Spray Theory and Applications
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Lecture – 37
Multiphase modeling - Selection of model- 2

Hello, welcome back. We will look at multiphase flow modeling; will continue our discussion of multiphase flow modeling. And will quickly recap before moving on to what it is that we want to do today.

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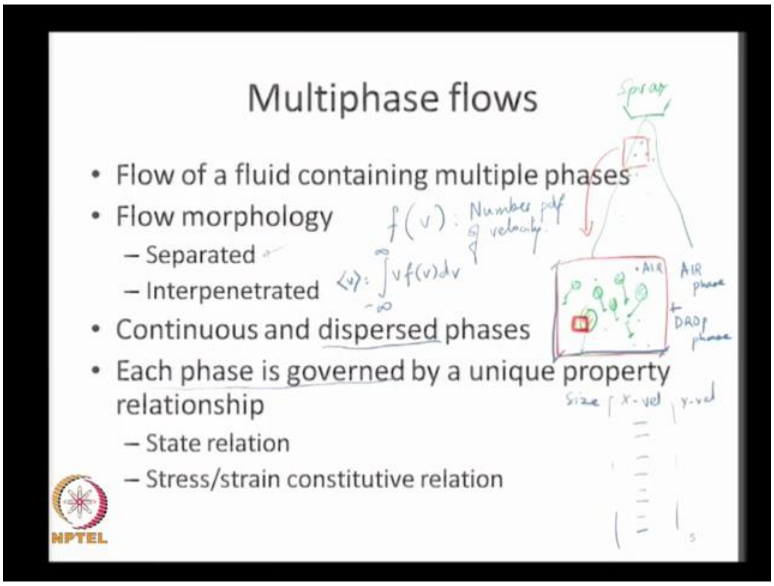

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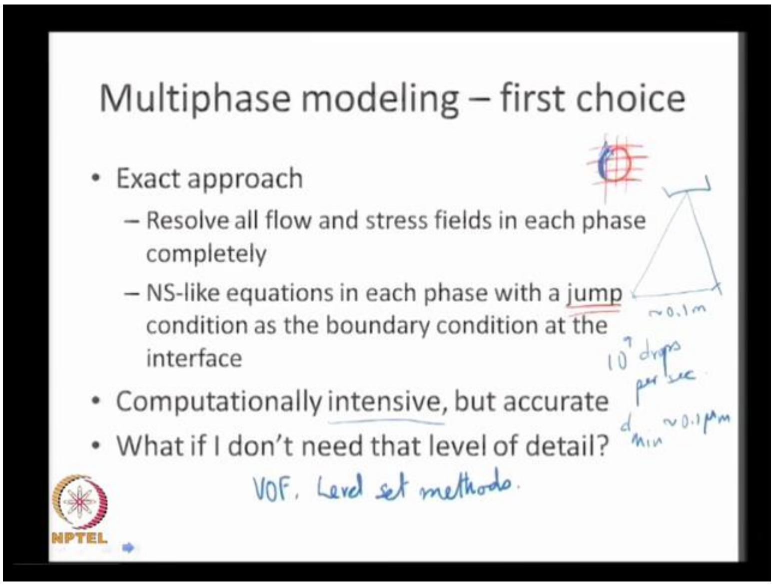

We will multiphase flow is typically flow of fluid containing multiple phases. So, you could have one phase or multiple phases. And we said there were four possible choices for the modeling itself. First is the exact approach, where you just solving the Navier stoke equations for each of the two phases. Now the properties, let us say if I am looking at bubbly flow the properties of air which is the material making up the bubbles is different from the properties of the water that is the continuous phase.

So, you only are allowing the properties to vary from what, from where you have air to where you have water and across the interface you have certain jump conditions, the jump conditions could mean discontinuity or it may mean continuity. So, essentially you have some interface balance conditions. Like for example, when I have an interface a bubble interface the normal velocity the fluid velocity in the air has to exactly v equal to the fluid velocity in the water.

So, this is an interface condition it is essentially kinematics that if material is coming in towards the interface from the air side and pushing the interface that interface has to intern push the water, simple. You may have other dynamic conditions like force continuity or stress continuity across the interface. So, you have to have like for example, if I have a certain sheer stress field in the bubble, inside the bubble, the sheer stress field just above the interface in the water has to exactly equal the sheer stress field in the air which is the material making up the bubble. If I will look at normal stress because of the curvature and because of their nature of surface tension the normal stress in the air is greater than the normal stress in the water if the curvature is inwards, curvature is positive towards the air. If the curvature is positive towards the water then the stress in the air bubble as to be less than the stress in the water phase.

Mind you these are all local balance conditions that is, they are valid at every point on the interface. So, if I write the Navier stokes equations with only the properties varying at different points in the domain basing on whether I have a bubble or water, and if I write these jump conditions across the interface and somehow solve them, this would be called the exact approach. So, you could actually do full scale simulation resolving every scale using this approach; people have done it. Like for example, if you look at boiling as a two phase process people have simple use this kind of approach to simulate a domain containing a few 100 to a 1000 bubbles. So, that is the limitation here because you are it is very, very computationally intensive, but you get a lot of information, you get information that is even inaccessible through experiments it is just so detailed.

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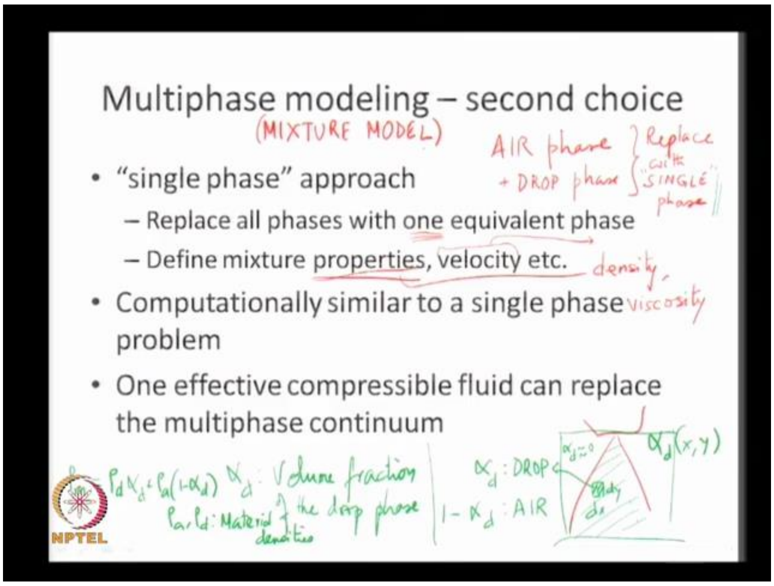

That is our first approach and we if I do not want that level of detailed what else can I do I move on to the second approach, which is I like I said my second approach is like swinging all the way to the other head of the spectrum, I am looking at the simplest approach possible. So, this is where I know there are two phases, I know there are bubbles that make up this that are embedded in water, but I am going to assume that all of this is one phase locally depending on whether you have a certain volume fraction of the bubble phase, depending on the volume fraction of the bubble phase, you end up assigning a property to this newly created fictitious fluid making up which is composed of both these phases. So, depending on the local volume fraction of air let us see again in the case of a bubbly flow example depending on the local volume fraction of air I assign a certain fluid property to this newly created mixture.

So the simplest way would be to use a weighted average like I show here down here, rho m is rho d alpha d plus rho a 1 minus alpha d where alpha d is the volume fraction of the drop phase or the dispersed phase bubbles in this case is, in the example I am discussing today, we have replaced drops with bubble so any kind of distributed phase and then the continuous phase.

So, I can use a similar linear weight age linear weighted average to describe any other fluid property say viscosity, etcetera. Now where I do this, this is not a huge assumption even the fact that I am replacing it with a third mixture fluid is not as limiting as the fact that we are only allowing one velocity field. So, essentially the moment I replace my two phases with a single phase and solve a single phase momentum and mass continuity equation, I am only going to arrive at one velocity field for this mixture fluid that inherently says that the local velocity at every point is the same for both the bubbles and the water.

So that is the main assumption underline the second modeling choice which is called the mixture model ok.

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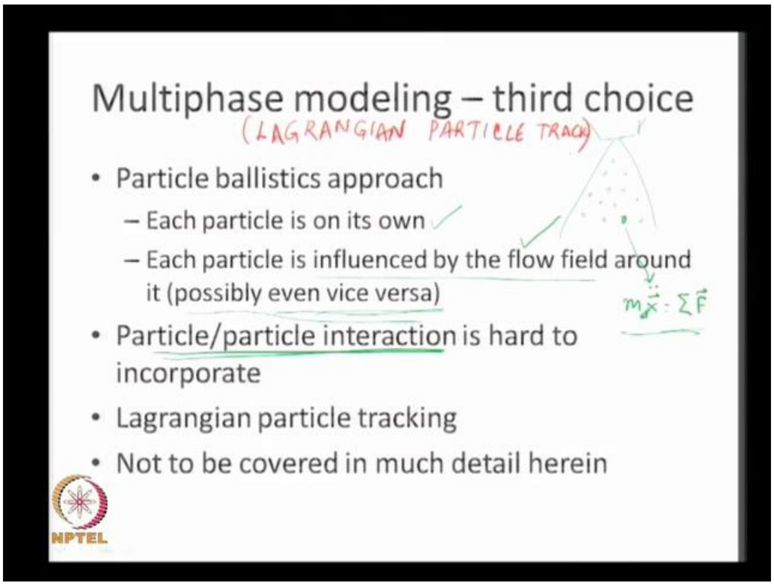

Now, for the third modeling choice is where we are using this particle ballistics approach, where I am going to treat every particle as being on its own. So, I am solving essentially Newton second law for the acceleration of that individual particle subject to the external forces that act on this particle. So, drag for example, drag from the continuous phase is one such example.

So, if I look at say like a go back to the exact model, and if I look at a domain containing about a 1000 bubbles, if the bubbles are not really interacting each bubble is spherical and each spherical bubble is moving with respect in response to the gravitational or buoyancy force that is trying to let us say drive this bubble upwards, and some kind of a drag force which is preventing it to, preventing it from a process of continuously accelerating then I can do a simple m x double dot equals the sum of all forces that is the kind of model that we are talking about here, this is refer to as Lagrangian particle tracking.

This is very easy to comprehend and very easy to implement for the particle from the particle point of view because, I am saying the particle is an entity that is moving subject to some external forces its nice and intuitive the problem in this approach comes from two parts, one if I am trying to model particle, particle interaction that is in situations where bubble, bubble collision.

So, if I have a lot of bubbles crammed in to a small domain then these bubbles are going to have to come and interact and that including that interaction in something like this is not easy, because every time two bubbles come there is a result that has to be followed by this event they either break up into more than two or they just collide and bounce of or they collide and coalesce these are all possibilities, and you have to know a priory what the possibility is depending on the initial conditions this kind of an approach is not easy to incorporate this physics is not he always easy to incorporate Lagrangian particle tracking simulations.

The second problem with Lagrangian particle tracking simulations is the reverse coupling that is we know that the fluid is influencing the particle. So, like you have a drag field that the drag force on a given particle influences the particles motion, but if I say the particle is of a finite size and mass and that particle finite size and mass intern influences the water or the continuous phase velocity field. So, to obtain the continuous phase velocity field, we have to solve the Navier stokes equations for the continuous phase and that continuous phase velocity filed being affected by the presence of the dispersed phase is not always trivial to handle especially, if I want to go fast the point particle assumption.

If I am dealing with only point masses, point particles that do not have a finite size it is at least doable there are ways of incorporating this Lagrangian modeling of particles in to and the forces due to the presence of these Lagrangian particles into the Eulerian equations of motion for the continuous phase, but if I say the particles are now of a finite size this becomes even more difficult. So, in other words we are heading more and more to towards the exact approach that becomes the challenge ok.

We will not discuss this in much detail from here on; I just want to present this as a viable way of modeling any kind of a multiphase flow in a spray situation.

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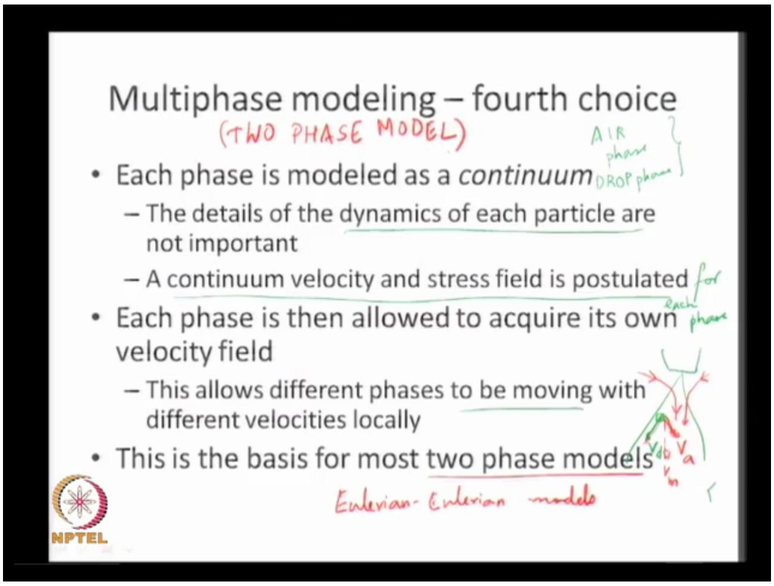

The fourth choice is where I know I have bubbles that are all distributed inside a continuous phase, and I also know that the first or the sorry the second modeling choice is not entirely accurate the second modeling choice requires meet to make the assumption that the bubbles and the water are moving locally at every point with the same velocity.

I somehow know that that is not completely accurate. So, the next level of sophistication from the second modeling choice or the mixture model is to allow the dispersed phase continuum, and the continuous phase which is already a continuum to have their own individual independent velocity field let us say independent, but they are coupled through some physics, but other than that they are allowed to freely move where they want to move subject to some coupling forces.

So, if I take all the bubbles that are distributed inside my domain inside my flow field I am going to ascribe the bubbles to be a flow, to be a continuum we have already discuss this and then that continuum is inter peritrated into my water continuum, and the water continuum as a velocity field the bubble continuum as another velocity field that are only related through drag and other forces ok.

This is what we will often called the two phase modeling approach, these two phase models rely on the assumption that I can take all the bubbles and postulate a continuum phase, comprising of all the bubbles. So, this is essentially the fourth modeling choice.

So, now we have learned four different ways of handling of modeling multiphase flows, the next question in your mind is I have a flow situation I am seeing some kind let us say a spray or a bubbly flow a boiling process, which one of these force should I choose this is even more important a question in the realm of computational fluid dynamics where most of these models are readily available in commercial c f d codes. So, you are free to choose one or the other without any cost of labour involved.

So it is only a question of fidelity of that model for that particular application, suitability of that model for that particular application. So, let us now look at how one would go about choosing the model suitable for a given application to answer this question.

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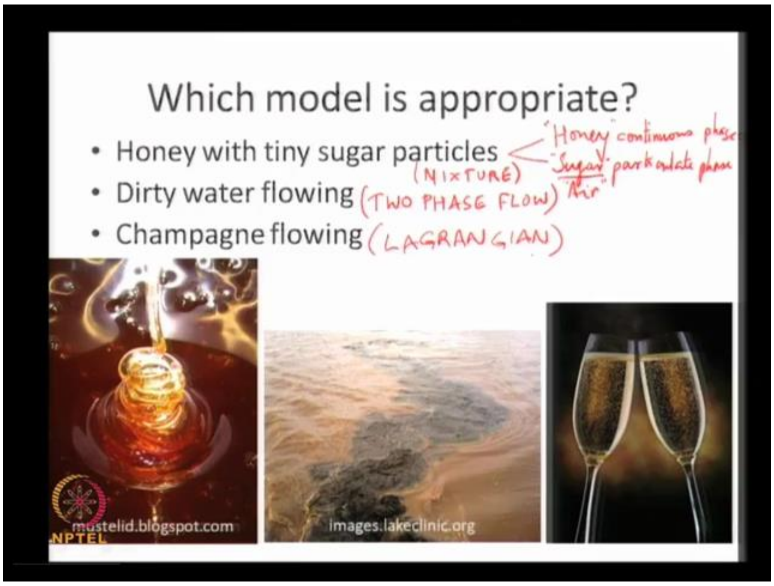

We will take three different modeling choice three different multiphase flow problems and then see what the physics involved these in each of these three multiphase flow problems and from there arrive at modeling choice methodology. So, the first one I want to take I want to consider is the flow of honey out of a bottle with tiny sugar particles involved in embedded in it.

So if I now treat this as comprising of two phases, honey which is a continuous phase and sugar particles which is, what I am calling my particulate phase. So, if I in all these of course, my exact model would work in the sugar is a bad example, but instead of sugar, if I had tiny bubbles that we are embedded in this honey then I essentially have little layer bubbles in this honey that are moving with the honey as it flows.

Now, that is a two fluid problem and clearly the exact approach which is my first modeling choice would work for any two phase flow problem, now in this particular instance the sugar particles are solid. So, it is in that sense I just pick this up because it is an example that appeal that we have seen actually visible with our eyes, but instead of sugar imagine we had tiny air bubbles ok.

Now, these air bubbles are my particulate phase, and if I now come to my second modeling choice which is the mixture model the fundamental assumption underlying the air is that in the two phase flow, the two phases are locally moving with the same velocity. So, in this particular problem which is that assumption valid? So, if I have honey flowing with some little tiny air bubbles embedded in it is it reasonable, and if I say have a lot of these air bubbles. So, if I have almost foamy structure in this honey lots of air bubbles strapped in this honey, but the honey is flowing and their the honey is dragging their bubbles is it reasonable to assume that the air and the honey are both flowing with the same velocity locally at every point, I think we can see that it is generally going to have to be true that is the bubbles cannot be moving with a vastly different velocity then the honey locally ok.

So, this would be a good case for the mixture model, and we will see quantitatively how to find this I want to show talk about the physical arguments underlying modeling choice then will talk of how to apply the mathematics to arrive at the physics accurately to describe the physics accurately.

The second example, I want to talk about is dirty water. So, let us say this is like a river joining the sea it is a bezel and you have these tidal currents that bring in silt I or take bring back silt, and is it reasonable to assume that the silt particles here are all moving exactly at the same velocity as the water especially, if I think of the situation where I could have the water flowing one way, but locally the silt due to its own inertia coming back towards counter to the current of water.

So, think of it this way, if I take this dirty water in a little beaker and I shake it laterally. So, in the when I move it in the positive into my right the water and the silt are both made to acquire a velocity in the positive direction, if I instantaneously stop this and start to move back this way. So, if I impose acceleration, acceleration deceleration is only a matter of sign. So, if I impose a sudden acceleration on this two phase mixture does the silt respond differently from the water, if the silt responds differently from the water then you cannot use the assumption that the silt and water are both moving with the same velocity everywhere, if I do the exact same experiment with the honey laden with tiny air bubbles.

So, I take a beaker full of honey laden with tiny air bubbles and lots of air bubbles, I move it to my right I stop instantaneously and start to move it to my left, would the bubbles move very differently from the honey locally the chance of that happening is much smaller than in the case of silt particles in water. So, in other words the silt particles together make up a silt continuum that is inter penetrating the water, and this silt continuum as an inertia that is substantial as an inertia that is enable to be completely suppressed by the drag forces and the coupling between the continuous phase in this case, water and the silt which is the dust or mud.

So, the fact that in this little thought experiment that I have this moving at a certain velocity, I pause instantaneously reverse the direction of motion the silt tries to respond differently the silt is still continuing to move to my right while the water moves to my left and in the process I should visibly be able to see the in the beaker little separated region, where I have more silt on the right hand side and less silt on the left hand side. This acceleration driven separation is essentially have a centrifugal separation process works I create an acceleration field and the disperse phase moves preferentially in relation to the continuous phase in this let say water, when this is the case, when the silt phase tries to do something slightly different from the water phase your mixture assumption is not completely accurate, your two phase flow modeling approach is more accurate because the silt has a mind of its own as an inertia of its own.

So, this dirty water problem is ripe for two phase flow modeling, the last example I want to talk about is the raising champagne bubbles in a in a little glass. So, I have a glass of some aerated drink its say and I soda I pore the soda into this glass and you can see bubbles raised from the bottom up to the top, and these raising bubbles. So, if I allow the bubbles to raise to the top these bubbles are going to intern create some kind of a circulation inside the water filed itself, and this circulation is a result of the bubbles raising to the top, but the spacing between the bubbles is sufficiently large in a I mean this is our common observation, the spacing between the bubbles is sufficiently large that particle, particle or bubble, bubble interaction is small. So, it is like each bubble is rising to the top subject to some gravitational buoyancy force gravitation induced buoyancy force and drag force from the fluid, and this bubble is really not interacting with the neighboring bubbles in any significant passion.

If you identify that this is the kind of morphology that you have in your two phase mixture that I have bubbles, but they are not really colliding and they are just sort of doing what they want to do, then your modeling choice should be the particle ballistics or Lagrangian particle tracking approach. Now mind you that the I am talking of the situations where they are exactly applicable, now if you use a commercial code they will advise you to use these models even in situations where they are not exactly applicable, because they have other sub models to handle the extraneous situations. So, if I have weak particle, particle interaction.

I could extent the exact Lagrangian particle tracking modeling to situations where I do have bubble, bubble collisions, but they are, but the forces generated due to these bubble, bubble interaction are small, but not 0 in comparison to the other two forces which is gravitational buoyancy and drag. So, when I have that situation I do, I mean there are ways of extending these slightly behind the regime where they are exactly valid.

Take for example, the mixture model I know that the mixture model is to be used in situations where there is no slip, there is no velocity slip, between the two phases that is when I know mixture model is nearly exact, but if I know there is a slight slip and it is small, there are what are called slip velocity slip sub models that one can use to posteriori, a posteriori understand what the amount of slip would be between the continuous phase and the dispersed phase, but as far as our exact as far as the mixture model is concerned it does assume that there is no slip.

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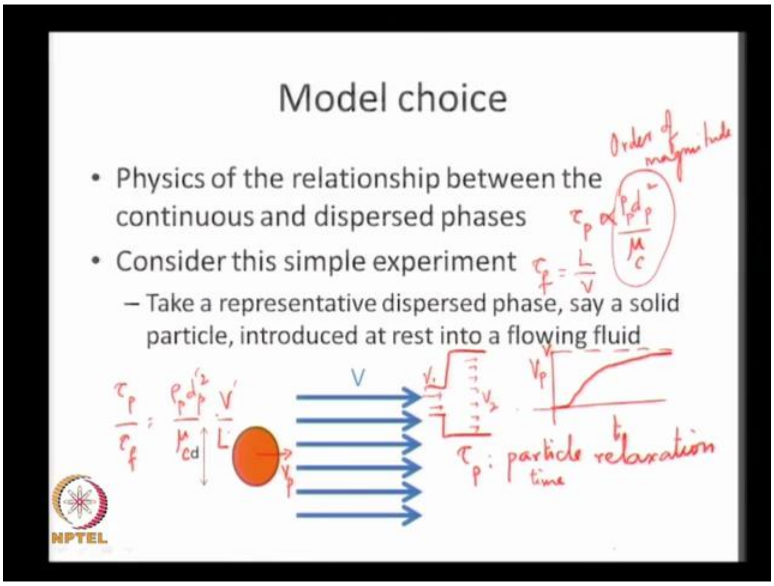

All right, so will now move into a quantitative description of how to arrive at these modeling choices? So, we have the first model which is the exact model that is valid in all two phase flow situations except it is computationally intensive, we have the second model which is the mixture model which is valid in situations where. Firstly, I can postulate a continuum for the dispersed phase. Secondly, the continuum of the dispersed phase and the continuum which is the continuous phase are both moving in a locally homogeneous flow field that is they are both they both manifest the same velocity locally at every point, if this be the case you can postulate a mixture model for that situation.

So let us see, let us do a simple thought experiment. So, let us say I have a certain particle size embedded in a flow filed, and I have certain the size of the particle d in a velocity in a fluid flow velocity v. If I essentially if I let the particle be introduced into this flow field at rest, there is an acceleration of this particle to the velocity of the fluid v. So, if I plot the velocity of the particle versus time or verses length along the cross section you will find that velocity reaches the magnitude v.

Now, you can see that this is typically what is referred to as a first order process; there is an exponential rise it is and with exponentially and asymptotically reaches the fluid velocity v. So, there is an acceleration filed locally that this particle is subjected to and the particle is responding in to that acceleration filed. I want to understand the time taken by the particle to respond to this acceleration filed, and that is what is often called the particle relaxation time that is if one of my dispersed phase particles I say particle it could be a drop a bubble or a solid particle it does not matter, if one of my dispersed phase particles is introduced at rest locally into basically is introduced in an acceleration field the time taken by that by the particle to reach the velocity locally at every point is proportional to this number, where rho p is the density of the particle d p is the size of the particle and mu c is the viscosity dynamic viscosity of the continuous phase.

Now, the proportionality constant here depends on the shape of the particle its spherical oblates spheroid cube things could be slightly different, but that constant of proportionality is not important, what I want to know is the magnitude the order of magnitude of this number in relation to. So, this is one of my length scales the other is what we will call the flow time, the flow time is essentially some bulk length scale in the problem divided by v.

So, if I am looking at the flow of dispersed phase in a pipe of length l, where the continuous phase is moving at the velocity v; that means, this continuous phase from the time it enters the pipe to when it leaves the pipe the residence time in the pipe itself is this number l over v, and if the particle relaxation time is much is comparable to the residence time itself or greater than the residence time then within the duration of time that the continuous phase resides inside the pipe the pa the dispersed phase, which as my bubbles will never reach local equilibrium locally homogeneous flow condition.

Because, I have this continuously flowing fluid, if I introduce a bubble there is this exponential process by which that bubble as to reach an equilibrium condition with the continuous phase, and that takes a time proportional to rho p d p squared by mu c, if that time is comparable to my l over v which is the time residence time itself is much is comparable to the residence time itself or greater than the residence time then within the duration of time that the continuous phase resides inside the pipe the pa the dispersed phase which as my bubbles will never reach local equilibrium, locally homogeneous flow condition.

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34:41) itself (Refer Time: 34:52) to my l over v which is the time residence time itself, then it will never reach equilibrium locally at every point.

So, this rho p d p squared by mu c is a time and l over v is another time, the competition between these two times tells me whether I should use mixture model or not. So, for example, if I right the competition in the form of the ratio what do I get rho p v over l. So, this is a dimensionless number and this dimensionless number has a meaning if this dimensionless number it is much less than 1, I am ready to use the mixture model mixture model as one possibility, if this dimensionless number is much greater than 1 I have to go to the two phase model.

Now in order for me to like I gave you a recipe for choosing between mixture model and two phase model, but understand that I need d p v and l rho p, I am assuming you know the properties material properties of your phases, but you also need an approximate size of the particle you are expecting you also need an approximate size of your domain which you may know easier then d p and you also need locally at every point what is the residence time. So, do not locally at every point, but you do need some estimate of residence time which is l over v.

This if you if I now take the situation where I am dealing with little tiny air bubbles in involved in honey embedded in honey and say for example, I allow the honey the air bubble laid an honey to flow through little sudden expansion, you know that the sudden expansion process involve the sudden deceleration for the on the fluid right. So, the fluid velocity is decelerated from some value v 1 to a smaller value v 2 here, and this deceleration is experience by the air bubble as well as the honey, the honey responds to it just like a single phase my question is do the air bubble also respond to it as fast as the honey does if they do. So, this is a competition such as this tells me whether the honey also responds to the air bubbles locally at every point the air bubbles also respond to the acceleration locally at every point just as a honey does, if so then the time scale of that that response time being small tells me that I am ready for again a mixture model.

So this is one competition so like for example, I showed you how the ratio of the particle relaxation time to the flow time is important the particle relaxation time can also be cast in terms of the local acceleration field, because if I go back to my thought exam thought experiment where I take this honey with lots of a little air bubbles in it I am moving it in the positive direction with the velocity v, and I instantaneously turn around and come back in the negative direction with the velocity v. So, that instantaneous acceleration or you know deceleration is experience by both the phases and I want to make sure the relaxation time of the air bubbles to a locally homogeneous flow condition as soon as this acceleration is experienced. So, from the instant the acceleration is experienced how long do the bubbles stake to reach a locally homogeneous flow condition back again, that is his that is a parameter that determines whether mixture model is applicable or do I need to go to a two phase flow model.

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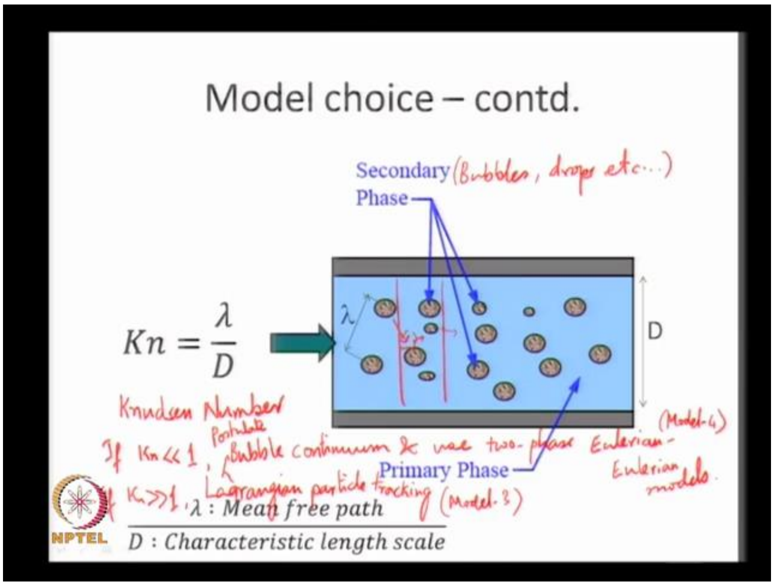

Let us look at another possibility, which is defined as which is based on the particle, particle interaction. So, this is intended to quantify the level of particle, particle interaction and I want to introduce a definition called the mean free path. So, if I take for example, a primary phase fluid say let us say my bubbly flow flowing through a pipe these are all the bubbles which I have called the secondary phase, the free path is defined as the distance traversed by one of these bubbles after it is just collided with a fellow bubble to where it collides with another fellow bubbles fellow bubble. So, it is essentially the distance traversed by one individual bubble between two collisions.

This distance is called the mean free path. So, for example, let us say this bubble comes now mind you these are these bubbles are all flowing. So, if this bubble, these two bubbles where to collide instantaneously here, and as the result of the collision let us assume there is no break up or coalescence or any of that let us say this takes off in this direction, there is another bubble that is also moving in some direction, some distance later these two bubble should again collide, if there is a relative velocity between the two bubbles that is one free path, if I sit in one small spatial region and sample all the free path. So, of all the in between individual pairs of collisions and do an arithmetic mean over all the free paths that gives me an indication of the average distance travelled between two collisions by any given bubble, this is called the mean free path.

Now, I will just pose to limiting cases if I have bubbles that are laden in honey, the honey is very viscous and all the bubbles are laden in the honey and there all moving what would be the mean free path for that kind of a situation its infinite in the bubbles never collide, if they do there is one chance collision the chance of another collision is extremely small so they travel a very long distance.

Now, I do not have to really go define what infinity is as far as a problem like this, whether it is experiment or computational simulation my infinity is the flow domain. So, if the mean free path is very large in comparison to the characteristic lengths scale which is my inlet to outlet total length I know I am in a regime where the chance of collision is very, very small.

So the mean free path divided by d a characteristic length scale. So, in this case I have indicated the characteristic length scale as the diameter, but one could also choose this length l as being the characteristic length scale, some indication of the physical size of the domain. So, if this ratio of the mean free path to the characteristic length scale is very, very small a; that means, something if it is very large it means something else this ratio is refer to as the Knudsen Number. If the Knudsen number is very small; that means, the means free path is very large, very small in comparison to the physical size of the domain that is his collisions are happening very, very rapidly, rapidly is a wrong word rapidly as a connotation in time beware of these things.

If the mean distance between collisions is very small in comparison to the total length, so over the life time of one of these dispersed phase entities in the length of the domain. It may have undergone several collisions; that means, what it means really for the case of for our modeling choice where does it make a distinction, if let us say I take the situation of all the bubbles and the continuous phase somehow being at rest. So, this is again a thought experiment everything is at rest, but they are separated by some distances if this bubble acquires suddenly acquires the slightly different velocity.

The question is in the absence of any drag forces how long can this bubble sustain its velocity, in the absence of any other external forces such as drag or buoyancy how long can this particle sustain its newly acquired velocity, if the frequency of collisions is very large it is going to have to give up some of its momentum to other fellow particles, that is there is a natural diffusion process of this momentum to other particles it cannot remain, it cannot keep its momentum and keep going in a certain direction.

That means, this dispersed phase distribution has all the attributes of a single phase fluid that is if there is a natural diffusivity of momentum, I could argue the same thing for let us say thermal diffusivity or let us say mass diffusivity concentration base diffusivity. So, if I have a very high frequency of collisions in the life time of the bubble inside my domain, it cannot live its life under only the action of external forces other fellow particles have a great influence on it that is a diffusional process; that means, I am ready to make the assumption that these bubbles I can postulate a continuum to replace these bubbles.

So, if I estimate the Knudsen number, and if this Knudsen number is very small I can postulate a bubble continuum, you can use two phase Eulerian; Eulerian models. If this Knudsen number is very large that is the bubble seldom collide; that means, my particle, particle interaction is weak if not completely upset, I am ready to use Lagrangian particle tracking. So this is my Lagrangian particle tracking is my model 3 in the hierarchy that I described, this is my model 4.

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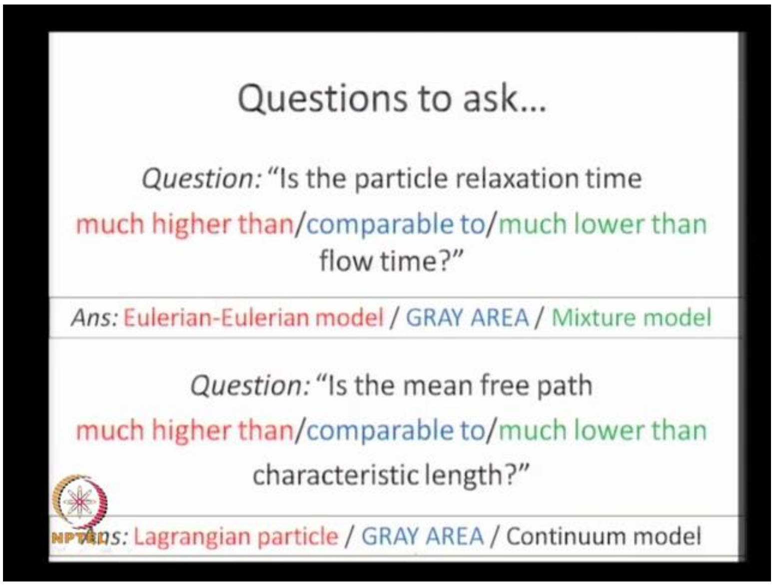

So, based on two questions you ask is the mean free path much higher than comparable to or much lower than the characteristic length, based on the answer to this you can use either Lagrangian particle tracking or a continuum Eulerian, Eulerian approach. If the particle relaxation time is much higher than comparable to or much lower than the flow time then you can use either a mixture model or the Eulerian; Eulerian multi phase model, there is always a little gray area in the middle when the Knudsen number is order 1 or when the particle relaxation time to flow time ratio is order 1.

Those are all those are all gray areas where modeling choice is not obvious and you have to have other justifications and reasons to choose one or the other, but like we have always said the exact modeling approach is always valid, but is very expensive computationally and as far as modeling choices 2 3 and 4 here, is a pair of questions you can ask and based on the answers to those pair to that pair of questions you can decide whether you need to be using a continuum model or discrete particle tracking or Lagrangian particle tracking model.

Will stop here will come back and continue our discussion of application of these multiphase flow models to sprays.