Spray Theory and Applications
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Lecture - 39
Droplet evaporation

Welcome back. We are going to continue our discussion of sprays. Now up and till now, we will looked at droplet formation process is, droplet breakup process is and droplet transport. Transport models involve different kinds of multiphase flow models. In all I mean in most spray applications if not all. Droplet evaporation is the next physical process that is of relevance you could take any process say starting with humidification.

If I want to let us say, a humidify a certain space in a where I am right now in Chennai of course humidification is not a problem, but if you go to a very dry area let say sort of a very hilly a mountainous region in winter you would need to humidify a room and the way that is typically done is, you have a small spray in one corner of a room the drops that are generated in this spray evaporate and cause water vapor to defuse in to the air causing the relative humidity in a room to increase slightly.

You could have other processes like, spread drying which is a process that we discussed earlier. As one that is use to make granular materials. So, you create slurry of whatever granular material you want to ultimately manufacture in a where, the slurry is essentially the granular powder imbedded or in liquid, in suspension in a liquid. This slurry when it is spread the water has to evaporate out or whatever or whatever is your solvent water or whatever is your solvent has to evaporate out and you are left with the granular material that settles down.

A third and one of the most important uses of sprays is in spray combustion, whether it is your typical air craft engine or car you know with an IC Engine internal combustion engine or power generation from liquid fuel sources, typically involve generating a spray evaporating the droplets that make up your spray. So, you produce to vaporized fuel whether it is, vaporized gasoline diesel atonal or aviation fuel the reaction. The combustion reaction that creates your heat is in the vapor phase that is the typical steps leading to energy release leading to this chemical energy release. So, one of the important processes in this whole chain of physical processes is Droplet Evaporation. And we will look at different models for droplet evaporation in today’s class.

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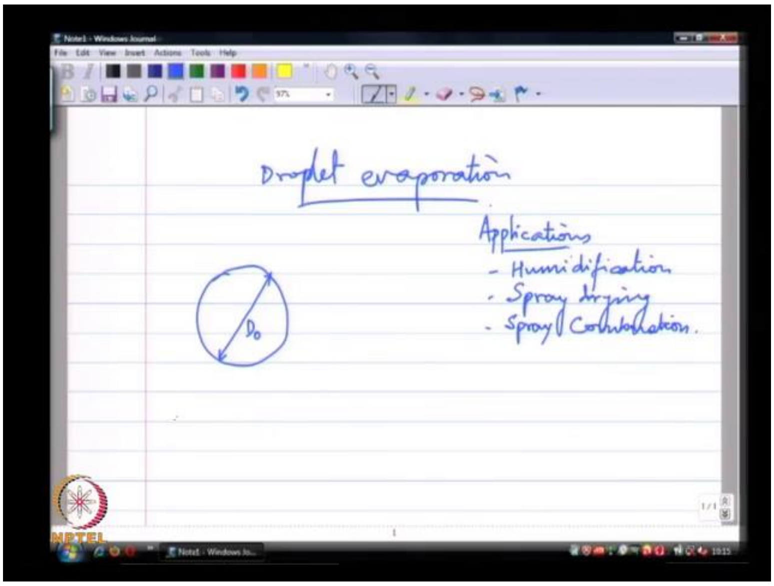

Droplet evaporation

Applications
- Humidification
- Spray drying
- Spray Combustion.

So, like we said applications are in humidification spray drying and of course, spray combustion which is a big part of why we discuss evaporation. So, we going to conceder an idealized model of a drop to start with, we will take a drop that is in suspension some drop of diameter D 0 that is, suspended in an infinite medium of some air or gas or some material.

How does this liquid get converted to vapor and how is this drop, what are the processes a companying this drop evaporating. So, before we go in to discussing how drops evaporate, we need to set the ground straight in our understanding of diffusion as a process.

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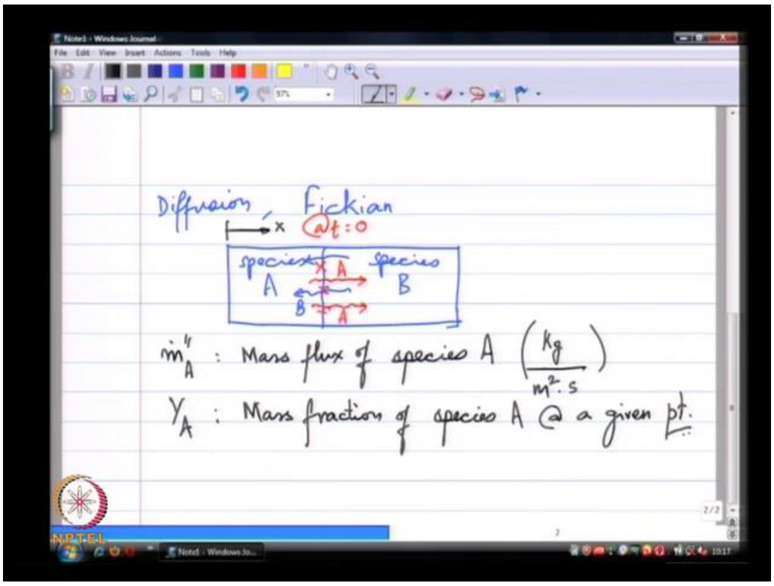

Diffusion, Fickian

$\dot{m}''_A$: Mass flux of species A $\left(\frac{kg}{m^2 \cdot s}\right)$

Y_A : Mass fraction of species A @ a given pt.

So, let us just look at diffusion more specifically Fickian diffusion. So, just to understand Fickian diffusion I am going to take a little one-dimensional simplified model where, let us say; I have some species A and species B; two species separated by little membrane. And it sometime t equal to 0, this membrane is removed. So, what you have happening is essentially the species A diffuses that way and the species B, diffuses the other way.

The transport of these species is defined based on a based on what we call Mass flux. And this has units of kilogram per meter square second. This is mass fluorite which is kilo gram per second per unit area. The two primes are basically, they denote per unit area. And will define Y A as the mass fraction of species A at a given point. So, if I define a coordinate system x this Y A is the mass fraction at a given point.

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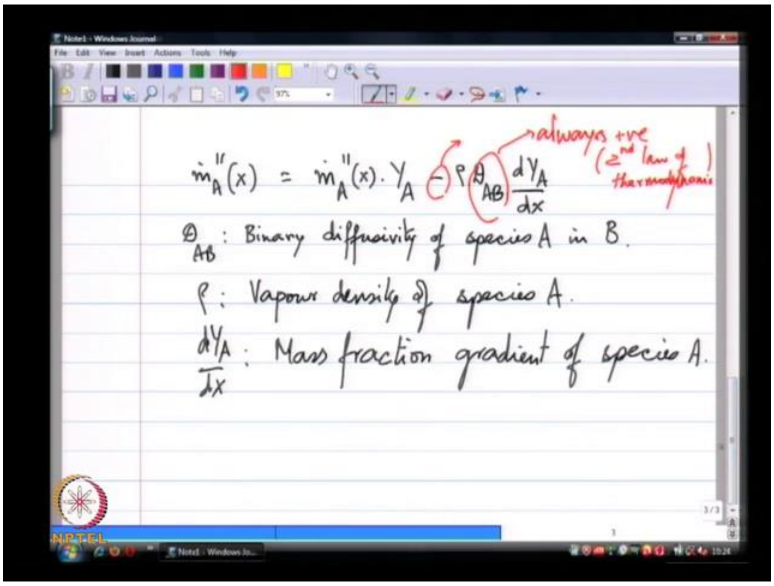

$$\dot{m}_A''(x) = \dot{m}_A''(x) \cdot Y_A \boxed{-} \rho \boxed{\mathcal{D}_{AB}} \frac{dY_A}{dx}$$

always +ve (2nd law of thermodynamics)

$\mathcal{D}_{AB}$: Binary diffusivity of species A in B.

ρ : Vapour density of species A.

$\frac{dY_A}{dx}$: Mass fraction gradient of species A.

If I now write down at any given point x what is the mass flux. So, notice how this problem is a one-dimensional problem and, if I ask the question what is the mass flux at a given point there are two reasons why you would have mass flux. One even if I had one is just pure movement of this species, this is what we call advection or convection advection is actually a better word to describe movement of mass; so pure movement of mass.

In other words, if this whole two species mixture A and B where all moving at some velocity in reference to O, O is my origin. So, you think of it this way, I mean if this whole pipe containing the two species was moving at some velocity v in as observed by somebody stationary at O that would amount to a mass flux at a given x, that would have to be accounted for. So, that is my first component of my mass flux and that is given by m dot double A times Y A; Y A being the mass fraction of A at that point in space. But there is also an additional component, if there at the whole pipe was itself stationary why would this species move to the right and why would B move to the left it is essentially due to concentration gradients, this is a model due to fic. And this comes in the form of an additional term d Y A dx is the concentration gradient, it is actually the mass fraction gradient of the species A. D AB is called the binary diffusivity. So, that is the diffusivity of species A in another species B. The rho is the vapor density of species A and d Y A dx is the mass fraction gradient of species A.

So, this part is now enough to tell me how a certain species transported. So, I could either have advection which is a first term on the right hand side or I could have diffusion, due to gradients in concentration and the total mass transport is some of these two, the negative sign here is only there to tell you the gradient diffusion happens against a gradient. So, diffusion of a species happens from region of high concentration to a region of low concentration. So, if Y A at x equal to 0 is higher than Y A at some x plus at 0 plus delta x the flux; that means, the gradient is negative in the x direction flux is positive in the x direction.

So, you essentially require this negative sign to make sure that the flux is positive in the x direction while gradient is negative in the x direction, another way of thinking about the, it is to ensure that your diffusivity is always a positive number in any equation. Whether it is your heat conduction equation mass transport equation or momentum transport equation which is basically, viscosity all these diffusivities can only be positive numbers because you already taken care of the fact that if the gradient is negative flux is positive by putting this negative sign.

This comes from your essentially your second law of thermodynamics you take a cornier statement of the second law of thermodynamics and may be you rephrase it in diffusion process mass momentum or heat the diffusivity has to be a positive number.

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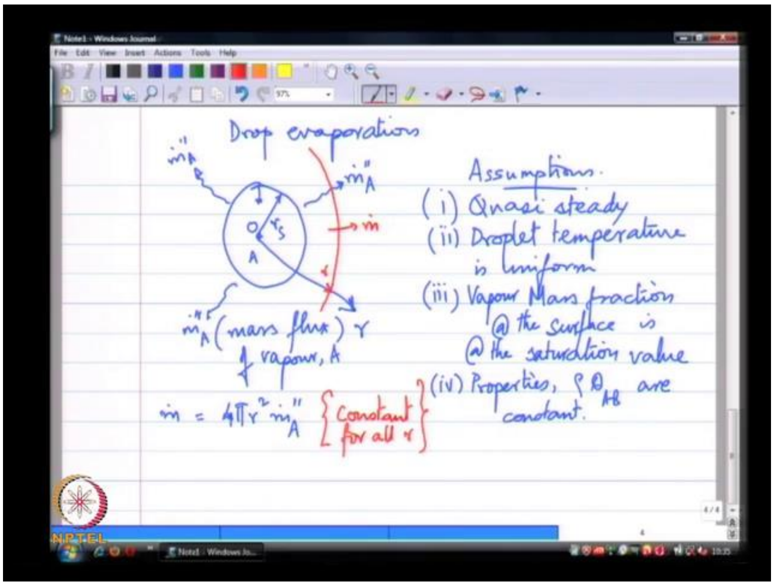

Drop evaporation

$\dot{m}_A''$ (mass flux) of vapour, A

$$\dot{m} = 4\pi r^2 \dot{m}_A'' \left\{ \begin{array}{l} \text{Constant} \\ \text{for all } r \end{array} \right\}$$

Assumptions.

(i) Quasi steady

(ii) Droplet temperature is uniform

(iii) Vapour Mass fraction @ the surface is @ the saturation value

(iv) Properties, ρD_{AB} are constant.

Now if I take, if I now come back to the situation where I have a drop. I place my coordinate system here, r s is the current radius of the drop, and in that r s formulation I can look at what happens in this r coordinate systems. So, instead of having we are still looking at a one-dimensional problem, but one-dimensional in a spherical polar sense where everything only depends on the radial coordinate not on the theta or phi coordinates. Theta is like your latitude coordinate phi is like your longitude coordinate in a typical r theta phi spherical polar coordinate system; we will reduce that down to a one-dimensional problem where everything all quantities are only functions of r your radial position from the origin o.

So, in this formulation we are going to make certain assumptions will first understand what the assumptions are, the first assumptions we are going to make is that the evaporation process itself is quasi steady. Quasi steady means that I do have let us say, some evaporation of this species A due to I have some flux of species A due to evaporation this flux divided by the vapor density is essentially, like a vapor phase velocity I gives you an order of magnitude of the vapor phase velocity. That velocity is much larger than the rate of reduction of this species the rate of reductions. So, there is as this drop evaporates the drop boundaries going to shrink. So, there is a certain velocity of this droplet boundary shrinking that velocity is very small in comparison to other flux based velocities and that is what we mean by quasi steady. And what it all, another way of looking at the same thing is that say for example, I take a drop and I put a small source of liquid at the center of the drop.

So, I have a small needle that I am injecting liquid into the middle of the drop, in order to maintain this radius constant at some value r s now, I am continuously going to facilitate evaporation of this liquid, so this is common observation. Put a drop of liquid on a table like this it is goanna be gone in say a few minutes. So, if I have a little needle precluding in to the drop from the bottom of the table a continuously replenishing liquid I am going to keep such that I keep the liquid at that same r s constantly, I am going to create a concentration of the liquid vapor away from this liquid drop and that concentration profile away from the liquid drop is going to be steady, that is not going to change with time.

So, this is pure stead in the case of droplet evaporation model that you are going to do today, where not going to remove that were not going to have a source of material in the center of the drop, but we say everything else happens as though there is a source of material and that source of material is replenishing it. That is our meaning of quasi steady. So, it is like I have a source, I assume everything else is steady, but based on that steady process we are going to remove mass form the liquid. Another assumption is that droplet temperature is uniform inside the droplet, and the third part is the mass fraction at the surface. So, this is the vapor mass fraction at the surface is at the saturation value will come back I want to make sure, we understand this number three, assumptions very clearly.

Number four, we are going to assume that properties, such as rho diffusivity are constant. So, just to that is a simplification that does not cost as much at this movement. Let us understand what three is, and to understand what three is, we need to understand how a droplet how a droplet evaporates in the first place. If I take a drop suspended in air this is a liquid drop the immediate and let us say this drop is in a room at some temperature to infinity everywhere.

So, the drop plus the air around are all at the same temperature t infinity. How does this droplet evaporate? The air in the immediate vicinity of this drop is going to have to be at the saturation value at this temperature. So, air at a given temperature is capable of holding a certain amount of moisture that is given by your Clausius-Clapeyron Equation in thermodynamics. So, that value of mass fraction that air can sustain at this given temperature and pressure, actually it also is called the saturation mass fraction. So, the air in the immediate vicinity of this liquid drop will always be at the saturation mass fraction, will always hold saturation mass fraction amount of vapor, but the air far away is at a mass fraction has water vapor at a mass fractions slightly less than this or it could 0. If it is perfectly dry air the mass fraction of the vapor far away is 0, but it is certainly cannot be more than this saturation mass fraction it has to be less than this saturation mass fraction.

So, whatever is the profile going from this saturation mass fraction value to the mass fraction value at r tending to infinity, sets up a gradient that gradient intern creates a diffuser flux. Any time we have a mass fraction gradient you have a certain diffused flux; that means, water vapor in the immediate vicinity of this drop is going to try to defuse away from this drop and that diffusion intern creates, I mean I am now talking of this is sort of a an equilibrium process, but you can think of this diffusion of vapor away from the liquid inter phase creating a need for more liquid to evaporate to saturate the immediate inter phase interfacial air. So, it is a continuous process although I made it look like a step vice process it is a continuous process were in this vapor is diffusing way and liquid is evaporating right at the inter phase to saturate the air in the vicinity.

This continuous process is pumping liquid in to vapor, vapor diffusing away and in some new course of time you do not have any more of a liquid drop remaining. So, you essentially evaporated all the liquid out and that liquid mass has gone into the air as vapor. So, how do we, essentially what we are saying in number three here, is that during this entire evaporation process the air in the immediate vicinity of the liquid inter phase is always at the saturation mass fraction value which is given by the thermodynamic which is given by the Clausius-Clapeyron Equation from standard thermodynamics. Now I say I use to water as an example to illustrate this, but the same argument works with every liquid in every other gaseous species.

Now, if I have a certain mass flux of vapor, so m dot m double prime dot A. A is the mass flux of vapor species A given that the m dot is nothing, but 4 pi r squared if I take any radius r the actual m dot crossing that sphere of radius r is given by 4 pi r squared the surface tension area of that sphere times my m dot double prime A. So, this m dot double prime and this m dot is constant for all r. This is our meaning of steady process that I have some liquid being replenish to the middle such that, I have a constant mass fluorite of species a at all radial locations r.

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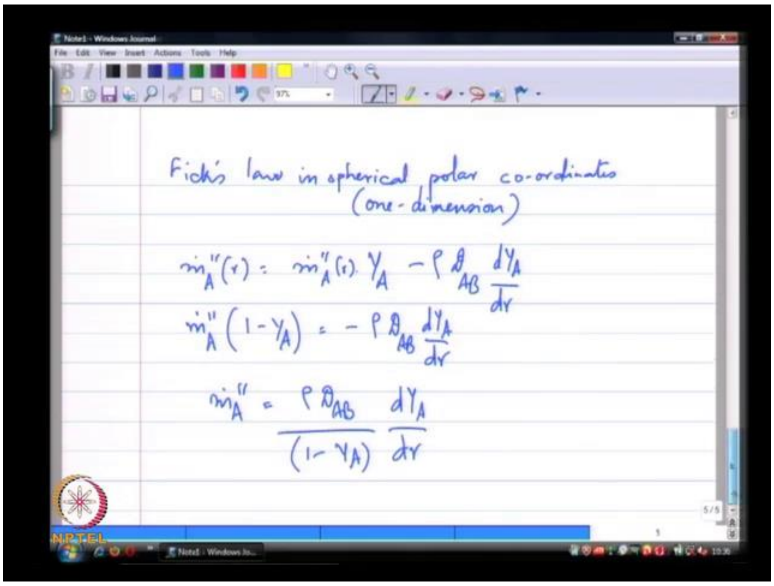
Fick's law in spherical polar co-ordinates (one-dimension)

$$\dot{m}''_A(r) = \dot{m}''_A(r)\, Y_A - \rho \mathcal{D}_{AB} \frac{dY_A}{dr}$$

$$\dot{m}''_A (1-Y_A) = -\rho \mathcal{D}_{AB} \frac{dY_A}{dr}$$

$$\dot{m}''_A = \frac{\rho \mathcal{D}_{AB}}{(1-Y_A)} \frac{dY_A}{dr}$$

So, first writing Ficks laws in spherical polar coordinates, in one-dimension we will find I can re arrange a few things here. So, for a given steady state concentration profile that I want to sustain I can replenish.

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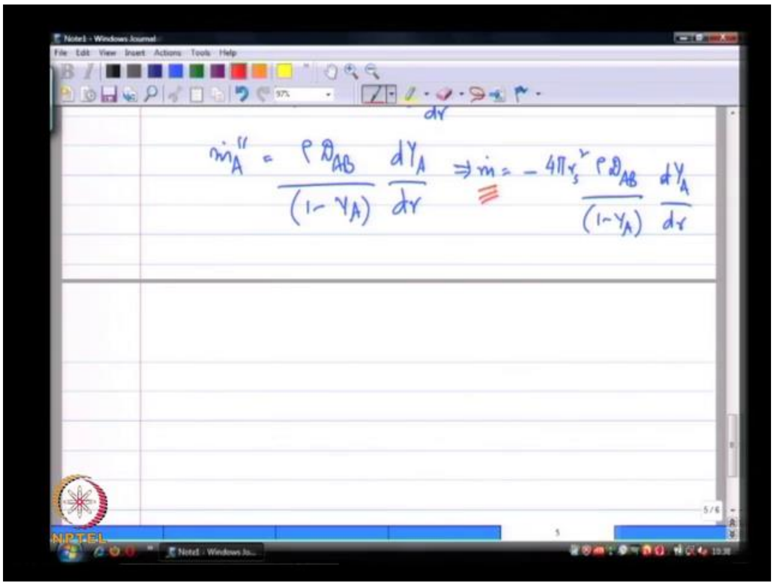
$$\dot{m}''_A = \frac{\rho \mathcal{D}_{AB}}{(1-Y_A)} \frac{dY_A}{dr} \Rightarrow \underline{\underline{\underline{\dot{m}}}} = -\frac{4\pi r_s^2 \rho \mathcal{D}_{AB}}{(1-Y_A)} \frac{dY_A}{dr}$$

So, if I use r s which my radius of the drop if I put in this much mass fluorite in the center of my liquid drop I will I will evaporate the liquid subject to, so if I know my mass fluorite that I put in. So, I know the mass fluorite that I put in, if I know m dot this equation essentially gives me a way to calculate Y a in the mass fraction as a function of

r. So, I am going to assume I know the mass fluorite I am putting in to the middle of these drop. So ok, I am going to assume for a moment that I know, m dot and calculate why a as a function of r. So, if I take this that gives me, let us make sure we understand the boundary conditions at r equals r s is a saturation mass fraction.

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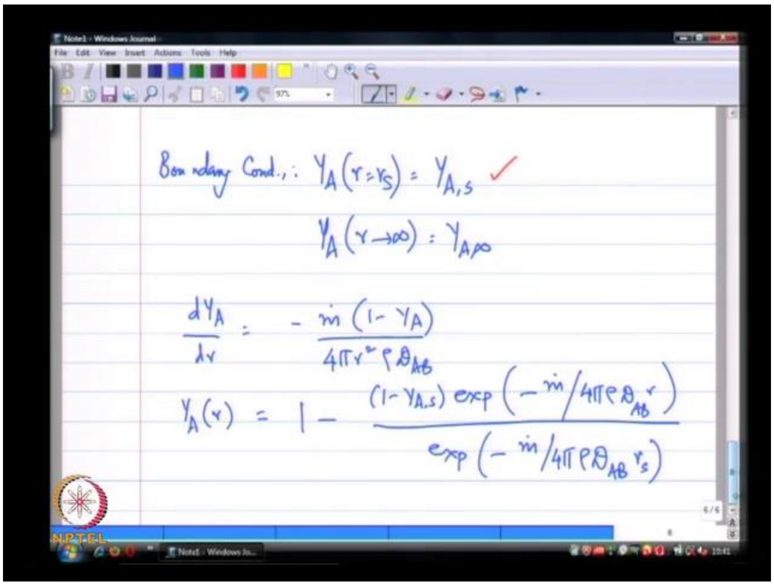

Boundary Cond.: $Y_A(r=r_s) = Y_{A,s}$ ✓

$Y_A(r\to\infty) = Y_{A\infty}$

$$\frac{dY_A}{dr} = -\frac{\dot{m}(1-Y_A)}{4\pi r^2 \rho \mathcal{D}_{AB}}$$

$$Y_A(r) = 1 - \frac{(1-Y_{A,s})\exp\left(-\dot{m}/4\pi\rho\mathcal{D}_{AB}r\right)}{\exp\left(-\dot{m}/4\pi\rho\mathcal{D}_{AB}r_s\right)}$$

So, notice how this is a one-dimensional if I rearrange this in the form of a differential equation can you say d Y A d r equals minus m dot into 1 minus Y A by 4 pi r s squared rho D AB or 4 pi r squared. If I say this is valid at all r at any r this is the mass fluorite flowing across the sphere, m dot is a mass fluorite flowing across the sphere at any radial location r. So, if I this is a first order ordinary differential equation I only need one boundary condition, but I know I have to use I have two boundary conditions I will choose the first one and will see why the reason we have, why we have this sort of additional boundary condition we will see the reason for that in just a moment, but if I use just the first boundary condition and solve this ordinary differential equation I can find Y A as a function of r and that comes out to be notice.

So, essentially the concentration profile or the mass fraction profile is a function of the mass fluorite itself, but I am thinking the mass fluorite is determine by what is a saturation mass fraction of the drop and what is the sature, what is the mass fraction far away from the drop these are the only 2 parameters that have to determine the mass fluorite I cannot determine the mass fluorite independent of those two.

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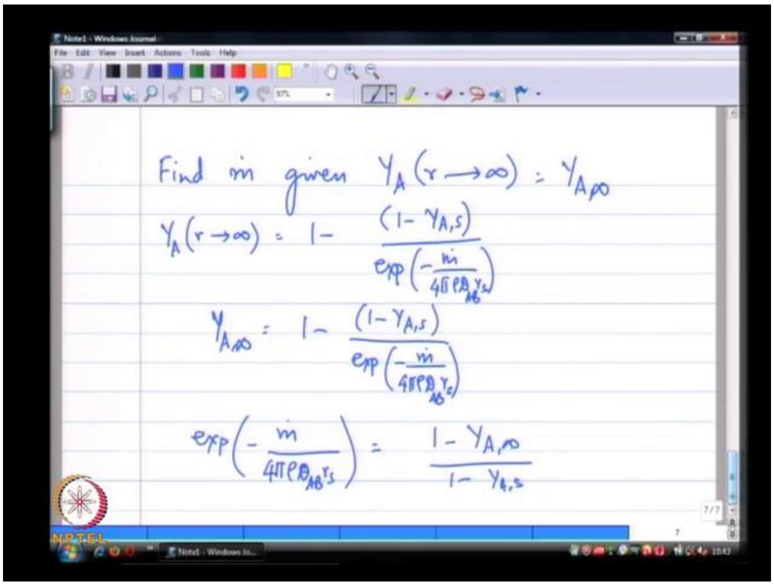

Find $\dot{m}$ given $Y_A(r \rightarrow \infty) = Y_{A\infty}$

$$Y_A(r \rightarrow \infty) = 1 - \frac{(1 - Y_{A,s})}{\exp\left(-\frac{\dot{m}}{4\pi \rho \mathcal{D}_{AB} r_s}\right)}$$

$$Y_{A\infty} = 1 - \frac{(1 - Y_{A,s})}{\exp\left(-\frac{\dot{m}}{4\pi \rho \mathcal{D}_{AB} r_s}\right)}$$

$$\exp\left(-\frac{\dot{m}}{4\pi \rho \mathcal{D}_{AB} r_s}\right) = \frac{1 - Y_{A,\infty}}{1 - Y_{A,s}}$$

So, if now use the second condition to find m dot given Y A as r tending to infinity equals Y A infinity, if I use that notice how I have m dot divided by 4 pi rho D AB r. So, I make sure I put this in parenthesis to be exact. So, this r goes to infinity which means m dot divided by that whole thing goes to 0 have an exponential of 0 1. So, Y A at r tending to infinity is 1 minus D AB r s and I know this is equal to Y A infinity if I rearrange a few things all have done is move the Y A infinity to the right hand side and move the rest to the left hand side from here.

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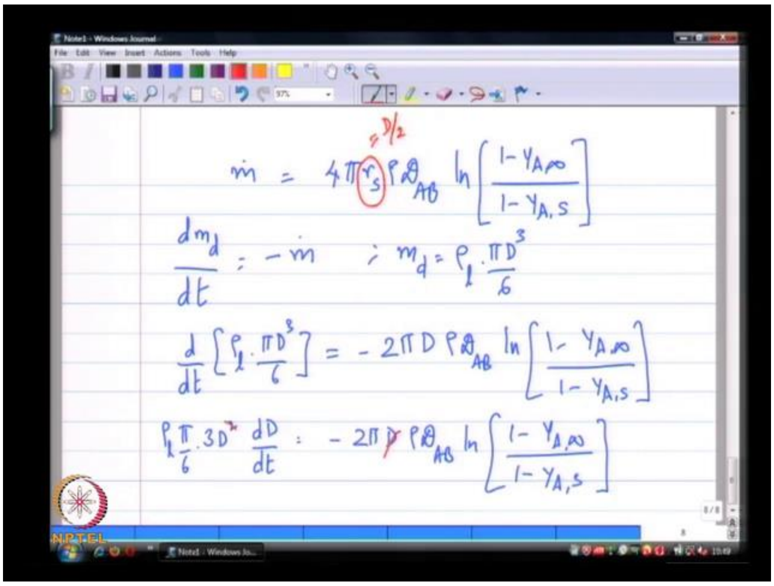

$$\dot{m} = 4\pi \overset{= D/2}{\boxed{r_s}} \rho \mathcal{D}_{AB} \ln\left[\frac{1 - Y_{A\infty}}{1 - Y_{A,s}}\right]$$

$$\frac{dm_d}{dt} = -\dot{m} \quad ; \quad m_d = \rho_l \cdot \frac{\pi D^3}{6}$$

$$\frac{d}{dt}\left[\rho_l \cdot \frac{\pi D^3}{6}\right] = -2\pi D \rho \mathcal{D}_{AB} \ln\left[\frac{1 - Y_{A\infty}}{1 - Y_{A,s}}\right]$$

$$\rho_l \frac{\pi}{6} \cdot 3D^2 \frac{dD}{dt} = -2\pi \cancel{D} \rho \mathcal{D}_{AB} \ln\left[\frac{1 - Y_{A\infty}}{1 - Y_{A,s}}\right]$$

I can write out m dot sorry this would be s on top and, this is the mass fluorite across any sphere of radius r for a given Y A infinity and Y A s. Y A s is the mass fraction in the immediate vicinity of the drop, Y A infinity is the mass fraction far away from the drop. We will just look at a couple of limiting cases, if Y A infinity is equal to Y A s that is if the air outside the drop is saturated with vapor and you can see that that ratio comes to 1 log of 1 is 0, so you have now mass fluorite.

You can take another sort of a an example if Y A infinity is actually greater than Y a s that is I have a drop that is slightly cooler because of which the mass fluorite because of which the saturation mass faction is less than what the what I have available outside I am going to have condensation otherwise this is in general negative number which means, I have evaporation. If Y A infinity is less than Y A s sorry, m dot being positive amounts to evaporation m dot being negative amounts to condensation. So, as of now we really have no distinction we have not made any distinction between evaporation and condensation as for as this process is concerned. So, this is the rate of mass laws form the drop for a given Y A infinity and Y A s.

Now, if I take a drop the drop itself is losing mass at this rate m d is given by rho l pi D cubed over 6. Now at some point in time we going to make the change to diameter, this r s is nothing but d over 2. In the next equation r s is the, radial location of the surface of the drop that is equal to the drop diameter divided by 2. So, if I now substitute what I have for m dot into this equation here for m d what we find is that d dt of rho l times pi D cubed over 6 pi D cubed over sorry, pi D cubed over 6 is the volume of a sphere of diameter d.

So, if I take this rate of change of rho l times pi D cubed over 6 to be equal to essentially I have this 2 pi D rho times diffusivity times log of 1 minus A infinity I want to integrate this in time. This d is diameter and the script D is our diffusivity let be clear about that. So, in the term and this square parenthesis we want to take the rate of change of the term in this square parenthesis with respect to time the only part which is change with respect to time is my d the rho l is of course, assume to be constant.

So, if I invoke that, I will find rho l times 5 over 6 times 3 D square times d D dt equals minus 2 pi D rho D A B, I can scratch out one of the diameters and when I do this simplification.

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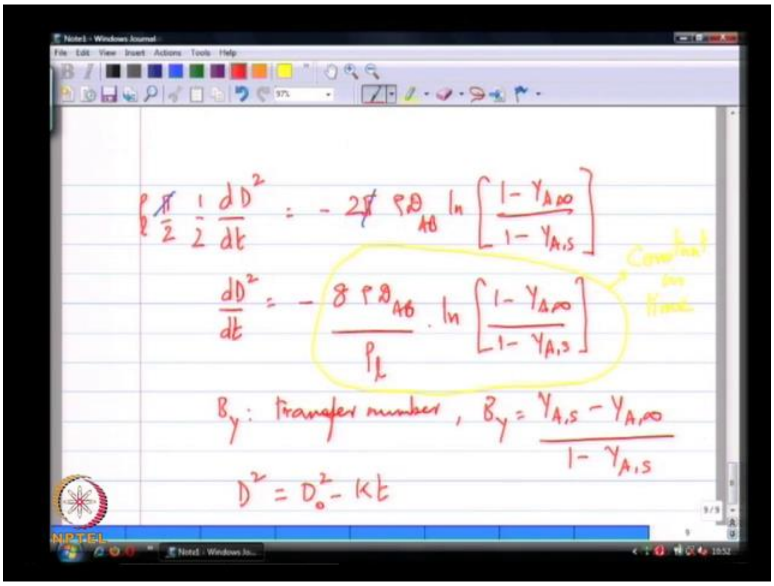

$$\rho_\ell \frac{\cancel{\pi}}{2}\frac{1}{2}\frac{dD^2}{dt} = -2\cancel{\pi}\rho\mathcal{D}_{AB}\ln\left[\frac{1-Y_{A\infty}}{1-Y_{A,s}}\right]$$

$$\frac{dD^2}{dt} = -\frac{8\rho\mathcal{D}_{AB}}{\rho_\ell}\cdot\ln\left[\frac{1-Y_{A\infty}}{1-Y_{A,s}}\right]$$

Constant in time

B_y: transfer number, $B_y = \frac{Y_{A,s}-Y_{A,\infty}}{1-Y_{A,s}}$

$$D^2 = D_0^2 - Kt$$

What I will find is this equation the d D squared dt this d D d can be written as half of d D squared and I have one half coming from here and when I rewrite this whole thing I have d D square d t, it is customary to define at a term called B y or the transfer number given by B y equals by 1 minus Y A, s and now I can show that d if I now, notice how all these all this part in the inside my yellow oval is constant in time. So, what this is telling me first of all is that D squared where is linearly with time. So, if I now complete this process I can write this as some D 0 squared minus K t.

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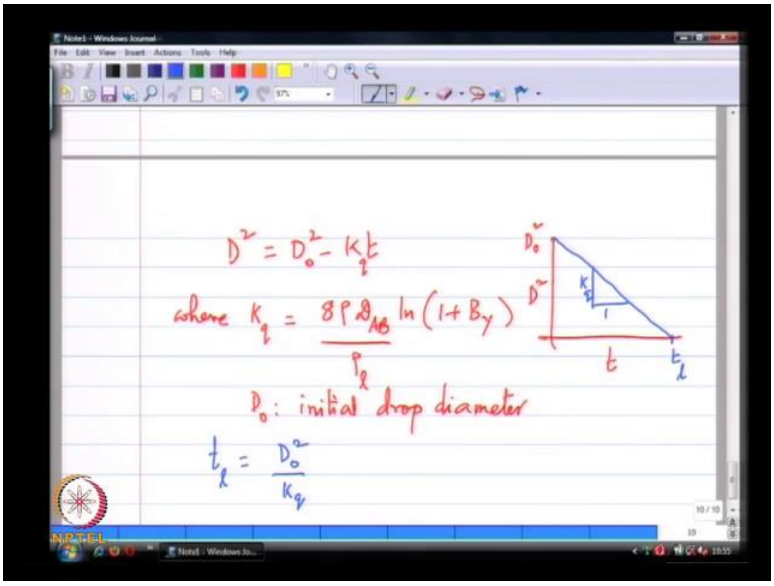

$$D^2 = D_0^2 - K_q t$$

where $K_q = \frac{8\rho\mathcal{D}_{AB}}{\rho_\ell}\ln(1+B_y)$

D_0: initial drop diameter

$$t_\ell = \frac{D_0^2}{K_q}$$

I will call this K q, I will tell you will for quasi steady K q equals you can easily show that with this definition of B y what is inside the argument for the natural logarithm is nothing, but 1 plus B y. So, when the saturation humidity and humidity at infinity are equal saturation mass fractions and vapor mass fraction far away, are equal the transfer number is 0. That is like saying I do not want any mass transfer to happen, and this K q is given by this equation and D 0 is the initial drop diameter if I subject an initial drop if I subject the drop of initial diameter D 0 2 an evaporation process, that evaporation process is going to proceeds such that if I plot D squared versus time the initial value is some D 0 squared then it is going to d K. Linearly such that, the slope is given by K were the value of this K q and you have a finite life time of the drop there is a finite point at which the drop reaches diameter 0 and that t l is given by D 0 squared over K q.

As simplified an analytical procedure as what we flowed to get to this kind of an equation it has been verified empirically in many different instances, that this t l the lifetime of a drop is very reasonably predicted very accurately predicted as a matter of fact and many different liquids and gaseous species. So, the assumption that we made that you have a quasi steady evaporation process is actually quite reasonable in even a realistic process where a drop may be moving slowly. Let us go back to our initial assumptions and see which once are like the worst culprits for a real spray, we are assuming quasi steady; that means, inherently we are assuming that the droplet is stationary with the air around being stationary. There is no movement of the air itself there is no convection of the air that is going to enhance this mass transfer process. So, if I take a liquid drop and have some flow of air around pass this liquid you going to tend to evaporate the liquid faster.

At the moment this analysis does not account for that, but as for as quiescent and single droplet evaporations concerned this is reasonable. But what is being found empirically also is that, even if you have a super imposed air velocity the phenomenology that the diameter square decreases linearly with time seems to hold very well. So, diameter squared still decreases linearly with time.

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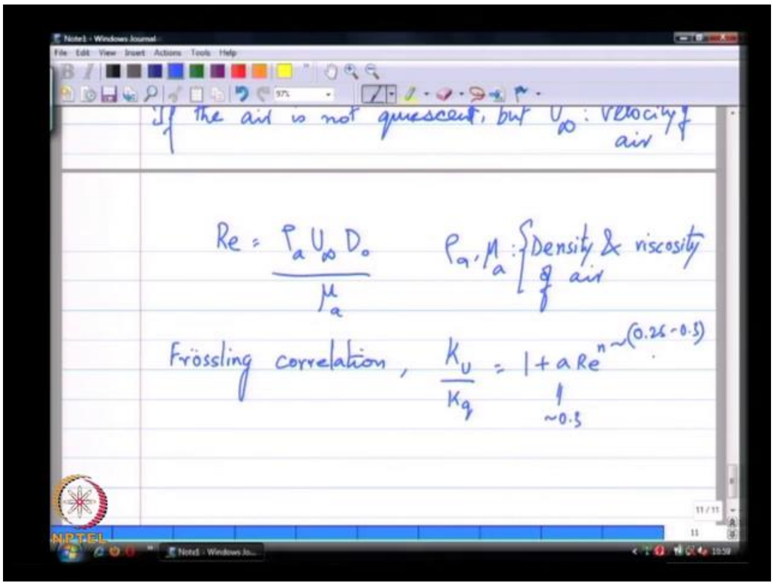

If the air is not quiescent, but U_∞ : velocity of air

$$Re = \frac{\rho_a U_\infty D_0}{\mu_a}$$

ρ_a, μ_a : Density & viscosity of air

Frössling correlation, $\frac{K_u}{K_q} = 1 + \underset{\sim 0.3}{a} Re^{n \sim (0.25-0.3)}$

If the air is not quiescent, but U infinity is the velocity of air, we can define a Reynolds number based on rho air U infinity D 0 divided by mu air and this a very famous very famous work by a German author, by name Frossling who develop this frossling correlation that says that, K in the presence of a u. So, K in the presence of some advection pass the drop divided by K quiescent is of the form 1 plus a times Re power n, k a a is I think typically about 0.3 and n is also about 0.25 to 0.3. So, it is a relatively weak dependence, but you can find a correction for the K q that you measure; K q that you estimate form pure analytically from pure thermodynamics and mass transfer K q is only a function of the diffusivity vapor density rho liquid density rho l and your transfer number B y.

So, you are able to correct for this K q in the presence of a finite Reynolds number by using this frossling correlation, but your phenomenology that your D squared the surface area of the drop still decreases linearly with time is a very good is been empirically shown to be reasonably accurate assumptions. So, all you have to do now is use this K u for your given Reynolds number calculates from the frossling correlation, but your K u will be and D squared would be D 0 squared minus K u times t. So, if your Reynolds number is a, of some finite value K u would be higher than K q that intern means the lifetime of your drop is lower.

So, if I am looking at whether I should use the frossling correlation or the other way is a is sufficient all I have to do is look at times scale associated with the evaporation in comparison to the times scale of transport to the drop itself, if the drop is moving slowly in otherwise quiescent air the time scale of movement of the drop may be small in comparison to D 0 squared by K q. That means, the drop is evaporating much faster than it is being transported. So, I can just ignore the frossling part in use K q alone, but if I have some finite time scale of m motion that is comparable to K q comparable to t l that we showed for the life time of the drop, this has to be accounting for.

We will stop here will continue our discussion of spray combustion in the next class.