten to the power of minus four T square plus T minus 600 minus 3 into ten to the power of minus four into 600 square plus 190.5 equal to zero. Divide throughout by 3 into ten to the power of minus four and make it into a decent quadratic equation. What do you get? Four… Very good.

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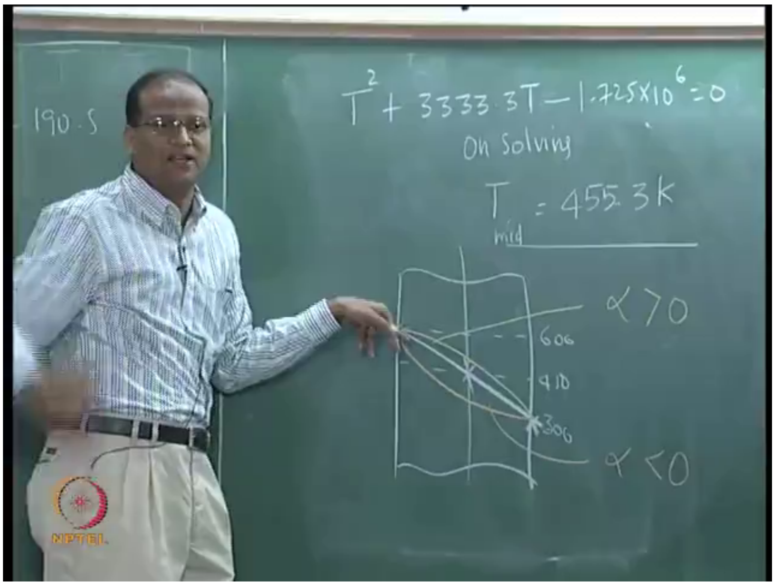

Now, let us write the quadratic. Minus… what is the quadratic? There is a provision in the calculator is it? But, what is it? What is the c term? … On solving, T mid is 455.3 kelvin. What is the big deal man? The lay man who has no knowledge of heat transfer will try to guess that it will be 600 by 300 by 2. But, it is off by 5 kelvin. You may say what is there in 5 kelvin. But, suppose the k and the variation of k is much more severe and the two temperature differences are too much, then the lay man approach will not work. You cannot simply take the arithmetic mean temperature difference and say that because there is a Physics associated with this problem, you have got, you should know the Physics in order to get this. Are you getting the point?

Therefore, this is 600, this is 300. Ok. 450, 600, 300. What did we get at the center now? Slightly more than this. So, a linear model for thermal conductivity gives rise to the yellow line. A linear model can also give rise to the orange line. The yellow line is for alpha greater than zero; this is for alpha less than zero. Thermal conductivity can also decrease the temperature right. Is this fundamental is clear? The beauty is the linear temperature profile is disturbed. Now, we have a quadratic or a parabolic temperature profile. This is consistent with the linear model for of a thermal conductivity. Right. Alpha equal to zero, it gives the linear model. Fine.

We got it numerically; we can also explain it intuitively. Watch here, since the temperature is the difference is the same, since the temperature difference is the same for a given heat flux for the linear temperature profile; there is a temperature gradient here. To transfer the same amount of heat at a higher temperature, if alpha is greater than zero the thermal conductivity is more. So, q is equal to k d t by d x because q is the same and k is more; the d t by d x must be lesser, it must be gentle for the case where alpha is greater than zero. By the same token, for the same q if alpha is negative, then the k corresponding to the constant k model has to be lower. therefore, the d t by d x has to be correspondingly higher to make up for the constant heat .

Suppose I had told you like this, you may or may not believe me. That is why I took a detailed this thing, where we numerically solved and proved that for alpha greater than zero, the mid plane temperature will be always higher than what is predicted by the arithmetic mean temperature difference. right. So, once linear, we can do we can have a quadratic fit for thermal conductivity and all this. So, now you know the way of handling this. Right. And often times, for large variations in temperature this constant thermal conductivity model is no good. The temperature…, if the temperature difference is small it is alright to use this. Ok.

Now, let us take the one dimension plane wall with heat generation. The heat generation can take place because of a nuclear fission or it can take place because of the chemical reaction and so on.

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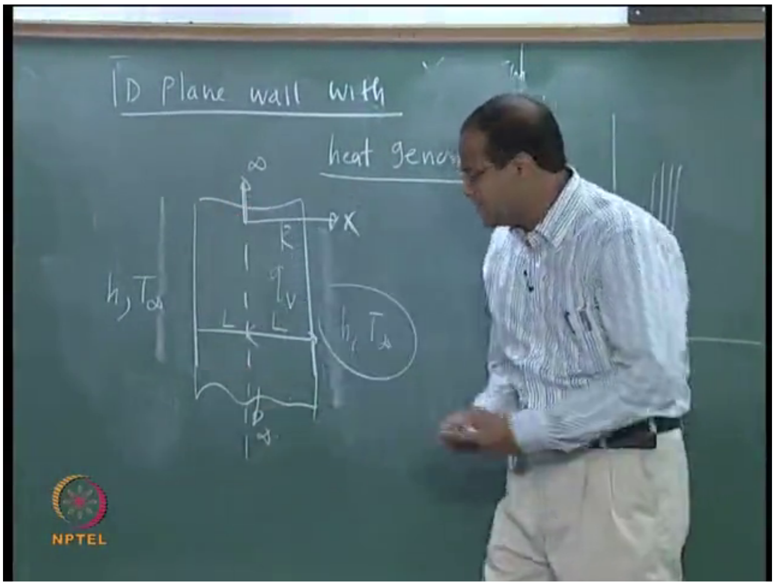

So, consider a plane wall like this. So, it is infinite in extent. Now, I have a material which has a constant thermal conductivity k. Let us not complicate it. But now, I have a fellow who is generating heat at the rate of q v watts per meter cube. But, to make matter simple, I am taking it infinitely deep in the direction perpendicular to the plane of the board. Volume will be L into 1. Ok.

Now, for convenience, I am starting x from here. Please look at the board. I am starting, origin is here. So, this is L, this is also L, the slab is of thickness 2 L. If I leave him like this what will happen? Without anything on the boundary? Both sides are insulated what will happen? It is continuously generating heat, temperature increase; no steady state. Therefore, I am cooling it on both sides. I am bathing it with a fluid, the cold fluid which has a temperature of T infinity and which gives a heat transfer coefficient of h.

Suppose, I have a fan or a blower or a pump which will pump the liquid, it has to be continuously removed. Otherwise, I am in trouble. So, I have h, not h one, h, T infinity on both sides. So, is the situation clear? Now, I want to find out what is the total heat which is transferred in the situation; number one. Number two, what will be the temperature at the surface at the interface between the solid between the solid and the liquid. And most importantly, what will be the maximum temperature anywhere inside the solid and whether the maximum temperature anywhere in the solid respects or obeys the design limits. ok.

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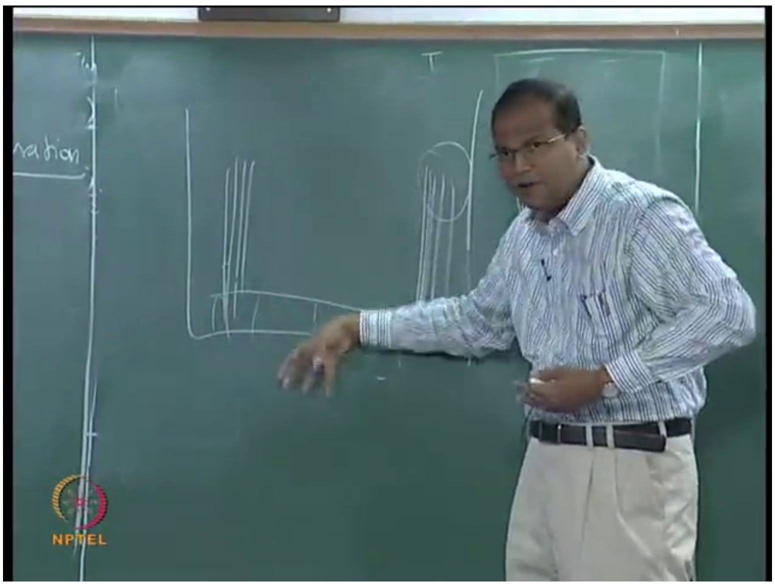

So, the Sodium can… In a nuclear reactor, you have lot of fuel rod like this. Right. If we look at the top view, one of this top view, there will be… what is called hexagonal fuel subassembly? It is called haf… each of this hexagonal fuel assembly; there will be lot of circular tubes. There will be sheet. Like that, you will have several hexagonal assemblies. Ok

There will be small gaps in between these tubes and there will be gaps between these two hexagonal subassemblies. So, that is called the inter wrapper distance. Then, there will be… because packing density cannot be hundred percent, there will be some gap in between. So, the Sodium will flow from bottom to top. I am talking about Sodium because Indian reactor is Sodium. So, bottom to top the Sodium will flow, it will pick up the heat. Then, it will go to a heat exchanger and then the secondary heat exchanger where another, secondary Sodium will pick up the heat from the primary Sodium. It will get heated. Then at the third stage, this Sodium will transfer the heat to the water. Water will become steam and then it becomes regular cycle. Ok.

Now, this pumping of Sodium, this is the primary place where the fission heat is taken away by the Sodium. So, this heat transfer process is accomplished by the Sodium pump. In the event of a Tsunami or an earthquake or in the event of a station blackout, this pumping system will fail. When this pumping system fails, the high values of h which we got with force convection will suddenly go down.

Then what happens is, this core, this temperature will dramatically increase. All right. If immediately after there is a station blackout, there will be operators who will ensure that control rods are control rods are lifted and so that the reactor goes through sub critical conditions. But, it is not like this. If you switch off this light, it will go. But, nuclear reaction is not like that. There is a half-life. It will continue to generate heat for the next twenty four to forty eight hours. It may not be at critical conditions. Therefore, it at a reduced power, new fission heat will continue to be generated.

Now in the absence of a pump, can natural convection sustain? Temperature will no doubt increase. But it will not, it should not increase to a level at which it leads to what is called the core meltdown temperature. When the core meltdown takes place, here they have all this … this they will have all this, what is called the core catch up plate made of stainless steel all. Once it is so hot it will just penetrate, just make a hole and go deep and then it will it will invade the soil and get in. Once it goes in, then it will horizontally spread. It will enter all your aquifers, this thing; it will enter the water, this thing, then with lethal radiation doses everywhere.

So, this core meltdown is what everybody is afraid of. So, ultimately it is a… the nuclear react… whatever happened in Japan, it is a heat transfer problem. It is a heat transfer. Your inability to transfer heat leads to all these. So, so heat transfer can be very critical. Now, let us do this.

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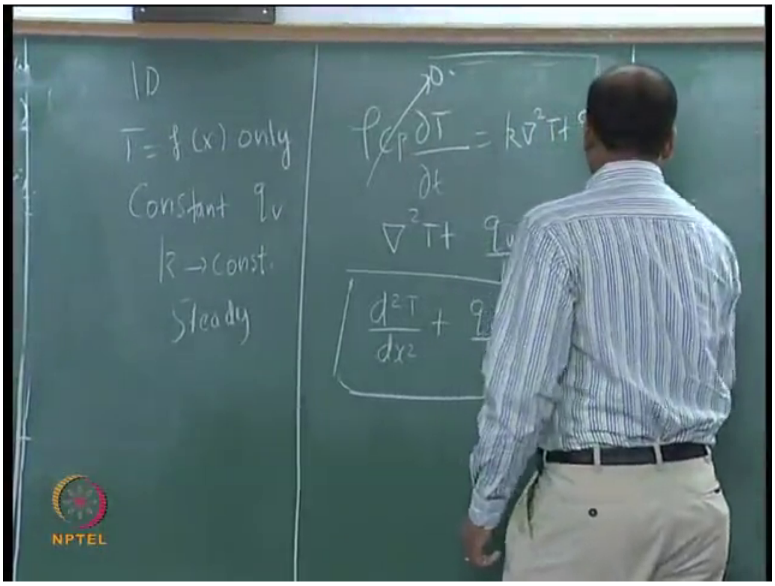

So, this is one dimension, T is a function of constant q v, then k is constant. Ok. Now… Ok. So, rho C p steady state. Steady state, which term can be knocked off? Left side. So, we have got… So, this will be the equation if the fuel rod is basically, infinitely deep in the other direction. For one fuel rod, what is the temperature distribution inside the rod subject to the convective boundary conditions outside?

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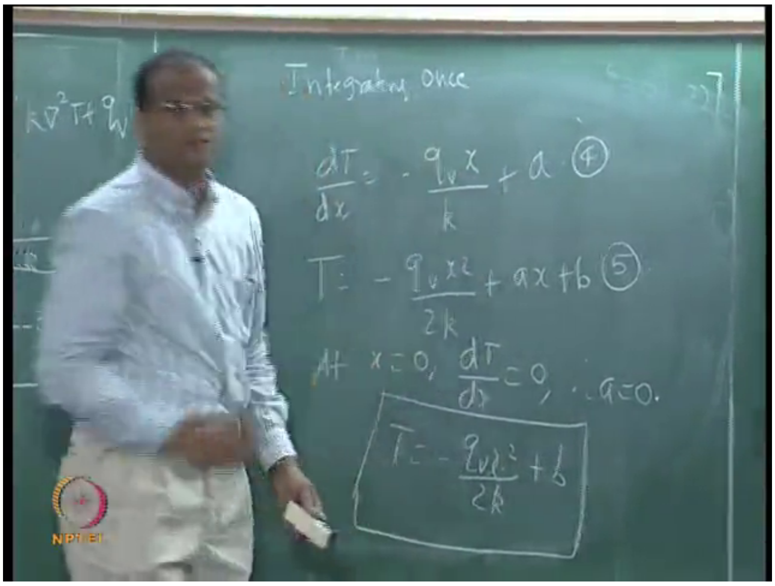

So, we can integrate it twice. Integrating once… Correct. Is it correct? Now, you will be able, you will be easily able to see why I took the x equal to zero at the center. At this, do you expect a symmetric temperature distribution about the center? If you have a symmetric temperature around the center, d t by d x at x equal to zero has to be zero. Therefore, which term will get knocked off here? a is zero. Ok. At x equal to zero… So, T… q v x squared. How will I get the b? How do I get the b? What boundary conditions? Convective boundary condition. You can apply minus k d t by d x at x equal to zero, x equal to minus l or x is equal to plus l will be equal to… ok.

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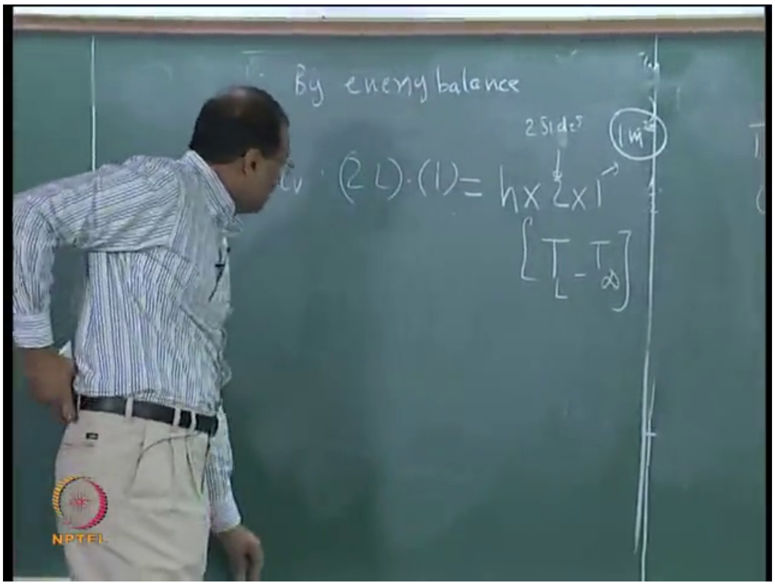

But, I have a simpler way of doing it. Whatever heat is generated by the fuel rod must be taken away by the fluid. By energy balance, what is the heat which is generated by the fuel rod? q v into one; 2 L is the thickness, one is this; watts per meter cube, meter meter square watts. This will be equal to h into… h into 2 into 1 into… What is 2 into 1? One is a one meter square; 2 is left side, right side.

So,. So, can I get the… h is known, q v is known, can I get T l straight away? Yes. But fine. But instead of doing mathematically, I am doing physically. If you apply the boundary condition also you will get the same thing. That boundary condition is energy balance. Is not it? right.

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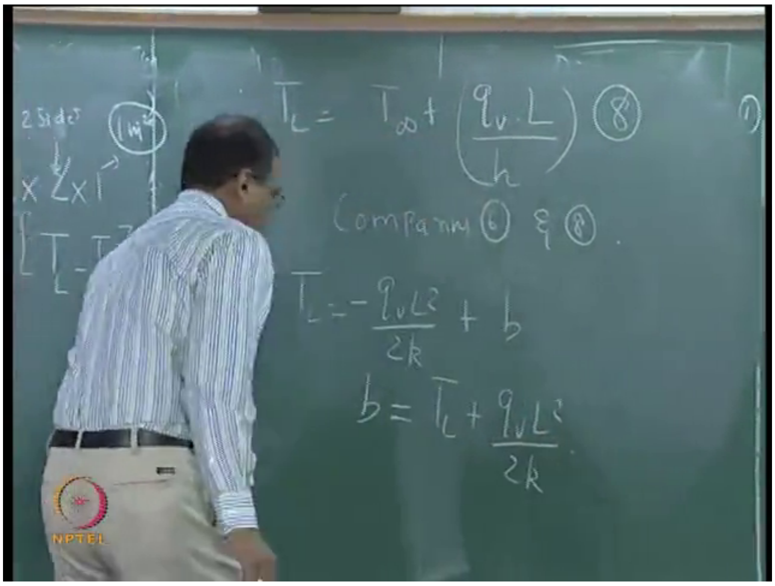

So, so what is this? T L will be equal to T infinity plus q v. Comparing six and eight… this is fine. right. So, this is known to us. Plus b, right. Is it correct or I am making a mistake? Is it okay?

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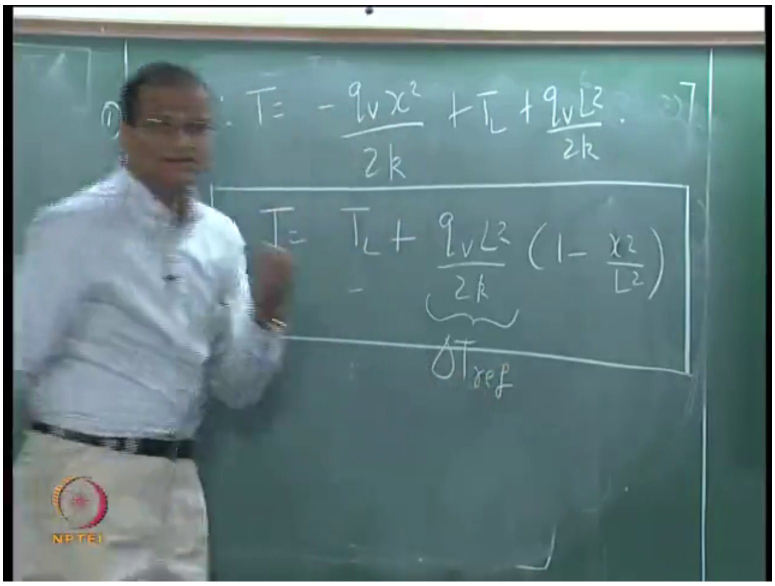

Therefore, T is equal to minus q v x square by 2 k plus T L plus q v L square by 2 k. What is the unit of this? Watts per meter cube meter square. So, numerator is watts per meter, this is watts per meter per kelvin. Therefore, the unit is Kelvin. So, this is something called the reference temperature excess. the reference temperature difference for the problem.

This gives you the power of the heat generation to increase the temperature inside the wall. Correct. This is called the temperature excess parameter. So, easily it can be seen that at x equal to zero… at x equal to zero the temperature is maximum; at x equal to l this term becomes zero. T is equal to T L. that already you know. What is T L? T L comes from the energy balance. Let us solve an example.

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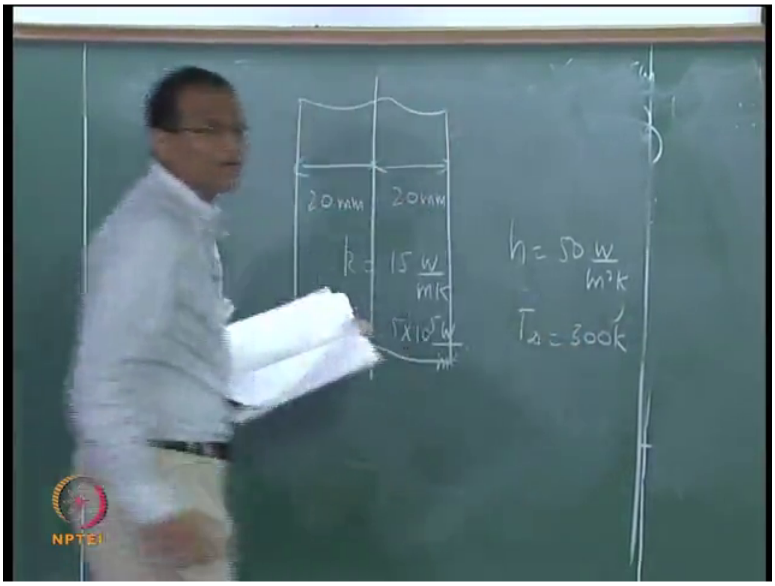

Problem number forty three: consider a one dimensional plane wall with constant q v. consider a one dimensional plane wall with constant q v. Consider a plane wall consider a plane wall with a constant q v. All the pertinent details are shown on the figure. All the pertinent details are shown on the figure. All the pertinent details are shown on the figure. Determine a. All details are shown on figure. Determine a) surface temperature and b) center temperature.

This is the representative. This is not, these are not the actual values encountered in a nuclear reactor. I did not choose to give you an example, but this will give you an idea.

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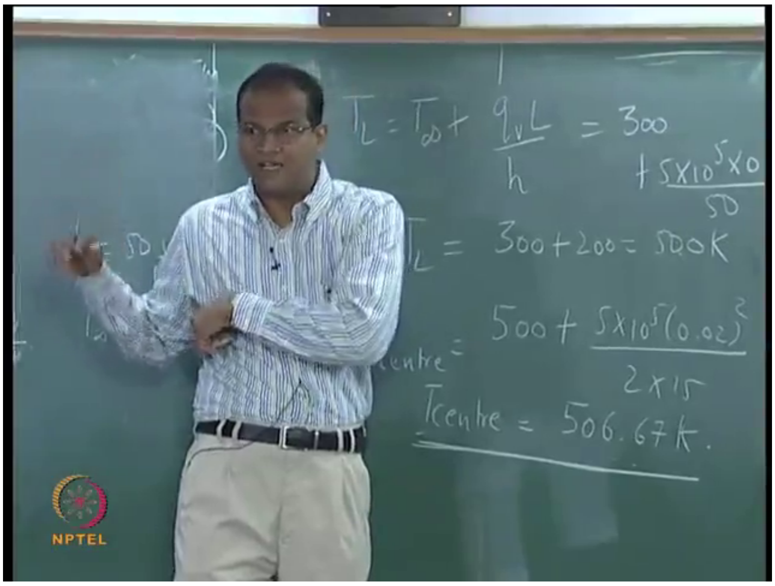

Now, you can get T L equal to T infinity plus q v L by h. right. That is what we got. How much is this? 500, 500 kelvin. 300 plus 200? Deepak you have used 0.04. That is not correct. 0.04 is 2 L; L is 0.02 right. Now, T center equal to 500 plus… What is T center? 506. 67. So, what does it what does it show? This…, this fuel rod or whatever is generating heat; you are having a cooling medium which affords a heat transfer coefficient of fifty which is available at twenty seven degree centigrade at three hundred kelvin to do the cooling. So, the conduction is better.

That is why from this surface to the center, the difference is only 6.67 kelvin. But, there is a mismatch between the heat generation and the capacity to absorb the heat by convection. That is why from three hundred, the temperature rises to five hundred. So, a temperature difference of two hundred degree centigrade is required at the surface in order to accomplish the heat transfer.

But, now, let us take a situation. Now, let us say this is force convection. The Fukushima reactor or whatever; this all… Let us say from fifty because the natural convection suddenly drops to five, what happens?

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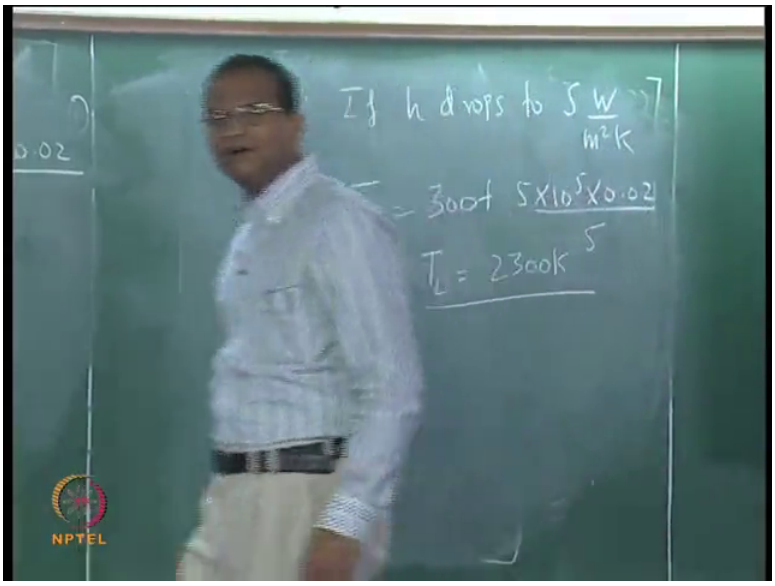

If h drops to five, how much is T L? 2300 kelvin. What will happen to the fuel rod? Do not worry about one dimensional governing equation inside the rod. At the surface itself... This is what happens, where if the pump fails. Of course, we assume lot of things, steady state and all that. But this will be the starting story. Immediately, some operator will put control rod, he will put boron and something, they will do all that. But, immediately the first thing is there will be a thermal shock. Right. The h drops, and suddenly the fuel rod temperature they will start rising like mad. ok.

Fine. We will stop here. In tomorrow's class, we will look at extended surface heat transfer. That is fine.