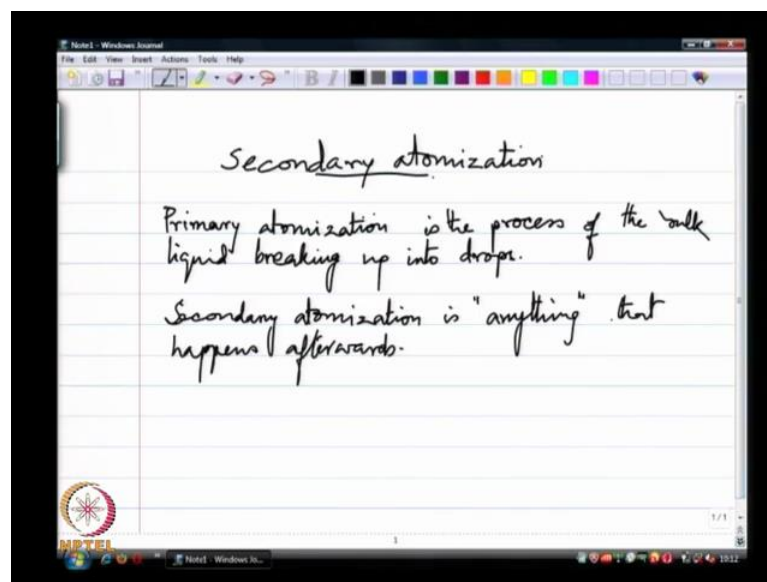


**Spray Theory and Applications**  
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**Lecture - 30**  
**Secondary atomization-Dimensionless parameters**

Good Morning. We will continue our discussion by starting to look at secondary atomization, and we will see briefly what secondary atomization is? What role it plays and then we will come back and look at some simplified analysis to start with, and you know what sort of techniques can be used to understand secondary atomization as a process.

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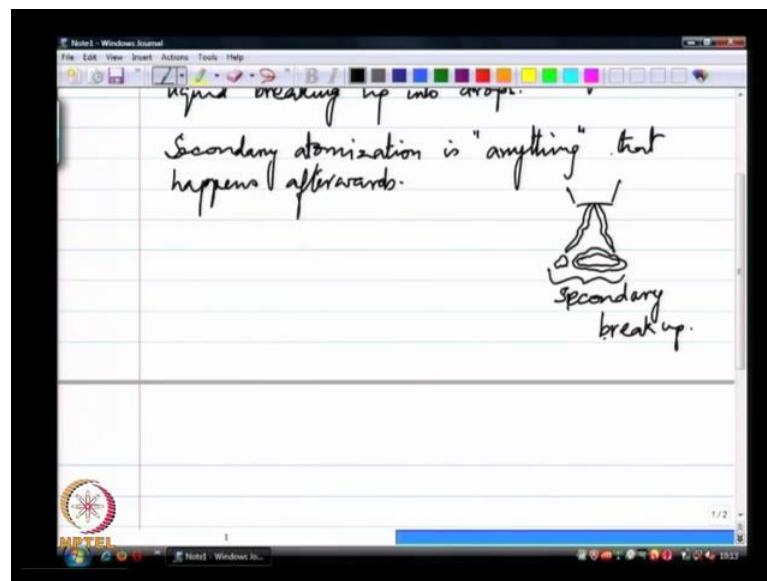
So, we looked at several different, we looked at least two different analysis techniques that we can apply to primary atomization. We looked at one in great detail called linear instability analysis.

So, this is where you have bulk liquid coming out of the spray nozzle and that bulk liquid is breaking up into drops, and these drops are somehow determined by the instability structure that occurs on this liquid sheet. That was the premise on which we learn the mathematical frame work in which we can do this analysis right. Now we want to study the life of these drops; what happens to these drops after they have formed is there a

possibility that they could breakup further if. So, under what conditions and if after they breakup and if they do breakup what is the result of such breakup process.

So, that is essentially what secondary atomization is like we said, primary atomization is a process of the bulk liquid breaking up into drops. Secondary atomization we will sort of loosely define anything that happens afterwards.

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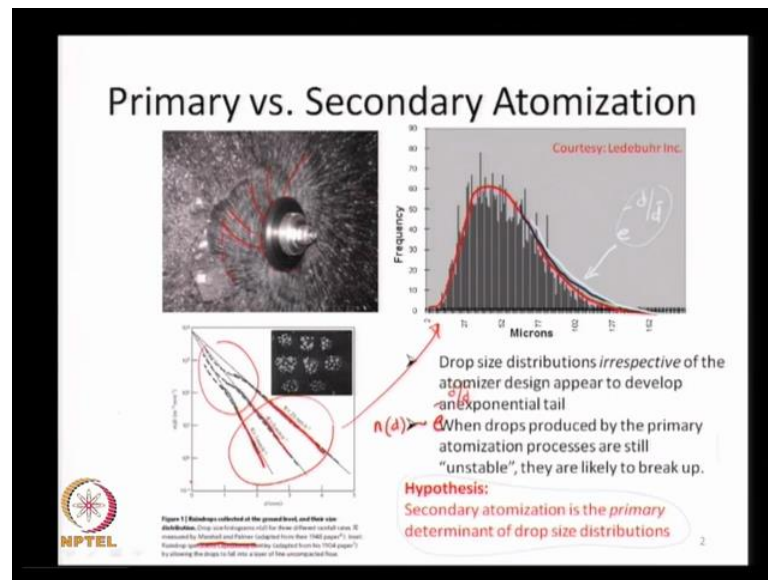


So, take a very simple example let us say I have this spray nozzle, and I have this liquid sheets. Let us say there is a sheet and there is a drop that has formed it may be a drop it may be like a rings structure depending on what sort of breakup happens. This ring is no longer in contact with the nozzle directly.

In other words if I try to spray a conducting liquid and I can measure the connectivity of each blob of liquid to it is to the parent nozzle, this blob that is formed which let say a toroidal or semispherical whatever, a sort of quasi spherical is no longer in contact with the main with the nozzles. So, the subsequent breakup of these structures is what we will term secondary breakup or secondary atomization.

So, there is a lot of secondary breakup that happens in a spray as you can imagine, because the drops that are formed from the primary breakup process may not and you know in most of the senses are not spherical, but the final resulting drops are mostly spherical; we will see why and how in a moment.

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So, this is the sort of our loose definition, so let see where we can take this definition forward. So, let say why do I want to study primary atomization? If I go back to that I will take a couple of very simple examples. Here is an example of a rotary atomizer; rotary atomizer as we saw very briefly is the case where we have disk. So, this is I am gonna just draw the outline of the disks say, oh no this is the disk here, the disk is spinning. And you can see there is a nice periodic structure in that sense of the liquid that is purring out and this periodic structure is spaced as azimuthally for it because of the serrations on the disk.

So, the primary breakup process is essentially a result of spilling liquid jet through those serrated edges and that spilling liquid jet breaking up due to capillary instability or due to some cross flow instability. So, I have basically a cylindrical jet that is being squeezed out of this rotation of the circular disk and this cylindrical jet is also moving in an azimuthally sense, which is similar to having an air across flow of air. So, I have a cylindrical jet with the cross flow of air that is gonna cause the breakup of this liquid jet.

We saw from primary instability form linear instability considerations that the breakup of any liquid jet produces drops that are on the order of the size of the liquid jet itself. So, if I have a round liquid jet undergoing capillary instability the size of the drops form from that process will be on the order of the liquid jet diameter, it cannot be much

smaller except if you have you know we saw in the spray instability with like a diesel injector, but in a case like this the primary drop formed are very big.

So, for example, on the right hand side here, I have the drop size distribution that the manufacturer are in this case ledebuhr claims to have measured in this rotary atomizer spray. The serrations in this particular case may be on the order of about 250 microns, which means the primary drops that are formed are on the order of about 250 microns. And if this is the distribution that was observed you can see that there are hardly any drops that are on the order of 250 microns.

So, even though the primary process caused drops that are on the order of 250 microns, there are no drops in the real spray where measurable of that size. So, the question then one has to ask is what happened in the middle; that is why secondary atomization happened.

Let us take one more example and see what we will learn. This is the size distribution measured in a real rain. So, this we are now talking of the grandest of all sprays a very heavy down pour of rain. So, if you take rain falling from the sky you have and if I had a way of sampling drop sizes in the rain, and in fact these people Marshall and Palmer; Palmer did that in this 1948 paper. They made measurements of the rain drops and showed that the number density which is now a probability density function of the drop sizes in a real rain follow a graph approximately like this.

A point to note is that is this exponential tail where you have you know the ordinate is logarithmic and they emphasize linear which means this is a case where, the number density goes as  $e^{-d/\bar{d}}$ . We have seen this in our studies of probability density functions. And different amount of rain fall, so this is 1 mm per hour, this is 5 mm per hour, and 25 mm per hour; 25 mm per hour is like a huge down just to give you an idea.

So, the indifferent instances the size distributions are different, but qualitatively they are all very similar in their shapes. You can even go all the way back to the very small sizes and you can see that there is a similar structure irrespective of the rate of rain fall. The point, we are going to focus on is the fact that this slope is linear as you get towards larger and larger drop sizes. If I plot this data in an axis like that essentially what I am

looking at is a long tail just like that. So, essentially this would be an  $e^{-d/\bar{d}}$  type tail.

So, whether it is a rotary atomized spray or rains which are obviously coming from very different sources. The behavior towards larger drops seems to be that you have this long exponential tail.

Now, you can show I just took examples that are very contrasting, but you can take real sprays coming out of an air blast atomizer, a simplest atomizer, and many other different mechanisms of atomization and you will find that there is a long exponential tail that of the probability density as the drop size increases. This is a standard feature of most sprays. So, the question done is like if all these are mechanistically so different why is the measure drop size distribution showing such similar features. The answer to that is that when drops are produced by primary atomization processes they are relatively still unstable.

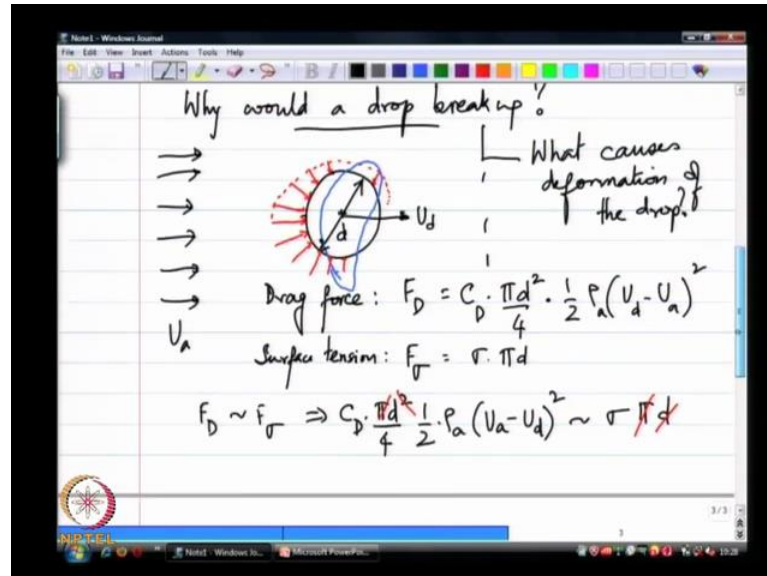
So, we really mean unstable in the sense of a linear instability analysis, showing a positive growth rate. And therefore, they are likely to breakup; so we will see the moment under what conditions are there likely to breakup and how. So, therefore, this is like I said the last part is just a hypothesis it would be very care full when you listen to hypothesis, that in most sprays secondary atomization processes are really responsible for drop sizes and their distributions much more than even the primary processes.

So, even if I did not do all the linear instability calculations, but I just assume that this spray nozzle is producing drop on the size of the liquid sheet, or on the order of magnitude of the phase diameter, in the case of a diesel injector or on the order of magnitude of the liquid sheet, in the case of either an air blast or a simplest atomizer. But did a more thorough job of the secondary atomization process is modeling; I might end up getting the better answer. So, I might end up getting a reasonably correct answer I should not say better.

So, that is the hypothesis under which we will learn secondary atomization. And clearly I did not present this hypothesis when I was describing linear instability analysis, because that is also an important process at least to know under what conditions jet is even likely to breakup. We cannot predict that or without the help of a linear instability analysis. So,

we will come back to the rest of this a little later let me close this. So, with that premise, let see why would a drop breakup?

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So, let us ask a question, say I have formed the drop and I am going to assume an initial drop is spherical. Let us say the drop is moving with the velocity  $U_d$  surrounded by an air flow that is at some velocity  $U_a$ . So, we are gonna to ask the question, why would the drop breakup? If I assume for a moment a well even before you ask the question why would the drop breakup we will answer the question what causes deformation. So, breakup is like an extremely went of a deformation process that I that has to initiate we initiated on the drop.

So, we will see what causes deformation of the drop, clearly for that we need a stress you need a force. So, what causes the force as you can see I have a drag force, the drag force on this drop is some drag coefficient times  $\pi d^2$  over 4, let me keep the notation right half  $\rho_a$  I know you are going to say  $u^2$ , but we have to be care full it is essentially a result of a relative velocity between the phases. So, in this one dimensional sense the relative velocity is just  $U_d$  minus  $U_a$  square, this is the magnitude of the drag force this is not getting indicating the direction.

Now, does it matter if  $U_d$  is greater than  $U_a$  or  $U_a$  is greater than  $U_d$  not really, because all I care about is the relative velocity I could have a stationary drop be impacted by a sudden by a velose, by high speed air or I could have the drop being injected at a

high velocity into quiescent there it does not matter. Now this drag force, if the drop was perfectly spherical and if the drop was let us say rigid that is now we are looking at a rigid sphere, I can draw like a pressure profile around the drop something like that you can clearly see that the pressure on the front part would be larger than the pressure elsewhere etcetera.

Now, if I have a perfect stokes drag then it will be a symmetric profile all around, but anything that has a finite Reynolds number, you are not going to see symmetric pressure distribution around the sphere. So, now, what does this pressure distribution do since we are now, since we are allowing the drop to deform this drop is likely to deform in the direction of the exerted force sort of like that, because they send the pressure force is now just trying to flat an out this into a pancake, and if you let this process continue all the way forward it is going to breakup. Now are there any other forces that are important yes the answer is we have a surface tension force.

So, on one side we have the drag force and we have the surface tension force which is given by  $\sigma \pi d$ , if  $\pi d$  is the circumference of this drop, if I have to deform this drop from the black shape to the blue shape I have to increase the surface area, and any increase in a surface area comes at the penalty of increased surface energy and that is work done against surface tension force which is of this magnitude.

So, if I allow these forces two forces to compete with each other which is what is actually happening, let us see what happens if I have a competition between these two forces, by the way this what is this  $\rho$ ? This  $\rho$  is that of the air let us be very clear about that. So, when these two forces are equal in magnitude are on the order of the same are on the order of the same magnitude, then you have something interesting happening in term it is basically like a fight between equals drag force trying to deform this drop and surface tension force trying to resist the deformation.

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The image shows a digital whiteboard with handwritten mathematical derivations and a graph. At the top, the equation  $\frac{C_D \cdot \rho_a (V_a - V_d)^2 \cdot d}{8} \sim \sigma$  is written. Below it, the equation  $\frac{\rho_a (V_a - V_d)^2 d}{\sigma} \sim \left( \frac{8}{C_D} \right)$  is shown, with the term  $\left( \frac{8}{C_D} \right)$  circled in red. To the right of this is a graph of  $C_D$  versus  $Re$ , showing a curve that starts high and then drops. Below the circled term, the text  $We \sim We_c$  is written, with an arrow pointing to the word "Weber No." written in red. At the bottom, the equation  $We = \frac{\rho_a V^2 d}{\sigma}$  is written.

So, and when would that condition occur when this mathematical condition occurs, and what you see on the left hand side is what we often called Weber number, and this 8 over  $C_D$  is another number for drag over is drag around sphere if I plot  $C_D$ ;  $C_D$  happens to be a function of the Reynolds number of the say Reynolds number around the flow, and you know you can look up any standard text books and you will see a graph that looks something like that, for  $C_D$  as a function of Reynolds number.

So, essentially  $C_D$  is like a number that comes from either empirical measurements or some source of some other source, the point is this that really the value of  $C_D$  is important, but not as important as the idea that when this Weber number is one the order of some critical Weber number. So, we will call this 8 over  $C_D$  as some critical Weber number. So, when this when the real Weber number around my off the flow around the drop is on the order of the critical Weber number, then I start to see deformation effects scraping is just surface tension force  $\sigma \pi d$  is much greater than the drag force, essentially I have a very very rigid sphere that refuses to deform because it is surface tension force is. So, high that it is able to keep the drop in time.

So, it is almost like a perfectly spherical body moving through air on the other hand if the drag force is very large in comparison to the surface tension force, it is as though I do not even have a liquid drop it is just going to be pulverized it is as though I have a for lack of another way of saying it like I have a little mass of gas, that is being impacted by



some other stream of fluid and this mass of gas is just going to disintegrate without knowing any idea, without having any effect of a restorative surface tension force, these are the extremes of what the Weber number looks in the Weber number space.

If the Weber number is very large; that means, the surface tension forces is insignificant in comparison to the aerodynamic force that is trying to deform the drop, and if Weber number is very small the surface tension force is. So, large that I would not have any kind of breakup the transition between these two regimes happens at this critical value of the Weber number.

Now, very often people ask me the question even if I write Weber number is  $\rho u^2 d / \sigma$  done a little bit of simplification  $u$  here is a relative velocity, but for the remainder of this lecture in the next few we are going to assume that the drop is stationary. So, in other words if I impose a coordinate system that is moving with the drop at the velocity of  $U_d$  then I all I care about is  $U_a$  minus  $U_d$ . So,  $u$  is that relative velocity measured in the frame moving with the drop, and if I write down the Weber number as  $\rho u^2 d / \sigma$  what is this  $\rho$  should it be  $\rho_a$  or  $\rho_l$  the answer it is it depends on what you want the Weber number to convey.

In this particular x position, I have shown Weber number as coming out of a competition between the aerodynamic drag force and surface tension force therefore, you should this becomes  $\rho_a$ . Now if I was trying to in and this is typically what you will define Weber number as, I mean where as if any sort of a dimensionless number should come out of physical arguments like this and we will do that we will look at other such arguments in the next couple of we will look at other such dimensionless numbers that will fall out of other fall out of similar arguments.

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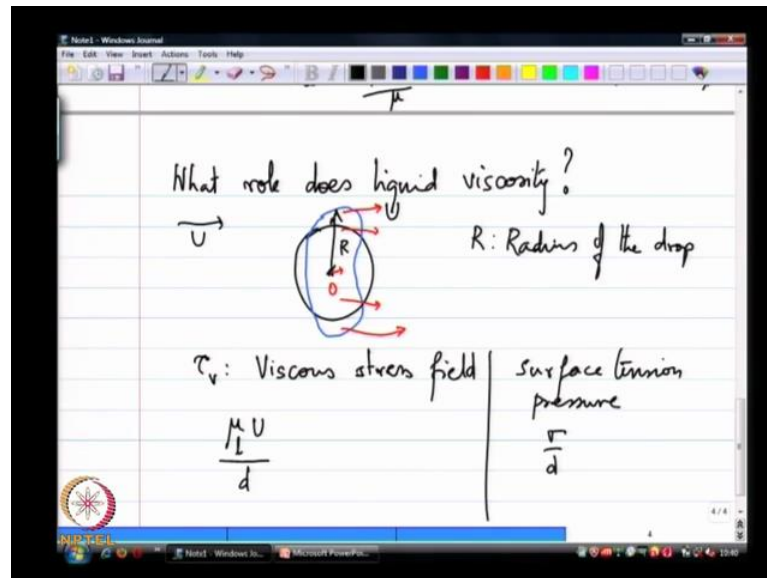
The image shows a digital whiteboard with handwritten notes. At the top, there is a small diagram of a drop with diameter  $d$  and velocity  $u$ . Below it, the equation  $\frac{\rho_a (u_a - u_d)^2 d}{\sigma} \sim \frac{\sigma}{C_D}$  is written, with  $\frac{\sigma}{C_D}$  circled in red. To the right, a red line graph shows  $C_D$  vs  $Re_d$ . Below the graph, the text "Weber No." is written in red, with an arrow pointing to the equation  $We \sim We_c(Re_d)$ . To the right of this,  $Re_d = \frac{\rho_a U d}{\mu_a}$  is written. Below these, the equation  $We = \frac{\rho_a U^2 d}{\sigma}$  is written, with "is important" written next to it. At the bottom,  $Re = \frac{\rho U d}{\mu}$  is written, followed by a question:  $\rho = \rho_a \text{ or } \rho_l? \mu = \mu_a \text{ or } \mu_l?$ .

If I take the next instance let me go back to my drops. So, we know now that Weber number is important, and that this Weber number has to exceeds some critical value and that critical value could be a function of the Reynolds number, if  $C_D$  is a function of the Reynolds number right the critical value is a function of the Reynolds number right, what is Reynolds number in this particular case it would be  $\rho_a u$ . So, I will use the  $u$  calculation.

So, I ask the same question I would answer it, but I will ask the question and then answer if I write Reynolds number as  $\rho U d$  over  $\mu$  is  $\rho \rho_a$  or  $\rho_l$  and is  $\mu \mu_a$  or  $\mu_l$  that depends on again what you want the Reynolds number to convey. We will see a couple of instances, if I want to talk of this Reynolds number I will make the distinction that this is the drop Reynolds number, this is the Reynolds number of essentially the drag coefficient acting on a spherical drop.

So, if I want to talk of this then  $Re_d$  is  $\rho_a U d$  over  $\mu_a$  because, I am looking at the drag on a steel ball of diameter  $d$  that is what this sort of  $C_D$  versus Reynolds number comes. So, if I want to understand how this critical Weber number behaves as a function of the Reynolds number, that Reynolds number is  $\rho_a U d$  over  $\mu_a$  which is the viscosity of the air. We will see another, but for the breakup of the drop itself is the Reynolds number associated with the liquid viscosity important.

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So, in other words let us ask another question what role does liquid viscosity play in this whole process, we saw the role of the liquid surface tension, let us see what role does viscosity play. So, if I take a drop and if I want to deform it slightly now these are all in the coordinate system moving with the drop.

So, the drop is stationary and it is just deforming as this deformation process happens I have to set up relative velocity field inside the drop. So, if every point in this drop is moving at exactly the same velocity there will be no deformation of the drop in the frame of reference fixed at this  $o$ , if I have to have deformation of this drop in the frame of reference fixed at some center of mass with the drop let us say, then different points in the drop have to be moving at different velocities, something like that the moment I have different points inside the drop moving at different velocities, I automatically I am implying that there is a velocity gradient field which means you are going to create a stress field viscous stress field.

So, let us see if we can estimate that viscous stress field. So, if again take a drop of the diameter  $d$ , the viscous stress field inside the liquid drop is what I now thinking of, is what I am now trying to model. So, that goes as  $\mu$  the liquid viscosity times any sort of a velocity gradient that you can set up inside the drop.

So, let us take the extreme case, where this  $u$  now we are looking at a stationary drop impacted by a velocity  $u$  right. So, if this  $u$  is the velocity of the farther most for further

point on the drop and I will take the extreme case where my  $\omega$  is at rest. So, over a distance  $r$  radius of the drop the velocity is going from some  $u$  to all most 0. So, the viscous stress is going to be on the order of that, we have only estimating the order of magnitude.

So, just to like if I put 1 mm drop in a relative velocity of field let say 10 meters per second, if I put plug in the numbers here that will give me an order of magnitude of the stress that is expected, since we are dealing with  $d$  for diameter in the previous example values is the same  $d$ , because we are only establishing an order of magnitude that factor of two really does not matter. So, this is the stress field, and the similar and the surface tension field surface tension pressure that is resisting this deformation if  $\sigma$  over  $d$ .

Now, notice both of these have units of stress force per unit area I can compare this stresses or I can compare the areas it does not really I mean I can compare the forces, or it does not really matter. So, all I will do is multiply it by like a  $\pi d$  squared over 4 or if I just multiply it by  $d$  squared. So, on both sides and it does not really change anything that.

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The image shows a digital notepad with handwritten equations in red ink. The equations are as follows:

$$\tau_v \sim \tau_r$$

$$\frac{\mu U}{d} \sim \frac{\sigma}{d} \Rightarrow Ca = \frac{\mu U}{\sigma} \checkmark$$


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$$\tau_v \sim \tau_a$$

$$\frac{\mu U}{d} \sim \frac{1}{2} \rho_a U^2 \Rightarrow Re = \frac{\rho_a U d}{\mu} \checkmark$$

I am about to do which is look when the viscous stress is on the order of the stress the restoratives surface tension stress or no let me be clear if I am comparing this viscous stress to the surface tension stress, what I am trying to say is that the surface tension stress is essentially destabilizing the drop, because of the oscillations that it is creating

the viscous stress is trying to damp out the oscillations of the drop. So, this particular comparison and if you notice is only intended to show what are the, if I have a surface tension pressure that is trying to somehow destabilize the drop.

Like for example, if I take a really type instability. So, just a cylindrical jet of liquid, cylindrical column of liquid infinite column of liquid in air, we saw from linear instability calculations that that column of liquid is unstable you could rather breakup into drops of a certain size based at a certain distance  $\lambda$ , that  $\lambda$  comes from the linear is the point of maximum growth rate. So, that is a process where surface tension is driving the growth of the instability that is the cause of the instability and that surface tension driving force, if that is much greater than the viscous damping force.

Then you essentially have a breakup process, if we if the viscous damping stresses are going to be much greater than the surface tension driving force, then you are going to change the rate of the growth and eventually could even stabilize the process we will see what happens.

So, essentially if I write this comparison at this point I want to sort of bring in all the different dimensionless arguments that you could make. So, you have this other number called the capillary number, which comes from a competition between the viscous stresses that arising the fluid during the course of deformation. If I do not have a gradient field set up inside the drop the velocity gradient field I do not have any viscous stresses. So, if the entire drop is translating I do not have any viscous stresses inside the drop, this is only that during this deformation process that I have gradient setup inside the drop, and because of the deformation process and on the right hand side here, I am looking at the surface tension forces that could be causing that destabilization.

So, if I say on the other hand the viscous stress is inside remember are proportional to the gradients in other words they are absolutely 0 unless there is a gradient field in the gradient field is what is actually causing the gradient field in our instance, it is the aerodynamic force.

So, I have this  $\frac{1}{2} \rho u^2$  which is the pressure force. So, if I look at the comparison between the viscous stresses and the aerodynamic stresses, what do I have I will ignore the factor half, because I am only interested in orders of magnitude know kind of the weirdness of the  $u$  of the see we are forming dimensionless groups, but you

never think of  $\rho a U d$  over  $\mu l$  until you write this comparison of until you use physical arguments. So, the Reynolds number with the  $\mu$  in the denominator is not really expressing the competition between the viscous stresses inside the drop and the aerodynamics stresses that are causing the deformation you would never be able to establish the competition between these two physical process.

Now, so I have this kind of a Reynolds number this capillary number and the Weber number that we have found already.

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The image shows a digital notepad with the following handwritten equations:

$$We = \frac{\rho_a U^2 d}{\sigma} ; Ca = \frac{\mu_l U}{\sigma} ; Re = \frac{\rho_a U d}{\mu_l}$$

$$Oh = \frac{\sqrt{We}}{Re} = \frac{\sqrt{\frac{\rho_a U^2 d}{\sigma}}}{\frac{\rho_a U d}{\mu_l}} = \frac{\mu_l}{\sqrt{\rho_a d \sigma}}$$

So, let just write those down, now as you can tell this is a very weird Reynolds number right. So, what people find it instead of dealing with the  $\rho$  and  $\mu$  coming from the different fluids people find it useful to define another parameter called the ohnesorge number, which is basically square root of the Weber number divided by Reynolds number, which if I do the calculation  $\rho a$  square root  $\rho a$   $u$  square root  $d$  divided by square root  $\sigma$  times the Reynolds number is  $\mu a$   $u$   $l$   $\rho a$   $U d$   $u$  cancels out.

So, what I have is  $\mu l$  divided by sorry, sorry see the point of some of these like derivations of dimensionless groupings without the use of Buckingham pi theorem which is what you typically learn in under grad fluid mechanics is very important, you would never be able to physically cast these kinds of groups without using physical arguments like this.

So, another fall out would be never say I will divide the numerator and denominator by  $\rho a$ , and replace this with the kinematic viscosity of sorts because they are coming from different they are of different fluids. Now, let see this is the role that viscosity place. So, I want to look at this dimensionless group to tell me how important is liquid viscosity this is very small; that means, the effect of liquid viscosity itself would be is negligible, where as for the same Weber number if I have a high ohnesorge number the effect of the liquid viscosity is now important.

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The image shows a digital whiteboard with handwritten mathematical derivations. At the top, the Ohnesorge number is derived as  $Oh = \frac{\sqrt{We}}{Re} = \frac{\sqrt{\rho_a V \sqrt{d}}}{\frac{\mu_l}{\rho_a V d}} = \frac{\mu_l}{\sqrt{\rho_a d \sigma}}$ . Below this, it states "Buckingham's Pi theorem:" followed by the list of parameters  $\rho_a, \rho_l, \mu_a, \mu_l, \sigma, d, V$  and the calculation  $N-3 = 7-3 = 4 \text{ dim. groups.}$ . At the bottom, a pressure ratio is given as  $\bar{P} = \frac{P_a}{P_l}$ .

Now, if I used simple Buckingham and pi theorem arguments, I will show you what is might happen. Where do we do in Buckingham pi theorem we essentially start out by making the list of all the parameters that are relevant, I do not think I am list out anything. So, if I have 1, 2, 3, 4, 5, 6, 7, parameters I can form  $n$  minus 3 which is 7 minus three or four dimensionless groups, have return three of them here 1, 2, 3 ohnesorge number is not the fourth dimensionless group, because it was formed out of Weber number and this definitional granules number.

So, I am looking for the forth group which in this case turns out to be the density ratio  $\rho_a$  over  $\rho_l$ . So, in other words I cannot form a  $\rho_a$  over  $\rho_l$  by any combination of these three groups primarily because  $\rho_l$  does not even occur anywhere any here. So, what does this tell us? So, we have now identified the four dimensionless groups that are important, now I could have gone through a similar physical process to identify this kind

of a group the way to do that is, what is the role of the liquid inertia in relation to the air inertia?

So, if I say the liquid is some density  $\rho_l$  it is mass is  $d^3$  proportional to  $d^3$  and it is velocity is  $u$ . So,  $\rho_l u^2 d^3$  times the mass is like a times the let say essentially is the measure of the liquid phase inertia, we want to look at this is essentially going to tell us we want to look at the competition between liquid phase inertia and gas phase inertia and this dimensionless group is one way to find that, now nothing in our arguments was restricted to drops in air if know this  $\rho_l$  I could be looking at the breakup of bubbles in an infinite medium of liquid for all this argument is concerned.

So, it really does not matter. So, I another way to think of this is I can take a combination of the ohnesorge number that I have defined here this definition and this definition of  $\bar{\rho}$ .

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$$\bar{P} = \frac{P_a}{P_l}$$

$$\bar{Oh} = \frac{\mu_l}{\sqrt{P_l} d \sigma} = Oh \cdot \sqrt{\bar{P}}$$

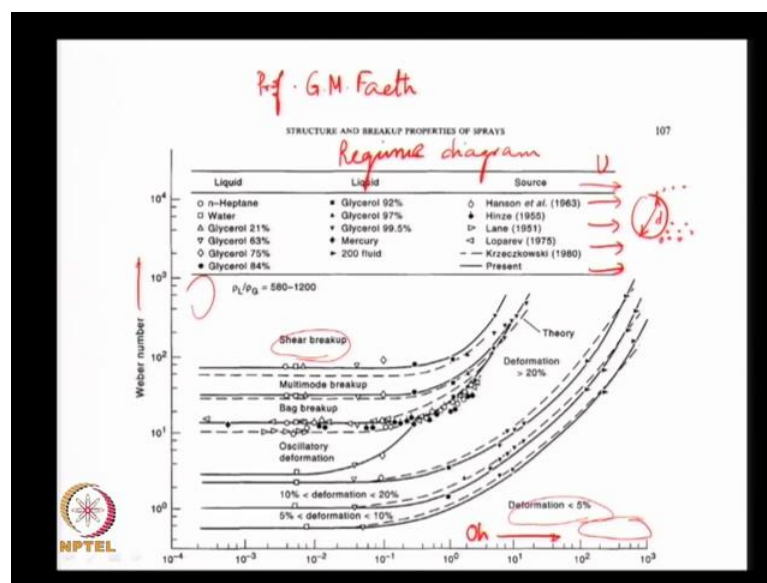
Time scale analysis.

And through some appropriate pairings of  $Oh$  and  $\bar{\rho}$  define another  $Oh$  bar which would be  $\mu_l$  divided by square root  $\rho_l d \sigma$ , which if you simply, I would just be  $Oh$  times square root of  $\bar{\rho}$ . So, once have four dimensionless groups I can find many, many more from just multiplicative and divide divisive combinations of all of these of the basic four.



But which once do I pay attention to physically you cannot get to that using Buckingham pi theorem you will need the physical arguments that we use before. So, if we go back we used forces to draw our analogies another way to do the same thing is using times. So, I could use what is called a times scale analysis to see what is it that the if the time what are the different orders of magnitudes of the different time scales and from there draw comparisons to which ones would be comparable which one should be relevant. So, we would not do that today, I just want to I will probably come back to that later on when we do need to talk of a secondary breakup.

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But for now I have identified these groups. So, what can we learn from that, the start of any understanding these experiments for the most part right? So, let see what experiments have showed in this particular instance what you have here on this access if it is not clear is the ohnesorge number when this is the Weber number that we define, this is the some very detailed data from all this different sources that professor g m faeth and his co workers put together in to this regime diagram.

What is a regime diagram, if I have Weber number and ohnesorge number and if I identify a certain range of ohnesorge numbers and Weber numbers? What do you expect would happen in that range? So, in different parts of this graph as you can see there is different physics happening we will see individually what each of those are. So, if I have

very low Weber number and very high ohnesorge number we will take the easier once to understand then we will come back to the rest.

So, I will draw schematic of a drop being hit with air at some velocity  $u$  this is some diameter  $d$  liquid viscosity  $\mu$   $\rho$   $\sigma$  etcetera, we have define the ohnesorge number, if I have a very high ohnesorge number on this order of magnitude and very low Weber number it means that there is the liquid viscosity is. So, high that it is practically preventing any deformation of the drop as you can see we say that it will move, but this is hardly any this break hardly any breakup expected and even hardly any deformation expected less than 5 percent.

We will go to the other extreme up here, very low ohnesorge number and very high Weber numbers this is all most like our like a blob of gas that is going to be impacted that surface tension and viscosity play hardly any role. So, that is where you get this thing called the shear breakup where I have a drop and I have tiny fragments being shed form the sides of the drop.

So, it is almost like I have flop around a spherical surface and that flow is just pinching of tiny drop. This is like our diesel spray as you can tell in this kind of a mode, of this kind of breakup the range of the drop sizes expected form the breakup process are going to be much smaller than the parent drop itself. And in the rest of the regime you have other physics happening that.

We will come back and discuss in the next class.