Spray Theory and Applications
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Lecture – 33
Multiphase modeling

Good morning, we will continue our discussion of multiphase flows and see one possible root by which we can model these sprays.

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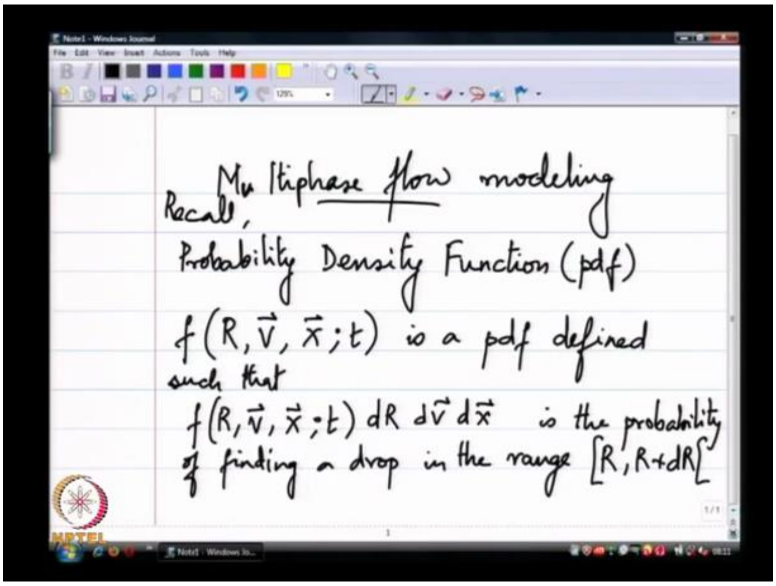

Before we go to the modeling multiphase flows, what I wanted to do is try to see if we can recap some of the information we learnt earlier and concepts will learnt earlier with regards to probability density functions. So, our old definition of probability pdf, if I define in f; I will use R for the radius of the drop and I will write the nomenclature. Defined such that this is the probability of finding a drop in the range R to R plus dR and velocity v to v plus dv and in the space in the spatial location x to x plus dx.

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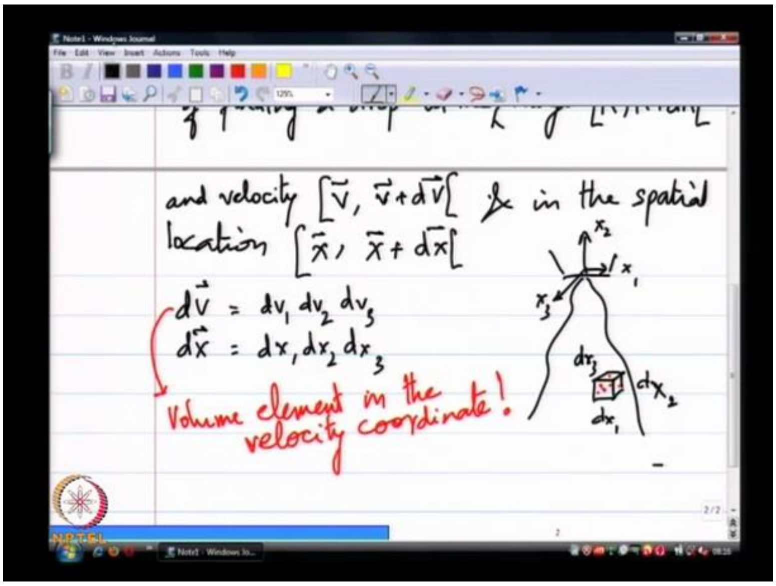

So that is this just quickly understand what this means, if take a spray and I will plant my coordinate system just since we are now talking about, real physical system and modeling real physical system. If I take tiny volume element and imagine this is a bunch of drops inside here. Then this f times this probability density f of R comma v comma x semi colon t; t is time of course, here this basically given by the probability of finding a drop in the radius range R to R plus dR and velocity range v to v plus dv. So, we should be clear about this vector v plus dv means, so I will write that v to v plus dv is just same as or rather let us be it is easier to define dv vector dV is equal to dv 1, dv 2, dv 3 which is the velocity components in the 3 directions.

So it is like a little tiny volume in the velocity coordinate. If I take a drop of radius r, I can easily understand what is the meaning of R to R plus dR, so it is like 50 to 51 microns, what the volume element in the velocity coordinate; this dv vector given by the product of dv 1, dv 2, dv 3; it means finding a drop of velocity let us say 10 to 10.1 meters per second in the x direction, some 1 to 1.1 meters per second in the y direction and let us say 5 to 5.1 meters per second in the z direction. So, drop satisfying all 3 conditions is what will fall inside this dv element. Likewise dx is easier to understand we have being trying to understand this since high school refer to this by x 1, x 2, x 3 makes more sense, this is dx 1 and dx 2 and dx 3.

So, this is easier to understand because it is physical space; it is the space spray like a little tiny volume and I am looking at all the drops that fall inside this volume, that is my starting point of all the drops that fall inside this volume, I want to look at the drops that have size R to R plus dR and have velocities between v to v plus d 1, v 1 to v 1 plus dv 1, v 2 to v 2 plus dv 2 and v 3 to v 3 plus dv 3, that is like a tiny. If you can think of this now as a volume element in the velocity coordinate, so this is I want to make sure we are all on the same page, we are used to spatial coordinate and volume in the spatial coordinate. We are now defining a velocity coordinate and defining a volume in that velocity coordinate. So, I can now define any other coordinate just like this and I can define a volume in that coordinate, the reason I have a volume in the velocity coordinate is because the velocity is a vector.

So, I require 3 components to describe the velocity vector and it naturally forms a volume in the velocity coordinate. Whereas if I took radius; radius is a scalar and I just need R to R plus dR, so essentially I am still defining a volume, but it is a volume in a one dimensional space. So, R to R plus dR defines a volume in one d space of this of the scalar quantity radius of the drop, dv defines a volume in the velocity coordinate in the 3 dimensional velocity vector space dv 1, dv 2, dv 3; dx defines a volume in the spatial coordinates vector space dv dx 1, dx 2, dx 3.

So what we have done is we are used to space as having coordinates, we are now introducing velocity as also being a coordinate that is the fundamental shift in the way we are going to look at this models and I think the reason will do that; we will come to that may be later on. So, the moment I have made this, I defined this multi variant probability density function f with R v and x as the independent coordinates. So, this is in seven coordinates and time as been a parameter, so we are not defining; I can define an instantaneous probability density function. So, at this current instant of time if I gather all drops that are in the spatial location dx, x to x plus dx and parts them into radius bins and velocity bins this gives me the multi variant probability density function.

Now, we looked at this even when we find multi variant probability density functions. The fact that the f at this point is clearly not as the same as f at and other spatial location, now f is a function defined in this seven dimensional space or 3 velocity coordinates, 3 spatial coordinates and the radius of the drop. So, typically we will use the word internal co external coordinate to define the spatial coordinate those are like obvious to us they have been we learnt from courtesan geometry days these velocity and radius are internal coordinates.

Now, I can continue on define many more internal coordinates as many as I need to completely determine the state of a single drop. So, for example, we talked about this in our earlier lecture if I have a mixture of let us say methanol and ethanol that I am spraying then the fraction of methanol in given drop is an internal coordinate of that drop, I can define a probability density in that internal scalar coordinate. So, I can now define concentration in a general sense of some n dimensional mixture as being an n vector in as being n vector and I can continue this argument for forward. So, I just want to bring the distinction between external coordinates and internal coordinates in the way we are going to look at sprays.

So, we want to write an equation that describes the evolution of f in space time and velocity of the coordinate. So, the velocity is now and independent coordinate just like space we are not going to make any distinction between velocity and space. It is like at this point I have a certain drop at this location moving at this velocity, this position and velocity are both characteristics of that drop, they are independent variables that there could be other dependent variables that fall out of this we will look at this.

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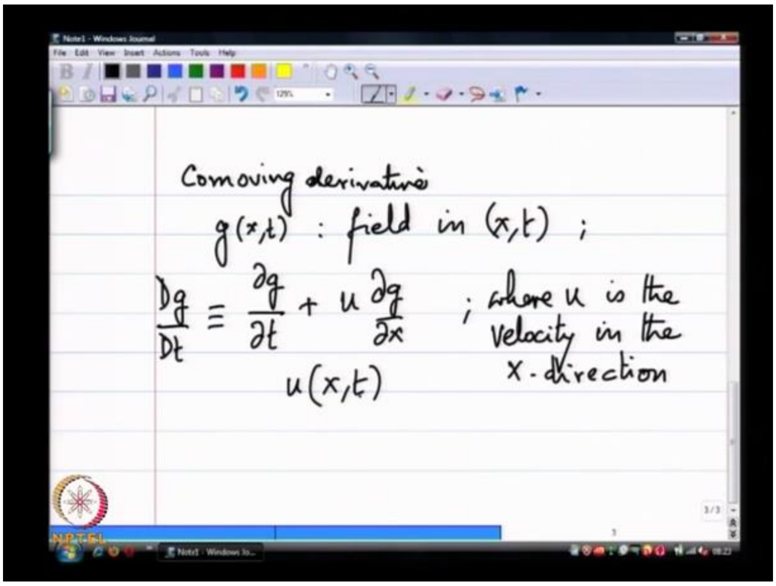

So, now let us go back to the idea of simple comoving derivative in see what; we are used to this say if I some function f or a some function g which is a function of x and time we write a comoving derivative of g, this is what we very often refer to as capital Dg Dt where u is the velocity; scalar velocity in X direction. This comes from basically defining g; as a field in x and time. So, essentially if I had this g is a property everywhere x and time and u defines the transport velocity at every point in space and time. So, u is also a property of x and time, that is generally how we understand fluid mechanics and this comes from basically defining velocity as a field not velocity as a property of a particle or property of a material.

So, if I define velocity as being a field variable depending on this spatial coordinate and time whatever material particle passes through that point in space in time acquires that velocity that is our basic assumption on which our hydro dynamics analysis equations are built. So, if I now take that forward and if I have x and t as my independent variables this is my comoving derivative.

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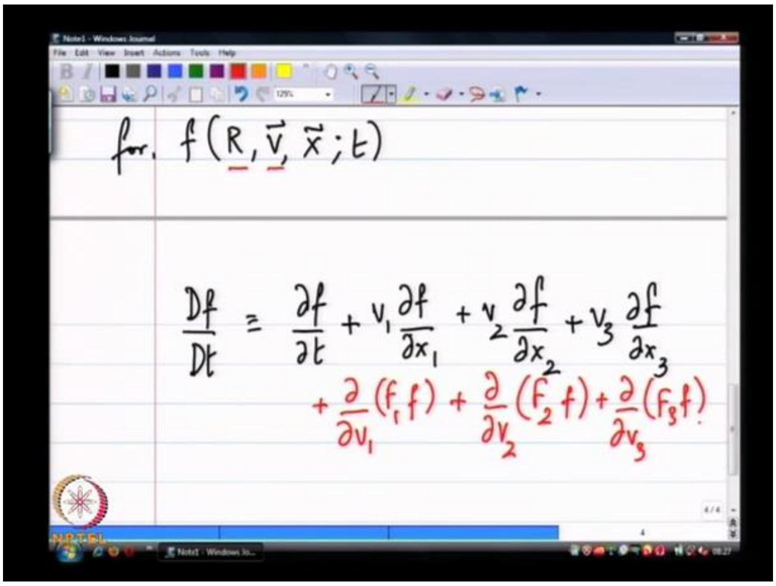

If I continue this forward if I define f like we have here is a function of R v x and t, so for this general probability density function how do we write the comoving derivative and what that does mean; this part is fairly straight forward. So, since f is function of the spatial location x, y, z or x 1, x 2, x 3; I could have an advection of this property f, f is just like our old g any property which is a field which is defined as field, which is defined as a function of x and time then we advected through that in that spatial coordinates and time by their respective velocity v 1, v 2, v 3.

So, if I have v 1, v 2, v 3 as a velocities at a given point of this material f of this property f then the partial derivative of f with respect to time plus v 1, d f, dx 1 plus v 2 d f dx 2 plus v 3 d f dx 3 gives me the rate of advection in space and time, but we now have additional coordinates in terms of v and R which are now defined as our independent coordinate. So, how do we count for those in required additional terms, what does this mean if F 1, F 2, F 3 is a force vector per unit mass acting on the population of drops?

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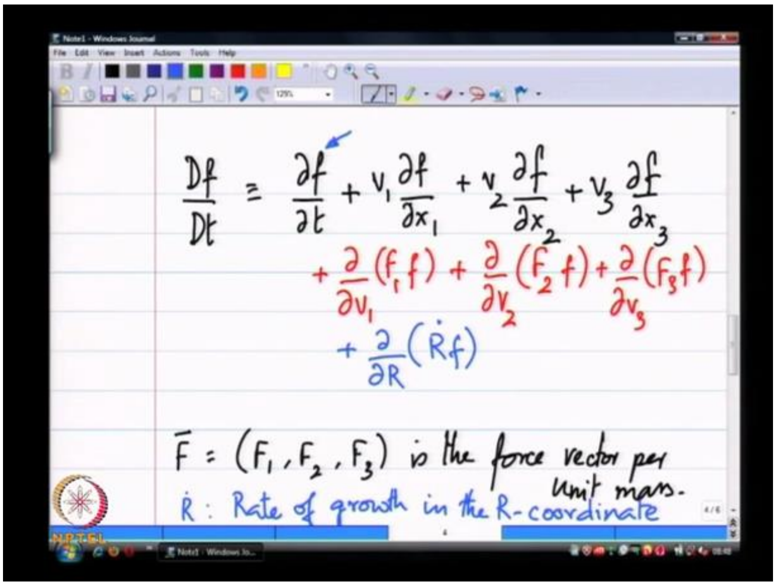

So, just like I require velocity which has units of meters per second to advect the probability density function in the spatial coordinate which has units of meter. So, this spatial coordinate has unit of meter, the corresponding velocity with which the property is advected has units of meters per second. If the internal coordinate has units of meters per second, the corresponding property; the corresponding let us say vector by along which this property is advected in the probability density function is advected in the velocity space will have to have units of meters per second.

So, essentially this force is; I mean F 1, F 2, F 3 is the force vector per unit mass which is what we know it is acceleration, if there is a force per unit mass acting on this population of drops that is going to cause movement of this particle in the velocity coordinate.

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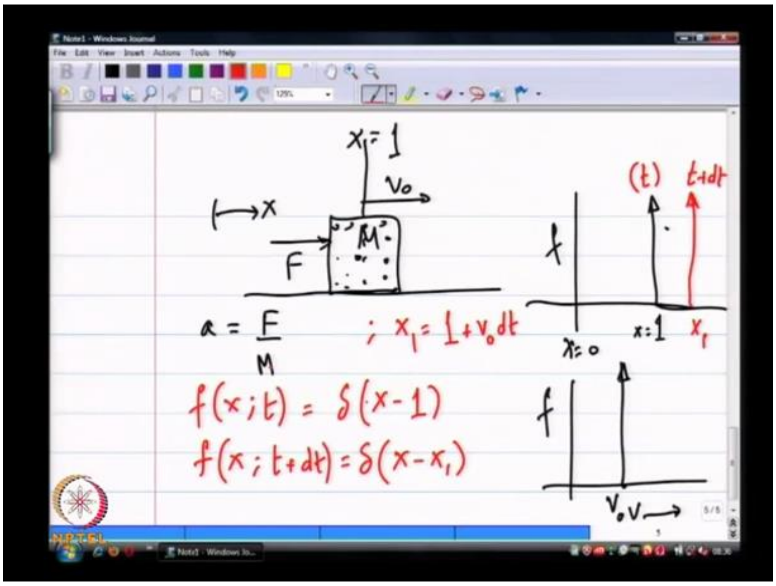

I want to make sure we understand this, let us take a very simple example one we are familiar with high school. If I take; let us say a block of mass M being pushed by a force F, let us say it is initially moving at velocity V. So, the acceleration we know is F over M and we only going to look at one dimension, one spatial coordinate. So, one way to look at say for example, let us currently at this location which is x equal to 1 let us say for x. So, if I look at the probability density function at this x equal to 1 or the probability density function in space, it is at direct delta function at x equal to 1 that is I have a mass at a x equal to 1 and everywhere else there is no mass, so we will just looking at a point mass let us say.

So, I have if I look at the probability of pi of finding particles or number density of particles, we are use to number pdfs and will define F also as a number pdf; the number pdf at every location other than x equal to 1 is 0 and rather than number probability at every other location is 0. So, if I take a location from 0 to 0 plus dx whatever dx may be, the number probability in that spatial location 0 to 0 plus dx is 0. The number probability in 10 to 10 plus 10 dx is 0, the number probability from 0.95 to 1.05 is 1.

So, I have one particle and there is probability of finding a particle around 1. So, in the limit of that dx shrinking to 0, we get our probability density function and that probability density function when plotted in the spatial coordinate takes a form of a direct delta function at 1 and basically it is like a very sharp (Refer Time: 24:16) going towards infinity at 1 with the area under (Refer Time: 24:20) this curve being bounded being equal to 1 that is the meaning of direct delta function and so this is the probability; this is the pdf in this spatial coordinate. If I plot the same pdf in the velocity in the coordinate, if it is initially moving at some velocity v 0.

So I am now disregarding space; I mean it is a little trivial talk of pdf of one particle, but I thought this example should illustrate at least the idea of an internal coordinate as being not very different form an external coordinate that is my objective here. I can look at if I have let us say the bunch of particles; we all understand how to create a pdf in the velocity coordinate. If I do a similar pdf of this one particle there is a probability of finding a particle at v 0 and no other velocity. So, I can at some velocity v 0 there is a direct delta function that says essentially says all my particles have one velocity v 0, all in this sense in 1, but that is a material to the pdf.

So, all my particles in the system have one velocity v 0 and you know I can actually make the argument that there is the mass containing you know many particle and are they all are moving at the same velocity, there are at all nearly the same physical location x. So, mean these things can also apply physically to that system, so now how do I move this pdf in this spatial coordinate by the spatial velocity. So, if the particles have a spatial velocity, if this is the pdf at time t; at some other time t plus delta t, this pdf shifts to 1 plus v 0 dt.

So, at some future time t plus delta t, t plus dt the same direct delta function as moved to a new location x 1. So, the probability density function in x coordinate is now given by delta, so if I define f of x; at time t this is given by the direct delta function at x minus 1, if the probability density function at t plus delta t plus dt is given by this x minus x 1.

So, this t plus so all we have done is velocity in a spatial coordinate has advected this pdf from being peak at x equal to 1 to some x equal to x 1.

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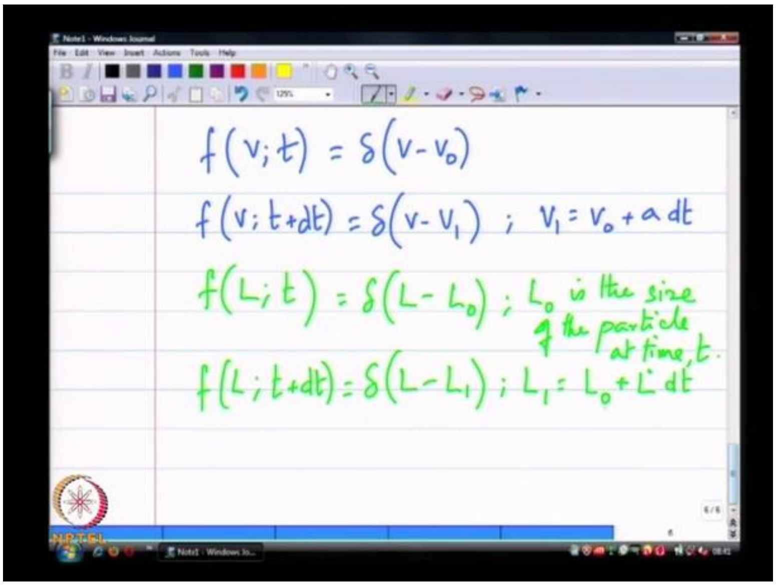

Similarly if I have a force acting on this system and the acceleration is F over M, the f of v; at some time t the f of v we are only writing a probability density function and single coordinate just to sort of understand what is happening and in this case the probability the multi variant or the by variant probability density function of x and v is the product of this two whatever writing here will show you that also. So, I know that at some time t, I had a direct delta function defining my probability density function in the velocity coordinate and the direct delta function was a peak act v 0, the value v 0. At some future time what would this look like, so just as we said we updated its position due to the velocity we say it we update the velocity due to the acceleration.

So, we are in this formulation the velocity is also coordinate along which motion is possible due to acceleration. I want to take one more last little bit of complication, let us say this block was growing. So, let us say initially this is of some mass m or I will say for example, of some size x some size l. So, cubic block or some size l let us say and I have a little you know mass accumulating on here at some L dot or some v dot, some volume is being added to the this block and as a result its size is also changing.

So, this is just like condensation or evaporation process in a droplet. I can now say, I can do something exactly similar. So, let us say initially it was some size l; 0, L 0 is a size of the particle at time t and if I now take at the next time, the same probability density function in the size coordinate if L dot is the rate at which that size coordinate is changing then L 1 given by L 0 plus L dot dt defines the new peak in my pd f. So, notice how size, velocity and spatial coordinate are practically indistinguishable except when a drop grows by let us say condensation the; we talk of not the drop growing, but the drop pdf being advected in the size coordinate space. Likewise if the particles are accelerated we talk of the particle pdf being advected in the velocity coordinate likewise space where use to idea of advection in space, we now introduce the idea of advection in the velocity coordinate and advection in the size coordinate.

Now, I can have advection in any other internal coordinate; I have methanol ethanol methanol is preferentially condensing on the drop at certain rate on drops on (Refer Time: 34:15), so I am taking a whole population of drops to really talk of pdfs. So, if I have some ethanol condensing on the population of drops then we rate of condensation of ethanol is going to advect this population of drops in the ethanol fraction coordinate.

Now in the granular material literature so when I am dealing with let us say crystallization or when I am dealing with powder material flow. If I take a single powder particle, in order to completely define the geometry of a single powder particle I may need to define more than its size, I mean it to define like number of facets or some polygonality or some I can come up with measures that characterize the single particle and let us say I take a whole population of this powder and I pound it and if I am taking what might be perfectly cubic particles and creating particles with more facets. So, this if I define a coordinate like average surface area per unit volume then pounding the cubic particles down to finer and finer particles which are non-cubic amounts to an advection in this area per unit volume coordinate.

So whatever force I am imparting, in this case I am not talking the physical force, but the act of pulverizing this powder is creating an effective advection force; in the area per unit volume coordinate. So, this is the different way of looking at looking at transport of a pdf in a given space, in this case we are looking at x v space, x v R space. So, we still have one more last term to define which is, so here R dot is the rate of growth in the R coordinate. So, if I now look at all these possibilities of advection in the different coordinates, let me take a very simple situation first; if I take just a bunch of drops that all are stationary, but are of different sizes. So, I have an initial f as a function of r, but they are all stationary and in both they are in stationary in position and in not moving, so

I mean clearly the stationary. So, if I now allow some sort of condensation process to be initiated or an evaporation process does not matter, it is a matter of just a sign.

So, R dot is a function of the R current radius of the drop then d d R of R dot times f defines the rate of advection of this pdf and d f dt which is the rate of change of the pdf explicitly due to time. So, this d f dt the partial derivative of; f with respect to time is where the pdf is explicitly evolving in time is due to R dot. So, if everything else in the black and the green red term going to 0, the only the explicit variation of; f with respect to time is due to plus the rate of change of rate of advection of the pdf in the R coordinate, together define the rate of change of this pdf in a system sense.

So, for you to completely understand this I would suggest you read an under graduate text book and understand this just as you said we would Eulerian Lagrangian frames of reference and where we use Reynolds transport theorem to go back in forth between the system and control volume view of a material. So, if you do that d f dt the total derivative is are systemic view of the material all of this right hand side is a control volume view of the material that is if I now define a control volume in x 1 x 2 x 3, v 1, v 2, v 3, R space this is the rate of motion in and out of that control volume.

Now, this alone is not enough, so if I now follow a certain group of drops and follow them; follow that exact same group of drops without paying reference to their velocities, their spatial locations and their size. So, I have marked a certain set of particles or drops and I am only going to follow those, there are we can write a balance law for that group of drops, this is just like our systemic balance law of force and force and momentum where we say the rate of change of momentum is equal to the force acting on the system.

So, likewise this left hand side or right hand side that I have is our control volume view and we have to replace the left hand side which is our systemic view of the fixed set of particles or drops with an equivalent balance law. So, what could be happening to these drops that would cause the probability density function in a systemic view to change, so I will just force this in some sort of physics sense will come back right the mathematics next time. I will just go again to a very simple view of two particles,

Let us say undergoing and inelastic collusion, so I have 2 particles moving towards each other with a velocity v and v in opposite directions with velocity plus v and minus v speeds v and v of the same mass coming towards each other and colliding. If the collision is perfectly inelastic, the velocity of this resultant entity is 0. So, these two particles come collide; let us say coalesce into one drop which going to become stationary. So, or I am going to treat this as you know two drops that are attached each other. So, the probability density function used to have two peaks at plus v and minus v now has become one peak at 0 and there is no external force acting on these particles right the forces are all internal to these 2 particles system.

So, everything on this right hand side bearing of course, if I take that instant where they were all at the same physical location. So, I am going to ignore the black terms for a moment, the black spatial advection terms for a moment you can imagine this happening in a very small neighborhood before and after the collision to say. The advection in all the coordinate is not is absent; you only have a collision driven change in the pdf. So, from a systemic view this is what is responsible for the change in the pdf from having two peaks at plus v and minus v to having one peak at v equal to 0.

So collisions between the particles of the system, so I have a system of particles collisions between the particles of a system is respond could be one force that is responsible for change in this pdf. We will look at what that means to the left hand side of this equation in the next class.