Spray Theory and Applications
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Lecture – 40
Droplet combustion

We are going to continue our discussion of droplet evaporation and lead into droplet combustion and spray combustion. We look at the different physical process is underlying evaporation followed by combustion, leading towards an understanding of spray combustion today.

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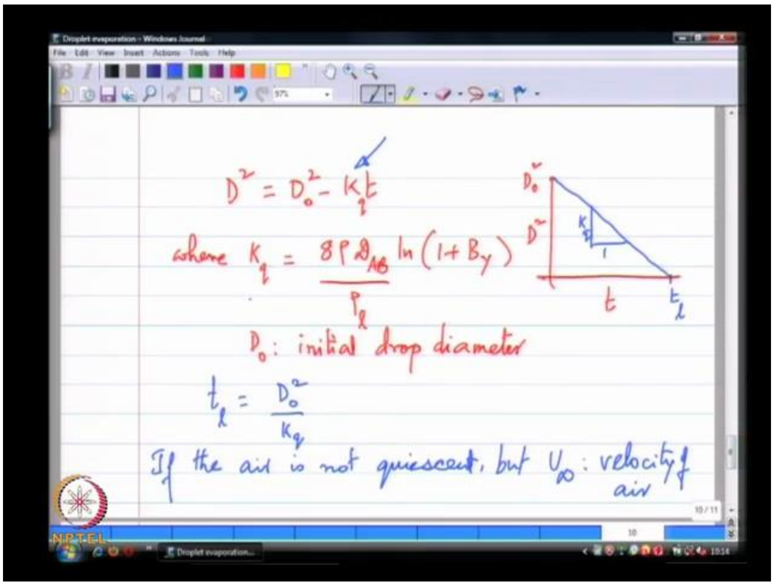

$$D^2 = D_0^2 - K_q t$$

$$\text{where } K_q = \frac{8 \rho \mathcal{D}_{AB}}{\rho_l} \ln(1 + B_y)$$

D_0: initial drop diameter

$$t_l = \frac{D_0^2}{K_q}$$

If the air is not quiescent, but U_∞: velocity of air

At the end of the last class, we had derived the famous D squared law where we showed how under quasi static conditions the square of the diameter scales linearly with time, decrease linearly with time and the rate constant is given by the K q, this K q which can be written explicitly in this form.

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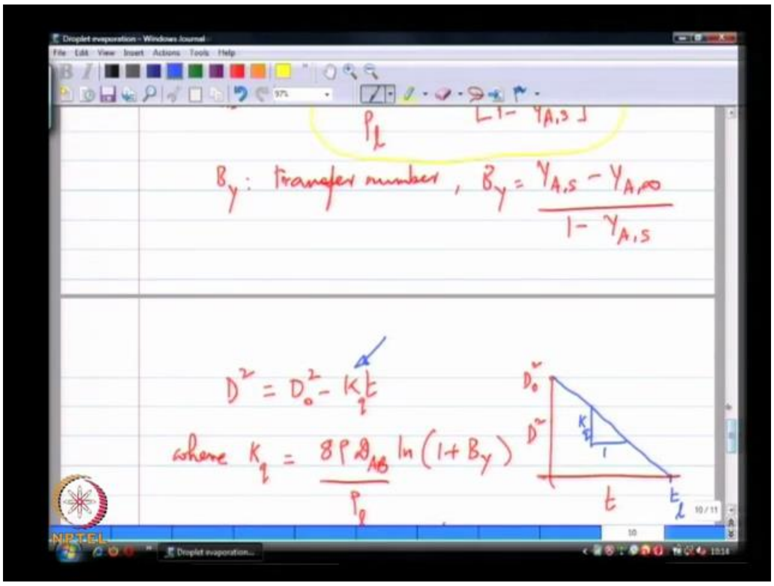

$\rho_L \quad [1 - Y_{A,s}]$

B_y : transfer number , $B_y = \dfrac{Y_{A,s} - Y_{A,\infty}}{1 - Y_{A,s}}$

$$D^2 = D_0^2 - K_q t$$

where $K_q = \dfrac{8 \rho \mathcal{D}_{AB}}{\rho_l} \ln(1 + B_y)$

Where B y is a transfer number that is based on the difference between the mass factions at saturation and mass fraction at infinite where in the ambient and essentially this ratio of these mass fraction combinations occurs in this natural logarithm and that determine the rate of D K. Is one point, I want to make here that the difference in the saturation concentration or saturation mass fraction versus the mass fraction of the water vapor at the infinite condition occurs inside the natural logarithm.

For this quantity to be very large, even if B y is very large since it occurs inside a natural logarithm, its effect is attenuated quite a bit. The point to make here is all the properties that influence droplet evaporation; it is the diffusivity that is the most important. The diffusivity of water vapor in air is going to significantly affect the value of K q much more than the value of the saturation concentration versus the concentration of water vapor in the infinite ambient. This has the direct consequence to the life time of the drop.

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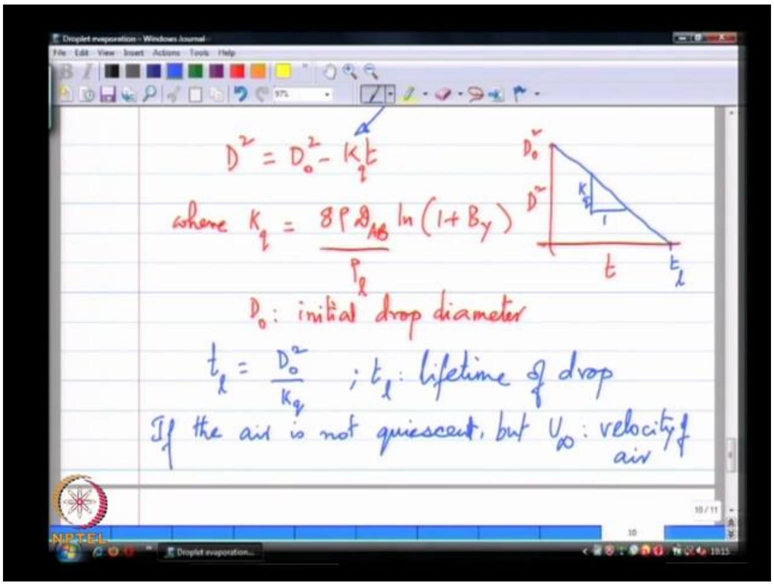

$$D^2 = D_0^2 - K_q t$$

where $K_q = \frac{8 \rho \mathcal{D}_{AB}}{\rho_l} \ln(1 + B_y)$

D_0: initial drop diameter

$t_l = \frac{D_0^2}{K_q}$; t_l: lifetime of drop

If the air is not quiescent, but U_∞: velocity of air

If t l is the life time of the drop, D 0 square divided by this K q essentially determines the life time and if you take 2 drop, one evaporating in a very very dry atmosphere and one evaporating under not so dry atmosphere, as long as B y is small in comparison to 1. You will see that D 0 squared is affected primarily by the diffusivity.

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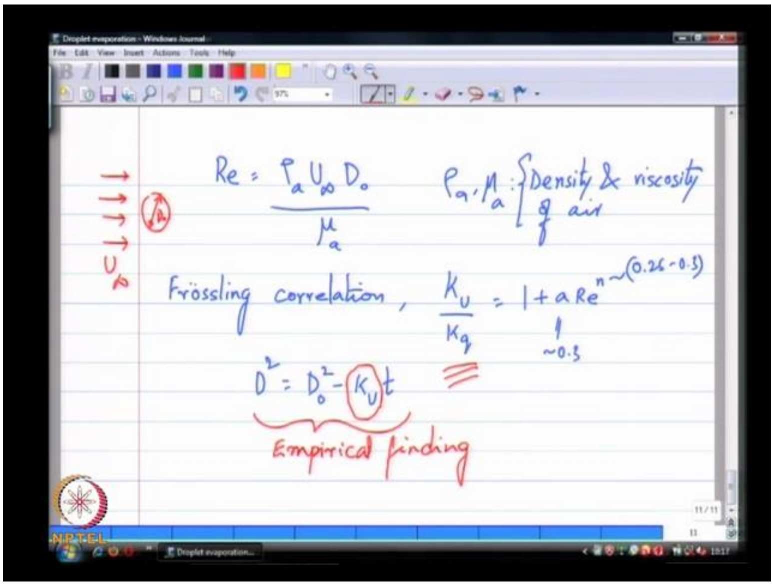

$Re = \frac{\rho_a U_\infty D_0}{\mu_a}$ ρ_a, μ_a : Density & viscosity of air

Frössling correlation, $\frac{K_v}{K_q} = 1 + a Re^{n \sim (0.26 - 0.3)}$, $a \sim 0.3$

$$\underbrace{D^2 = D_0^2 - \boxed{K_v} t}_{\text{Empirical finding}}$$

And then we went on to include a correction to this correlation, correction to this analytically derived D square law to include effects of advection. If I have, if I take a drop and follow pass the drop at u infinity, the drop itself some diameter D 0; I can define a Reynolds number given by that quantity and the rate of evaporation of this drop in this co following air stream is also excepted to follow a D squared law with modified rate constant K u, and K u is given by this relationship.

Now this has no theoretical basis, it is essentially an empirical finding. While the finding that D squared decreases linearly with time for a quasi static, in a quasi static evaporation case is entirely based on theoretical arguments, in and advection case the fact that D square law seems to hold well is purely empirical finding. It is been validated in many many different instances that the square of the diameter decreases linearly with time even in cases where you do have an advection.

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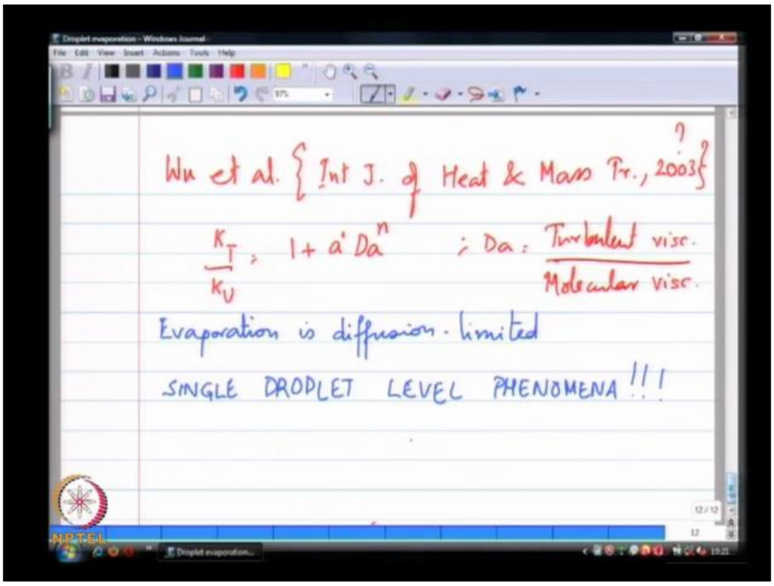

Wu et al. { Int J. of Heat & Mass Tr., 2003? }

$\frac{K_T}{K_U} = 1 + a' Da^n$; $Da = \frac{\text{Turbulent visc.}}{\text{Molecular visc.}}$

Evaporation is diffusion - limited

SINGLE DROPLET LEVEL PHENOMENA !!!

Now, there has been other modification for this, recently there was a work by Wu et al in the International Journal of Heat and Mass Transfer, I believe in 2003, although I am not sure of the year; where they provided a modification to the frossling correlation in terms of a turbulent fluctuation, ratio of turbulent viscosity to molecular viscosity. In other words, if I went back to my advection problem, I have drop of some diameter D 0 in a co stream of in a luminal flow of u infinity mean flow u infinity I have get a certain modification of the K value.

Now if on top of this, I had a u prime which is a turbulent fluctuation that would require the additional modification using a dam collar number. Now why am I taking of all these in the contest of sprays, in the contest of droplet evaporation; primarily because evaporation is at the end of the day diffusion limited. The process of mass begin moved away from a liquid vapor interface into the bulk ambient is purely by diffusive process and this diffusive process is accelerated if I have some kind of a turbulent fluctuation in a mean flow field, if I have purely laminar flow pass the drop I am only going to have to rely on molecular diffusive process is. This ratio of turbulent diffusivity or viscosity to the molecular viscosity or diffusivity is essentially is another parameter that accelerate this diffusion process. And therefore, the D square low rate constant has to be modified for the presence of turbulent diffusivity in the process.

Now these are all still single droplet level phenomena. At the end of the day all we have discussed as far as face change is concerned is in the contest of single droplets. Now is this how accurate is this in the contest of sprays.

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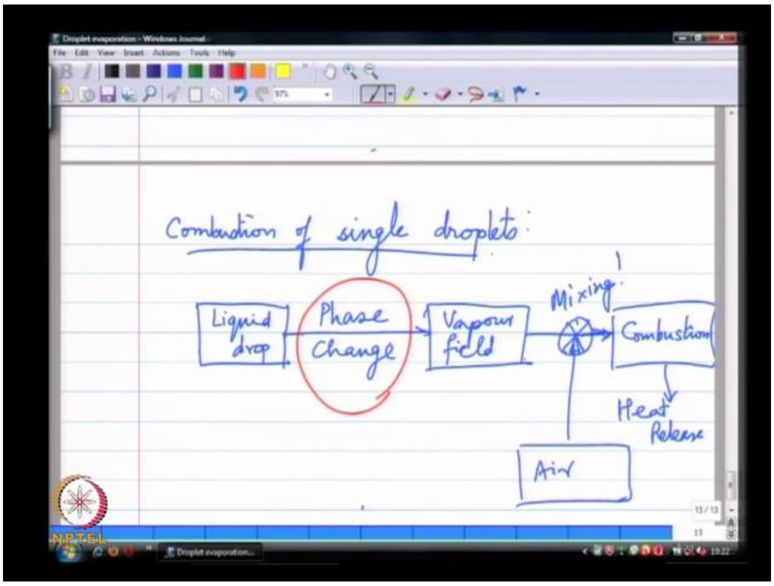

To understand that let us continue our discussion of single droplet phenomena for into the case of combustion of single droplets. We looked at evaporation of single droplets, now we look at combustion of single droplets. And the process essential is where you take a liquid drop under goes a phase change process to produce a vapor field and this vapor field uses oxidizer and there is a mixing process here, follows is preside combustion which is essential gives rise to heat release.

Up until now we look at this physical process. Now this physical process is significantly enhanced if I am in the presence of warm temperature field. Like for example, if I am spraying kerosene into a flame, the flame has already created a high temperature filed around this liquid droplets and that high temperature filed is going to accelerate this process. Now that process involves three phases.

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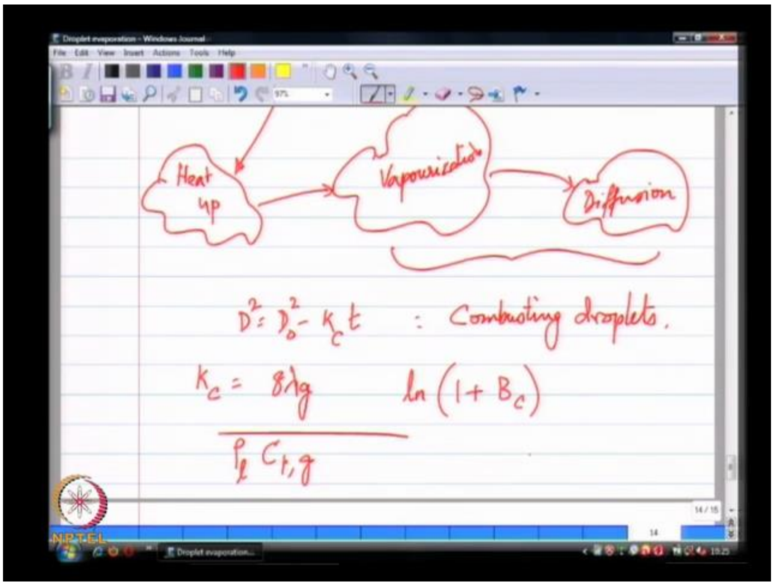

$$\underbrace{\text{Heat up} \rightarrow \text{Vapourization} \rightarrow \text{Diffusion}}$$

$D^2 = D_0^2 - K_c t$: Combusting droplets.

$$K_c = \frac{8\lambda_g}{\rho_l C_{P,g}} \ln(1 + B_c)$$

If I now take this phase change itself, it has three parts to it. One it is called the initial heat up followed by vaporization followed by diffusion. Now if I take all this process is including heat up, vaporization and diffusion. If purely include vaporization and diffusion in a hot field where the source of energy for this vaporization process is my, is the temperature filed around the droplet then I can still write a D square law and I will call this K combustion and that K combustion is given by.

Essentially in the context of combusting droplets one can write a modified D square law. Again these are all the original D squared law is theoretical derived version, but it empirical validated in many many different situation where those theoretical arguments may not hold water rigorously that is the beauty of this law.

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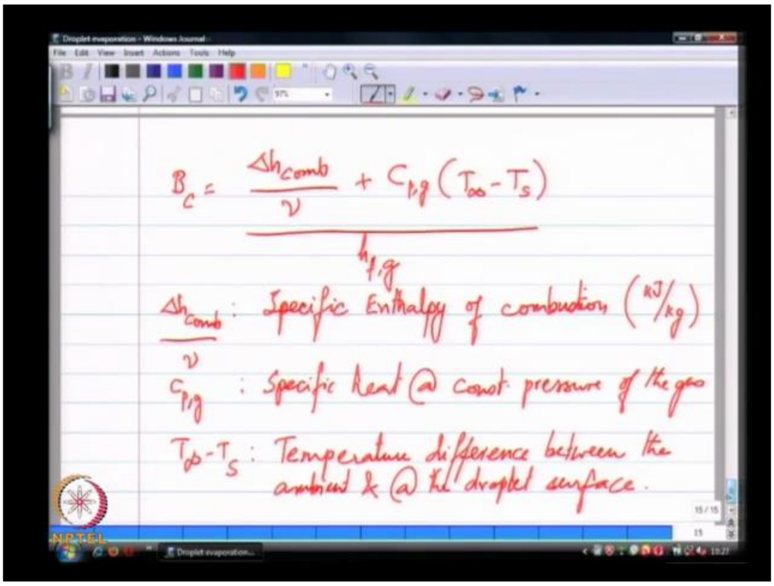

$$B_c = \frac{\frac{\Delta h_{comb}}{\nu} + C_{p,g}(T_\infty - T_s)}{h_{f,g}}$$

$\frac{\Delta h_{comb}}{\nu}$: Specific Enthalpy of combustion (kJ/kg)

$C_{p,g}$: Specific heat @ const. pressure of the gas

$T_\infty - T_s$: Temperature difference between the ambient & @ the droplet surface.

Now you, I can write a B c equal to delta h combustion is enthalpy of combustion or let just say delta h over nu is a specific enthalpy of combustion typically in units like kilo joules per kg C p g is the specific heat at constant pressure of the gas phase T infinity minus T s is the temperature difference between the ambient and at the droplet surface.

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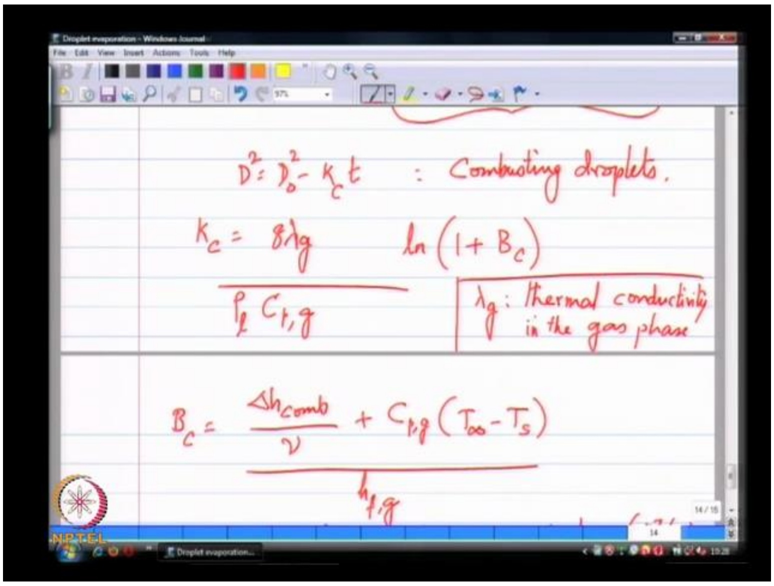

$D^2 = D_0^2 - K_c t$: Combusting droplets.

$$K_c = \frac{8\lambda_g}{\rho_l C_{p,g}} \ln(1 + B_c)$$

λ_g : Thermal conductivity in the gas phase

$$B_c = \frac{\frac{\Delta h_{comb}}{\nu} + C_{p,g}(T_\infty - T_s)}{h_{f,g}}$$

Essentially if I look at this and of course, I have to also define my lambda g here the thermal conductivity in the gas phase. If I look at this expression for K c the only process that limits this evaporation the rate of evaporation is now controlled by lambda g where in the past we have diffusivity there is important assumption underling this called united Louise number assumption.

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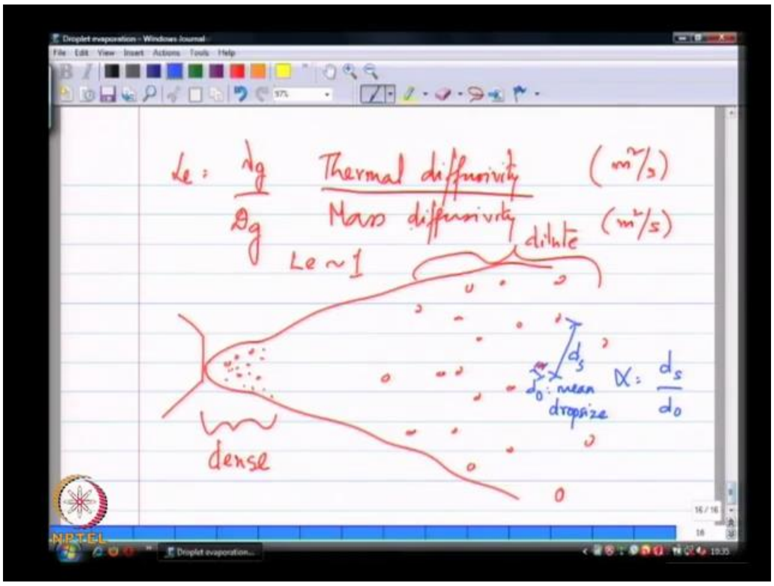

$$Le = \frac{\lambda_g}{\mathcal{D}_g} \quad \frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}} \quad \begin{matrix}(m^2/s)\\(m^2/s)\end{matrix}$$

$$Le \sim 1$$

Louise number is essentially the ratio of thermal diffusivity also known as thermal conductivity to mass diffusivity. Notice all unites and all this meter square per second in a psi terms. The thermal diffusivity to mass diffusivity ratio is what is responsible for the rate of thermal diffusion or thermal transport from the ambient gas to the droplet which is also the same as limiting process in term of the vapor diffusing away from the liquid into the ambient gas. So, when we take this Louise number and say Louise number is order 1 essentially that thermal diffusion process and the mass diffusion process are control by the same a parameter that is on the same order of magnitude.

And therefore, under this condition whether the process is limited by thermal diffusion lambda g, lambda sub g; thermal diffusion or to the liquid drop or mass diffusion away from the liquid drop the two are the same order of magnitude. You will see that in the rate constant K c mass diffusivity which occurred in the previous arguments is now replaced by thermal diffusivity, it actually not thermal conductivity, but thermal diffusivity. I think this is conductivity, but if you go through the arguments we had a row D B D a v that row as be absorbed into giving us the diffusivity as the net result.

Now we are at the point where I am able to introduce the droplet into a hot atmosphere and estimated its life time or the rate of release of vapor from that droplet. This is followed by the vapor phase reaction. We are not gone discuss the reaction part as it is purview of is out of the purview of and current discussion, what we are going do is understand that the reaction process where you have fuel let us say gasoline vapor reacting in the oxygen in the air come and to produce heat the product CO 2 and water vapor is essentially gas phase reaction. So far as physic of the process is concern it is no different from methane combustion were you have c h four reacting with let say oxygen in the air producing CO 2 and water vapor and heat. The process of methane combustion and the process of gasoline vapor combustion are physically identical all though the parameter space in which they operate could be different.

For the movement we gone leave out of the discussion we want to understand the effect of the heat release on the droplets or the rate of evaporation of droplets which is what we just post in the form of modified D square law. Following this evaporation process you have a heat release and that heat release now going to control the temperature field around this droplet. Now we have understood the process of the vapor release and we said that vapor reacting with the oxygen is similar to any other gas phase combustion reaction. How does now various point sources of this vapor which is essentiality what my spray behaved.

If I take a real spray, I am going to start with dilute spray and work my towards the dense spray, but how does the dilute spray look like and how does the dense spray look like as far as what are the difference between this two different kinds of spray as far as my argument for the combustion process is concerned. If I think of spay for a movement the lost of drop they are produced and this drops, I am drawing the cartoon just to show the difference between the dilute and the more dense spray. As you can see the one difference between the dilute spray and the dense spray is this distance to the droplet size. If I take ratio of D s to D 0; D 0 is like my mean drop size. If I take a ratio of D s to D 0, if I look at the limit were alpha is very large that is the droplet spacing is very large in comparison to size of the droplet, I am going to create vapor filed which is essentially super position of evaporation process is happening from multiple drops. Each drop is evaporating in some mean temperature field completely unaware the presence of the other drops.

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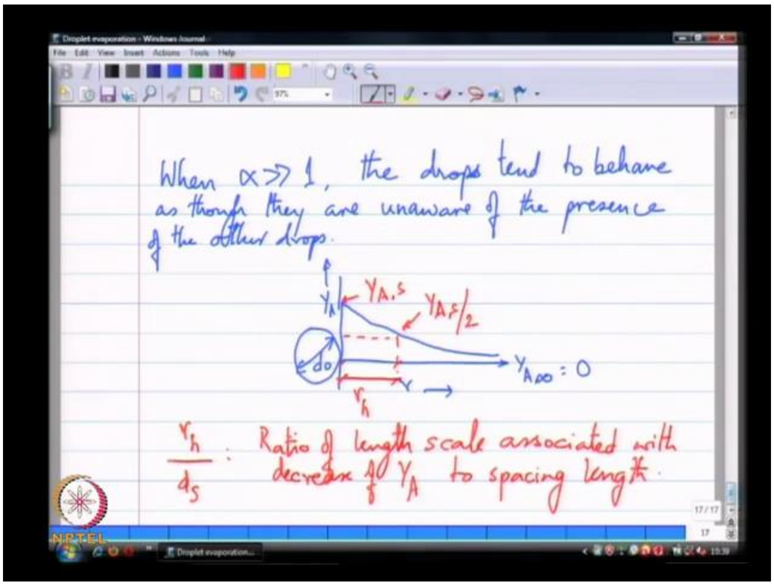

When $\alpha \gg 1$, the drops tend to behave as though they are unaware of the presence of the other drops.

$\frac{r_h}{d_s}$: Ratio of length scale associated with decrease of Y_A to spacing length.

This is the limit when alpha is much greater than one as though they are unaware of the presence of the other drops when alpha another way of think of this if i take a individual drop of some diameter D 0 I am going to create and plot the mass fraction of the function of the radial distance i K. Now I am going to create an exponentially decreasing mass fraction and I am going to assume for the movement that is Y A infinity is 0. If I plot Y A as a function of the radial distance, I have an exponentially decaying distribution and that comes from my earlier relationship that I had derived which is given by this relationship this is my Y A as a function of radial position you can look at this there is an exponential D K of Y A as a function of radial position it actually not the exponential D K its slightly complicated function your Y A goes as it is e power minus m dot or r and m dot itself. We found it determine by this parameter.

Essentially for a given Y A infinity and Y A s I have m dot that scales as r s and this Y A as a function of r scales as that r s over r. If I now plot this I can think of a length scale over which I can think of a length scale over which the mass fraction goes down by a factor of half. So, this is analogous to our half life in other typical D K processes. I can think of the length over which the concentration gradient or the mass fraction gradient takes the mass fraction from the saturation value. This is my saturation mass fraction value to a value that is half the saturation mass fraction. This distance I will call this half the r savage r savage also is important in terms of the ratio of r savage to droplet spacing itself I called D s base. Again if this r savage over D s I call this some second ratio beta if beta is much less than 1; that means, r savage is small in relation to D s.

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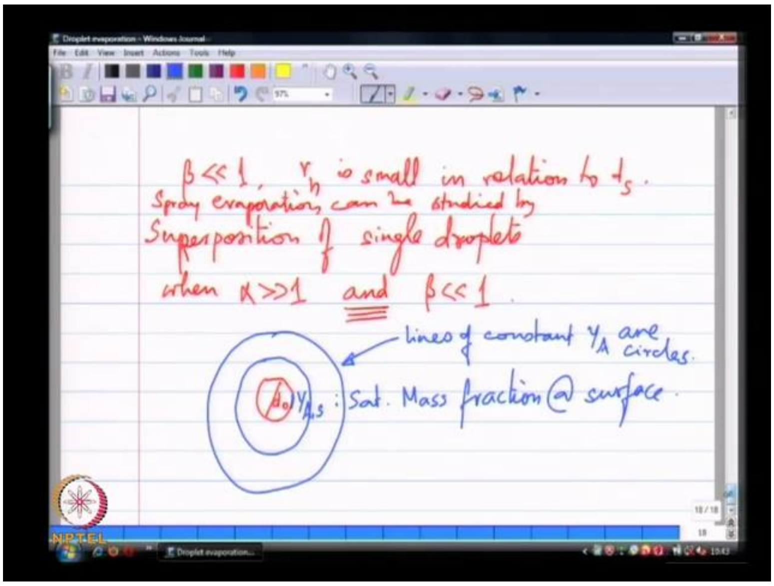

$\beta << 1$, r_h is small in relation to d_s.

Spray evaporation, can be studied by Superposition of single droplets when $\alpha >> 1$ and $\beta << 1$.

lines of constant Y_A are circles.

$Y_{A,s}$: Sat. Mass fraction @ surface.

That is each drop is again uninfluenced by the presence of the other drops this is important because what when alpha is very large and beta in this case define as r savage to D s when this are very small. I have regime in which I can treat individual droplet as though it is there by itself we have all the theory and empirical evidence that we have gurnard from analytical study as well as from all the empirical date away gather those can be extended and applied to spray combustion situation if this ratio is in a right regime. Now as far as dilute spray combustion is concern this does hold true.

This alpha and beta do default to the appropriate limits in the dilute spray. When the droplet spacing in general very large you are quite ok, in that regime, I am going to take the vapor concentration field. So super position when alpha is greater than 1 and you have to ensure that both of these are valid you cannot just take one or the other and assume that what you find and assume that you can use single droplet studies super pose to study a spray. If I want look at evaporation in a spray evaporation can be studied by super position of single droplets when alpha is greater than 1 much greater than 1 and beta is much less than 1.

If this is not to be the case then I have to look at interference effect. If I now look at single droplet say of some diameter D 0 the vapor surface is at concentration Y s, Y A s.

This is a saturation mass fraction lines of constant y a will now look like circles around a single droplet this is not sup I mean this is coming from that fact Y A only a function of radial distance and not anything else if I take 2 droplets.

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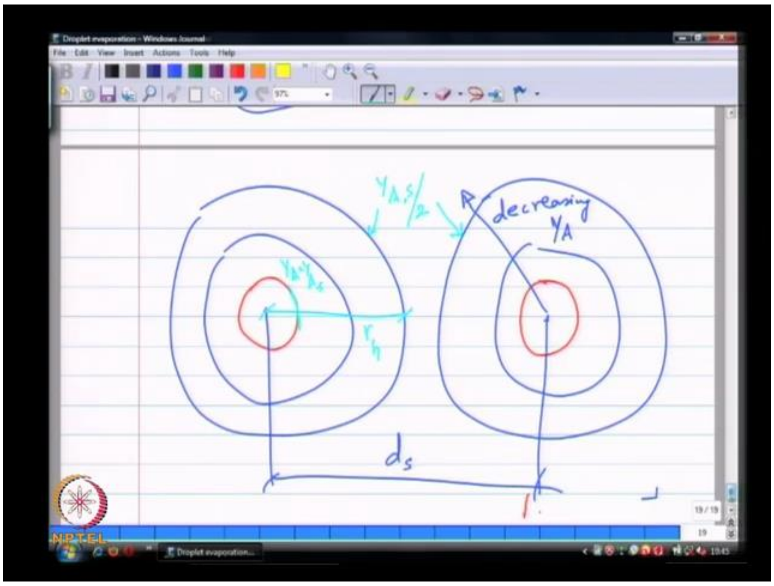

And the do the same exact thing, I take one droplet here the lines of constant Y A are still circles around each drop and these are decreasing Y A this is each circle is contour of constant Y A and of decreasing value of Y A away from the centre. If this if the droplet spacing is very large in comparison to size of the droplet or this r savage. This is where if you think of if this is contour of Y A equal Y A comma s and if this is a contour Y A comma s over 2 both of these and if they intersect the all I am saying is they will not intersect by large distance what we are saying is that r savage is much greater than is much smaller than D savage. The spacing between the drops and in this case I can use superposition of single droplets 2 create a vapor filed around the drop. If I now look at multiple droplets coming together this may not necessarily hold true.

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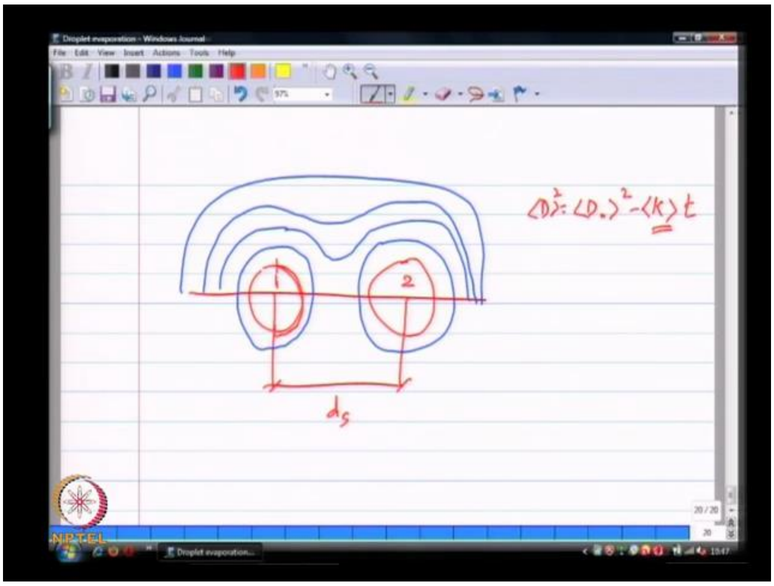

If I take a situation where 2 droplets are placed at some D s again, but you can notice how D s is less than in previous schematic and if I draw lines of constant Y s Y A, now the 2 drops are not entirely independent of each other. The first drop the rate of the evaporation of the first drop is affected by the presence of the second drops. If I call this my first drops and this my second drops the 12 o’clock position to 4 o’clock position, 12 o’clock to 6 o’clock position on the first drop sees at different concentration gradient in relation to concentration gradient that 6 o’clock, 12 o’clock thus the other half the drop seize and this cause different rate of evaporation on the 2 drops which means that I still may be able to use like a D square law, but this is now a function of the droplet spacing like a mean droplet spacing.

And this is again an area of active research to extract information that relates to multiple droplet effects; in other words when I have drops that are overlapping in terms of their contour of constant mass fraction when I have significant overlap in those contour than each droplet is not evaporating independent of the rest which also means that I do need to think of multiple droplet interference effects or at least 2 droplet interference effects. Now this particular they have been several studies on this to think of to study evaporation effects or interference effect in the evaporation field.

Now just to extract the effect of the droplet spacing on the rate of evaporations, now again like we the where we need this information is where if we want to move away from over so called exact formulation in the multiphase model. If we want to give up the exact formulation and look at either a mixture or oilarian multiphase or Lagrangian interaction we do need this kind of information. Especially in Lagrangian models you are now not just limited by 2 droplet coming together specially you are now limited by the spear of influence of each droplet coming under the influence of the neighboring droplet. The interference effects when you include evaporation in the laganzine model are significantly higher than in a non evaporative Lagrangian model.

If I want to use, if I want to look at employing multiple phase models to study sprays propagation and combustion I do need this level of information at the micro scale constitute level. At 2 droplet level, I do need to know what the droplet spacing dose. At the larger level when I know the droplet spacing on average, I can use this 2 droplet inference effect 2 get order epsilon information where order 0 is saying pure super position no interference effects order epsilon is where you are saying that alpha is no longer much larger than 1, but or rather beta is no longer 0, but is some small quantity in relation to one we will than the able to estimate the effect of beta in relation to beta begin 0 which pure super position of signal droplet studies.

We now looked at what dilute spray combustion spray evaporation looks like dilute spray evaporation look purely like super position of droplet than extending it more and more towards the nozzle we say when the droplet spacing becomes significant in comparison to that r savage, I do need to consider evaporation effects interference in the evaporation processes is an; and D then when I come even closer to the nozzle is it possible that I have really no evaporation at all that is I produce a very dense concentration of droplets. And due to some process if the air inertial air between the droplets if the inertial air completely saturated with vapor then my transfer number essential become 0 in that case when the transfer number become 0 I no longer have any evaporation process possible.

We now sort of moving from the dilute regime closer and closer to the nozzle dilute regime is essentially super position of signal droplets I am saying that very close to the nozzle it is possible that you really have no evaporation at all past a given point where saturated the inertial air with vapor. Given this whole range of physical manifestation let us look at what spray combustion look like.

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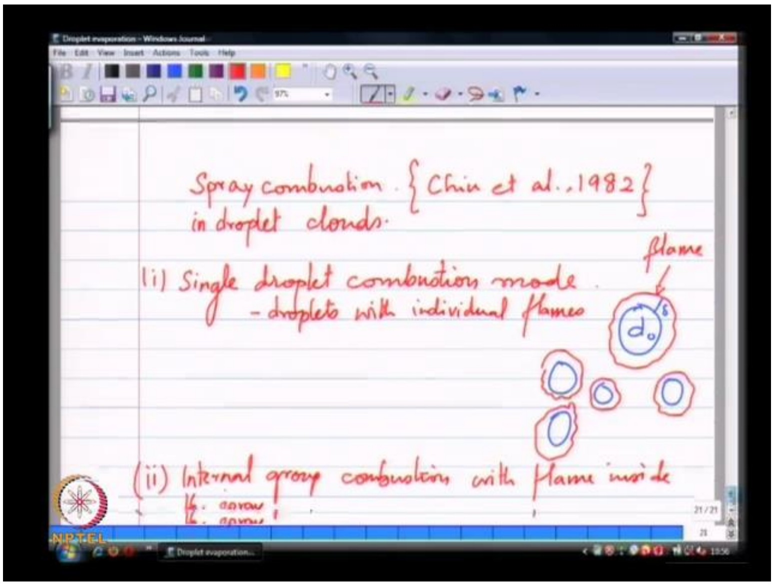

Spray combustion in droplet clouds. {Chiu et al., 1982}

(i) Single droplet combustion mode.
- droplets with individual flames

(ii) Internal group combustion with flame inside

We now go to look at our last point of argument which is to look at spray combustion this is adapted from a very important work by Chiu et al in 1982 where they passed the different modes of combustion in among droplet clouds into four possible regimes. The first is essentially what we are familiar with as a single droplet combustion mode where you have droplets with individual flames? So, for example, I have a droplets and this droplet as a flame around it and multiple droplets each has a flame around it this is possible if the droplet are separated by a distance. Now the position away from the droplet at which this flame is stabilized, is given by the point where the mixture ratio between the fuel vapor concentration and the air concentration is appropriate.

For example, very close to the droplet I may have very high concentration fuel especially a fuel paper especially if I have very volatile fuel and away from very far away may not have enough vapors defused to create zone which is ready for combustion. The delta the stand of distance between the flame and the droplet is given by essential the region where the mixture ratio is close to the equivalent ratio one could think of the equivalent ratio as begin sort of the most appropriate concentration of fuel equivalent ratio being one is the most appropriate concentration of vapor and air for now we are only going to use quality at arguments just to understand different modes of spray combustion. This is what we see individual droplet with a tiny flame around it.

In fact, it is common observation that when we have a kerosene lamp you occasionally see a spark fly off and that spark is essentially either a burning droplet or an incompletely burnt droplet that still has some chemical energy remaining in it to cause a visible flame. So, this kind of individual droplets flying off is what we are use to see in like a kerosene liquid fired flame when you may have a occasional spark fly off the second regime is what is called internal group combustion inside the spray.

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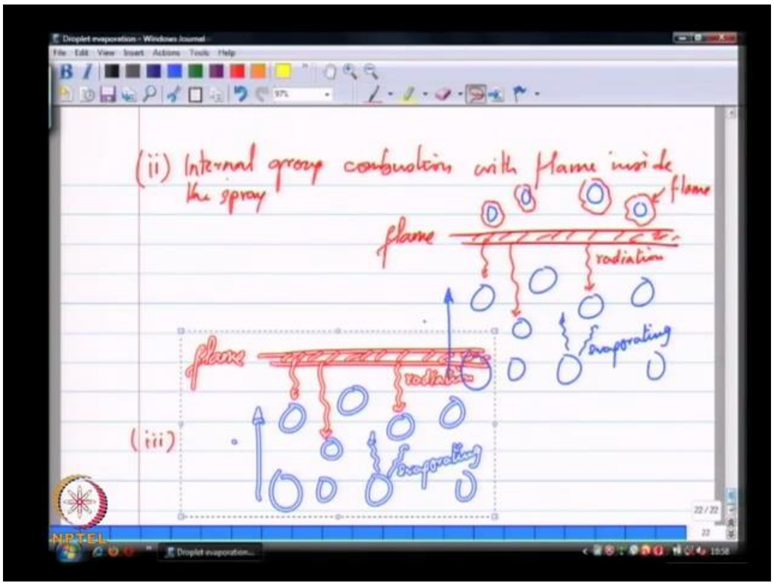

(ii) Internal group combustion with flame inside the spray

Let us see what this look like this is the case let say where I have a bunch of drops and the drops are evaporating in the presence of flame, but flame is sitting far away from the drops and this flame could be radiating heat or transporting heat somehow to the droplets which are intern evaporating. These droplets are evaporating because there is heat transport and the inertial air is un-saturated at that temperature. You do have the evaporation process that is taking this vapor release process forward, but the interstitial space is sufficiently small that I do not have a flame residing inside the spray now it is possible that some of these droplets which are being transported in the direction are incompletely evaporated. I may end up with a tiny spray around it a tiny flame around it.

This is the case were we gone from a pure dilute regime as we come as we make a way towards the nozzle you may have a visible flame, but some droplets are incompletely evaporated and escape the flame boundary causing individual sparks which are these droplets with burring droplets essentially, but the bulk of the vapor release is happening inside the flame regime inside the flame zone the third is similar to this except I now have. I have this same u infinity process I have some transport of droplets but I can now clearly identify a region inside which the spray is to dense for evaporation.

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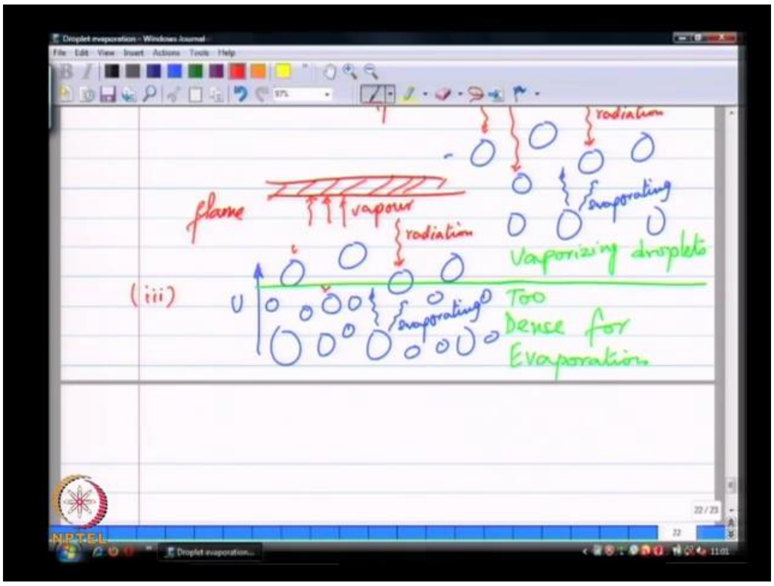

This region is where I have a very high mass, very high number concentration of droplets to the point where the droplet spacing is less than r savage the distance required to avoid interference effects between droplets and that when you are in this too dense for evaporation regime you tend to have this are non vaporizing droplets that during the course of let us say typical spray which has diverging angle which has the angle of diversion to the cone encompassing all the droplets.

The cross section area available flow increases and as a result the droplet spacing is increasing and in that increasing droplet spacing sense you tend to create vaporizing droplets. I have green boundary a line that I have drawn in green color here now mind you these are schematics these are just a sought of a physical mode for our understanding mathematical modeling at this level involves lot more is a lot more involved, but this is essential before we go to that level. This too dense to evaporation regime followed by vaporizing droplets followed by a flame zone that is completely separate and the flame zone is only supplied vapor.

The flame zone is purely sustain by fuel vapor transported into the into the flame with no droplet escaping the flame this is our ideal situation in any kind of spray combustion application the fourth regime which is often called and this is called this mode or spray combustion is called external group combustion.

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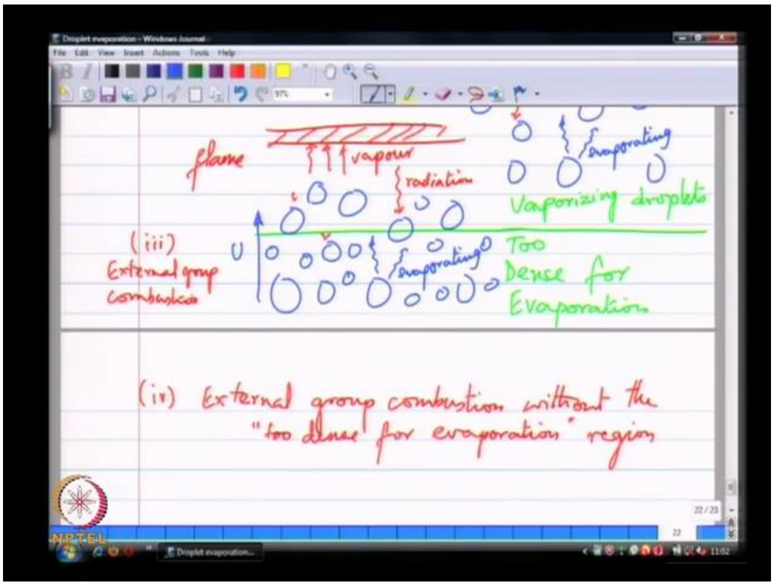

(iv) External group combustion without the "too dense for evaporation" region

And this is also an external group combustion the fourth mode is also external group combustion without the too dense for evaporation region not going to draw schematic, essentially if I take the vaporizing droplet region and extend it to all the way to nozzle. Even very close to the nozzle I have not saturated the interstitial air if that be the situation we would essentially have vaporizing droplets animating from the nozzle. And this vapor this vaporizing droplets supply fuel vapor to the flame and flame itself is completely separated physically separated from the droplets. And all the droplets are completely evaporated by the by the time they get to the flame.

We will continue this discussion in next class, to look at how one would mathematically modal these different regimes and then conclude our discussion with some application of spray combustion.