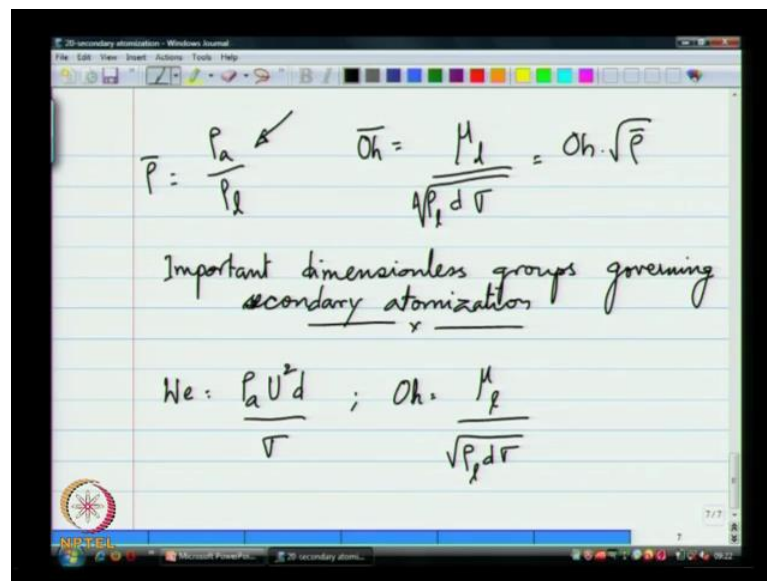


Spray Theory and Applications
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Lecture – 31
Secondary atomization-Modes of breakup- 1

Good morning, welcome back we going to continue our discussion of secondary atomization. To see what are the important parameters that govern this process, we went through two different kinds of analysis at the end of the last lecture, one using force scaling analysis and another using Buckingham pi theorem, to give us set up dimensionless groups that can be used characterize the behaviour of droplets in an air field, in a gas field. What we are going to do is start from there and say.

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Handwritten notes on a digital whiteboard showing dimensionless groups for secondary atomization. The notes include the following equations and text:

$$\bar{P} = \frac{P_a}{P_l}$$

$$Oh = \frac{\mu_l}{\sqrt{\rho_l d \sigma}} = Oh \cdot \sqrt{\bar{P}}$$

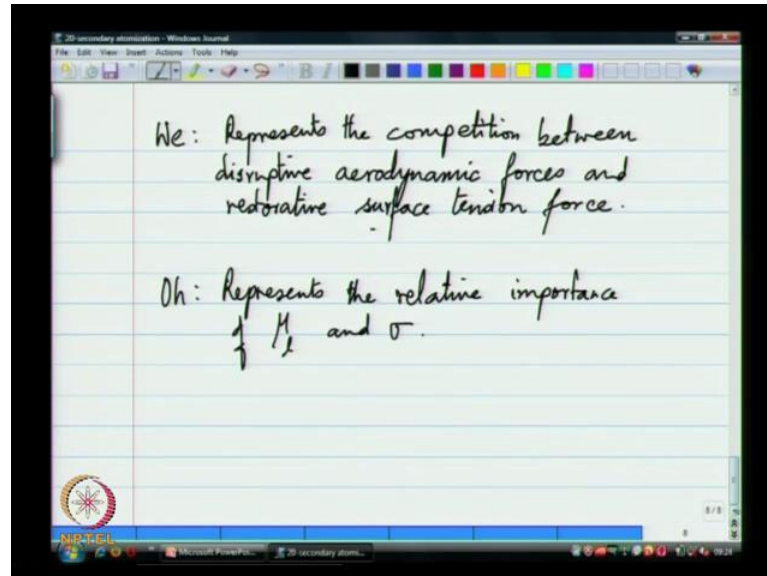
Important dimensionless groups governing secondary atomization

$$We = \frac{\rho_a U^2 d}{\sigma} ; Oh = \frac{\mu_l}{\sqrt{\rho_l d \sigma}}$$

I could have the important dimensionless groups governing secondary atomization, are we have a Weber number, which we showed and we are going to like we said at the end of the last lecture that there are many combinations of these dimensionless groups. Weber number, capillary number, Reynolds number and you can see ohnesorge number which is formed by a combination of this Weber number and Reynolds number these are all possible candidates for characterizing the behaviour of drops in a gas filed. So, typically for secondary atomization, we found that this Weber number and ohnesorge number defined based on the liquid properties seem to be used in the literature to

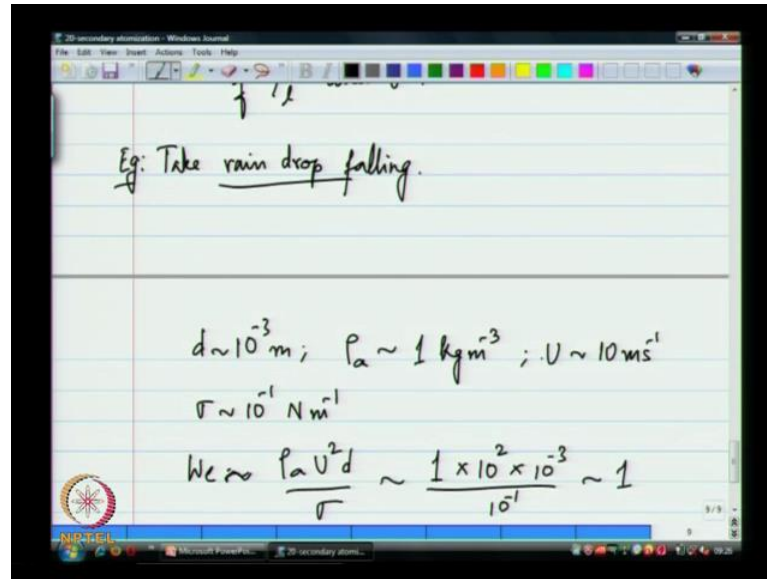
characterize the regimes in which these drops breakup we will look at that in just a moment.

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So, now I just to sort of summarize, Weber number as this group shows it represents the competition and restorative surface tension force, ohnesorge number it represents the relative importance of μ_l and σ . So, essentially in this formulation the in the way the ohnesorge number is written here. The liquid viscosity is trying to damp out any oscillations while surface tension is trying to cause the breakup of the drops. So, we want to understand the different roles that the same physical properties surface tension place in the 2 different term phenomena. Let us take a typical example, and see what these numbers look like.

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Eg: Take rain drop falling.

$$d \sim 10^{-3} \text{ m}; \rho_a \sim 1 \text{ kg m}^{-3}; U \sim 10 \text{ m s}^{-1}$$
$$\sigma \sim 10^{-1} \text{ N m}^{-1}$$
$$We \approx \frac{\rho_a U^2 d}{\sigma} \sim \frac{1 \times 10^2 \times 10^{-3}}{10^{-1}} \sim 1$$

So, let us take an example of, a rain drop falling. So, let say d is about 1 mm rho a, I am only go and do an order of magnitude analysis. So, I will start by writing d is about 10 power minus 3 meters, rho is about 1 kg per meter cube rho a.

So, we will assume for a moment that the drop has reached at terminal velocity condition roughly about 10 meters per second. This is sort of an over estimation, we will see what this number looks like and sigma for water is about 10 power minus 1 Newton per meter approximately, it is a about between 10 power minus 2 and 10 power minus 1. So, given this the Weber number an order of magnitude of the Weber number, the order of magnitude of approximately a 1 mm rain drop falling through air at a terminal velocity of about 10 meter per second, gives you Weber number of about 1.

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$d \sim 10^{-3} \text{ m}; \rho_a \sim 1 \text{ kg m}^{-3}; U \sim 10 \text{ m s}^{-1}$
 $\sigma \sim 10^{-1} \text{ N m}^{-1}$
 $We \sim \frac{\rho_a U^2 d}{\sigma} \sim \frac{1 \times 10^2 \times 10^{-3}}{10^{-1}} \sim 1$ ✓
 $\mu_l \sim 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}; \rho_l \sim 10^3 \text{ kg m}^{-3}$
 $d \sim 10^{-3} \text{ m}; \sigma \sim 10^{-1} \text{ N m}^{-1}$
 $Oh \sim \frac{\mu_l}{\rho_l d \sigma} \sim \frac{10^{-3}}{\sqrt{10^3 \cdot 10^{-3} \cdot 10^{-1}}} \sim 10^{-3} \text{ to } 10^{-4}$

What it means is that the aerodynamic force. So, if I take a rain drop falling at 10 meters per second. The question I am asking is this lightly to breakup and if I have this air essentially in the frame of reference fixed around the drop there is air moving around it. So, if was to do this very sort of a regress fluid dynamic analysis this drop would have to deform into something like that that may not be it, there it may further deform by flattening out into a pan cake and if this where to happen this process continues the lid breaks up. So, let me draw those separately, if I take a spear initially under some air flow, after some deformation this may become something like a pan cake and the moment you have this sort of a flattening out this process is only going to be accelerated further until you start to get some kind of a breakup.

You can sort of imagine this and we will see more details on this in a moment, but what if the calculation that we just did showing us that the value of Weber number being order of magnitude 1, means this aerodynamic force that is trying to deform the drop and the surface tension force which is trying to bring it into a spear are very close to be of the same order of magnitude. So, that competition keeps the drop in a nearly spherical shape at this condition.

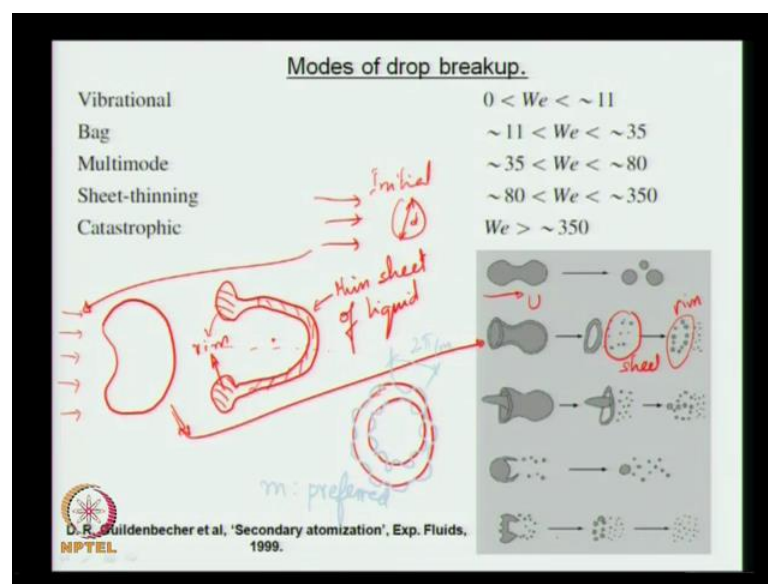
So, in other words if the aerodynamic force is much greater than the surface tension force. For example, if I had a larger drop that would cause the Weber number to be higher or if I have a lower surface tension fluid that would cause the Weber number to be

higher under those situations, the drop is more likely to deform. Let us do the ohnesorge number calculation for the same fluid, for the same situation. Ohnesorge number requires μ which is about 10^{-3} kg per meter per second, ρ which is about 10^3 kg per meter, cube d which is 10^{-3} and σ which is about 10^{-1} . Ohnesorge number for this would be μ divided by square root $\rho d \sigma$, which in this case is about 10^{-3} divided by square root $10^3 \cdot 10^{-3}$ into 10^{-1} . Which is on the order of 10^{-3} to 10^{-4} , somewhere in there.

What this shows is that the role of viscosity in damping out oscillations is very small because the ohnesorge number is on the order of 10^{-3} to 10^{-4} . What it means is if the drop were to go in to oscillations, those oscillations would sustain for a fairly long period of time. So, imagine a real rain drop falling, you would expect if you took images of this rain drop in the center of in the frame of reference fixed with the rain drop the drop would also show an oscillation it would not just be a fixed sphere. Because the ohnesorge number is very small, if the ohnesorge number was very large on or at least order of magnitude 1, I would expect the drop to be nearly spherical as it is coming down.

So, this is a situation of about order of 1 Weber number and ohnesorge number being order 10^{-3} etcetera.

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So, let us see, what these really mean in the case of a spray. And so we will first look what Weber number means in the case of a spray, these this is a simple classification from the work by guildenbecher et al, from a paper title secondary atomization. What they show is that they as several different regimes of Weber number under which one could expect the single droplet to behave differently at over the range of Weber numbers.

The first and simplest is where the Weber number is between 0 to 10. Let us say broader eleven is what they say this is the case where you would expect a drop. So, in the vibrational mode, I would expected drop you undergo some kind of a deformation. So, initially it could be a small deformation. So, this leads into this and this in turn leads in to this. Now this is not process by which the drop is essentially going from going sort of quayside statically forma nice spherical drop into a dumbbell and then breaking up it is a vibrational breakup mode that is this drop is undergoing a set of vibrations, but those vibrations are amplifying in amplitude are increasing in a amplitude.

So, the amp in other words I may have this vibration initially small, but because the air is speeding energy in to this vibrating drop this amplitude of the vibration could increase and I could imagine very simplistically, that if the amplitude of vibration becomes on to the order of the radius of the drop. Then you have a situation were from here you come to something that looks like that and it could pinch off. I could have a nice spherical drop, but because of the modes of vibration of this drop it could undergo a breakup by amplification of the vibrational amplitude.

So, and that is pictorially shown in this first part. So, essentially if you imagine in this particular instance, there is very little preferential direction of the air motion. So, for example, I would have to imagine in all these cases that the drop is introduced in to an air stream the drop is introduced into an air stream of some velocity u and that gives rise to the Weber number basically. So, initially the drop is addressed and you let go of it you have this relative velocity between the drop and the air that causes this kind of a behavior if your Weber number is very small. If your Weber number is slightly larger then, the kinematics of what you expect would happen starting from here is that there is an initial formation of this drop in the middle.

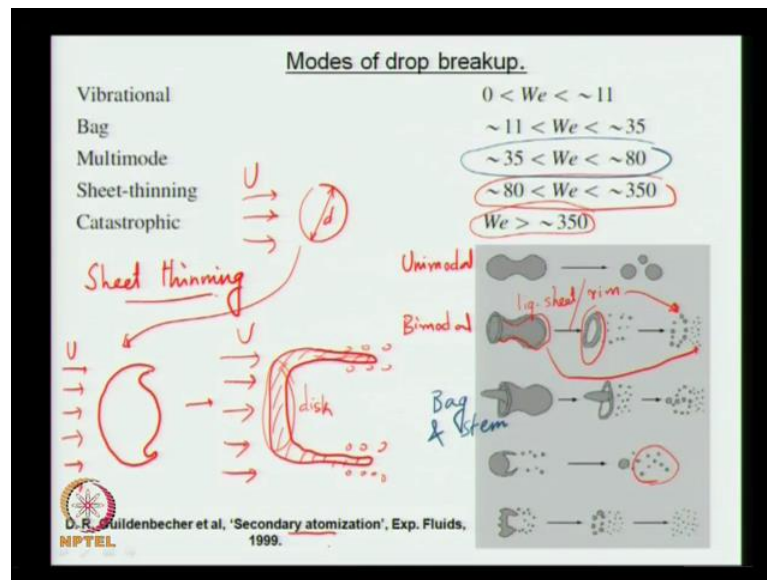
So, you have an air that causes this nice and spherical drop to deforming to what looks like a jelly bit and from there the progression is towards what looks like a bag. So, I will

just draw this arrow to here just to I do not want to redraw the same figure. So, this middle part could elongate with the thick rim on the outside here, something like that. So, this is a thin sheet of liquid with an axis symmetric rim now, you will have imagine this is axis symmetric. So, now these are all sort of in schematic representations, but they are actually based on experimental observations, so that fairly accurate. So, what happens at this point is that this drop that is been stretched out into this thin rim, into a rim plus a thin sheet of liquid attaching into the rim, the thin sheet of liquid is now looking more like a bubble, which is sustain by this high slightly higher pressure on the inside and that shatters into a bunch of drops.

So, these drops you see are coming from the sheet breaking up and then these larger drops that you see are coming from the rim breaking up. So, you can straight away see that if might drop in the initial state. So, this is my initial state is in the Weber number range say about 10 or 11 to about 35. I am going get this kind of a breakup of the single drop and which means in each drop breakup, I get 2 different physics coming into play, one due to the sheet breaking up and that sheet breakup gives rise to very small fragments of drops and the second the rim breaking up. Now in our earlier lectures we discuss linear instability analysis.

So, if I take you know and we looked at the linear instability of a cylindrical column, I can look at the linear instability of a toroidal ring. So, if I were to create a liquid toroidal ring which is what this is and one can make these observations you know from drops breaking up, that you can actually see a toroidal ring form and if I create a liquid toriodal ring and do a linear instability calculation of that liquid toroidal ring, there is a certain preferential breakup mode that will show up. So, if this is my liquid toroidal ring, there is a certain azimuthal mode. Let us say $2\pi m$ is this theta. So, there is a certain m which is preferred. So, there is a particular value of m that will have the maximum growth rate it is just like any other prefer it is a like any other linear instability problem, where a certain mode is likely to grow faster than others and that particular value of m .

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So, if I look at the rim again, the particular value of m that you see causing the breakup of this rim determines the number and size of the drops form from the breakup of the rim. So, you have 2 different physical phenomena causing the breakup, causing the final result, impacting the final result in the case of the bag breakup process. One the sheet which is this liquid sheet breaking up to gives these fine fragments and then the rim breaking up to give you the larger drops. And the result is that in this particular case you are more likely to get a bimodal distribution of droplets form a single droplet breakup event because they are originating from 2 different physical processes.

In the first vibrational breakup, you are more likely to see a unimodal breakup, that is if the preferred breakup process in the vibrational mode is for each drop to breakup into 2 half's. The preferred breakup process does not mean it exactly splits into half, but if I were to look at population of drops at that same Weber number condition, then on the average I expect the drops to produce the mean diameter of the dot a drops produced from secondary process to be about half the size of the primary drop, half the size in mass or you know when you do the calculation it you will find what the scaling is in the diameter to direct.

Now, form there I will skip this multi mode, because that requires a little more thought,, but we will moving to sheet thinning. Now, if you there is a supporting video to go with this paper that you can go to this general website and look at where they have videos of

this process. So, what I am describing is actually from an experiment from high speed emerging and experimental observation that you can see for yourself. The sheet thinning process also starts with a same initial condition.

So, a drop of some diameter d at rest in a wind of velocity u , what do I expect will happen in this if the Weber number is slightly higher and now in this range. The first deformation process from here is where I start to see a thin rim form on the side. So, this is where the side of the drop is being stretched forward as suppose to the middle of the drop being pushed forward which was the case in a bag breakup mode. The natural progression from here on is where you create a very thin long stretch sheet. And so it almost like I am shearing drops of the sides, this is more like our high Weber number diesel or jet atomizer. And so I create this thin elongated liquid sheet breaks up into final drops which are shown here and a few large drops from the breakup of this disk like structure.

So, I still may end up with what looks like bimodal distribution, but it is significantly more concentrated number wise in the smaller drop rates. So, the mean drop size of all the doter drops form from the breakup events of a population of parent drops is going to be smaller as the Weber number increases. That is the expectation in that is what is observed. The last breakup process is what is often called a catastrophic breakup process, which is where the Weber number is greater than about 300 and something. And this is actually very difficult to investigate in there is not much information available on how drops behave at this situation. In fact, the word catastrophic is descriptor of v of nothing more than lack of information lack of our information about this regime.

But there is some experimental evidence of drop sizes that will form,, but specific physics of how the breaks up at this high Weber number is something that is I think not completely understood, as we will understood as the others. And this middle region that we skip as we went forward is called a multi mode breakup regime, which is where you have a transition from the middle part be stretched being pushed like we said the bag breakup.

So, just to show, so it is being this idea of the middle being stretched into like a balloon to the sides being stretched in to thin sheet. So, the multi mode breakup shows a little bit of both and therefore, you see that this is often called the bag and stem breakup. So, you

Now, in all these processes we will have to physically understand how this is happening. So, like the way to sort of imagine this all of these kinds of stretching elongation and then breakup processes are happening in time. That is the drop should have enough time for one of these processes to take the atomization process forward. Take for example, the sheet breakup in the sheet thinning regime, you have to have sufficient time to create velocity gradients inside the drop to stretch the drop out into a thin sheet and the time over which the drop is stretched into a thin sheet is much smaller than the time over which we drop can retract back into a spherical drop. So, you can think of this Weber number as being a combination of as being a competition between the times scales as well. That is the viscous time scale or the surface tension time scale which is trying to restore this drop back into a spherical shape is much longer than the aerodynamic or the viscous time scale or aerodynamic time scale which is stretching this drop out to this thin sheet. So, one process is so much faster than the other that it does not have time to catch up.

Regime chart

Liquid	Liquid	Source
○ n-Heptane	● Glycerol 92%	○ Hanson et al. (1963)
△ Water	● Glycerol 97%	● Hinze (1963)
▲ Glycerol 21%	● Glycerol 89.5%	● Lane (1991)
▽ Glycerol 63%	● Mercury	● Logarev (1975)
○ Glycerol 75%	● 200 fluid	— Krzeczowski (1980)
● Glycerol 84%		— Present

$\rho/\sigma = 580-1200$

Sheet thinning breakup

Shear breakup

Deformation > 20%

Multimode breakup

Bag breakup

Oscillatory deformation

Vibrational breakup

10% < deformation < 20%

5% < deformation < 10%

Deformation < 5%

Glycerol Rain

Small Oh

$Oh = \frac{\mu}{\sqrt{\rho \sigma d}}$

$We = \frac{\rho U^2 d}{\sigma}$

Drop breakup regime map

G.M. Froth, 'Structure and breakup properties of sprays', Int. J. Multiphase flow, 1995.

So, that is the meaning of the Weber number. So, what faith did in 1995 and this is sort of a collation of a lot of work that presided a faith structure and breakup properties of sprays paper in international journal of multi phase flow. What faith did is to pass all this information into a regime chart. So, we will start some this corner because it is the easiest to understandably, if I have order 1 Weber number, so this is Weber number order 1.

So, this is are rain drop that we discussed earlier Weber number wise, but instead of raining water if it was raining glycerin which is let us say a 1000 times more viscous. Then this is essentially where our glycerin rain would fall. Order 1 on the ohnesorge number and order 1 on the Weber number and what this chart tells us is that under those circumstances the drop would hardly deform. So, the deformation of this drop would be less than 5 percent. So, it would oscillate, but the net deformation would be less than 5 percent of the radius. So, there is hard there is no expectation of any atomization. You can go towards higher and higher Weber numbers and even if you have somewhere a drop that is at a very high Weber number, but correspondingly high ohnesorge number minds you these axis are all logarithmic. So, high ohnesorge number let us say ohnesorge number about 10 power 3 would allow for very high Weber number and still a stable drop.

So, if I want to create a nice and stable drop I have to have a high ohnesorge number. Even if I have a for every given Weber number, let us just write down what these are. So, we know what we are meaning, notice that diameter of the drop d accurse in the denominator in the ohnesorge number and in the numerator in the Weber number; that means, for the same fluid μ l ρ l σ and the air density being the same and you being the same the smaller drops 10 to have a higher ohnesorge number and a lower Weber number. So, we do not have to necessarily look at different property fluids even in the same spray some drops in other words smaller drops will tend to be in this corner here and the larger drops would tent to gravitate towards the other parts of this graph. So, let us take our own little rain drop where we side the ohnesorge number is about 10 power minus 3 and Weber number is about 1.

So, this is where we were, this is our typical rain drop the deformation is expected to be between 5 to 10 percent, but; that means, that it is likely to oscillate with the net oscillation amplitude being limited to about 10 percent. So, now, breakup again, as you

go higher and higher up on the Weber number oscillatory deformation starts to occur when your Weber number is on the order of about 10 this is what we call vibrational breakup mode in the previous slide. And if I would look at further increase in if we look go back let say between 11 and 35 you expect a bag breakup and you can imagine what 11 and 35 would look like on the on a log scale, this region over which one would expect bag breakup is that on the Weber numbers scale, as I keep growing up by transition into multi mode breakup and then the shear or sheet thinning breakup. Notice how so essentially, the limits that we discussed in the previous slide correspond reasonably well with an axis if I were to draw the axis right through here.

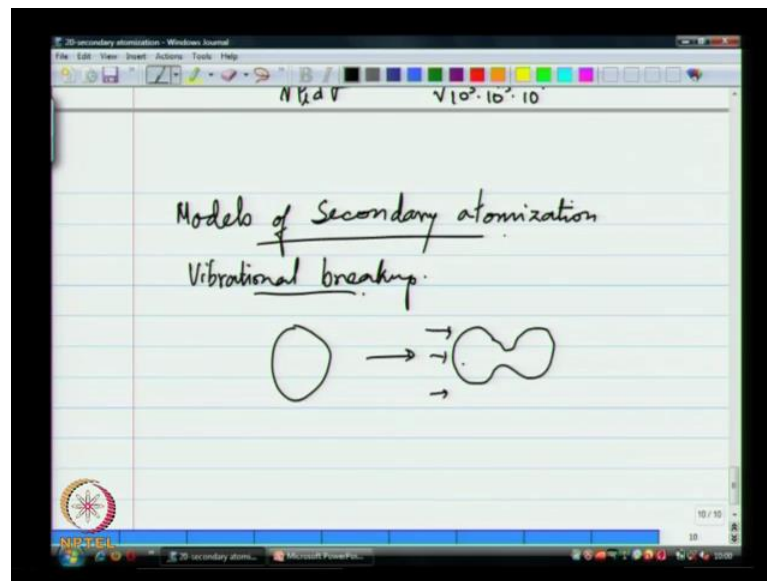
So, these let say start of the oscillatory breakup till write there, then the bag breakup multi mode and then above that you have a little bit of sheet thinning and further up is your catastrophic breakup. So, these correspond reasonably well with the values that we discussed in the previous slide, but as I increase my ohnesorge number clearly, these values are not going to be the same we can see where if I have ohnesorge number order 1; these values is for oscillatory deformation is no much higher bag breakup multi mode and etcetera.

So, the values that guildenbecher et al gave in their paper are mostly for very low viscosity fluid, which is typical of any sort of a fuel like gasoline or diesel or evocation kerosene these are all low viscosity fuels and in a given spray with ohnesorge number is expected to be usually small on less than order 1, for less than order 1 drops you expect that the sheet that the limits are what guildenbecher et al, work the but for larger ohnesorge numbers one would have to look at the complete regime chart from faith. So, this as you can see from the class of liquid that were studied. This kinds of information where experiments from various sources are all collated on to 1 graph is called a regime chart a regime chart a like this is very useful, because for my given situation I know what to expect in for my behavior.

Now, at a given point in the spray I may have wide range of drops and these drops may be travelling with the arrange of velocities, we have looked at joint pdf of sizing velocity and when you have a pdf of sizing velocity one can imagine that you actually have a pdf of Weber numbers and ohnesorge numbers. So, I can take the dimensional information form let say $p(d)p(v)$ data, that we looked at before and parset into Weber number ohnesorge number joint pdf.

So, at a given point in the spray given spatial location in the spray, I have a whole distribution of Weber numbers and ohnesorge numbers. And these drops are now likely to breakup by varying physics at the same point in the spray. So, the question is, how do I model this breakup process? So, I will we will say take 1 or 2 very trivial examples in a lead in to something that is probably applicable to real situation. The simplest model is what is called a tailor analogy breakup model.

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I can only start talking of modeling this process, only after I know what the physical what the physics looks like. So, I do a bunching of experimental measurements parset in to a regime chart and now I am very to look at modeling this process. So, if I take the simplest to model is actually the vibrational breakup, where I have this drop leading in to sort of a dumbbell like mode and this dumbbell like from mode is likely to breakup due to oscillations. Will continue this in the next class and look at other models as well along with tailor analogy breakup.