Spray Theory and Applications
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Lecture – 36
Multiphase modeling – Selection of model – 1

Welcome back. We will continue our discussion of Multiphase Flow Modeling. We were currently looking at taxonomy of multiphase flow models, to look at what are the different approaches and what level of detail do they give us.

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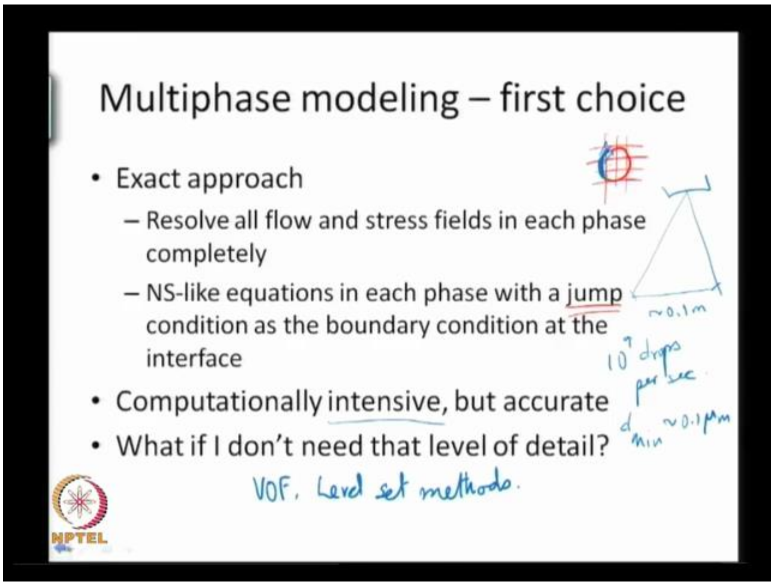

Multiphase modeling – first choice

- Exact approach
  - Resolve all flow and stress fields in each phase completely
  - NS-like equations in each phase with a jump condition as the boundary condition at the interface
- Computationally intensive, but accurate
- What if I don't need that level of detail?

VOF, Level set methods.

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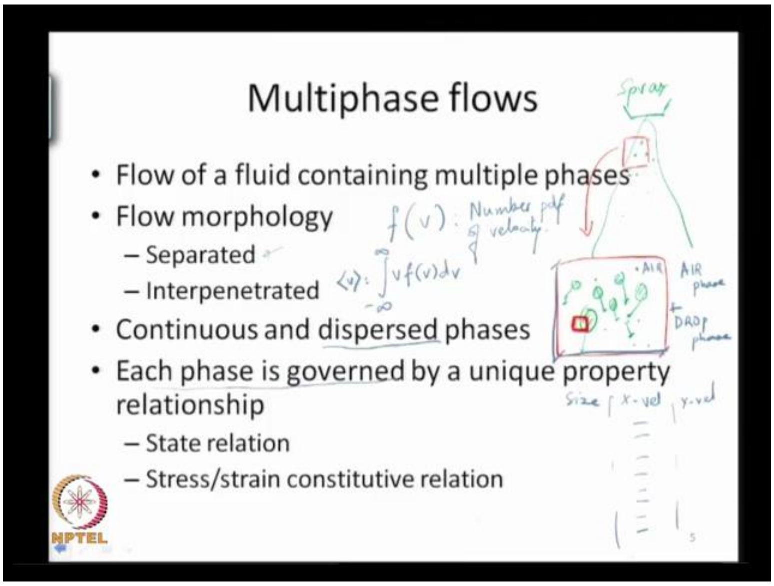

Multiphase flows

- Flow of a fluid containing multiple phases
- Flow morphology
  - Separated
  - Interpenetrated
- Continuous and dispersed phases
- Each phase is governed by a unique property relationship
  - State relation
  - Stress/strain constitutive relation

So, I will start of where we left off we started looking at phase as an entity that is separate from the discrete particles that make up that entity. So, we said each phase is governed by a unique property relationship either state relationship or and our stress strain rate constitutive relationship. And when we apply the different, we start to the most exact model where we talk of resolving all the flow and stress fields in every phase writing a set of jump conditions to just to go from one phase to another. This is the highest level of detail we can get in a continuum model.

So, where the water and air are both continua this is the highest level of detail we can get. And this is typically what is implied in volume of fluid and level set methods where we want to look at every entity every drop or bubble or whatever is the multiphase flow problem on hand and resolve the flow field inside every drop and every bubble. And if I write the set of the jump conditions appropriately like, we said last time I can also model break up and I can also model coalescence in some conditions.

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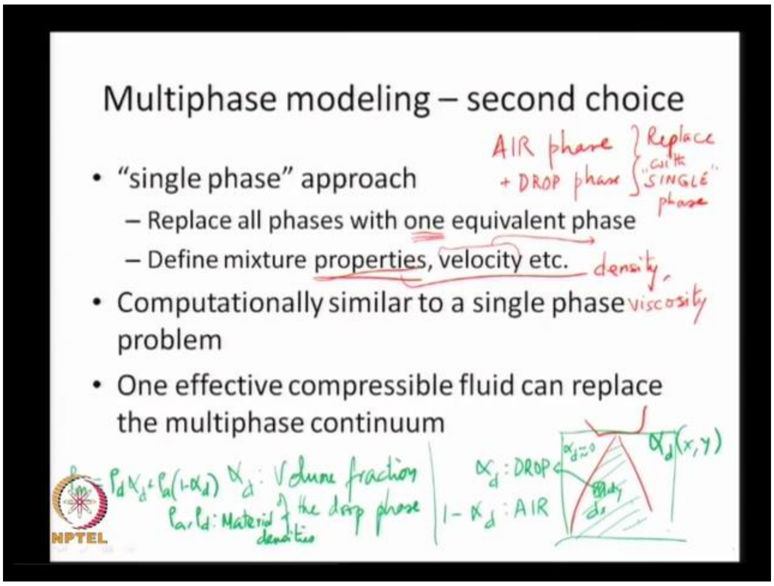

Multiphase modeling – second choice

- "single phase" approach
  - Replace all phases with one equivalent phase
  - Define mixture properties, velocity etc.
- Computationally similar to a single phase problem
- One effective compressible fluid can replace the multiphase continuum

AIR phase + DROP phase } Replace with "SINGLE" phase

density, viscosity

$\rho = \rho_d \alpha_d + \rho_a (1-\alpha_d)$ α_d : Volume fraction of the drop phase

ρ_a, ρ_d : Material densities

α_d : DROP

$1-\alpha_d$: AIR

We will jump to the second choice and we will jump all the way to the other end of the spectrum. So, we are start we looked at exact modeling as our so called first choice, and second choice I have here is where we looked at all the way to the other end of the spectrum and which essentially involves using a single phase approach. So, if I take a spray, a spray is composed of drop phase and air phase we have already defined what these terms mean.

So, instead of talking of drop phase and air phase as being to separate phases we look at the two as being as composed of one phase having one unique property relationship, but of having a variable density. So, say for example, in a certain part of the spray; if the volume fraction of the drop phase is higher than the density correspondingly in that local part of this mixture of the two phases would be higher. I mean air is about 1 kg per meter cubed and water is about a 1000 kg meter per cube.

So, if I have a larger volume of the in a certain space occupied by water drops or liquid drops that local region is going to look like a fluid having a density close to water or if it is the volume fraction is closed to 100 percent or if the volume mean it never gets to 100 percent. But let say, the volume fraction is high locally that amounts to a high density fluid and in regions where the volume fraction of the drop phase is low, it amounts to a region of low density.

So, essentially we replacing all the phases in the problem and in this case we are looking at a drop fluid or a drop phase and air phase, and replacing the two phases together with one equivalent phase. And that one equivalent phase has mixture properties say, when I say properties in the first thing that come to my mind is density viscosity of that phase, these are as we will see a little later on. These are not trivial especially viscosity is not trivial to define.

So, we will figure out a way to define it at least we will figure out what the state of the art is in trying to define viscosity of this of this mixture phase and then you can define other dependent variables like, your velocity pressure etcetera at the mixture level. So, this is I have now defined a third fluid. So, we had two fluids my air phase fluid drop phase fluid and now I am defining a third fluid as a composition or a mixture of these two fluids. And we will only ride balance loss math balance and momentum balance we only write momentum balance for sure, for only the third fluid we will write mass balance for the first and second fluids.

So, where we are the simplification here is that as for as momentum balance is concerned this is a single phase, but each phase has to balance it is mass. Now I want to get the physics of how this taxonomy works to you first, after I go through this we are going to look at the mathematics associated with each of these modeling choices, but our first objective is going to be to understand the physics of what is underline each of these models. Let us the first one was the exact approach, so while we are talking of what is underlying the what are the assumptions underlying the modeling choice we will also understand what are the computational requirement and what are the grid requirements for each of these modeling choices.

Let us quickly summarize the, our modeling choice number one, was where I choose a grid, so fine that I am able to resolve the flow inside every drop including the smallest of drops. And I choose my time scale, so small that I am able to capture the flow or all the droplet level phenomena whether it is coalescence break up or whether it is some kind of a circulation inside the drop whatever be it I want to capture all of that at the I want to cap my time scale is such that is I capture all of that. That is my first modeling choice what we call the exact approach.

The second modeling choice is a replacement of the two phases. So, I take the air phase I replace the two phases with the single phase. So, first, let us first understand the resolution requirements the moment I use the word drop phase. The moment I use the word drop phase, I am looking at a scenario where I do not see individual drops I am seeing this smeared white fluid in the case of a picture taken from a spray let us say, Ok. So, at what length scale and time scale would I see this smeared white fluid, it is like if I took a picture of a very fast moving spray I would just see these sort of streaks of whiteness at what length scale and time scale would I see those would I see that as the picture of my spray if my length scale is larger than the largest drop. First of all I cannot see, if my length scale of resolution is so small that I am able to resolve any of the drops including the largest drop, I am not in a at a level of picture where I am seeing this smeared into a phase.

So, if the largest drop in my system is say, 100 microns I cannot allow my grid to become much smaller than that, 100 microns. I have to remain on the scale of 100 microns likewise, on the time scale I cannot I am looking at this smeared entity, so I do not want to go to the level where I am capturing every individual drop oscillation. Say for example, this is the multiphase model where you are looking at a phasic interpenetrating phasic continuum approach is only valid when I am looking at this smeared out picture.

So, at that level I have the drop phase and the air phase and every position in the spray. If I take a little spray every little position in the spray, has a volume fraction alpha of the drop phase and 1 minus alpha of the air phase. So, alpha d is the volume fraction of the drop phase. So, this volume fraction now becomes our defining variable in terms of telling us how much of each of the phases is in any given special location. So, if I know alpha d as a function of let us say x and y simply, to start with relief time out for a moment. It really does not make any difference if I look at this entire spray alpha d takes on a different value between 0 and 1 at different points in the in the spray.

Now, I do not really know this red line a spray is like a smeared density. So, in this I know alpha d in some domain including the spray. Now potentially alpha d here, is very small is very close to 0, because there are hardly any drops in that parts, but if I take a domain I know alpha d as a function of x and y in there and I can define a mixture density.

Student: Sir what happen if the (Refer Time: 12:29).

Show you, I mean just a marker, right? I can have alpha 1 alpha 2 alpha 3.

Student: (Refer Time: 12:37) product since (Refer Time: 12:39).

If I have 10 phases also each one of the volume fraction is above 0 to 1.

Student: Each one of (Refer Time: 12:52) so, there (Refer Time: 12:55) more than.

If I have a 3 phase problem I have at least.

Student: Two volumes.

Two independent volume fraction alpha 1 plus alpha 2 plus alpha 3 equal to 1.

Student: (Refer Time: 13:02).

Plus rho a times 1 minus alpha d, where rho a and rho d are the material densities. These are actual properties of the material, that are measurable with any sort of a let us say water density can be measured by a hydrometer, air density can be measured by other means; so these are measured material properties. But my single phase which is the composition of the air phase plus the drop phase, now has a new property called mixture density in this mixture density I can write as rho d alpha d plus rho a times 1 minus alpha d for the case of a two phase problem.

So, this rho m now becomes a function of alpha d. So, if alpha d is a new state property of my system. So, alpha d is a different and different point that is a property just like let us say, it temperature in a heat transfer problem alpha d is a property that I know at every point in my system and locally at every point I know rho as a function of alpha. So, this in some sense is rho is another property alpha is another property these two a related through this kind of a relationship, this is looking a lot like our state property relationships or more specifically like our ideal gas law. Ideal gas law relates pressure to temperature through density pressure density and temperature.

Here is a simple way by which that; density of my single fluid is related to another property of that fluid called alpha. So, I can now define other properties also similarly I do not want to go in to that with before we going to more detailed mathematics associated with this model, but essentially what we have done is we replays these two fluids or we replays the spray with one continuous with one continuum fluid. Let us say I called this is the single phase fluid, but you may call it the spray.

This spray has variable properties at different parts of the spray alpha d is different in different parts of the domain and rho the density of this is different and different parts the domain. So, this is got all the look and feel of a compressible fluid that is essentially if you get down to the mathematics any sort of a multiphase problem model as a mixture essentially becomes a continuum problem of a continuous fluid of a compressible fluid. So, once we do this I have replaced all of the properties I have replaced the air plus drops in everything the single fluid now I just have to solve my balance laws for that one single fluid.

So, that is essentially you know where we solve a mass and momentum balance for that single fluid, but in trying to solve for the mass balance of that single fluid I have to account for the fact that, I have two phases that are composed that make up this mixture and that I have a mass balance of the mixture, but the mass balance of the mixture is exactly the same as individual mass balances of the two phases separately. So, we will see what this means and in terms of the mathematics a little later on, but for now we have replaced our fully complicated droplet laden flow problem as though it is a single phase with the variable density, variable viscosity, variable other properties. But what it means is by in doing the single phase sort of assumption. The single phase has a unique velocity at every point just like a single phase fluid flow problem.

Now, in all these discussions we are going to ignore that these ignore any kind of turbulence related effects that is a vast area in itself I just want to introduce the modeling complexities and the modeling taxonomy in the context of laminar flows. So, if I have a laminar flow, in pipe single fluid water flowing on a pipe I know the velocity uniquely at any point inside the pipe. Just like that, if I know the single phase flow I know the velocity uniquely at every point and that velocity is the velocity of this single phase and that single phase being composed of the air phase and the drop phase means, that the air and the drop phases are moving at every point in this domain with the same velocity. So, as for as our modeling assumptions are concerned this is the most restrictive in some sense that is, that we are saying in writing this mixture model that at every point in my domain the air phase and the drop phase are moving with the same velocity that is the only way under which I can only manner in which I can make this assumption that I am dealing with the single fluid.

So, before you embark on any kind of a modeling exercise you have to first understand the assumption and see if those assumptions are valid for the physical system you are trying to model. So, we will look at that either later today or in the next class. So, this is our second choice we will come back and revisit the mathematics of each of this choice later on.

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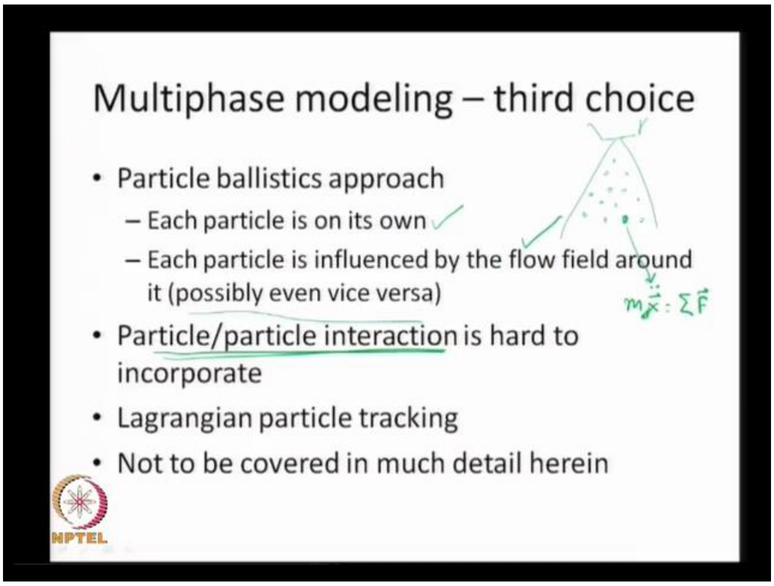

Multiphase modeling – third choice

- Particle ballistics approach
  - Each particle is on its own ✓
  - Each particle is influenced by the flow field around it (possibly even vice versa)
- Particle/particle interaction is hard to incorporate
- Lagrangian particle tracking
- Not to be covered in much detail herein

$m\ddot{\vec{x}} : \sum \vec{F}$

The third choice is what we will call the particle ballistics approach. So, like for example, I know that I have a spray I will draw this one more time and I know the, it is composed drops. And each particle is on it is own. So, I can write for each particle I can write an equation like m x double dot equals sum of all forces.

So, if m d is the mass of that drop I can write an equation that governs each particles motion of this form. This forces what are all the forces that could act on this particle one, they drag force from the surrounding air; two, if there are other particle colliding with this particle and pushing it or creating an impulse force creating any kind of an additional force to accelerate or decelerate this, in this my one particle under consideration. Or I could have other external forces gravity some sort of an electromagnetic force, if I am looking at other effects etcetera.

So, the idea is I take one particle and I know the set of forces acting on that particle alone I can write a set of equations for that particle and I write each I write an equation like this for every particle that is in my domain as soon as I do that I have as many equations as I have a number of particles in the spray. So, typically again we go back to this number let say 10 power 9 particles I have to write 10 power 9 such equations and solve them.

Now, at some level this is still intractable, but there are some assumptions that underlie that are use to simplify this kind of a modeling approach where instead of solving 10 power 9 equations for 10 power 9 particles, there are ways to group them into packets this is the term that often used and in this lagrangian particle tracking literature where these packets are now allowed to propagate forward in time. So, packet is not one drop, but sort of a collection of drops. So, the reason I do not, I mean what I want to do here is understand the different limiting cases and I can always do sort of a combination of these models. So, a packet is like a volume fraction that is being advected by itself under the action of other external forces. So, this is like, phasic assumptions super laid over a particle ballistics model.

So, you can do all of these other combinations and every combination gives rise to a slightly different view of the spray, but what I want to is restrict myself to air we know the limiting models the different exact and approximate models and how the other models arise out of combination of this. So, essentially the art of this kind of modeling approach is the extent of this particle-particle interaction. So, I am not looking at the packet approach that I discussed earlier, I am looking at the individual particle level approach. And at the individual particle level approach like I said, each particle is on it is own each particle is influenced by the flow field around it and possibly even vice versa.

So, if I have considered the case of one particle sort of moving in a stationary medium. So, that one particle just like let us say cricket boll thrown from the outfield from near the boundary is on it is way to the wicket keeper and the particle is influence by the air, but the air is also influenced by the particle by the ball. So, the balls motion after the ball has completed it is flight from the fielder to the wicket keeper you still have some ruminant motion of air somewhere along it is path.

So, if I was to throw successive balls from the fielder straight to the keeper along the same trajectory the succession of balls do not take on; do not experience the same force as the first ball thrown. This is essentially the meaning of the vice versa interaction. That is, the particle influences the air which intern could influence the other particles slightly differently. If I am interested in where the particles are going, why I am interested in this vice versa interaction because that altered air field could affect the particles slightly differently than the first particle.

So, I can write a set of equations for the particles of this form and the air phase is now modeled as a single phase fluid, but each particle is a source of momentum if it is moving and giving up some of it is momentum. So, this is essentially the physics of how particle tracking works both one way coupling. So, that is each particle is influence by the flow field alone would be called one way coupled model, one way coupled particle tracking modeling.

If you allow the vice versa thing to happen also that would be a two way coupled particle tracking model. I do not want to go in to that go in to this kind of a modeling approach in any modeling more detail, only because of the following reasons if I want to look at the particle level model just like I have discussed it again I keep going back to my length scale time scale requirements. At what length scale and what time scale can I say, I can write a equation like m x double dot for that particle equal to some of forces, first I need to be able to see individual particles, right. If I do not see individual particles I cannot right it what I mean by see is if I took an image of the spray at some resolution.

I can see individual drops at some special and temporal resolution we have already discussed this, likewise a I have to have a grid fine enough where I am seeing individual particles or rather fine enough that I am seeing individual particles, but course enough that I am not going into every particle same goes with the photography requirement they are exactly analogue us. So, if I now take this m x double dot equal to some of forces the grid on the air phase associated with writing a model like with this for the particle phase is such that I am able to see individual particles, but not a collection as a fluid at the scale at which I am resolving and typically, this is only valid far away from the nozzle where I can actually see individual particles.

So, if I was spray nozzle in the dense spray region very close to the nozzle the requirements, the resolution requirements should be very stringent for may be for me to be able to see individual particles. So, the region where a model like this is tractable is far away from the dense region where I have gotten into a region where particles are in individual flight particles or drops are in individual flight. So, this is in the rarified spray region that this kind of a model is accurate and tractable.

But in most spray applications while this model is easy to implement and actually very useful in most spray applications especially, if I am interested in the dense spray physics something like this would not be very useful. But if I am, if we it a sufficient for me to know where the particles are headed, after all the particles are formed and after the particles are in this rarified region, rarified spray re regime then a modeling like this would be quiet ok.

So, these are the sort of underlying assumptions, we may talk of to mathematics of this little bit later on, but for now will say we will go over to the forth choice.

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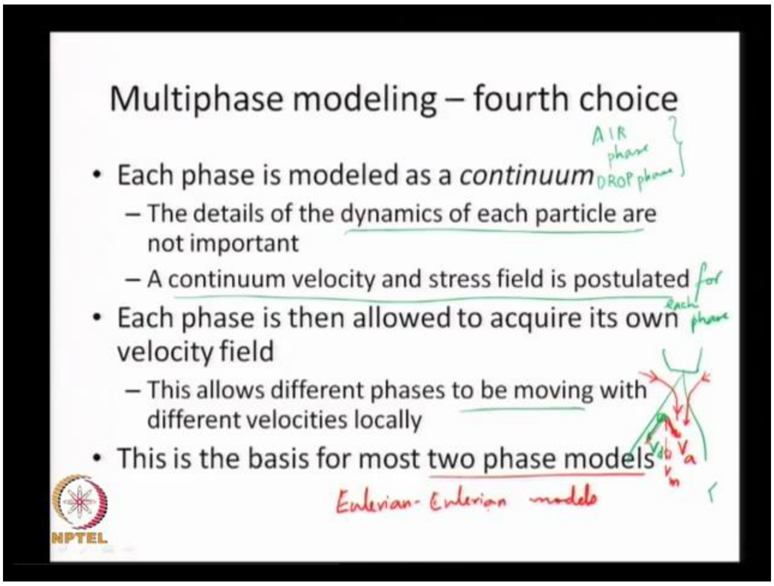

Multiphase modeling – fourth choice

- Each phase is modeled as a *continuum* AIR phase DROP phase
  - The details of the dynamics of each particle are not important
  - A continuum velocity and stress field is postulated for each phase
- Each phase is then allowed to acquire its own velocity field
  - This allows different phases to be moving with different velocities locally
- This is the basis for most two phase models

Eulerian- Eulerian models

A fourth choice is where I am looking at each phase as a continuum. So, here I have the air phase and the drop phase we already defined these terms. So, the moment I write drop phase, you know the sort of resolution at which you have to take this picture you also know the great requirement. It is at a level where I do not see individual drops I am only seeing this smeared fluids both temporally and specially.

So, this is a requirement this is a model the fourth choice is a model where each phase is allowed to be modeled as a continuum, except that these two continua or interpenetrating. That is at every point I have this alpha d a marker or a volume fraction that tells me, the fraction of the region fraction of the spatial region that is occupied by the drop phase and 1 minus alpha d as the fraction of that spatial region that is occupied by air.

And the again like I said that, the dynamics of each particle are not important that is the that is the whole idea of going to this phasic level view of my spray and a continuum velocity and stress field is postulated for each phase this is the distinction. If you go back to our mixture level or a single fluid level approach we said that, each that the two fluids together are a mixture are mixed into a single fluid and your only defining a velocity for that single fluid.

Whereas, in the fourth approach we are defining a velocity and a stress field for each of the two phases in this case we are only dealing with two phases air phase and the drop phase, we are allowing each of the two phases to be moving with it is own velocity field and we are not restricting them to the locally homogenous locally moving with the same velocity. So, what this actually does is now the drop phase can be go where it wants to go and the air phase can go where it wants to go.

So, if I take a spray for example, say for if I take a region here the drop probably want to go in this direction, but the air may want to go in a slightly different direction the reason for this is actually due to entrainment. So, the air as this draw as the spray is like a jet and the jet entrains air from the outside and essentially this is like a representative stream line. And these representative stream lines mean that the air at this point may want to come in towards the middle of the spray, but the drops may want to go in a slightly different direction. in our locally homogenous flow assumption underlying the mixture model, we are not going to be able capture these two distinct velocity fields where only going to get sort of a mixture level velocity field which may look like that.

So, if I know for a fact, that different parts of my spray have drops moving very differently from the air phase and I have reason to believe that we will see a little later on how to a certain whether this is indeed the case before doing the modeling. So, like if I have 4 choices for modeling we want to see without doing a modeling or without doing all 4 models which one is appropriate and we will see that model in choice a calculations a little later, but for now this allows each of two phases to be moving with a different velocity. This is what are often called two phase models or also called Eulerian-Eulerian approach Eulerian-Eulerian models.

Now I have a stress field this stress is of course, the pressure field as well as the shear stress field. So, I have both the normal under shear component of the stress that I have that are included, now for the drop phase what does this pressure component mean. So, if you think of just the drops without the air what does pressure mean in this drop phase, let us be first very clear it is not the hydrodynamic pressure inside every drop because we have discarded that right of front inside the moment I define a drop phase I am not focused on individual drops I am in this smeared out region.

So, when would in this drop phase. So, essentially at this level the drops may not liquid drops they may be steel bolls I have absolutely no problem thinking of just having steel bolls has being the entities making up my drop phase. What does pressure mean in these drop phases, is essentially is like a conjunction of drops in once special region that causes a certain normal stress. So, if I have locally at some point in the domain a high collision frequency of these drops high normal that results in like a normal stress in the drop phase, that is what we mean by normal stress in this drop phase.

It is not easy to imagine this kind of normal stress in the context of liquid drop making up the drop phase, but if you think of these as granular objects like think of these as really steel bolls it is a little easier to understand. That I have a lot of I have a collection of these steel bolls that that tend to are somehow compressed and the microscopic properties, inside the bolls are causing like a macroscopic stress. In this smeared out picture the microscopic stress in each of the individual bolls is resulting in some macroscopic stress in this smeared out phasic level picture.

What does shear stress mean for the drop phase? Again if the drops are not at all interacting that is a case for noxious stress because at the phasic level, even though if I have drops that are not at all colliding or very low probabilities of collision then, the shear stress of this phase is essentially very low. Because the whole idea of shear stress is where you immediately think of a simple quiet flow experiment, if I put this fluid in between two parallel plates and move one parallel one the move the top plate, what is the force required to move the top plate.

If the drops are hardly colliding the force required to move the top plate would be zero, the bottom plate would have to somehow communicate it is presence to the top plate for the force on the top force to move the top plate to become non-zero. That only happens by this momentum diffusion process which happens through collisions. So, if I have very low probabilities of collision; that means, the top plate can be moved at zero force which means it is a case of an inviscid fluid.

So, this idea this idea of viscosity of this drop phase is related to the collision frequency and the collision frequency as you can see is related to the overall volume fraction. So, it is volume fraction itself is low they are not exactly related, but you can imagine that if the volume fractions are low collision frequency is may go down, but I can think of a case where the volume fraction is low, but the collision frequency could be high it just depends on the local velocity non homogeneity that exists at the droplet level, but for now all I want to stress is that this velocity field is at the phasic level so this is the mean velocity of a collection of drops. So, this smeared fluid collection of drops that make up this sheared, this smeared fluid and the stress field is also a property associated with this collection of drops at this phase level.

So, essentially if I have the phase level description I can define a normal stress under shear stress of this continuum and that is related to the microscopic collision probabilities in collision frequencies. So, these are this is basically a what we would called a two phase model and this is also referred to in the literature as Eulerian-Eulerian modeling approach.

We will stop here and will continue our discussion on how to choose between these 4 models during the next class.