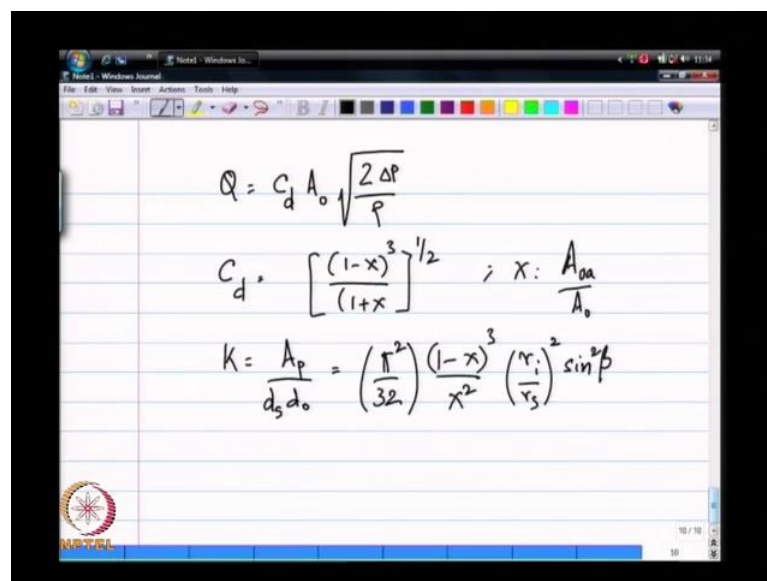


Spray Theory and Applications
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Lecture - 28
Design of pressure swirl atomizer-3

Let us come back, we were working through a design procedure for the pressure swirl or simplex atomizer and we arrived at the following.

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The image shows a digital notepad with the following handwritten equations:

$$Q = C_d A_o \sqrt{\frac{2 \Delta P}{\rho}}$$

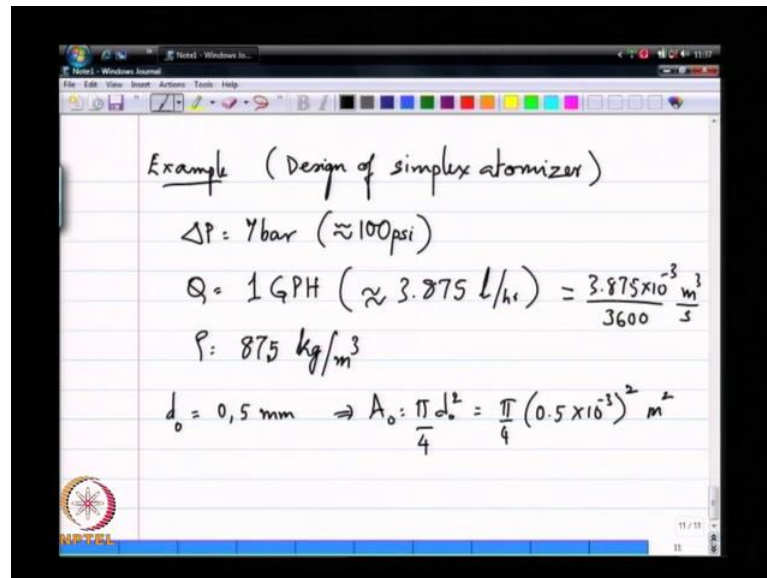
$$C_d = \left[\frac{(1-x)^3}{(1+x)} \right]^{1/2} ; x = \frac{A_{oa}}{A_o}$$

$$K = \frac{A_p}{d_s d_o} = \left(\frac{\pi^2}{32} \right) \frac{(1-x)^3}{x^2} \left(\frac{r_i}{r_s} \right)^2 \sin^2 \beta$$

We had an expression for C_d , which is our expression coming from the basic definition itself. Then we found that C_d is equal to this quantity where x is A_{oa} divided by A_o . And from there we have K which is equal to A_p over $d_s d_o$ being equal to π^2 over 32 $(1-x)^3$ divided by x^2 r_i divided by r_s square $\sin^2 \beta$.

For a given exit orifice diameter r_o , if I assume r_o I can calculate C_d and from the next expression, I can calculate x and calculate K . Now, I have some estimate of A_p , the relation between A_p d_s and d_o I do not yet have unique values for those. Let us see if we can go through an example to actually get to some unique values.

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Example (Design of simplex atomizer)

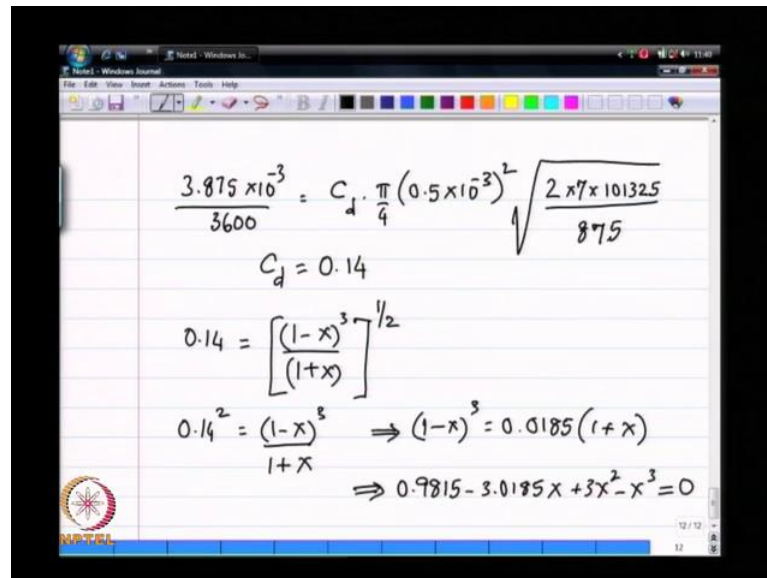
$$\Delta P = 7 \text{ bar } (\approx 100 \text{ psi})$$
$$Q = 1 \text{ GPH } (\approx 3.875 \text{ L/hr}) = \frac{3.875 \times 10^{-3} \text{ m}^3}{3600 \text{ s}}$$
$$\rho = 875 \text{ kg/m}^3$$
$$d_0 = 0.5 \text{ mm} \Rightarrow A_0 = \frac{\pi d_0^2}{4} = \frac{\pi (0.5 \times 10^{-3})^2}{4} \text{ m}^2$$

What do I have? Let us say I want to design a pressure swirl atomizer to operate from a supply of about 7 bar. Just to give you some flavor for English units as well, it is approximately 100 Psi. And I want to flow rate of 1 gallon per hour which is approximately 3.875 liters per hour of some very light weight fuel. That is all almost like z A essentially. This is what is given to me.

I said; can you design a nozzle that will do this. What do I start with, I start by assuming an r_0 which I am going to say is about 0.5 mm for a flow like this and r_0 0.5 mm implies a d_0 is pi; I am sorry, if I choose the diameter to be 0.5 mm, I am going to work this example out in metric units so we can all follow through. Q, in these same units is 3.875 into 10 power minus 3 divided by 3600 meter cube per second.

My first step was to start by assuming an r_0 , I have done that I have chosen at d_0 of 0.5 mm.

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$$\frac{3.875 \times 10^{-3}}{3600} = C_d \cdot \frac{\pi}{4} (0.5 \times 10^{-3})^2 \sqrt{\frac{2 \times 7 \times 101325}{875}}$$

$$C_d = 0.14$$

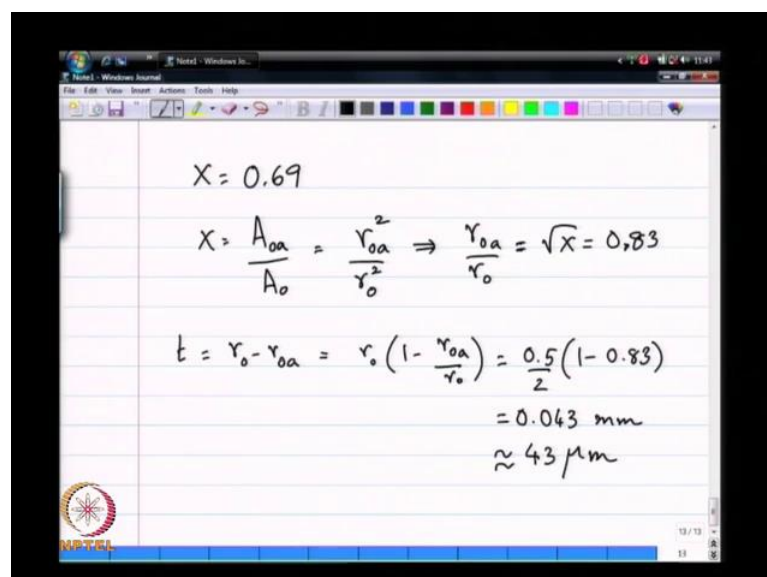
$$0.14 = \left[\frac{(1-x)^3}{(1+x)} \right]^{1/2}$$

$$0.14^2 = \frac{(1-x)^3}{1+x} \Rightarrow (1-x)^3 = 0.0185(1+x)$$

$$\Rightarrow 0.9815 - 3.0185x + 3x^2 - x^3 = 0$$

I can now calculate a C_d , Q is 3.875×10^{-3} divided by 3600 equal to C_d times the supply pressure ΔP is 7 bar, 7 bar when converted to Pascals is 7 times 100 and 1325 Pascals. The density happens to be 875, and from here I can get a value of C_d usual come out to 0.14. Now if I use the expression that relates C_d to x , if I simplified, I am trying to solve for x , I have make some rearrangements, I get an expression involving x alone.

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$$X = 0.69$$

$$X = \frac{A_{0a}}{A_0} = \frac{r_{0a}^2}{r_0^2} \Rightarrow \frac{r_{0a}}{r_0} = \sqrt{X} = 0.83$$

$$t = r_0 - r_{0a} = r_0 \left(1 - \frac{r_{0a}}{r_0} \right) = \frac{0.5}{2} (1 - 0.83)$$

$$= 0.043 \text{ mm}$$

$$\approx 43 \mu\text{m}$$

From here, I can find there is only one real root of x happens to be 0.69. I abbreviate that

it is not really needed that I keep all those decimal places.

X happens to be 0.69. Now this is typical that like in this case, I am finding that seventy percent of the cross sectional area is taken up by the air core and only 30 percent of the cross sectional area is available to the liquid film. Let us make some simplifications, x by definition is the cross sectional area of the air core divided by the cross sectional area for the flow. This would be r_0^2 divided by r_0^2 ; r_0^2 divided by r_0^2 is square root of x which in this case is 0.83.

If I want to calculate the film thickness t is r_0 minus r_i , I will just divided multiply by r_0 and this is what I have r_0 is 0.5 mm divided by 2. I have chosen the diameter to be 0.5 mm times 1 minus 0.83. I end of getting film liquid film that is only 40 microns in thickness. The resulting drop size is also going to be much lower is going to be on the order of 40 microns at the very highest, most of the drops area going to be smaller than this or on the order of 40 microns. Here is a nice way to control the saw drop size where you had orifice that is 500 microns, the drop size is only on the order of the 40 microns.

That is the advantage of the basic advantage of using a simplex design. Let us continue our thought. This is not you know the end of it. We have a value for x; we have to find the rest of the design variables. If I go back to the expression, I had for K which is given by this it is actually K squared is give is equal to this right hand side.

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$$K^2 = \frac{\pi^2}{32} \frac{(1 - 0.6853)^3}{0.6853^2} \left(\frac{r_i}{r_s}\right)^2 \sin^2 \beta$$

$$K^2 = 0.0205 \left(\frac{r_i}{r_s}\right)^2 \sin^2 \beta$$

(Take sq. rt.)

$$\frac{A_p}{\frac{\pi d_o^2}{4}} = 0.14 \left(\frac{r_i}{r_s}\right) \sin \beta$$

0.5 mm

If I substitute that I have K squared equals. I can take the square root of this. In fact, K squared if I do this computation, it comes to be 0.0205 times r i over r s squared times sin squared beta.

All I now have is I can relate this A P over d s d 0 to r i over r s and sin beta. All I have done is taken the square root on both sides. I have all of these design handles at my disposal beta r i, r s, A P, d s, d 0, etcetera and I have one more relation between these variables. You can notice that in this particular instance r s and d s cancel out leave a factor of 2, d 0 is something we identified up front that is 0.5 mm. Essentially I have an equation; I have a way of relating r i to A P that gives me this flow rate.

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The image shows a digital notepad with handwritten mathematical derivations. The first part shows the equation for $\sin \theta$ as a ratio of two terms, with several terms crossed out. The second part shows the result $\theta \approx 90^\circ$. The third part shows the equation $SMD \sim t^{0.4}$ with a bracket indicating it is derived from 'Linear instability' and 'Empirical data'.

$$\sin \theta = \frac{\left(\frac{\pi}{2}\right) C_d \sin \beta}{K(1 + \sqrt{x})} \left(\frac{r_i}{r_s}\right)$$

$$= \frac{\left(\frac{\pi}{2}\right) \cancel{0.14} \sin \beta}{\cancel{0.14} \left(\frac{r_i}{r_s}\right) \sin \beta (1 + 0.83)} \left(\frac{r_i}{r_s}\right)$$

$$\theta \approx 90^\circ$$

$$SMD \sim t^{0.4} \begin{cases} \text{Linear instability} \\ \text{Empirical data} \end{cases}$$

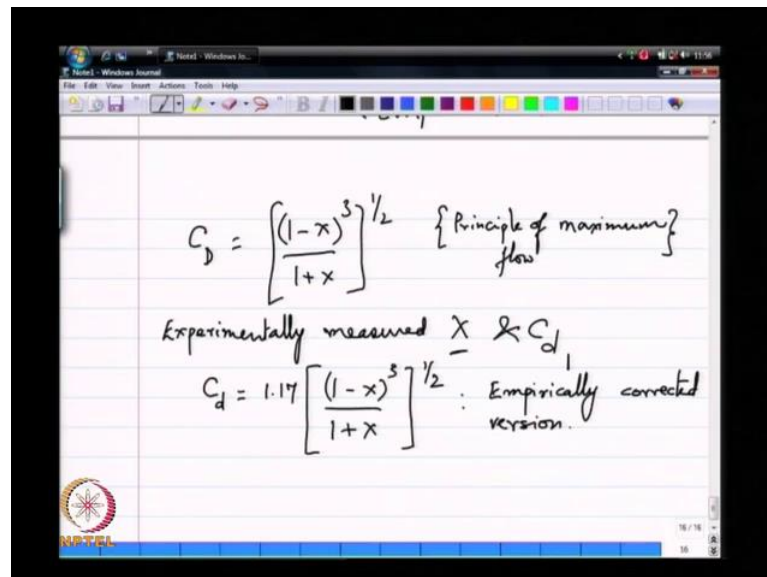
Then lastly if I take sin theta, I can make the substitutions, I know C d from the earlier calculation, beta is a design variable that is still that are disposal time K that we have calculated from the previous step K is this 0.14 r i over r s sin beta.

If I make that substitution this canceling does not happen automatically, just numerically happens to happen in this particular example. You will see that this gives us a value for theta that is almost 90 degrees with this particular. Sin theta comes out to be very close to 1; I prefer using the expression for tan theta because that allows you can without loss of ambiguity find theta that goes beyond 90 degrees. This is how we started with certain set of inputs in the form of the pressure the source pressure Q the volume flow rate density and i assuming a certain exit orifice diameter we were able to go through the

calculation to get all the way to the spray angle.

We were able to find the sin theta or the cone angle for a given set of input conditions now the, we said we are also able to relate the SMD to the film thickness. This can be done either through linear instability analysis will looked at ways of doing this or through empirical data now the empirical data path have shown that you can find the SMD to approximately scale as t power 0.4. You have essentially 2 ways of going from the film thickness that you get from x all the way to the sauter mean diameter in the spray as well as the cone angle.

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The image shows a digital notepad application with a toolbar at the top. The screen contains handwritten mathematical expressions. The first expression is $C_d = \left[\frac{(1-x)^3}{1+x} \right]^{1/2}$ followed by a bracketed note: { Principle of maximum flow }. Below this, it says 'Experimentally measured x & C_d '. The second expression is $C_d = 1.17 \left[\frac{(1-x)^3}{1+x} \right]^{1/2}$ followed by a bracketed note: Empirically corrected version. In the bottom left corner, there is a small circular logo with a star and the text 'KAPTEL' below it. The bottom right corner of the notepad shows '10/10' and a small icon.

$$C_d = \left[\frac{(1-x)^3}{1+x} \right]^{1/2} \quad \{ \text{Principle of maximum flow} \}$$

Experimentally measured x & C_d

$$C_d = 1.17 \left[\frac{(1-x)^3}{1+x} \right]^{1/2} \quad \text{Empirically corrected version.}$$

Now, let us go back to some of these expressions that we derived actually that fell out of the analysis you remember this is what we had for C_d and this is what we obtained from this. So, called principle of maximal flow maximum flow in reality if you take A , if you take experimental data measured x and C_d .

If I take a range of spray nozzles and I am able to measure C_d on those spray nozzles by simply measuring ΔP and Q and measuring x by getting the air core diameter and what we find is that this. In fact, does show a linear C_d . In fact, does show linear scaling with this grouping of x except it is off by above 15 percent in absolute value. This is the empirically corrected version from the theoretical expression it is not of considering that all of what we did was starting with Bernoulli's equation and some simple physical arguments to have a design that is within 15 percent is quite remarkable now another

correction we need to make has to do with the fact that you have the we make the Inviscid flow assumption.

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$$Re = \frac{\rho U d_o}{\mu} \quad ; \quad \mu: \text{liquid viscosity}$$

Jones' correlation:

$$C_D = 0.45 \left[\frac{\rho U d_o}{\mu} \right]^{-0.02} \left(\frac{L_o}{d_o} \right)^{-0.13} \left(\frac{L_s}{d_s} \right)^{0.05} \left(\frac{A_p}{d_s d_o} \right)^{0.52} \left(\frac{d_s}{d_o} \right)^{0.23}$$

Wide range of atomizers, mainly power generation.

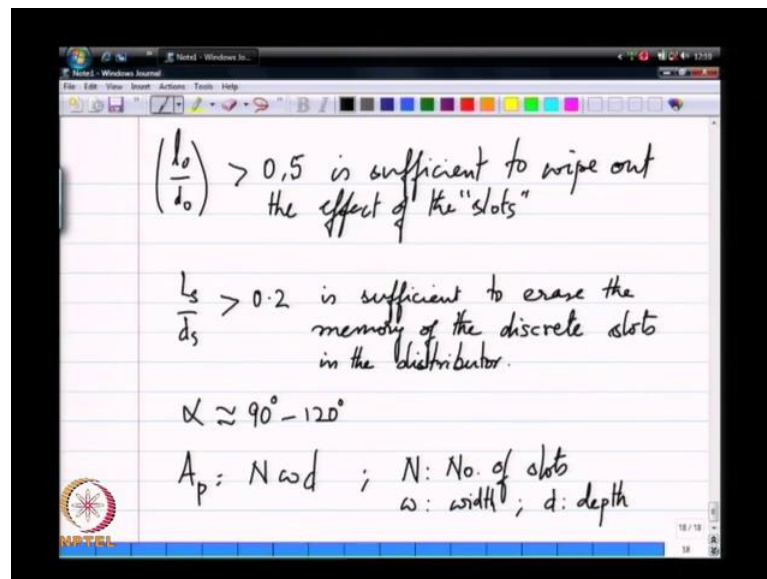
So, we ask yourself the question, we will bring in the effect of viscosity, but in the form of Reynolds number μ is the liquid viscosity u is of course, the speed it is not the axial velocity there is an expression due to Jones which includes this effect of viscosity where C_D is given to be 0.45 this is obtained on a wide range of atomizers mainly from the power generation sector and this is one of those correlations that has with stood the test of time. Now I want should do draw your attention I want to draw your attention to some of the exponents in this correlation C_D is essentially what we had for Reynolds number raise to the power minus 0.02.

In fact, the smallest in magnitude of all of the a different exponents in the correlation and that clearly shows that liquid viscosity has no effect on the flow rate it is quite surprising in we talked little bit about this has to why it shows this insensitivity one reason is that as the liquid viscosity increases you have essentially increase fluidic resistance because of which the axial velocity decreases and your swirl velocity is effected even more than axial velocity because of which the film at the exit is now thicker.

You have to contrast to sort of effects that are acting to oppose each other fluid viscosity like always is trying to decrease the flow rate and the same fluid viscosity is trying to increase the film thickness which effectively increases the flow rate. These 2 effects

counteract to give you a very weak exponent for the viscosity. In fact, if I write this correctly way C_d is μ power 0.2, 0.02. It is as the viscosity increases C_d increases to the power 0.2. If any as the viscosity increases the flow rate to the nozzle increases, all being only by about only by that power 0.02. This is observed in experiments quite a bit. In fact, this correlation is based on experiments. Now there are other terms in this other dimensionless terms in this correlation.

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Let us talk a little bit about those deferent terms l_0 over d_0 also does not have much of an effect on the flow rate. What role does it play in the design itself you essentially want to have a sufficiently long orifice for a given d_0 such that memory of the slots is wiped out? You have the fuel coming in the form of discrete slots. Let us say in have 3 or 4 or 6 or like I showed in the schematic 2 slots that are variety degrees a part on the pitch you do not want your spray to still have remnants of the number of slots that brought the fluid in to the swirl chamber. If you did that you would see essentially if I had 6 slots coming in, I would if I took a circumferential measurement of the fuel flux the liquid flux I would see 6 angular positions where the flow rate was higher than at the other places which is not good for uniformity of the spray.

I want the length of the orifice to be sufficiently large that memory of these slots is wiped out other than that it really does not play much of a role and just is a rule of thumb anything that is greater than about 0.5 is sufficient. I will put slots in quotes; let us look

at the other one which is L_s over d_s . If we go back this was our length of the swirl chamber divided by the diameter of that swirl chamber as you can see from this correlation, it really does not have much of an effect on the flow rate again, but it does have an effect again on the circumferential uniformity of the spray that you get now some of these exponents should not come as a surprise to us when we go back to our Inviscid theory, if you just think of the nozzle as an Inviscid the as if you go back to your nozzle as though your only spraying and Inviscid fluid the only time pressure or velocity change is if there is if change in the area of cross section other than that length of the orifice diameter of the length of the orifice certainly has no bearing there because you do not have any pressure drop associated with the long pipe in flow of an Inviscid fluid.

As a result what you find is that all theses length involve terms involving the length really have very weak exponents and this argument is based on Inviscid theory, but if you back to Jones correlation it is shows that the effect of viscosity itself is small which means even further that is an Inviscid fluid approximation is sufficient for a real fluid. All of our logical arguments which we will work for inviscid fluids will also work for real fluids.

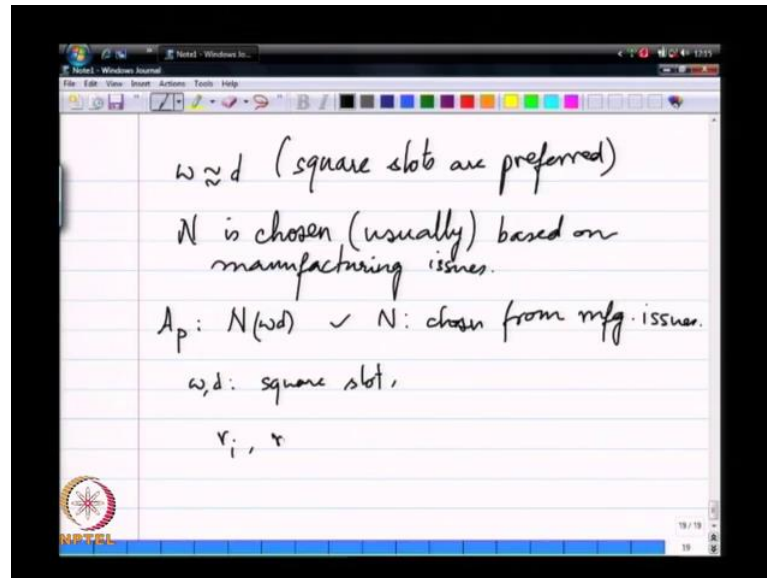
Therefore, the length of the orifice really has no bearing as for as the flow rate is concerned it has a very weak bearing, but it does have an effect on the circumferential uniformity of the spray. As far as L_s over d_s is concerned again the idea is only sufficient to wipe out. Just the fact that I had these discrete slots come a bringing in the fuel I do not want those discrete slots to still have an effect as the fluid goes through the nozzle and comes out in the form of a spray.

Essentially this L_s over d_s being greater than about point 2 is sufficient to ensure that that effect is wiped out likewise again in the same this thing α is usually on the order of about 90 degrees to about 120 degrees α happens to be the convergence angle going from the swirl chamber to the exits orifice now again without any surprise C_d is most influence by this ratio $a P$ over $d_s d_0$. If I want the particular C_d that is the parameter that tells me what it is that I that is my design group.

I can find a value for this $a P$ over $d_s d_0$, but I do not still have a way of identifying individual dimensions how do I do that say $a P$ is n times w times h we wrote this down n is the number of slots w is the width of the slot and h is the height or depth of the slot I

think we said d , but there now you can again see the only parameter that makes a difference to the flow rate is a P which is again n times w times d fluid mechanically it makes more sense to have w approximately equal to d .

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You are bringing in the fluid without any choice of choice of an aspect ratio choice of preferred asymmetry to the fluid flow inside the swirl chamber. Essentially square slots are preferred n is chosen usually based on manufacturing issues say for example, I go through this design process and I am able to identify a value for A_p . I can now choose n let us say as number like four in which case w times d becomes, I can find a value for the cross sectional area and from the from the argument that I wanted to be square, I can find individual dimensions if n instead of being four became 6 then w times d is now smaller which means I have to now machine smaller dimensions.

You choose n such that those machining tell you that your current machining practice is able to machine those dimensions not just that you also do not want very very small fuel passages inside the nozzle because most of these nozzle especially once that going combustion applications could be in a high temperature environment. You do not want any sort of reaction to be initiated inside the spray nozzle although that is unlikely because it is being cooled by cold fluid all the time, but you certainly do not want is some of those slots becoming clogged with dirt coming from the coming past your fuel filter most of these spray systems have a filter on the line before you come to the spray

nozzle now the filter is only able to tolerate certain class of particles and the smaller ones do get through and the smaller particles are still likely to clog up slots in your spray nozzle even though the slot is wider than any one individual particle.

Let us say, I have a fuel filter that is able to keep out particles on the order of 10 to 10s of microns and if the slots are now let us say 100 of microns in size, I could have these 10 micron particles actually agglomerate and clog up particles even though the slot is A is on the order of 100s of microns. Essentially n is chosen from manufacturing issues I mean determine n I can find w and d from the fact that I need a square slots I can find r_i and r_s now if you go back to most of these the correlations that we had for C_d and K you have this ratio r_i over r_s that occurs many times over.

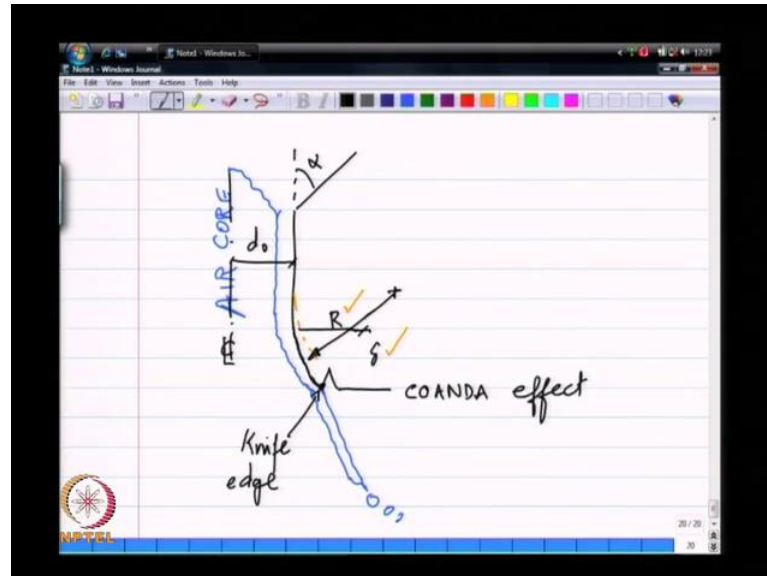
The closer you make that equal to 1 the more efficient the usage of energy because if I take a slot that is coming in at certain radius r_i with respect to the center, but I will have swirl chamber that is let us say much larger than this r_i than r_i the ratio r_i to r_s is essentially going to create a fluid motion inside swirl chamber that is not useful to me, I have a swirl chamber that is much bigger, but I am bringing in the fluid at a at an intermediate radial location if my thumb is the center line and if my index finger is the wall of the swirl chamber, I want to be as close to the wall as possible the only manufacturing problem with that is if I go to these very first figure that I had, if this is r_s if this diameter is 2 times r_s it would be very difficult to draw a square slot where r_i becomes very close to r_f .

Some practical issues related to manufacturability over take efficiency considerations and therefore, you have to live with r_i over r_s typically on the order of about 0.6 to 0.7 you cannot really go beyond that. Now $\sin \beta$ also has a bearing as you can see, now this is the angle at which the fluid is brought in to this swirl chamber I will go back to my very first sketch. If the fluid is brought in at some angle β then, the $u \sin \beta$ is the tangential velocity and $u \cos \beta$ is the axial velocity that is coming in. β is a nice direct way of independently controlling the spray angle because I can now control the swirl momentum flux to the axial momentum flux I can control the ratio of the swirl momentum flux to the axial momentum flux.

β gives me a direct handle on the swirl on the spray angle the cone angle much more than any of the other variables. That is the handle that is often used, but in reality again

coming from manufacturing issues we find that there is a nice ingenious way of achieving a spray angle will talk about that for just a movement.

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If this is my diameter d_0 let us say this is α , I have the swirl chamber up strain of the figure of the sketch that I drawn here, and the liquid is coming out let me draw this in the liquid is coming out in the form a film in this may be the position where the air core ends.

All of this is the air core now what is usually done here at the exit it is that feature that looks like a trumpet is added on. It is essentially have some very large radius and Axisymmetric about the center line, when you add this feature smoothly to the exit orifice what you find is that this film sticks to the exit orifice and by mass conservation constraints; it actually becomes thinner. Because the cross sectional area available to the flow is now increasing the film by mass conservation becomes slightly thinner although that is not the intended effect it is a desired desirable side effect. Now if I take these, if I create a sharp edge at this corner this film is no longer able to make the sharp turn it is essentially what is called Coanda effect in surface tension literature, just as if I am trying to pour water out of a glass jug the water tries to dribble around the radius edge of the glass jug if I have a sharp edge on the glass jug it is able to perfectly flow out it is entirely using surface tension.

I can now have this liquid film depart from their and what I have is what I have

happening we more quantitatively accurate, what I have happening down here is that I have start to get a spray. By controlling this radius r and this depth δ we can see that if I take a certain radius, the deeper I go, the more radius the more of an angle I create which widens the cone angle automatically. By controlling δ and r there is a very nice elegant way of controlling the cone angle I can do whatever I want upstream and I can dyeline cone angle at the by controlling the exit geometry which is always good from manufacturing perspective. Again one more little trick that is often employed, which is kind of lead is that in order to further facilitate this Coanda effect I can create very sharp exit orifice.

This is almost sharp corner knife edge. I can get a very precise spray angle for a for any flow rate that is up strain a upstream all I need is that I have sufficient centrifugal force and or surface tension Coanda force one of the 2 is enough I can, if I have if I do not have enough surface enough centrifugal force upstream in the nozzle surface tension may be sufficient to cause this liquid film to stick to the wall as it exits the nozzle just like water being poured out of a jug. I can control, this is how typically the spray angle is controlled in a commercial pressure swirl atomizer. You designed the spray nozzle for a given flow rate and film thickness.

The one parameter that I still need control over from the geometry up stream is the film thickness which then has a role to play in the Sauter mean diameter. I am able to dyeline a certain flow rate Sauter mean diameter cone angle for this spray for and then satisfied the given set of constrains. This is although this is how the design process for a typical pressure swirl atomizer has evolved and most of these literature goes back to at least 40 years the Chu et al paper that I discussed is more resent, but it is more of a validation of the literature from back then in terms of the insensitivities of the flow rates to the parameter that we discussed. But if you are spray applications engineer today, one would aspect that you would not have to go back to the drawing board to do a design of this of go through a design process such as discussed today it would be sufficient if you go to a selection table to choose a nozzle.

We did discuss that as well in one of earlier classes, that most of most applications today is essentially a selection not a design excise.