# Spray Theory and Applications
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## Lecture - 29
## Design of pressure swirl atomizer- 4

Welcome back, we are going to continue our discussion of design of atomizers. In the last two class we looked at simplex or a pressure swirl atomizer, the different parameters are go in to the design, the degrees of freedom like we said the other day and the constraints that you have to have your design satisfied. So, that is essentially the objective of any design exercise that you take advantages of the degrees of freedom and deliver a certain performance constraint. We are going to continue that discussion today to look at drop size, if you go back; we really left of at a point where we are able to get a film thickness we did not go much further than that.

So, we are going to today look at drop size as a constraint, that you could place on the performance of an atomizer you will see how to make sense of it.

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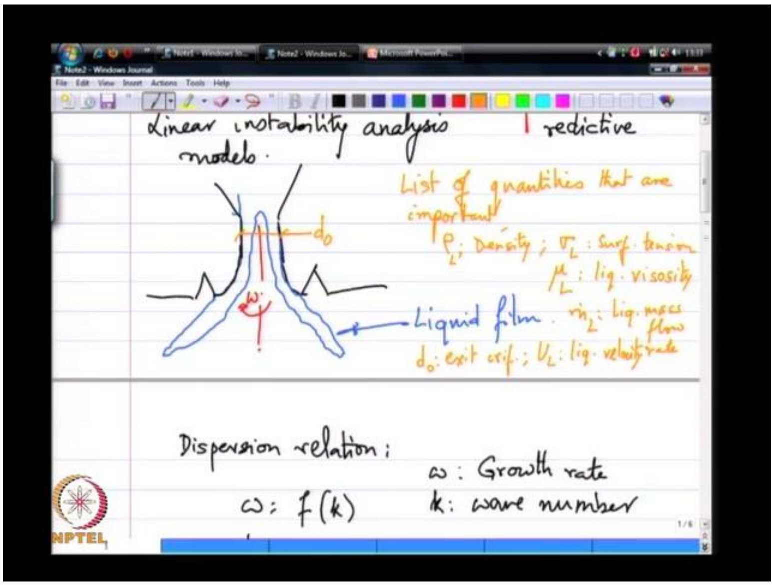

Linear instability analysis models. | redictive

List of quantities that are important

$\rho_L$: Density; $\sigma_L$: Surf. tension

$\mu_L$: liq. visosity

$\dot{m}_L$: liq. mass flow

$d_0$: exit orif.; $U_L$: liq. velocity rate

Dispersion relation:

$\omega$: $f(k)$

$\omega$: Growth rate

$k$: wave number

So, we are going to continue our discussion of design. Now the one of the ways in which drop size has been factored in to spray equations is through the use of linear instability analysis we looked at that. Say for example, in a simplex atomizer, you may have let us say this is the exit part and if you did the reentrant trumpet phase at the exit like we said, you essentially have liquid film that is exiting your spray nozzle and this liquid film itself could be swirling. So, if you liquid film is swirling and the swirling liquid film is, subject to some disturbances, it could cause the liquid film to break down.

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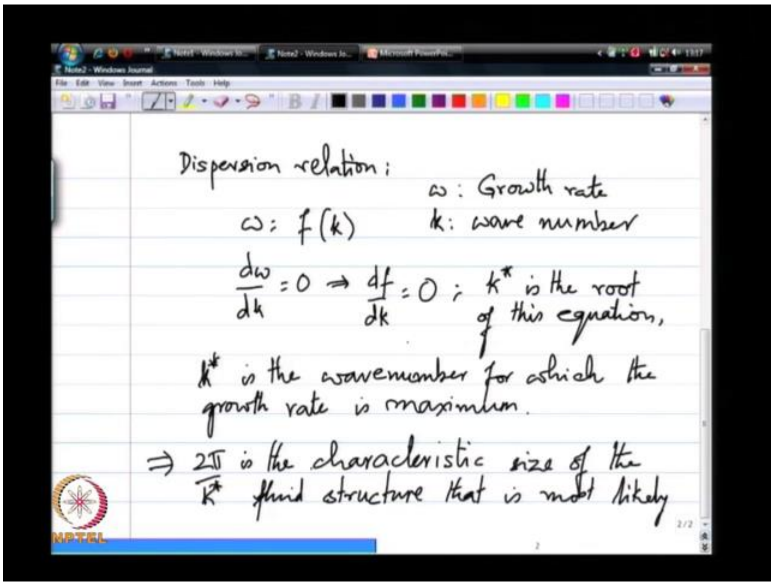

Dispersion relation:

$\omega$ : Growth rate

$k$ : wave number

$\omega = f(k)$

$\frac{d\omega}{dk} = 0 \Rightarrow \frac{df}{dk} = 0$ ; $k^*$ is the root of this equation,

$k^*$ is the wavenumber for which the growth rate is maximum.

$\Rightarrow \frac{2\pi}{k^*}$ is the characteristic size of the fluid structure that is most likely

So, the way we suggested we could model this is one of the classes, is you can develop a dispersion relation governing the linear instability, the growth of small instabilities on this system, that dispersion relation gives you omega as a function of k; k is your wave number we looked at least 3 examples, where we derive this omega the growth rate as a function of k, and we find the value of k at which df dk is 0. So, that this means k star is the wave number for which the growth rate is maximum which means that in a very short distance away from the nozzle, that particular wave number is going to dominate the structures the fluid structures in the liquid sheet or in the liquid film. So, that gives me which also implies the 2 pi over k star, is the characteristic size of the fluid structure that is most likely going to break off.

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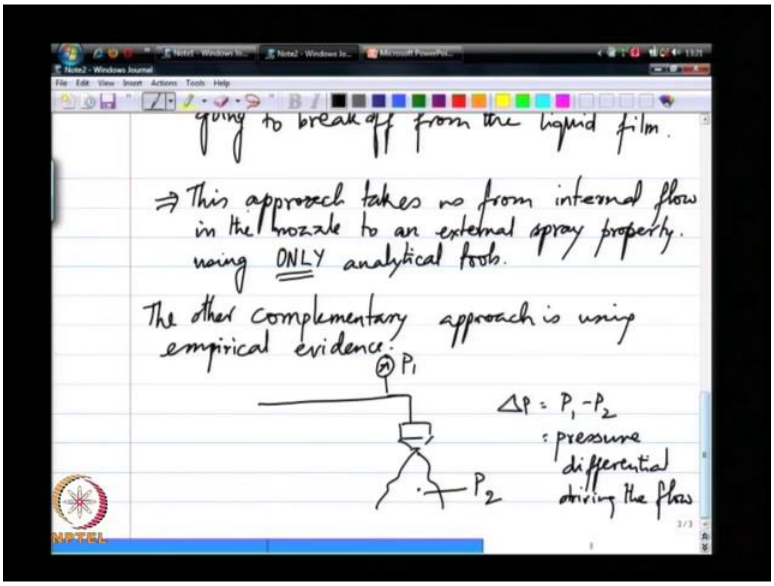

going to break off from the liquid film.

⇒ This approach takes us from internal flow in the nozzle to an external spray property. using ONLY analytical tools.

The other complementary approach is using empirical evidence.

$\Delta P = P_1 - P_2$

: pressure differential driving the flow

So, here is a way I can relate the nozzle performance or the parameters of the associated with the flow inside the nozzle, to a structure that could likely break off from the liquid sheet, which could further coalesce to from a drop for the downstream. So, to start with I have this linear instability modeling allows me to go from internal flow inside the nozzle to external flow. We looked at the model due to define an Morazán and Shue at all where you are able to take internal flow, the swirling flow inside swirl chamber and predictive film thickness, a film swirl velocity, film axial velocity and those will form inputs to your linear instability model, which could give raise to a dispersion relation that intern give right to drop size.

So, this is following a track of almost completely analytically modeling the system. So, we are only sticking to looking at the system as being enveloped by an analytical model. The other approach, the complementally approach has been to rely purely on experimental data. So, while this approach takes us from internal flow in the nozzle to an external spray characteristic, such as drop size. Using pure and using only analytical tools for the most part either than that 1.17 factor that we put in front of a CD equation if we go back to the last lecture.

Everything else which purely analytical and the model work even without the 1.17 correction factor, we are just off by about 15 percent which is not too bad. The other complimentary approach is using empirical evidence. So, what we have do in this approach, I have let us say a fluid line in which I am able to monitor the pressure, I call this P 1 and this fluid is coming in to some sort of a spray nozzle, that is creating the spray. Let us say the pressure here is P 2, so delta P, P 1 minus P 2 is the pressure differential driving this flow.

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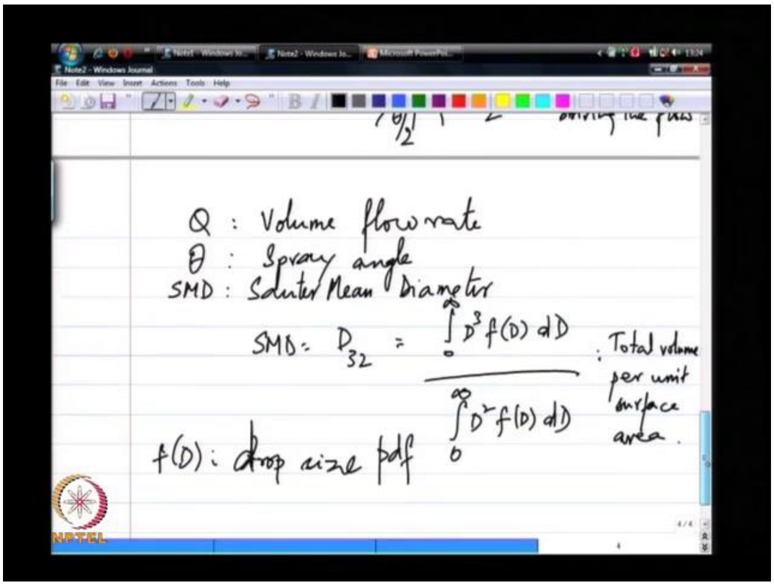

Q : Volume flowrate

θ : Spray angle

SMD : Sauter Mean Diameter

$$SMD = D_{32} = \frac{\int_0^\infty D^3 f(D)\, dD}{\int_0^\infty D^2 f(D)\, dD}$$ : Total volume per unit surface area.

f(D) : drop size pdf

So, in return for the pressure differential I could get some Q, which is a volume flow rate. I could get a spray angle theta, so I call this theta over 2 and I could get some SMD which is the Sauter mean diameter. So, if you go back our own notation from PDFs SMD is essential D 32, which is defined as D cube, times f of D, d D, divided by 0 to infinity D square f of D is my drop size pdf and D 32 is defined as the third moment which is integral 0 2 infinity D cubed f of D, d D divided by 0 the second moment, which is 0 to infinity d squared f of D, d D.

Now, clearly you can see this has units of diameter and this usually has a special relevance when it comes to sprays because this is the total volume per unit surface area, we looked at this early on and the total volume per unit surface area has a relevance to a lot of spray applications.

So, I can now let us say we looked at Q the volume flow rate, in terms of the pressure and the Q (Refer Time: 12:17) is not directly related to the pressure, but the way we found Jones correlation for example, gives us a way to relate c d the discharge coefficient to everything else that is related to the spray geometry. So, I can similarly find a correlation for the spray angle in relation to the other geometry features, we in fact, did that although we found an analytical expression based on Inviscid flow theory, that seems to work reasonably well and we also found that really speaking in a commercial spray nozzle, they do not rely on the fluid mechanics inside the nozzle to give us a certain spray angle, they used squared effect which is much more effective and provides much finer control over the spray angle at the exit.

What we are going to do today is look at Sauter mean diameter and how a Sauter mean diameter is intern related to the properties inside the nozzle. So, again like I said there are two approaches the first approach is using purely analytical tools, such as Inviscid flow theory and linear instability theory, but does go fairly close to predictions fairly close to measure values, but the best that for most design level tools would be to use the empirical correlation.

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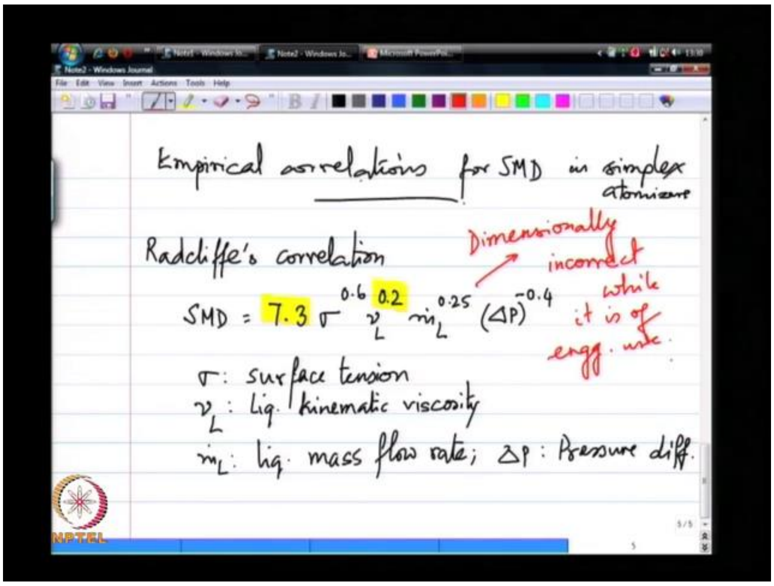

Empirical correlations for SMD in simplex atomizers

Radcliffe's correlation

$$SMD = 7.3\, \sigma^{0.6}\, \nu_L^{0.2}\, \dot{m}_L^{0.25}\, (\Delta P)^{-0.4}$$

Dimensionally incorrect while it is of engg. use.

$\sigma$: surface tension
$\nu_L$: Liq. kinematic viscosity
$\dot{m}_L$: liq. mass flow rate; $\Delta P$: Pressure diff.

Since, we are going to look at few such empirical correlations today. So, what sought of Empirical correlations is available for SMD in simplex nozzle, in pressure swirl nozzle. The one of the first and early works is due to this man Radcliffe, who gave us this formula, so I just to get the nomenclature right sigma surface tension, nu L is liquid kinematic viscosity, m dot L is the liquid flow rate mass flow rate and delta P is a pressure difference. Now we look will study this little carefully and see what we can learn, the first think that Radcliffe is trying to tell us is that surface tensions seeing to have a significant defect as far as the sheet breakup is concerned, because the exponent associate with that has the highest magnitude 0.6, liquid viscosity on the other hand has a small positive exponent 0.2, which says that as the viscosity of the liquid goes up the drop size is expected to go up although not very strongly; as the mass flow rate goes up on the same atomizer.

Now, how does the mass flow rate goes up on the same atomizer it goes up because delta P goers up, there is no other way because this is only a single fluid atomizer. So, as delta P goes up mass flow rate goes up and as the result the increase mass flow rate gives us Sauter mean diameter that is slightly higher, but the delta P is raise to the power minus 0.4. So, really speaking this is supposed to be valid over a large range of atomizers that go beyond just a single atomizer, but if I were to apply this over one single atomizer, I have to figure out how delta P is related to m dot L and that is usually using the Jones correlation using CD, where we can relate the volume flow rate to delta P or square root of this delta P more precisely and that gives us somewhere to relate all these parameter.

There is one serious problem and short coming with writing an equation, like this that is the fact the equation like this is dimensionally incorrect. That is all of your quantities sigma nu L, m dot L and delta P or all dimensional quantities. So, this number 7.3 is expected to have some weird units to it, such that the right hand side all of the quantities of a right hand side raise to those powers, give you a length scale give you units of micro meters or some drop size unit, which is what is on the right on the left hand side of this equation.

So, one of the biggest problems with an equation like this is that it is dimensionally in correct, while it is of engineering use meaning I can convey information to a fellow engineer through an equation like this, but I will have to give the fellow engineer a lot of other information about what unit system should be employed for sigma for nu L for m dot L etcetera otherwise the number 7.3 would be different, it would depend on the choice of the unit system.

So, there were right around the 40s, 1940s and 50s, there were a lot of these kinds of correlations that were developed that gave us dimensionally in correct correlations, that in order to reconcile we have to go to some sort of a non dimensionalization. So, that was the need for non dimensionalization comes from needing to convey Sauter mean diameter like information, without needing to convey a system of units, that without being constraint to a single system of units.

So, let us see how to do that. If I go back to my simple model of a liquid film exiting the simplex nozzle and I want to make a list of all the quantities that are my designs variables. So, in this simple problem let us say list of dimensional qualities, that are important it starts d c rho the density of the liquid, sigma L which is surface tension, mu L which is liquid viscosity, m dot L which is liquid mass flow rate, U L which is like a liquid characteristic velocity and let us say d 0 which is that distance exit orifice diameter, these are all quantities that one would expect will play a roll. So, I am going to transcribe theses back on the other end here.

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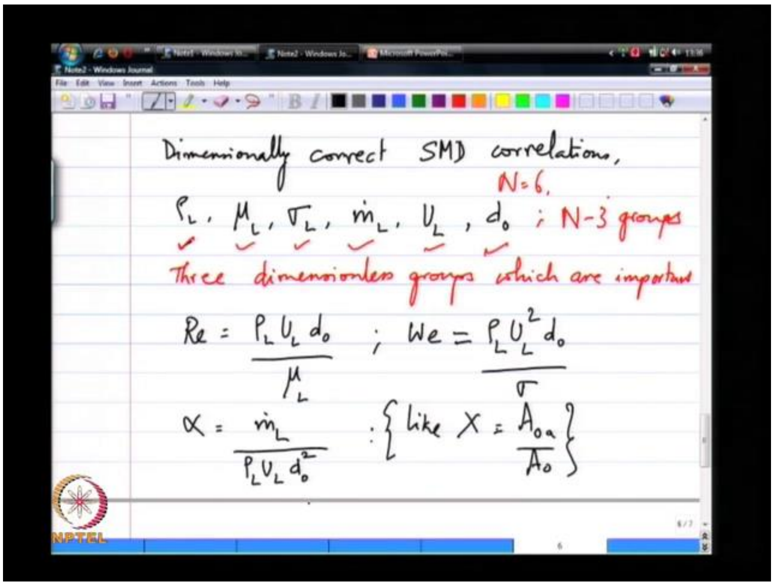

Dimensionally correct SMD correlations,

$N = 6.$

$\rho_L, \mu_L, \sigma_L, \dot{m}_L, U_L, d_0$ ; $N-3$ groups

✓ ✓ ✓ ✓ ✓ ✓

Three dimensionless groups which are important

$$Re = \frac{\rho_L U_L d_0}{\mu_L} \; ; \; We = \frac{\rho_L U_L^2 d_0}{\sigma}$$

$$\alpha = \frac{\dot{m}_L}{\rho_L U_L d_0^2} \; : \left\{ \text{like } X = \frac{A_{0a}}{A_0} \right\}$$

So, deriving dimensionally correct SMD correlations, involves taking this list of quantities and finding a set of dimensionless groups that are important. So, if I go through this I have 1 2 3 4 5 6, if I go through the dimensionless grouping, I am expecting to find 3 dimensionless groups which are important how did I come up with these 3, I am expecting to find N minus 3 groups and for this case N equal to 6 and I have three primary dimensions.

Now, I can go through the. So, called Buckingham pi theorem analysis, or I will I will just write down the result because this is standard UG class room exercise, I have here Reynolds number, Weber number which is given by rho L and the last one. In fact, is like our CD. I called this alpha which is essentially m dot L divided by rho L, U L, d 0 square, coming to think of this the denominator here has units of mass flow rate.

It has units of kilograms per second; the numerator is the actual mass flow rate m dot L. So, if you go back this essentially represents is this is like our x, which is area of the air core divided by all though not exactly the same it is like our x essentially, it gives us a the effect of the air core at the exit. Now if you go back we can now figure out a way to correlate all of these variables to their respective dimensionless groups.

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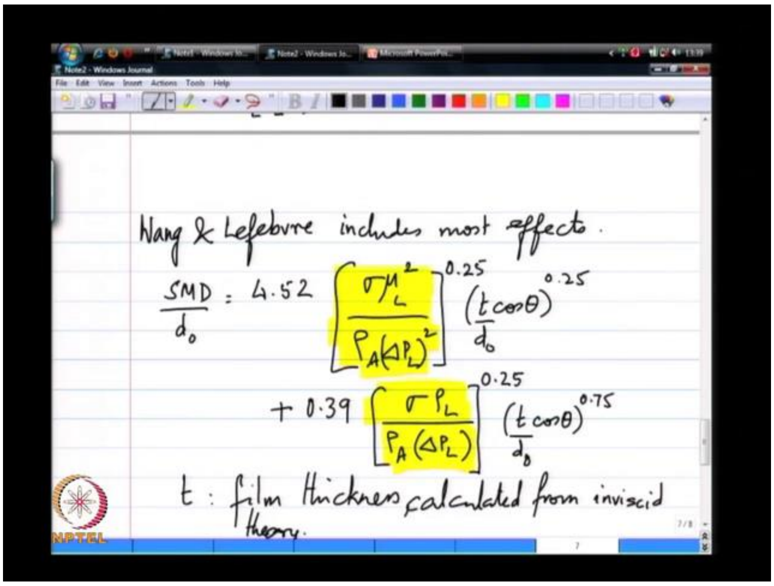

Wang & Lefebvre includes most effects.

$$\frac{SMD}{d_0} = 4.52 \left[\frac{\sigma \mu_L^2}{\rho_A (\Delta P_L)^2}\right]^{0.25} \left(\frac{t \cos\theta}{d_0}\right)^{0.25} + 0.39 \left[\frac{\sigma \rho_L}{\rho_A (\Delta P_L)}\right]^{0.25} \left(\frac{t \cos\theta}{d_0}\right)^{0.75}$$

t : film thickness calculated from inviscid theory.

And the one due to Wang and Lefebvre is the most general in this regard, and what that gives us t here is a film thickness, that can be obtained from inviscid theory. Now these quantities that you see in this parenthesis, actually I need to face that, so that case be the dimensionless film thickness, these quantities inside the square parenthesis, this part and this part are essentially dimensionless groups that are functions of the Reynolds number and Weber number.

So, some combination of Reynolds number and Weber number give us and alpha gives us these groups. So, there is 2 different approaches to predicting spray performance, one or drop size in a spray actually, one the use of in viscid flow theory inside the nozzle to give us the boundary conditions to go into a linear instability analysis and the length scales that are predicted from the linear instability analysis, giving us initial estimates of drop sizes. The second approach is to purely go to an empirical correlation based approach, where we relate the input parameters to the nozzle to the parameters of the performance of the nozzle meaning SMD.

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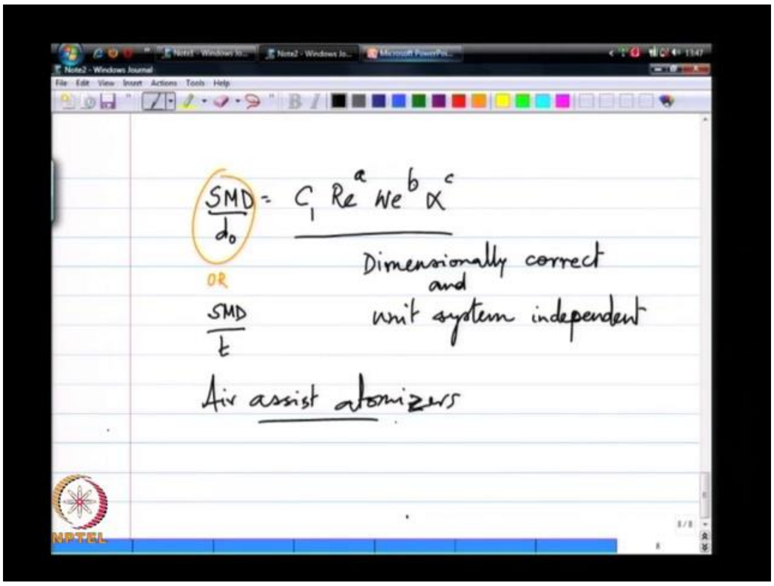

$$\frac{SMD}{d_0} = C_1\, Re^a\, We^b\, \alpha^c$$

OR

$$\frac{SMD}{t}$$

Dimensionally correct and unit system independent

Air assist atomizers

So, SMD I can write down a correlation of the form SMD over d 0 or SMD over t film thickness which is so I can either choose this, equals C 1 times Re power a times Weber number power b alpha power c. One of the biggest advantages of writing this down is that this would be dimensionally correct, not just dimensionally correct but unit system independent and unit system independent because I can choose whatever system of units for the dimensional quantities; the Reynolds number comes out to be the same.

Now, there are different studies out there where you can take wide range of spread measurements and correlate this quantity SMD over d 0 or SMD over t, plotted verses Re and Weber number and alpha and these coefficients C 1, a b and c can obtain through regression. Now, let us move on this is as far as understanding spray performance, for simplex nozzles are concerned. So, let us talk about air assist atomizers now, air assist atomizers are much involve wider class that includes this so called Prefilming air blast atomizer we looked at this design in one of our earlier lectures.

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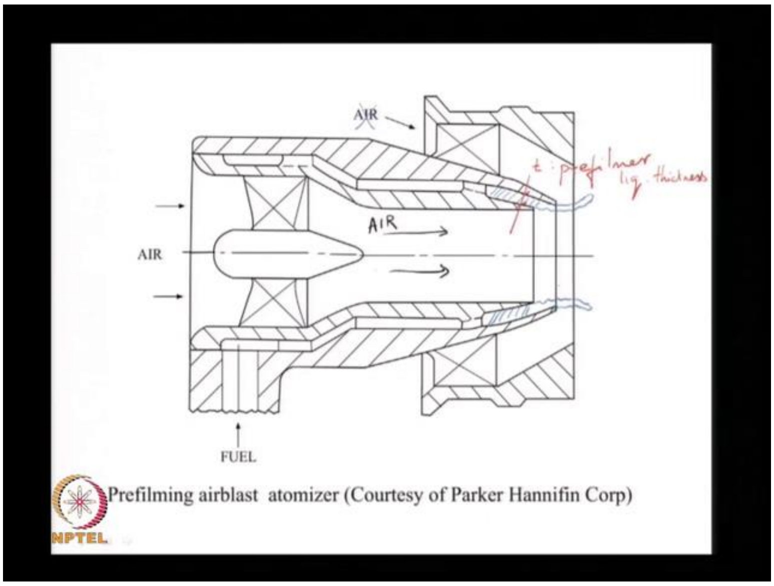

Prefilming airblast atomizer (Courtesy of Parker Hannifin Corp)

So, essentially what we have here is air coming down the middle and liquid that is spilling over from this passage. So, we have a liquid film that is spilling over, that you have that is impacted by the air on the inside and outside. This is our simplest air assist atomizer; let us take a case where there is no air on the outside. So, we are going to just for the sake of our argument will keep it simple we will look at air only on the inside there is swirling and we have a liquid film coming out.

So, right away I know that one of the indicators of drop size is going to be this thickness t that is call the prefilmer liquid thickness. The prefilmer liquid thickness is going to determine the thickness of the liquid film that is spilling out of that annular passage and of course, we have the liquid the air velocities and liquid velocities. So, if I go through the same argument as to making a list of parameters that are important.

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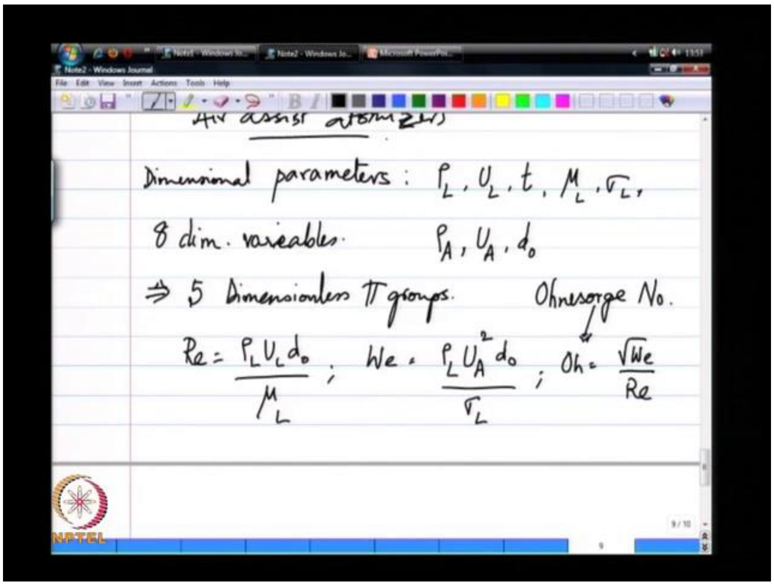

Air assist atomizers

Dimensional parameters : $\rho_L, U_L, t, \mu_L, \sigma_L,$

8 dim. variables $\rho_A, U_A, d_0$

$\Rightarrow$ 5 Dimensionless $\pi$ groups. Ohnesorge No.

$$Re = \frac{\rho_L U_L d_0}{\mu_L} ; \quad We = \frac{\rho_L U_A^2 d_0}{\sigma_L} ; \quad Oh = \frac{\sqrt{We}}{Re}$$

I have liquid density, liquid velocity, liquid prefilmer thickness, liquid viscosity surface tension potentially, but in addition I also have the air side properties, the air density, air velocity, the airside length scale like a diameter of that inner air passage actually I could even include air viscosity.

But we are going to experiments I have showed that that effects small, but let us just say will keep it simple and this is going to give us 1, 2, 3, 4, 5, 6, 7, 8 dimensional parameters, which means I am expecting 5 dimensionless pi groups. So, I am going to write down the ones that people have shown to have a relevance, the first one is of course, will make it simple and define this based on d 0.

Now, I also want to point out that Ohnesorge number is very often defined as the square root of the Weber number divided by the Reynolds number. But if I have Reynolds number and Weber number as 2 independent pi groups, then Ohnesorge number is only formed by a combination of these 2 pi groups and therefore, cannot be regarded as a third independent pi group.

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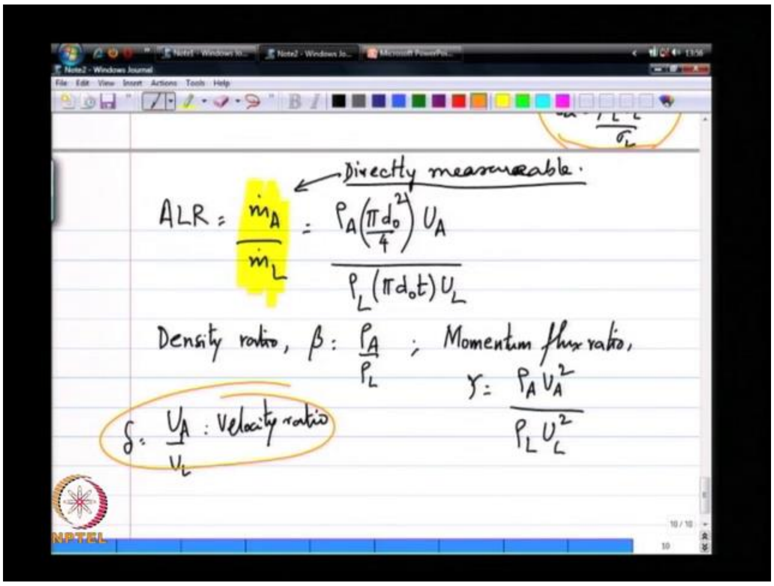

Directly measurable.

$$ALR = \frac{\dot{m}_A}{\dot{m}_L} = \frac{\rho_A \left(\frac{\pi d_0^2}{4}\right) U_A}{\rho_L (\pi d_0 t) U_L}$$

Density ratio, $\beta = \frac{\rho_A}{\rho_L}$ ; Momentum flux ratio, $\gamma = \frac{\rho_A U_A^2}{\rho_L U_L^2}$

$\delta = \frac{U_A}{U_L}$ : Velocity ratio

Now, I am expecting 5 I have written 2, there are clearly others first one that is important here is what is call the air to liquid ratio, which is the mass flow rate of air to the mass flow rate of the liquid which if I write in terms of the liquid densities and liquid velocities can be written in that form. Although it is easiest to treat this group in terms of the mass flow rates itself, because this is directly measurable any time you do a test you have mass flow rate measurements and therefore, it is much easier to relate them directly through the air to liquid ratio.

Now, there are also other dimensionless groups, call the density ratio or the momentum flux ratio. So, you have mass ratio, momentum flux ratio, density ratio these are all variables that are these are all pi groups that are independent of the others that of that have already been present at you. Now I can form a capillary number just like that, but that would be a pi group that is already that can be formed from the Reynolds numbers and Weber numbers.

So, there are not dimensionally these are not independent pi groups, but the ones that I have listed here the Reynolds number, Weber number and the rest of the groups here are all independent of each other, I cannot form one from simple combinations of the others. But if I define UA to UL which is simple velocity ratio, I can form this using beta and gamma, so if I take the gamma divided by beta square root give me this delta. So, this velocity ratio would not be an independent pi group, but beta gamma and the air to liquid ratio are all independent pi groups.

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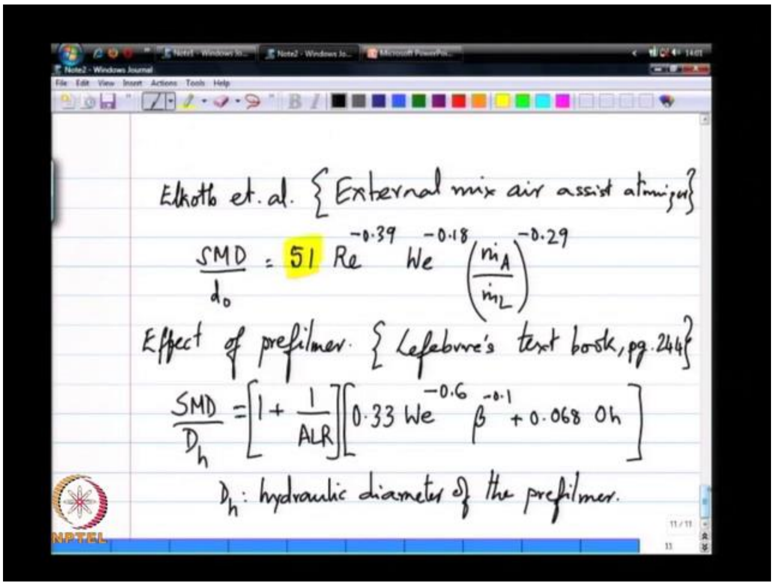

Elkoth et. al. {External mix air assist atomizer}

$$\frac{SMD}{d_o} = \boxed{51}\, Re^{-0.39}\, We^{-0.18} \left(\frac{\dot{m}_A}{\dot{m}_L}\right)^{-0.29}$$

Effect of prefilmer. {Lefebvre's text book, pg. 244}

$$\frac{SMD}{D_h} = \left[1 + \frac{1}{ALR}\right]\left[0.33\, We^{-0.6}\, \beta^{-0.1} + 0.068\, Oh\right]$$

$D_h$ : hydraulic diameter of the prefilmer.

So, how do I relied this to performance we will take a couple of examples of correlations, here is one correlation due to Elkoth et al, that is dimensionally correct as you can see there only involve dimensionless groups and this number 51 does not depend on the unit system used, that is that is the sort of acid test of what is a correct correlation. Now as the Reynolds number increases expect the drop size to decrease, as the Weber number increases the drop size decreases, as the air to liquid ratio increases drop size decreases, again sort of intuitively correct trends appear in this correlation; when I include the effect of that.

So, this is for a classic external mix air assist atomizer, if I want to look at the prefilmer effect of the prefilmer in the air blast system, you can look at Lefebvre’s text book for correlation based on his work D h is the hydraulic diameter of the prefilmer passage.

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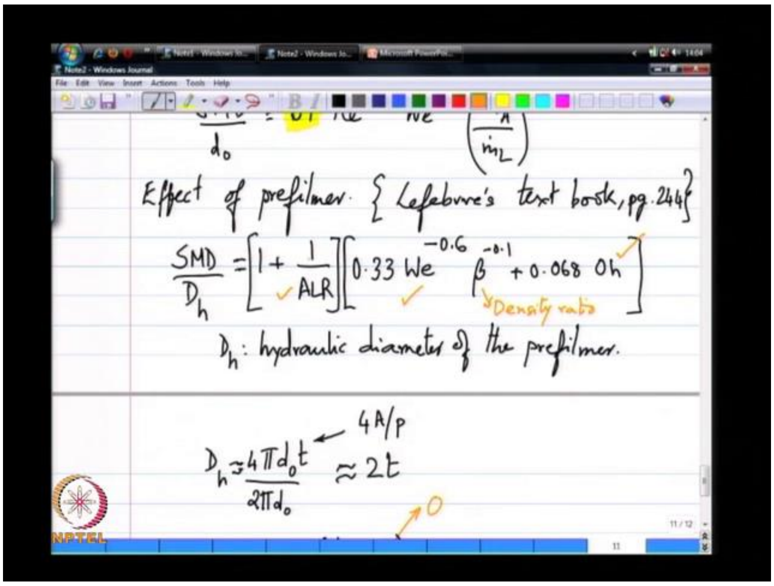

Effect of prefilmer. {Lefebvre's text book, pg. 244}

$$\frac{SMD}{D_h} = \left[1 + \frac{1}{ALR}\right]\left[0.33\, We^{-0.6} \beta^{-0.1} + 0.068\, Oh\right]$$

Density ratio

$D_h$: hydraulic diameter of the prefilmer.

$$D_h \approx \frac{4\pi d_0 t}{2\pi d_0} \approx 2t \quad \leftarrow 4A/P$$

So, if I take the prefilmer, this is the area of the prefilmer, the hydraulic diameter is approximately equal to 2 times t.

So, this is essentially 4 times area divided by perimeter, that gives us for a circle D h would exactly be equal to the diameter of the circle, the numerator here is the cross sec 4 times, the cross sectional area available to the fluid flow which is pi d 0 pi times, t is the thickness of the prefilmer, the denominator is the total weighted perimeter which is actually equal to pi d 0 on the inner side, plus pi d 0 plus the tiny thickness pi d 0 times pi plus pi 2 t, the weighted parameter in this case is that.

So, if I ignore the thickness in favor of the diameter I get 2 times pi d 0. So, that hydraulic diameter being equal to twice the thickness is reasonably good estimate, where the where the prefilmer thickness is small and comparison to the to the inner core diameter d 0. Now there are 2 aspects of case that you have to take in to account, this first of all is a very comprehensive correlation obtain from experimental data, that includes the effect of Ohnesorge number, beta the density ratio and Weber number as well as the air to liquid ratio.

So, we made a list of 5 parameters, the only one that is not included here is the momentum flux ratio and for this set of experiments, they are not reported the effect of momentum flux ratio. Essentially if you look at this correlation, as the air to liquid ratio increases, this one over air to liquid ratio decreases which means the Sauter mean diameter is going to decrease and even for an infinite air to liquid ratio, even if you have a very high air to liquid ratio this pre multiplier 1 plus 1 over ALR only attend towards one.

So, what we know there is that there is a limiting performance associated with increasing the air ratio that does not yield benefits beyond a certain value of ALR. So, typically for a classical air blast or a prefilming air blast that is at about 10 percent. So, beyond about 10 to 15 percent you really more air does not give you the benefit of that increase energy input in terms of reduce drop size.

The second part that we want to observe is this Weber number and the power associated with that which is minus 0.6, that as the Weber number increases the drop size is going to decrees for the same air to liquid ratio and as beta the density ratio increases, again just for our own understanding density ratio is the density of the air divided by the density of the liquid, as beta increases the Sauter mean diameter is going to decrease, as the power 0.1 minus 0.1. Now at density ratio is important in an aircraft or a power generation application, where this air blast atomizer is spraying in to a high pressure chamber.

Essentially beta is a reflection of the chamber pressure, because the liquid density does not change much between atmospheric condition to the high pressure condition, rho l remaining the same rho a goes up proportional to the pressure in the chamber, which means that the same atomizer is going to give you a finer drop size inside the combustion chamber than measured outside in an atmospheric pressure test (Refer Time: 50:02) And lastly there is this almost linear dependence on the Ohnesorge number, that as the Ohnesorge number increases, the Sauter mean diameter is going to increase, but the pre-multiplication factor is very small and notice that this is an additive effect.

So, in comparison to that pre-multiplication factor the effect of viscosity is very small. So, liquid viscosity in general does not play much of the role in this spray process, which is why inviscid theories give us reasonably good answers, when we use it for real spray applications we will continue this discussion in the next class.