Spray Theory and Applications
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Lecture - 32
Secondary atomization-Modes of breakup-2

Good morning, welcome back. We are going to look at, we are going to extend our discussion of secondary atomization look at a couple of different modeling approaches and then look at generalized modeling approach that you can do for not just secondary atomization system, but something beyond that. The simplest of the breakup regimes for secondary atomization is the Weber number base model which is and the first of those is the vibrational breakup regime for Weber number about 0 to 10.

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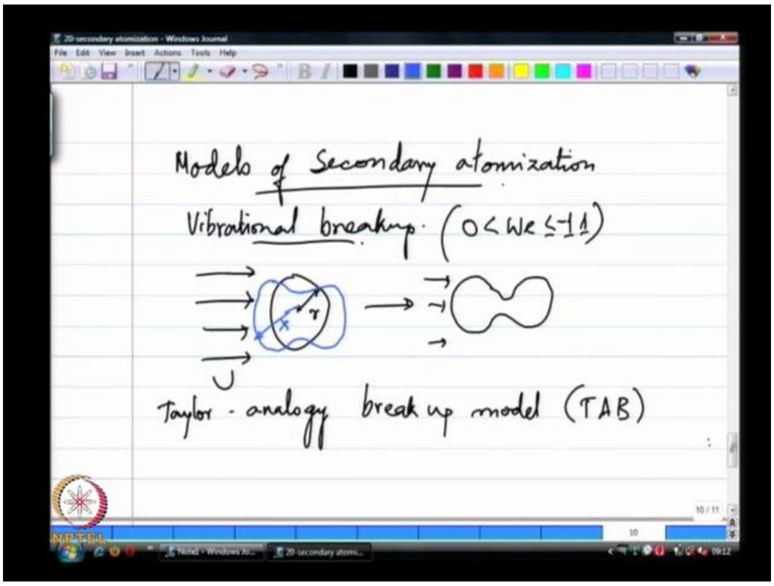

This is what we found yesterday. And for this regime essentially, the physics associated with breakup of a drop in a stream of some velocity u, is that it starts to oscillate and this oscillation can grow in time and eventually cause drop to break up. Let us look at, and the simplest of models that is used to describe this regime of breakup is called the Taylor analogy breakup model; very often called that Tab model. The approach in this model is quite simple, you take a drop of some radius r and superpose over it a disturbance some x.

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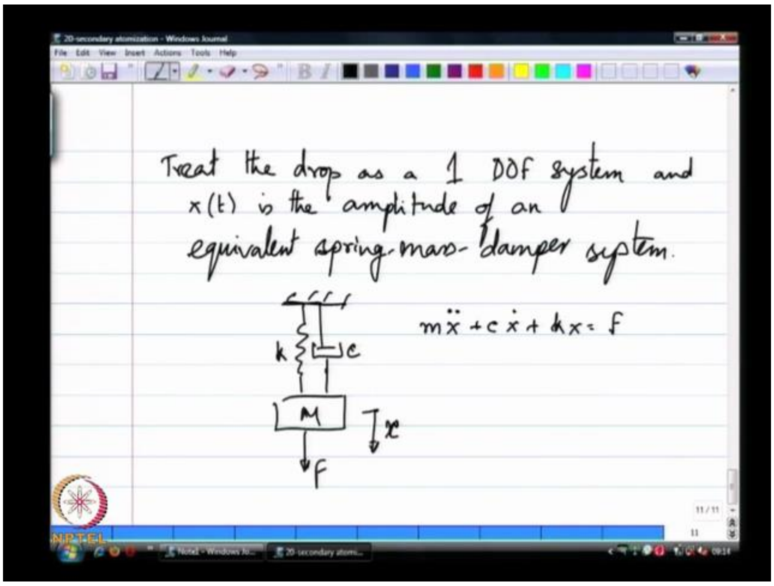

So, you treat this system as a 1 dimensional, as a 1 degree of freedom system; is an amplitude response to an equivalent spring mass damper system. So, you can think of the drop. So, the moment I have perturb the drop, we know that we seen videos of this it is going under go oscillations, and if I impart and impulse force forcing to the drop it is going to oscillate and then come to rest. The reason for the damping is due to the liquid viscosity.

So, I mean it has all the features of regular spring mass damper system, just for the sake of completeness I will write, I will draw a schematic for a spring mass damper system, one kind of a spring mass damper system. And an equation if x is the way is the variable characterize in the degree of freedom, then I have m x double dot plus c x dot plus k times x equals sum F. So, I have a force F acting on it.

So, I have the a drop of a certain mass, the surface tension force acts like spring stiffness and the viscosity acts like a damping force, in the origination of the viscosity is responsible for damping in the system. So it as we are going to make this equivalence.

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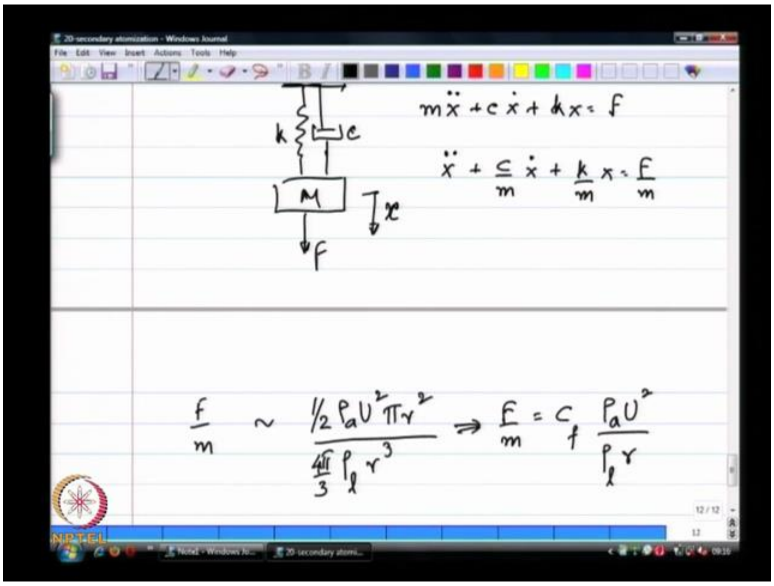

And write down some scaling variables for each of these. So, I will just for the sake of simplicity I will divide all of this by m and will write a scaling for each of these f by m goes as the force acting on this drop is due to the relative velocity.

So, if half times rho a U squared times pi r squared is the a force acting on the drop its sort of a magnitude of the force, divided by the mass of the drop itself which is rho l times 4 pi by 3 r cubed. So, I am going to write this F by m as sum c f times rho a U squared by rho l r.

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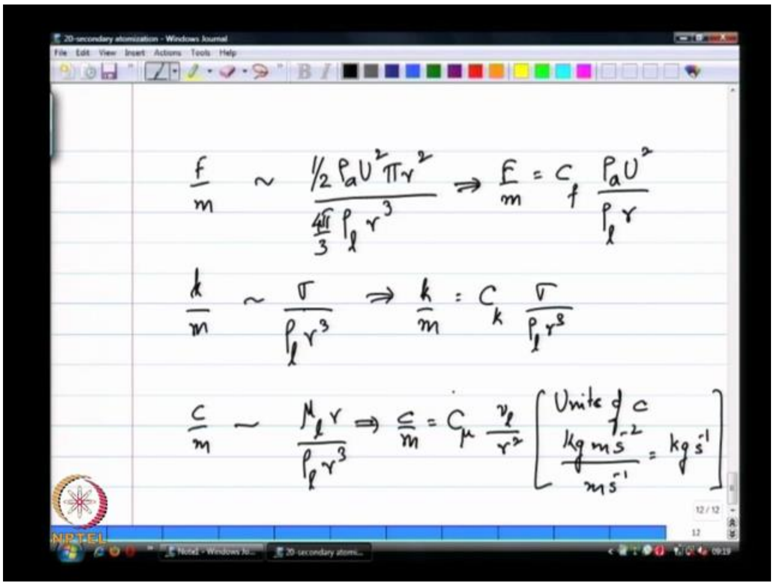

Likewise k by m has units of force per unit mass, and in this case k has the same units as sigma which is Newton per sigma being surface tension. We use this before so actually, sigma divided by rho l r cubed that is the scaling of this k by m, or I can write k by m as some c k times this dimension less group. And then I also have c over m; c over m as units of or at least phi as units of force per velocity. So, let us just check the units on c its kilogram meters per second squared per kilogram, so per meter, per second as units of kilogram per second.

So, I am going to reconstruct, if I take c to be dependent on the liquid viscosity. And so, liquid viscosity times r the drop radius has the same units as kilogram per second because the unit of liquid viscosity dynamic liquid viscosity is kilogram per meter per second. So, mu l times r has the same units as c, divided by our rho l r cubed. So, from here I can write c m is some c mu times nu l over r squared.

So, I have now taken a model system which is a spring mass damper system and taken the model constants which are these F by m k by m and c by m and related them to my real system which is an oscillating drop in a wind. I have to have some way of estimating the c l, c f, c k and c mu that is one of my remaining tasks, and secondary part of remaining task is what do I do with the results predicted from this model, and how do I related back to the drop itself.

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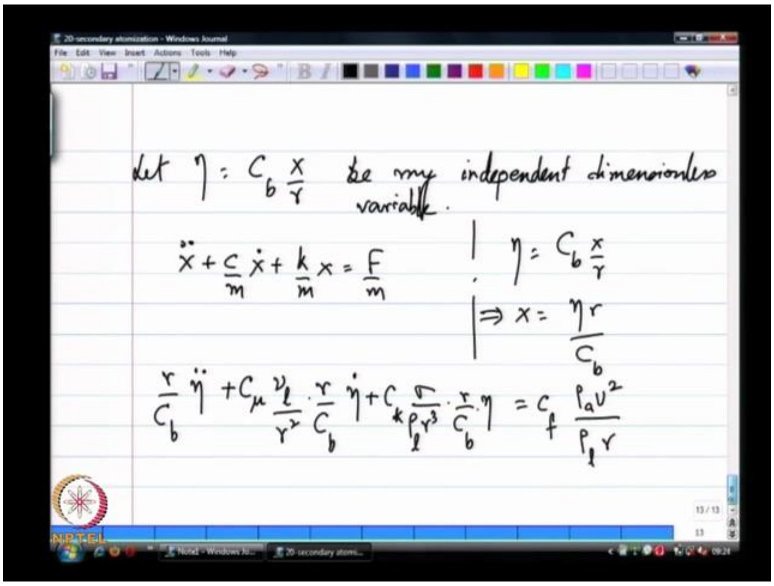

The way will look at the second part. So, will now define and eta which is some c b times x over r, I am going let this eta be my independent dimensionless variable.

So, I can effectively take my equation which is and non-dimensionalize this whole thing in terms of eta a cast and replace f by k by m with these values which as c prime, which as c f k, c k and c mu. If I do that what do I find? So, if eta is c b; c b is some number c b times x over r. So, it is like saying that when this eta. Let us say for example, c b is 1 when eta which is my oscillation amplitude reaches 1. The non dimensionalized oscillation amplitude is equal to the radius the drop is likely to break.

So, I can have a number for c b which says for example, if the drop reaches 90 percent of the radius, it would basically break the drop up. Or c b is a parameter in the problem, but it is only used to caste, our dimensional variable x in terms of a non-dimensionalized form. So, let us see what do I find here c b over r let me just write rewrite this part eta is c b x over r, which automatically means x equals eta r over c b.

So, x double dot is r over c b times eta double dot plus c mu nu l over r squared. Nu l is of course, are kinematic viscosity which is mu l over rho l m. I am just using standard nomenclature we followed all along, and then still I have x dot which is r over c b eta dot plus k over m is c k sigma over rho r l cubed, times x which is r over c b eta this equals f over m which is my c f rho a rho a u squared rho l over r.

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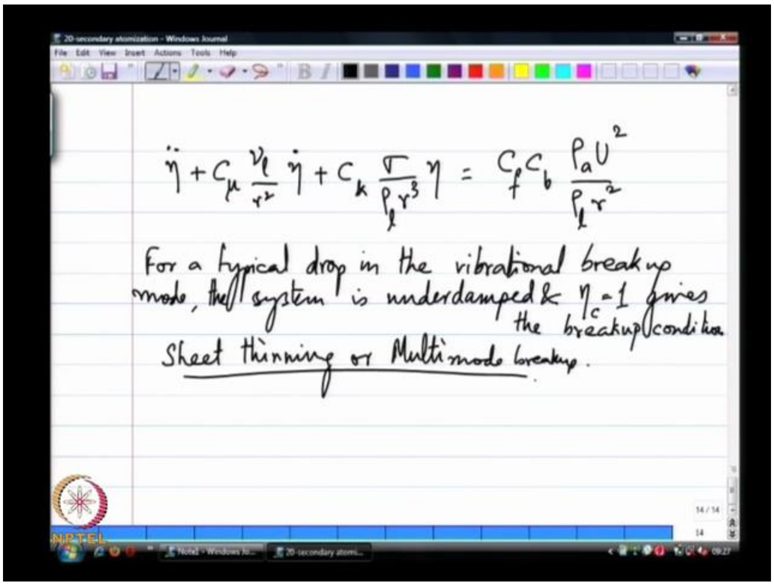

I can factor out this r over c b from all of the equation or terms on the right hand side.

And what do I have? Now, this is a linear second order ordinary differential equation eta and if I for the right set of values. I mean we do not have to assume, say for the right side of values for a typical drop the system is under damped, which means if I plot eta is function of time given my forcing function rho will be able to find the eta for a given forcing function rho a U squared will be able to find eta is the function of time.

And you said threshold value of eta at which the drop is likely to break up. So, this is the simplest of models that can use to study secondary atomization given the set of fluid properties surface tension density you have model to relate to break up characteristics. In the higher Weber number regime say in the sheet thinning or multimode break up.

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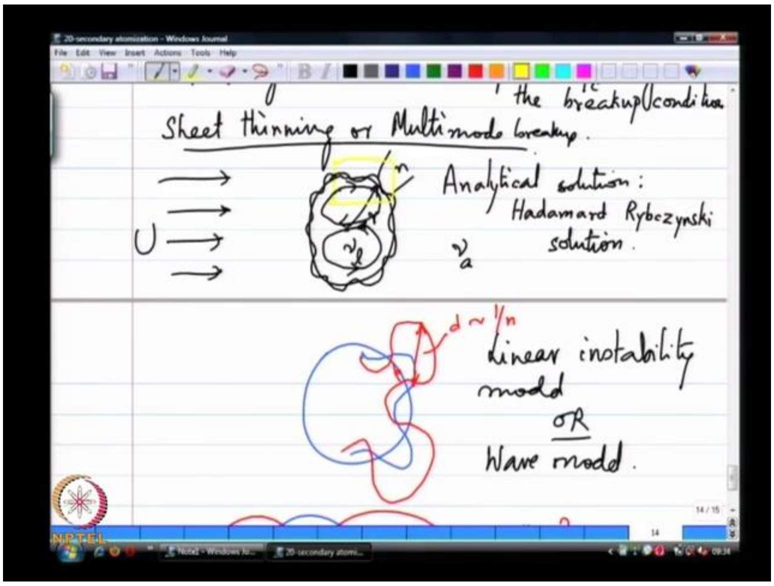

In the sheet thinning or multimode breakup what you essentially have is a drop in a cross flow. If I do a linearize stability analysis of this drop. So, essentially if I do the full linearize instability analysis. So, far spherical drop of liquid in a cross flow, you can do exactly what we did with the cylindrical jet or plainer liquid sheet except, the mathematics would be much more cumbersome, in fact the analytical solution for this flow. So, consider a drop of radius r of some of viscosity nu l in a fluid of some viscosity nu a. The analytical solution is given by what is called the Hadamard Rybczynski Solution. So, this is a case of analytical solution. For the case of laminar flow past, the liquid drop where you set up if you can imagine a vertex inside the drop. So, this solution gives you the complete fluid mechanics what is happening inside the drop and outside the drop in the laminar regime.

So, this is your mean flow condition. At this mean flow, if you now if you perturb this mean flow where you in on a surface, you have a certain n in the direction. That I have indicated and another m in the so it is like a latitude direction longitude direction. You have a certain m second azimuthal wave number in the other direction. So, if I do a full 3 dimensional linear instability analysis of the Hadamard Rybczynski Solution subject to these m n wave numbers, you will find 1 m star and n star which correspond to the most unstable pair of wave numbers. What that means, is that if I subject this drop to a cross flow and that means, I have a certain number of waves that will preferentially grow on this drop and as they grow, I might have this wave grow and eventually cause pinch have pinch out a drop from the side that corresponds to this wave number.

So, this is what we seen even in the jet break up problem in the cylindrical jet break up problem the wave number that corresponds to the most unstable point is responsible for the drop size. Likewise here, you can do this entire calculation and show that the wave number that is responsible azimuthal wave number that is responsible for the drop the most unstable point is responsible for the drop size. This is called again a linear instability analysis based model for secondary atomization often also called the wave model. But the wave model is little more than just a lineraize instability analysis of the haramard rybezynski solution the wave model makes some assumption.

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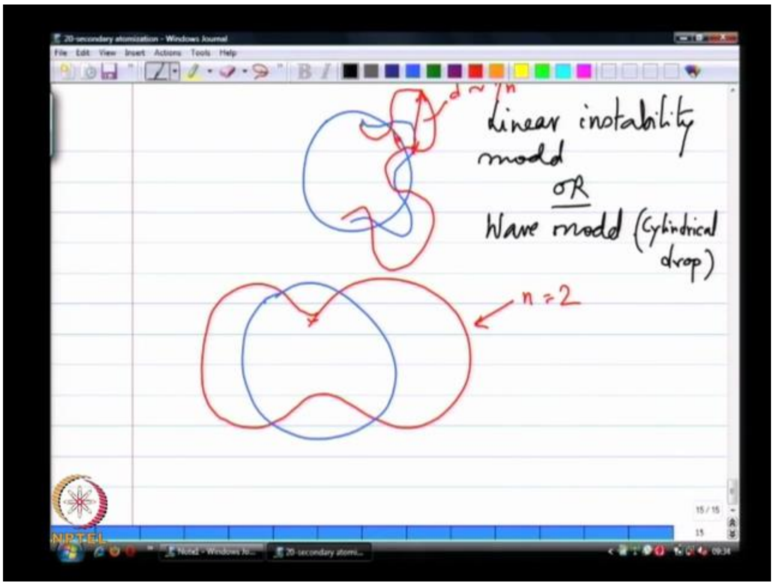

So, if I take this drop, which is initially spherical. The red line, that I have drawn corresponds to m equal to 2, that is it corresponds to if we go back to our linearize instability, corresponds to the case where there are 2 lobs that are formed. And the higher the value of m the smaller is length scale d, this d is proportional to 1 over m or n whichever is the; will use n I think. So, as n increases or more precisely as n star, the most unstable wave number increases the size of the drop that you are pinching of becomes much smaller than the size of the parent drop itself, which basically means that the fact that the parent drop as some initial curvature is no longer as important.

So, this is practically like repulse on a lake. So, if I look at a small part here, I might as well be looking at repulse on a lake, if the waves, if that little yellow region contains a large number of repulse already the fact that, the drop is curved not as important. So, this is the assumption underline this wave model, is wave model assumes not a spherical drop, but a cylindrical drop. So, it looks at wave number in just 1 plane assuming no waves in the other plane and from that gets an m star which is the responsible for the most unstable wave number.

So, linearize instability analysis is a very powerful tool because it allows, you to the study perturbation from the mean flow all way to predicting the final performance characteristics, which is I mean, there is no theoretical reason to believe this we already talked about this except to say that empirically it seems to hold true in a wide range of this kinds of spray situations. I want to show you a slightly more generalized way of talking about this kinds of modeling.

So, we looked at taylor analogy breakup model, which is basically a spring mass damper equivalent, I replace the parameters in my spring mass damper equation, with what I know from of properties of my drop, from there I am able to predict what is going to happen. So, how can I go about, writing a general model for a generalize situation.

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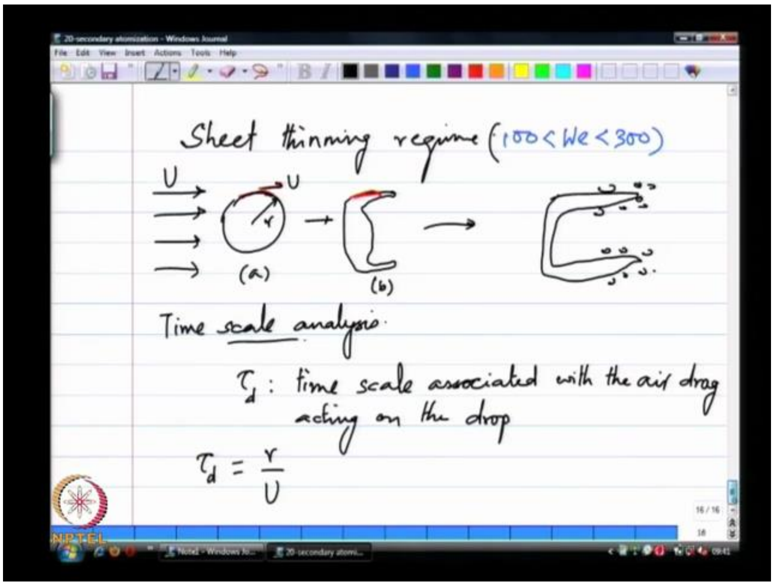

Say for example, the simplest case would be; I will take the example of the sheet thinning regime, where I start with a drop in a cross flow, this drop seems to produce a sheet like that, is this is further elongated may I have some drops being shield. I want to write now a model of this process. How do I do it; the start is of experiments, you have to have some understanding of what is happening.

So, the fact that has drawn this sequence of cartoons means that I have known this is what is happening in this Weber regime. Which is like let us say about between some 60 and some 300, I know this is something this is been observed to happen in this range of Weber number. How do I go about understanding, how do I go about writing a model for this. The start of writing a model is of course, experiment like I said and then will talk of tool called time scale analysis. What do I mean by time scale analysis, there are many different physical process is occurring in this in this phenomenon, each one is happening on a slightly different time scale and we want to get estimates of those time scales and then see how under what time scale can we expect the sheet thinning break up to happen.

So, if I start with a breeze of velocity U, flowing past circular drop of some radius r and if I am like I said for me to get these observations experimentally, I have to be zoomed into a single drop. Correct for me to be zoomed into a full picture of a single drop, I am automatically saying my length scale is the diameter of the drop. That is the length scale on which I have to fix my observation, to see these phenomena. If I fix my length scale to be much smaller then this, say like I had like the yellow window I draw in the previous graph, if I instead of looking at the whole drop, if I only look at tiny section there, I am looking at repulse on a lake, I am not looking at the drop breaking up. If I look at a much larger length scale, I am not looking at the physics of what is happening at the drop let level I am looking at a particle shattering.

So, if I know that, at that length scale let us see the radius or diameter does not matter. The first cartoon going from this a to b, is where I am observing a deformation in this part. So, this part is being drag forward, due to the shear stress from the air. That is the start of this sheet thinning, at least the way I have written this sequence of images. So, what is the time scale associated with this drag force, time scale associated with is air drag acting on the drop. Is at I want to have, visible deformation on this scale with a velocity u. So, in other words that red part is moving forward at the velocity u and the on a scale that is approximately the drop size, I want to be able to see this.

So, I will use r the radius as my length scale. So, the time scale associated with drop let deformation due to air drag is this r over U. Now, let us come into the drop and see what this deformation is doing inside the drop.

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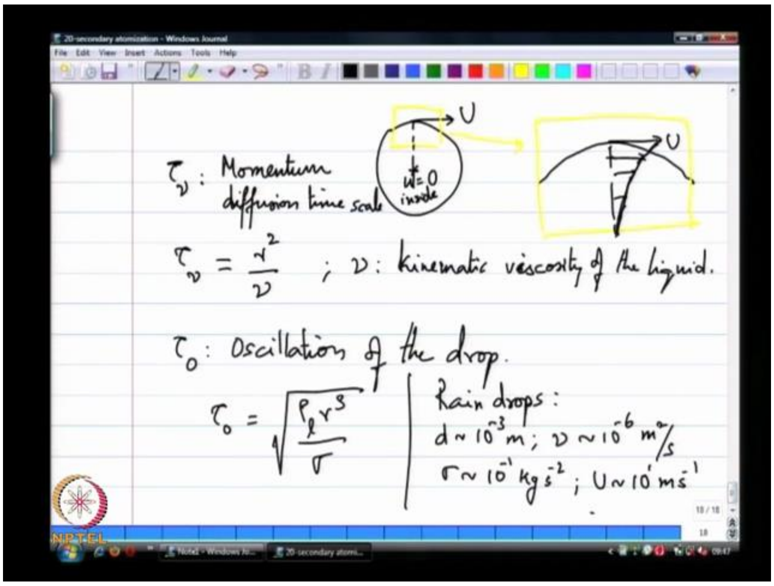

So, I will take a slightly simpler version of this. If I have this drop and this part is moving at a velocity u, if I look at what is happening inside the drop in this region, initially the velocity everywhere was 0. Initially the entire liquid inside the drop was at rest, as soon as this starts to become dragged what do you expect will happen, I am now writing a zoomed out picture to get what the velocity profile inside here will look like. Essentially, you start to create what looks like a boundary layer. Now this is still. So, in other words I am saying that, this is my center, but it is very rapidly decaying, this is a after a short time after I have initiated this process now; that means, the short time after I initiated process, this momentum at the drop free surface is diffused into the drop, due to the liquid velocity.

So, this momentum diffusion time scale I will call this t nu. Is given by r squared over nu where nu is the kinematic viscosity of the liquid. That is in a time r squared over nu for whatever r important to the free surface to reach the center of the drop. So, if I have a certain velocity important to the top of the drop in a time given by r square over nu that momentum would have come into the middle of the drop, the middle of the drop would have experienced some effect of the air outside the drop in this time, until then if you are in the middle and the free surface on the top of the drop is moving you would not and whatever for nu. I would expect diffusion; I would expect momentum know it that is essentially the meaning of this momentum diffusion time scale. At times much less than r squared nu, if you are in the center of the drop you would not know that there was an air outside that was causing the free surface to deform. There is a third time scale which is due to the oscillation of the drop.

So, what is this? If I take the drop and if I just give it an impulse, that drop is going to oscillate. And these oscillations have a certain time scale associated with it, how do I get that time scale, I know that this looks like again like a spring mass system. So, if i ignore the damper part of the spring mass damper essentially the oscillation is due to the mass of the liquid inside the drop and surface tension. So, this is given by sigma over rho l r cube, rho l r cube over sigma the surface tension. So, let us take our rain drop just estimate these 3 problems. So, we said a rain drop this 3 time scales, these was the values that we had, we going to assume r is above since we are r chosen r to be length scale I will leave it at that.

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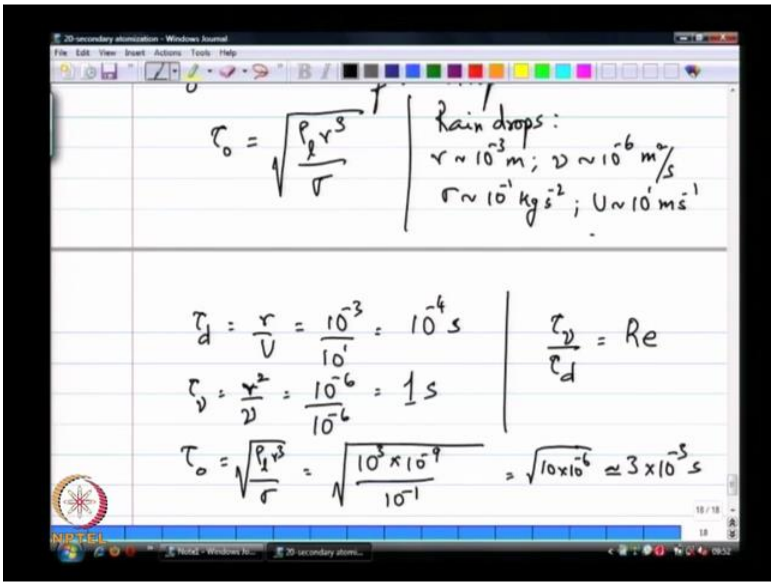

The drag time scale is r over u.That is the air outside trying to drag the free surface and you will be see visible deformation in 10 power minus 4 seconds from the start, that is the meaning of this, if we take the viscous time scale.

So, the oscillation time scale time scale is this rho l, approximately 10 power minus 3 seconds, we doing only a order of magnitude analysis from these this are the only 3 time scales that matter in the problem in any drop break up problem. So, if you look at what is happening here you will see visible deformation on the drop in 10 power minus 4 seconds, surface tension reacts in about 10 power minus 3 seconds viscosity is going to take much longer to react. So, it is like if you are in the middle of the drop, you will hardly know what is happening outside.

So, if you look at this, if you look at just this 3 time scales the relative computations between this 3 time scales tell you which dimensionless parameters are important if you take the drag time scale and compare that to the oscillation time scale, you end up with essentially the Weber number. When that drag number time scales are on the order of the oscillation time scales you end with the Weber number, times density ratio in this case. If you compare that drag time scale to the viscous time scale you end up with the Reynolds number. If you compare the viscous time scale to the oscillation time scale you end up with the ohnesorge number, I will leave that you as a home work to derive these 3 under the situation when the drag time scale is comparable to the viscous time scale or more precisely, the ratio of the viscous time scale to the drag time scale is your reynolds number. So, instead of 1 millimeter drop, like we had, if I take a 0.1 millimeter drop which is a 100 micron drop, clearly r is now factor of 10 lower for the same viscosity nu which means your viscous time scale is 10 power minus 2 seconds.

So, that is 10 power minus 2 seconds the effect of what is happening on the free surface of the liquid drop is felt in the middle, if the viscous time scale is the lowest of all these 3 time scales, will just take an extreme case. If the viscous time scale is the lowest of all these 3 time scale, what you expect is whatever happens at free surface is immediately felt in the middle, whatever happens due to oscillation is transmitted everywhere in the drop immediately; that means, this is where the situation where the drop is almost behaving like a rigid sphere, meaning that there is instantaneous transfer of information due to diffusion, but it is. So, fast that if I try to move the free surface of the drop the whole drop starts to move.

So, if the viscous time scale is very small; that means, you have the case for the viscous forces trying to hold drop together, if I want to break up a drop I have to create a velocity gradient inside the drop. The moment I create a velocity gradient, I have created a stress field inside the drop. If that stress field is not defused sufficiently fast this drop is likely to break up, if the stress field is defused sufficiently fast then I cannot break up this drop I viscosity will spread any kind of a differential motion I create. So, fast that it is essentially going to bring it back to a rigid body translation motion.

So, if I try to move 1, I have a drop if I try to move 1 part of drop preferentially, if this part also starts to move in the same direction I can never break it up due to aerodynamic forces alone. This is the physical meaning of secondary atomization and these are contributing time scale that any problem for any given drop situation look at these 3 time scales. So, if I take let us say standard p d p a data set. I have a data set of some n drops each one moving at some velocity of some diameter in some air velocity. Let us say I know the air velocity from some other source of information say the smallest size class of the drops at that point. Now I can take that information and for each drop in the problem estimates this 3 time scales. And for each drop based on the 3 time scales, I can tell what each of those drops likely to do, is it just going remain like, it is under go rigid body translation, is it going to break up.

So, is it due to vibrational breakup, which is essentially the oscillation time scale being comparable to the viscous to the drag time scale or is it going to break up by the sheet thinning mode where the viscous time scale is much smaller than the oscillation in the drag time scales that is essentially what happens on the surface, stays at the surface for drag the surface forward that surface is being stretch to a thin sheet with the with the middle of the drop remaining where it is, that is what we do carton for the sheet thinning break up. That is where the drag time scale is much faster than the viscous time scale and the oscillation time scale.

We will stop here, we will move on to a discussion of multiphase flows and understanding sprays as the random process from next time onwards.