Conduction and radiation

Prof. C. Balaji
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Module No. #01

Lecture No. #03
Solid angle, spectral radiation intensity

In yesterday's class we defined a blackbody and then we proposal arguments to look at the attributes of a blackbody namely, it is a perfect emitter, there is a radiation isotropy of radiation coming out of blackbody, that is the intensity same in all the directions. So, it is a perfect emitter in every direction. It is a perfect emitter in all wave lengths and the enclosure characteristics do not affect the radiation coming out of a blackbody, it is only the enclosure of temperature of the blackbody, which holds the key to determine the intensity of radiation which is coming out of a blackbody.

And finally, we figured out that the radiation strength, from a blackbody is not only function of temperature, it monotonically increases the temperature; it cannot be an inverse function, it cannot be inversely proportional to temperature. Then, we started introducing solid geometry because, radiation is going in all the directions and therefore, it is an advantage for us, to use its spherical coordinate system. So, from xyz coordinates you can go to r theta phi coordinates, where r is the radius given by root of x 1 square plus y 1 square plus z 1 square with respect to 0 0 0, with respect to the origin, they cannot theta and phi there are two angles, which are entering the problem one is the zenith angle, how it is displaced from the vertical?

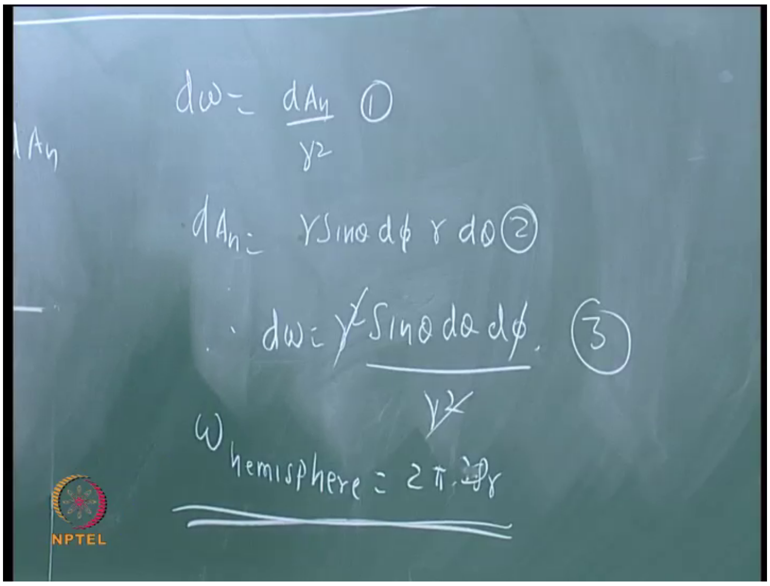
That is a zenith angle theta, it is also frequently used in solar energy, people who are working in solar energy, will know that zenith angle is very much useful there. And there is also the azimuthally angle, where you are looking at this angle. So, zenith angle is basic with respect to the vertical, the azimuthally angle is with respect to the horizontal. So, xyz can be transformed to r theta phi, then we saw the difference between plain angle and solid angle.

The solid angle is basically a important concept to study radiative heat transfer because they are always interested in radiation heat transfer between surfaces. Whether it is a combustion chamber of an IC engine or whether it is a radiant super heater of a boiler basically there are various surfaces, one surface is hot, the other surface is cold. For example, the hot surface is where the radiation is coming out and the cold surface is the surface on which water tubes are there, hot water or steam is flowing through those tubes, then the job of the radiant super heater is basically the radiation from one side, will heat up this steam of the water, which is on the other side.

So, in all these cases we are looking at radiation heat transfer between finite surfaces, we are not always interested in the small body, which is completely intercepted by a hemispherical bucket or hemispherical basket or whatever. Therefore, if you're interested in radiative heat transfer between surfaces and these surfaces are the finite area because, radiation has a tendency to spread in all the directions, it is important for us to know the directional orientation of one surface with respect to the other surface. How is the receiving surface oriented directionally with regard to the emitting surface?

In order to do this, not only the spherical coordinate system useful. The definition of solid angle is also become imperative or it becomes essential just like plain angle is tn by r the solid angle is dAn by r square under the spherical coordinate system.

(Refer Slide Time: 03:34)



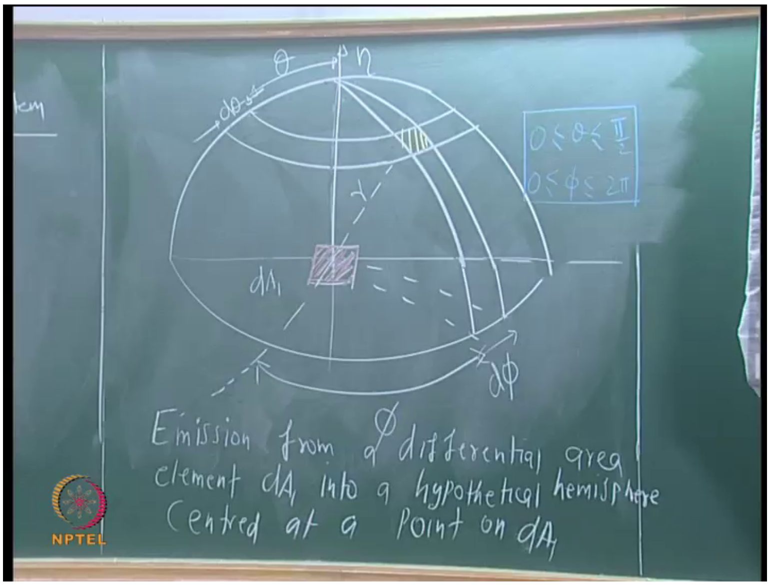
We just broke down dAn into r sin theta d phi into r d theta, then the d omega became sin theta and d theta d phi, r square and r square gets cancelled and the omega for hemisphere is two pi steradian. Why are we not talking with sphere we always talking with hemisphere.

Because, we are always instead in the surface so, radiation from a surface to, but this is not the be-all and the end-all of everything. For example, the radiation from the atmosphere, if you consider a volume, then radiation will travelling in the upper hemisphere and lower hemisphere also. So, when we come to radiation participating media, particularly with radiation in the atmosphere and so on. It need not be radiation of the atmosphere; it can be radiation from the combustion gasses in a combustion chamber. Once, you have water vapor and carbon dioxide, they cannot keep quit. Carbon dioxide is a culprit for greenhouse gases, is a culprit for global climate change and all that. So, if you have carbon dioxide and water vapor, then the arguments will slightly change instead of talking about hemisphere we will talk about full sphere.

But the, first part of the story is, we wants to calculate radiation heat transfers between surfaces. So, the surfaces are at different temperatures, their characterize with different plains whatever, they are oriented in different directions, there can be several surfaces, what is a net radiation heat transfer between any two surfaces? This is a fundamental engineering question, which we want to answer. Slightly, more complicated version of this is apart from radiation, there is also convection and conduction which are taking place. So, it becomes truly multimode heat transfer problem, under these conditions how we handle radiation, either when it is alone or when it is in conjunction other modes of heat transfers.

We already saw that, even at low temperatures, radiation can be significant if natural convection is present. Therefore; we focus on radiation between surfaces.

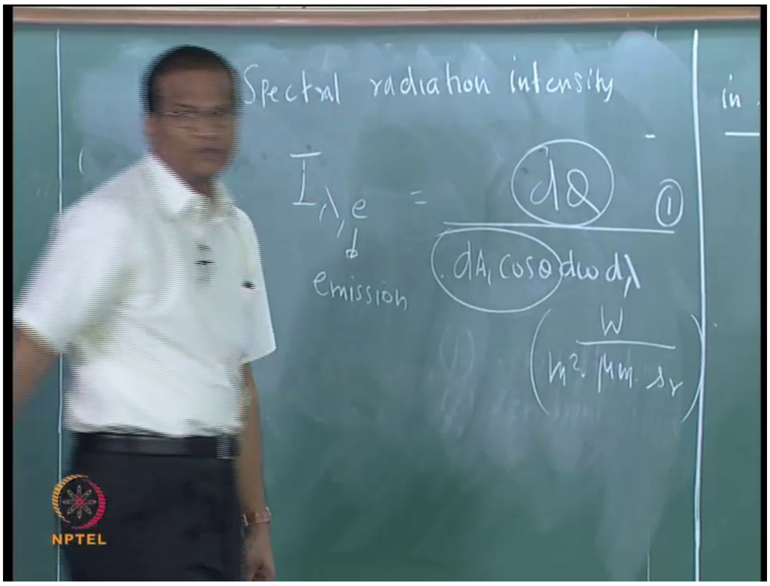
(Refer Slide Time: 05:37)



Now, basically the dA 1 is a surface which is emitting the radiation, it is elemental. There is another elemental surface, this is given in red color, this is given in yellow color. So, that is dAn. dAn is elemental surface, which is intercepting the radiation from dA 1. So, the radius, if you join the central of the dAn and the center of dA 1 it is r impensibly, if you join any point on dAn to any point on dA 1 it is also be very close to r because dA 1 and dAn are after all infinitively small surfaces. Otherwise analysis will cut, it is r then the other two angles are basically the zenith angle is coming. So, this angle is theta and this angle is d theta and then if you projected on to the horizontal plane. So, this is d phi and d phi and this is phi.

Now, theta can varies from 0 to pi by 2 for the hemisphere, theta can varies from 0 to pi for the full sphere, we're looking at the hemisphere, so it is up to pi by 2 and the azimuthally angle is 2 pi 360 degrees, it can be 2 pi. So, the caption for this could be emission from differential area dA 1 into a hypothetical hemisphere center at a point on dA 1, which is at a radius on which is r that is a figure caption.

(Refer Slide Time: 07:40)



Now, with this on background, we introduce a quantity called radiation intensity. Radiation intensity is given by I, the subscript lambda denotes that it is a spectral quantity, there it is concerning the radiation intensity in a small wave length interval d lambda about lambda. That is why it is lambda, e denotes the emission. It is applicable for reflection other things also, but we have to start somewhere. So, I will consider emission. So, this is called as spectral radiation intensity.

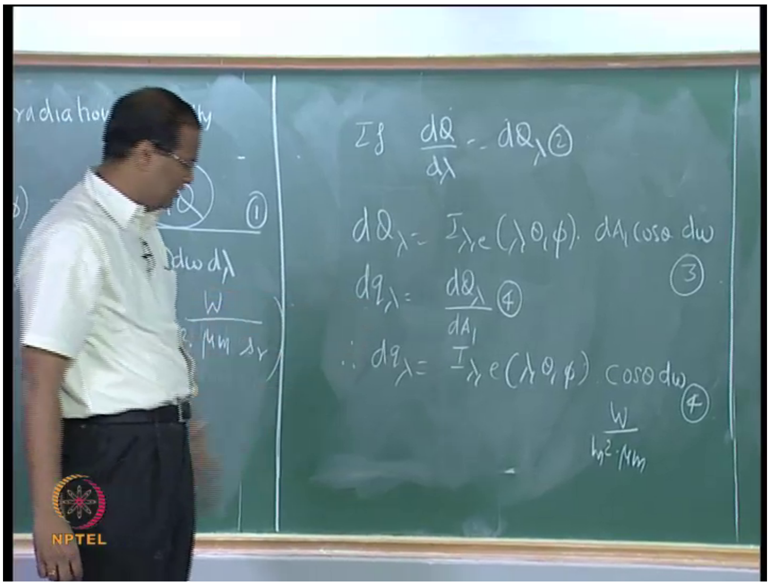
In conduction and convection heat transfer, we have q flux, flux is watts per meter square. Already we have watts per meter square, where is a need to introduce another intensity, why do you think radiation people have introduced this I, already there are so many things to bother us, why cannot we just work with flux, why did people deem it fit or necessary to introduce a new quantity called lambda that is called I.

You want to have quantity which can vary with wave length. So, that is spectral, but why intensity is spectral, basically there is also direction involved. The answer to this question is, it becomes necessary for us to introduce a spectral radiation intensity because, radiation falling on a surface, can come from all possible directions, the radiation emitted from a surface can also be in all possible directions. Therefore, it is important that the directional nature of the intensity of the radiation is taken into count the flux is very difficult, if you work with watts per meter square

Therefore you define a radiation intensity which takes cares of these directional effects and in conjunction with your definition of solid angle spherical coordinate system; it becomes much easier and convenient for you to handle radiation heat transfer problems. Therefore, the first step in technology is to define the radiation intensity. So, the radiation intensity can be defined as this and find out what the units of radiation intensity are I will define it formally and mathematically.

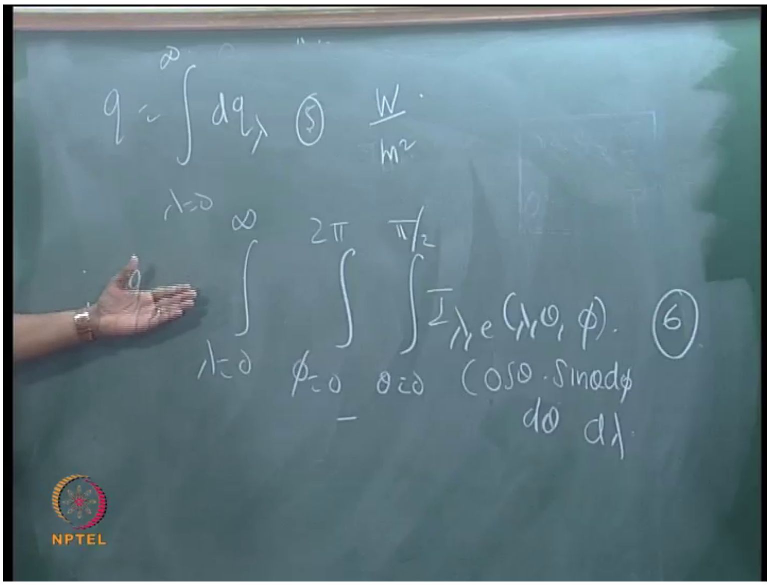
So, the spectral radiant intensity the spectral radiant intensity or emission given by I lambda e is a rate at which radiant energy is emitted per unit area normal to the surface in the direction theta pi, per unit solid angle d omega around theta phi in a unit wave length interval d lambda about lambda. So, the spectral intensity of an emission is the rate at which a particular area is emitting radiation that can be dA 1. So, it is a flux quantity per unit area normal to this in the theta phi direction the theta phi direction is given by this. So, that is why I have dA 1 cos theta. So, if this is the area I take care of this because this surface will not know dA 1 it will see only the projected area, this is the receiving surface it does not care about dA 1 it is always cares about dA cos theta, that is why solar collector also rays depending on the latitude and all these, if you want to track the sun, you want to be normal. So, that you capture the maximum rays. So, it is the amount of radiation which is leaving this, the amount of the rate at which radiant energy is emitted by this area per unit area normal to this in the theta phi direction about an elemental solid angle in the theta phi, direction per unit wave length interval d lambda about lambda

(Refer Slide Time: 15:12)



So, the units will be watts per meter square per micrometer per steradian. So, I lambda e must now be a function of lambda. What is d omega, sine theta, d theta, d phi, solid angle will depend on phi. So, now, what are the units of d q lambda watts per meter square per micrometer.

(Refer Slide Time: 17:58)

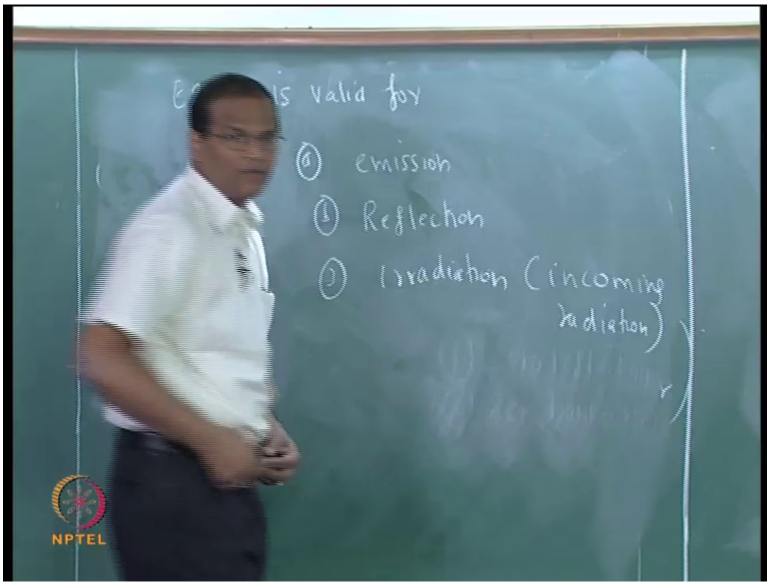


So, what are the units, it is d q lambda into d lambda is it or the variable is d lambda now the equation number 6 is very important. So, we will encounter double triple integrals in this course. So, the importance of equation 6 is as follows, if somebody gives me a distribution for I lambda e it is possible for me to integrate and find out what is a value of q. So, it lets me calculate the q between surfaces, provided I lambda e is known, but who will give me I lambda e that is a separate story.

Somehow, if somebody gives me I as a function of lambda theta and phi, it could also be some of the variable temperature. So, we have a mathematical framework for converting I to q, the q is based on the actual area because, q has got the units of watts per meter square, but I is based on the projected area because it is dA 1 you have to get dA 1 cos theta because, I lambda takes care of the directional effects.

So, while I is based on projected area, q is basically based on the actual area. Therefore, this is a fundamental expression. So, this is valid. So, this is valid regardless of whether you have emission reflection or incoming radiation. The incoming radiation is called irradiation.

(Refer Slide Time: 21:36)



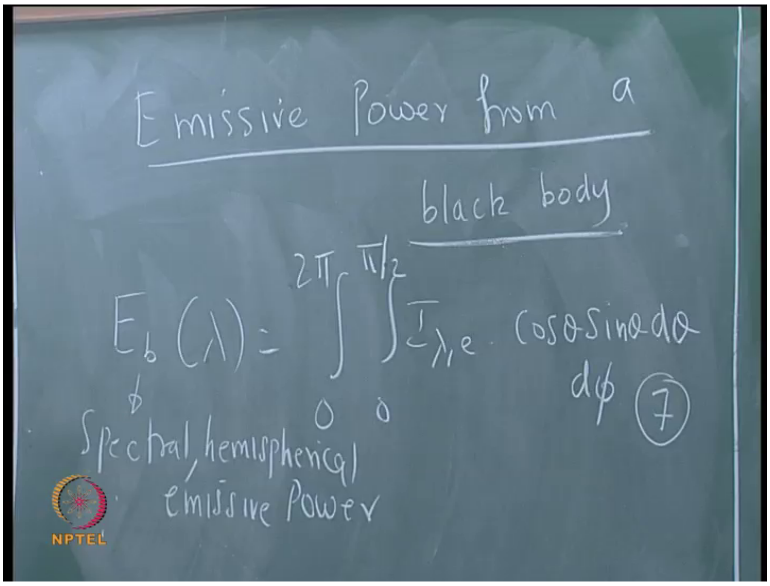
Therefore, I can say that equation 6 is a generic expression to convert I to q, it is applicable for radiation which is emitted from a body or radiation is incident on from a body or radiation which is reflected from a body. Now, we will get back to our blackbody and see how we can define the emissive power of a blackbody, based on the mathematical framework, we have proposed here and then we then we will come to a stage, where if you know what I lambda e is for a blackbody it is possible for you to calculate spectral flux, directional flux, total flux and all these things.

That is the quest for finding I lambda e. So, many scientists are found, first they figured out the incorrect distribution Rayleigh and jeans. And, finally Planck figured out the correct I lambda e, if the correct I lambda e is put into this framework, then automatically the Stefan Boltzmann's law will emerge and if I lambda e is differentiated you get the wine's displacement law, all the fundamental laws will comes from one fundamental expression for I lambda e.

So, the search, I lambda e produced so many Nobel. So, many physicists miserably failed celebrated physicists Rayleigh jeans. So, many British physicists miserably failed and they tried to hold on because, they tried to explain I lambda e from classical physics, the quantum jump came when, Planck use a quantum hypothesis to derive the correct hypothesize or propose the correct distribution for I lambda e even today we cannot say that this is only distribution which is correct, it may be disproved later on. But, the arguments seem to be sound and it is only distribution which agrees with experiments.

Therefore, the Planck's distribution must be correct, till it is found incorrect, boundary layer theory is correct it has a shell 19 naught 5 hundred and phi years it has been useful, but it does not mean that, the life ends with boundary layer, there may be somebody who is much smarter than Planck who will propose.

(Refer Slide Time: 25:21)



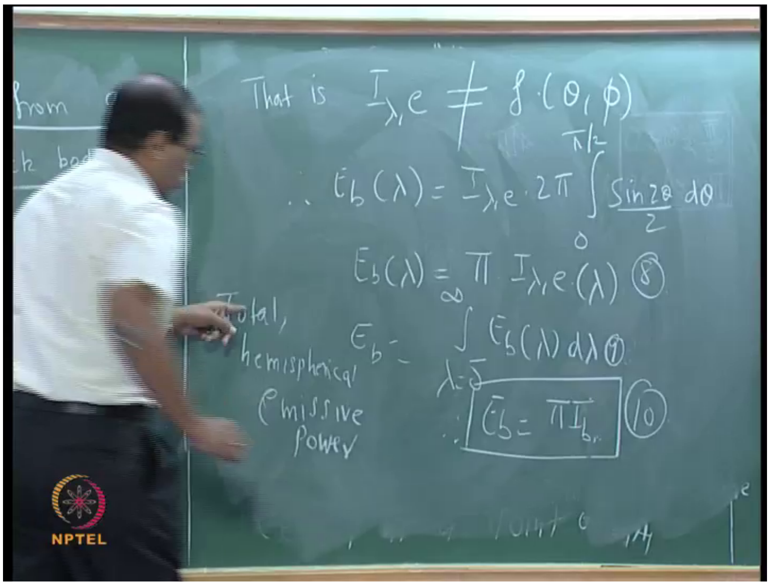
Now, we will look at the emissive power from a blackbody. So, Eb lambda I use expression e for the emissive power, Eb lambda is basically given by 0 to pi by 2, I keep using this throughout this course, this refers to the integration with respect to theta, this refers to integration with respect to phi.

So, I lambda e, what is equation number 7 it is called as the spectral hemispherical emissive power. Why it is spectral? Because, it has still the function of lambda I am not integrate with respect to lambda. Why it is called hemispherical? I integrated with respect to hypothetical hemisphere by doing two integrations, integration with respect to theta, and integration with respect to phi.

What are the units of the Eb lambda watts per meter square per micrometer, meter is unit for a length, and micrometer is also unit for length. So, we may incline to put, watts per meter cube, it will give some silly numbers. So, we want to put watts per meter square per micrometer because, meter is too big for us.

Blackbody is a perfect emitter, it emits same in all the directions and all that, why should the Ib inside the integral, pull it out of the integral and simplify the expression. A blackbody is a diffuse emitter; that means, the I lambda e is not a function of theta and phi all the directions, it is the same, why should that stupid I lambda e b inside the integral sign, pull it out, just do the integration with respect to the two angles.

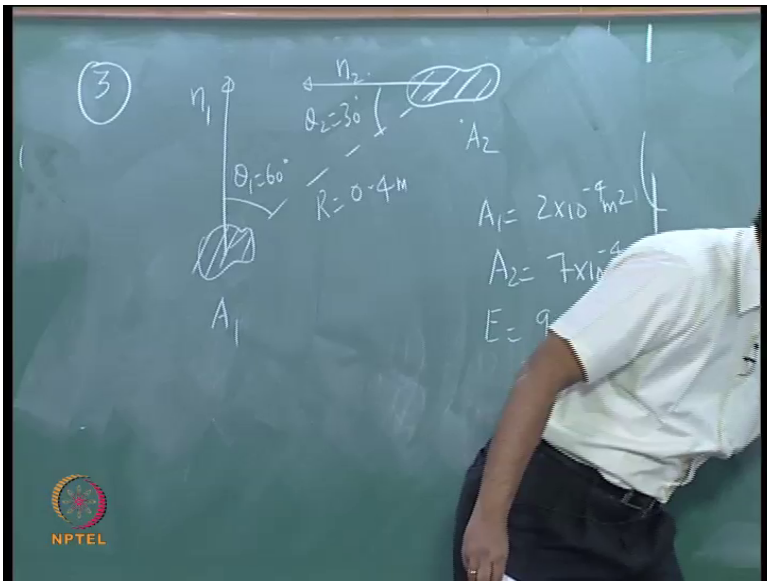
(Refer Slide Time: 28:42)



A blackbody is…now, the beauty is after having define the solid angle, after having introduce spherical coordinate system and after having introduced I lambda e, we have an excellent framework with which we can calculate the flux, the quantity of engineering interest is on the left hand side of the equation, whether it is Eb lambda or Eb, right side is I right side is physics, left side is engineering. On the right side, whatever they figured out, we take that I and do all these integrations and get the spectral quantities or the total quantities. So, this is basically total hemispherical emissive power, total means the integration is with respect to the wave length, hemispherical means the integration is with respect to the angle.

So, the total hemispherical emissive power or simply the total emissive power is 3 integrations perform. You must remember, that in Stefan Boltzmann's law 3 integrations have already been done, 3 integrations are incorporated theta, phi and lambda. The fundamental, the primordial relation with Stefan Boltzmann's law is the fundamental I lambda, which a blackbody supposed to emit, which can be verified by experiments and which has been proposed by theory, that we are going to see in the ensuing classes. Now, let us solve some two problems. So, two important points you have to note, first, if you look at radiation heat transfer between surfaces, we are talking about hemisphere and not about sphere. Second, surprisingly the result has pi and not 2 pi.

(Refer Slide Time: 34:13)

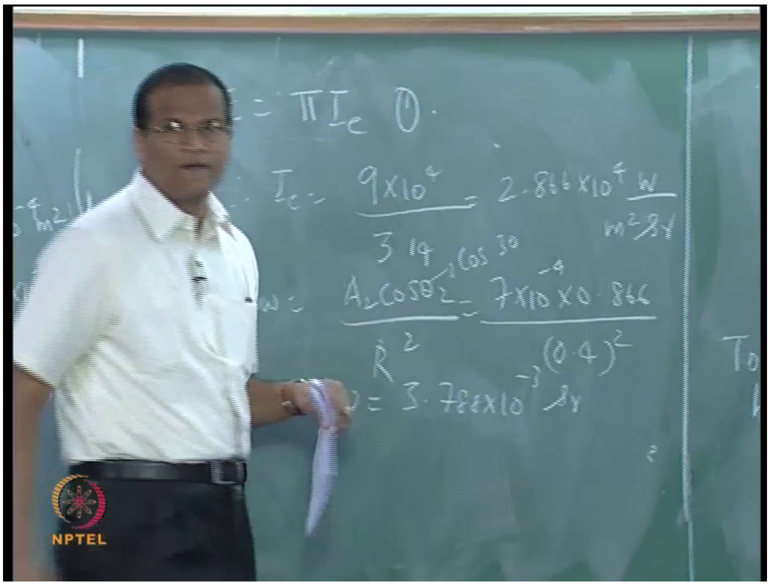


So, we are looking at a simple problem, whose solution will re-enforce the basic concepts, which we have studied in the last two classes. So, there are two elemental surfaces A 1 and A 2, the areas of A 1 and A 2 are given on the blackboard, A 1 is 2 into 10 the minus 4 meter square and A 2 is 7 into 10 the minus 4 meter square, this is a fellow who is emitting and this is the fellow who is receiving. So, the power of emission from this surface is given in terms of E is 9 into 10 to the power of 4 watts per meter square, that micrometer business is taken care because one integration with respect to lambda’s already been done.

So, what is the radiation or the heat flux due to radiation, which is falling on or which is received by A 2 that is simple So, it is basically a geometric problem, though it is a thermal problem, we have broken down the thermal path, by already declaring 9 into 10 to the power of 4, what should be the temperature and what should be the characteristics of A 1 to ensure that 9 into 10 to the power of 4. We are discussing only the geometric aspects so, that our concepts are clear.

So the problem statement is consider a small surface of area A 1, which emits diffusely, diffusely means, it is same in all the directions with a total hemispherical emissive power or you can say with an e of 9 into 10 to the power of 4 watts per meter square. Remember, e is total hemispherical emissive power. The question is what is the rate at which a small surface A 2 oriented as shown in the figure? Intercepts, that is receives this radiation. So, you have to look at the quantities relating I and q, I will tell you briefly with steps involved, you have to look at the relationship between I and q, you must invoke the definition of solid angle.

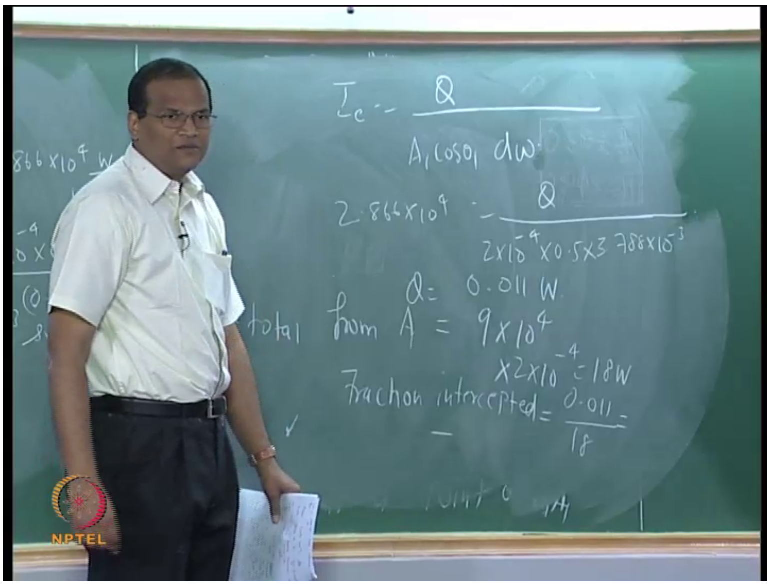
(Refer Slide Time: 38:50)



If it is diffused, I know that the relationship between e and I e is equal to 5 times I, I am using the terminology I e, because to denote emission. So, this is equation 1. So, d omega the solid angle is A of the receiving area multiplied by the cos theta 2 divided by r square this R. So, this is 7 into 10 the minus 4 into point cos 30 divided by 0.4 hole square. That is the 2.866 into 10 to the power of 4 watts per meter squared per steradian is the uniform diffuse radiant intensity which is going in all the directions.

If it were to be intercepted by a hemispherical basket which replaces A2 then that 9 into 10 to the power of minus 4 watts per meter square will be the final value, but unfortunately this A2 is only a part of that hemispherical basket outside this A2 and whatever is part of the basket is outside the A 2 is also equally entitle to receive this radiation.

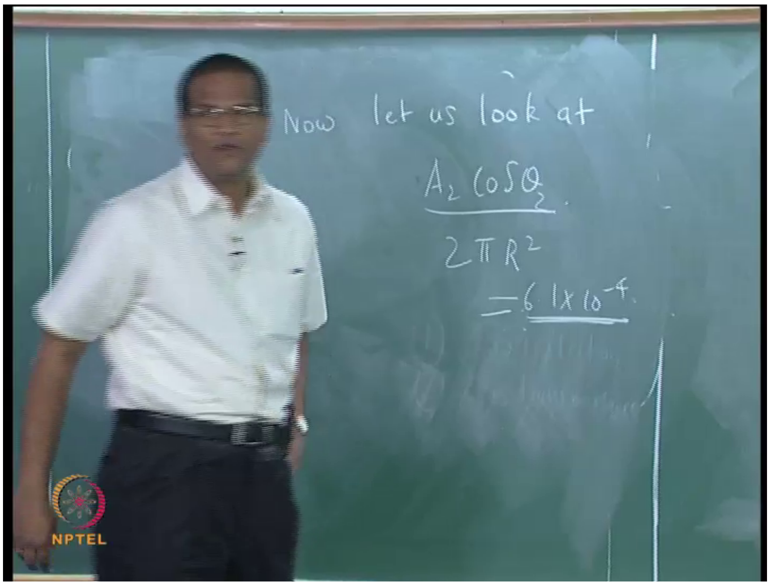
(Refer Slide Time: 42:31)



Now, I e q by A 1, I am freely using A 1 and A 2 instead of using dA 1 and dA 2 because in the problem I told that, A 1 and A 2 are elemental, by elemental I mean that between any two points the angle is the same and the radius is the same. But, I can make A 1 and A 2 finite areas and I can give assignment number two. So, you have to subdivide into various area and then use coordinate geometry find out the r, find out the theta and then integrate it, that is how we do, when we solve engineering radiation problems, that is -hat this fluent also does, s 2 s model is there. There is something called s 2 s, surface to surface model, for radiation in fluent which does that. So, now 2.866 into 10 to the power of 4.

So, it refuses 0.0111 watts. But, if you want to have a real physical feel, if you want to have a more comprehensive picture of what is the state of the affairs is, then you will have to see what is the cube total, from A 1 which is coming out of the surface. What is it 9 into 10 to the power of 4 into 2 into 10 to the power of minus 4, 18 watts because in between is vacuum, there is nothing which absorbs the radiation. Therefore, the total radiation energy which is coming out of A 1 is 18 watts, out of 18 watts only 0.011 watts is intercepted. So, fraction intercepted, is 0.011 6.1 into 10 to the power minus 4.

(Refer Slide Time: 45:43)



Now, A 2 cos theta two is area which is oriented normally to A 1 and divided by 2 pi r square is the total area of the hypothetical hemisphere, of which A 2 is a part. Now, we can see that, this will be tantalizingly close to that 6.1 into 10 to the power of minus 4. To cut a long story short, if A two is to be a hemisphere, it will capture all the radiation which is coming from A 1, but since it is a small area it receives only that much amount of radiation and in order to quantify that, we put solid angle and this thing and definition of I lambda e and all that, this leads us to the concept of what is called view factor or shape factor which we use in radiation, how much is a second surface oriented with respect to the first surface, if the second surface were to be a hemispherical basket it will have a view factor equal to 1.

So, the ratio of its area to the area of the hemispherical basket, will give an approximate idea of the view factor, of course the view factor is a little more complicated theta, one theta two and all that will come and you have to subdivide the A 2 into several areas and each of these areas may have a different elemental view factors, but this gives you broadly an idea what the view factor is all about. Therefore, the orientation of an object with respect to the orientation of an object which is receiving the radiation with respect to the object is emitting the radiation is critical; in ultimately deciding how much of radiation will be eventually intercepted by the object number 2.

We will do a small problem in tomorrow's class, how much is the energy intercepted between 30 degrees and 60 degrees and phi is equal to 0 to 45 degrees and so on. So, that you get comfortable with those integrations. Now, what is that I? How to find out this I? In the next few classes we will go through some very tricky and mathematical derivations to get the right distribution.

Thank you.