Spray Theory and Applications
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Lecture – 27
Design of pressure swirl atomizer – 2

Welcome back. We will continue our discussion on design of atomizers. We will start by looking at simplest of all which is the pressure swirl atomizer and try to understand a formal design process that we can used to design a simplex atomizer. Now before we go much further it is always a good exercise to first have a schematic of what you want your atomizer to look like, and various degrees of freedom that you have available to you at your disposal, say you know whether is an orifice size or any other dimensions. There, these are all degrees of freedom that you have available at your disposal.

And you have been given a certain set of constraints that you have to meet. So like for example, you may have a certain supply pressure that is available to you and you want your atomizer to spray a certain fluid at a certain flow rate these are all design constraints that you have to obey at the end of your design process.

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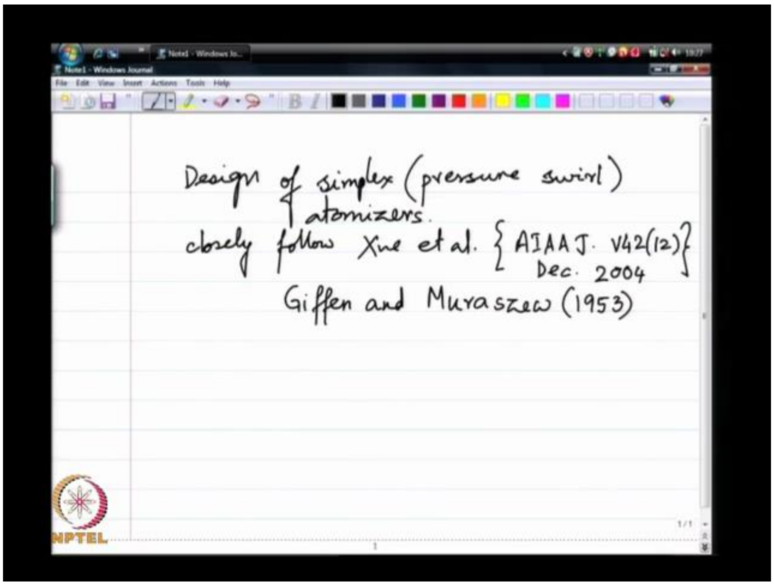

So, we will start our discussion of design of simplex atomizers. This is going to closely follow a paper by Xue et al, I will give you the reference it is AI double A journal volume 42 12 from December 2004. In fact, their process also follows the original work in this field by these two people, this is a classic paper where the first design process where of simplex atomizer where was presented.

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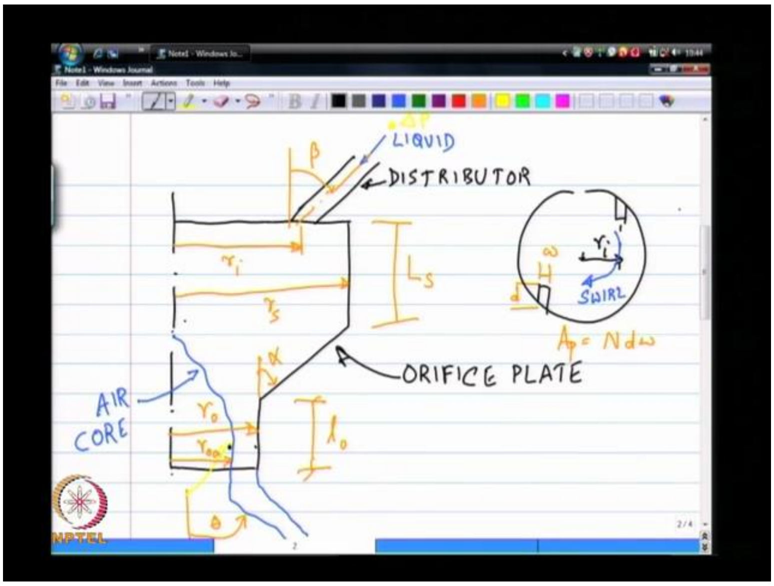

Now, let us first draw a schematic of this whole of what it is that we are talking about. We will start by looking at the schematic itself, the atomizer that in question as two parts to it. Usually, there is a fuel distributor. So, this we will call this part distributor and this part is often called the orifice plate. So, the simplex atomizer at it is heart as two parts to it. Now, the distributor element has a set of tangential slots and in this case, we have shown those to be at some angle beta. And, they are bringing in fluid into this passage here as we as just I will sketch this just to show you, this is all like the passage that is enclose by the distributor and the orifice plate; we want to erase and this wanted to show you the basic idea of how those works.

So, all of there a part that I hatched in yellow could be wetted by the liquid that you are trying to spray. So, that is the geometry we are looking at. Now their distributor element brings in fluid at some angle beta as well as it having a swirl component. So, if I look at the top few of the distributor plate essentially it could be a set of slots. Let us say I am showing in this case two slots that are coming in at some of axis location r i, the of axis location r i decades the amount swirl. Now I am shown the swirl chamber here to be slightly larger than r i itself. So, the distributor slots are not exactly at the very tangential edge of the swirl chamber which is sort of more like a practical situation.

Now, in this case, essentially; what we form because the liquid coming in is at tangential has a tangential velocity you create a swirl in this plane and that swirl sets up a free vortex inside this region. For a free vortex we saw that, we total angular momentum is conserved, because there is no dissipation because the flow is irrotational, for a free vortex the angular momentum is conserved which I can write as follows.

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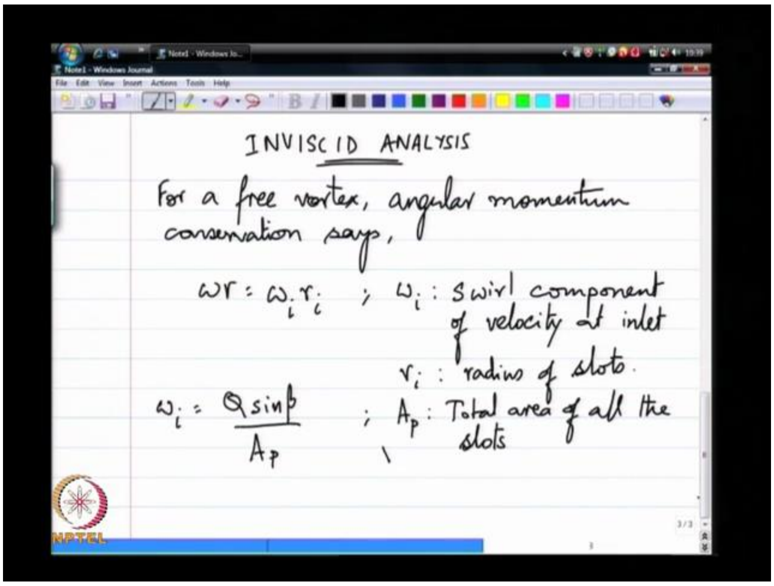

If I want to know what the swirl velocity is at any r location, and if w i is the swirl component of the velocity at the inlet, r i is of course the radius of the slots in the distributor. Now like I have shown in this picture two slots but, I could have multiple slots, so at any r location I have w i. Now just to clear what w i is; w i would be Q times sin beta divided by A p. A p is the total area of all the slots put together.

For example in this case, if w is the width and d is the depth N time d times w is A p. Before we go much further let us write down the list of givens, what are all the degrees of freedom that are available to us in the design process and typical constraints. For example, the degrees freedom available to me are N, d, w, r i, r s, I will add a few more - if this is L s and if this is l o. So, L s is the length of the swirl chamber, l o is the length of the exit orifice and of course r o that I have shown here which is radius of the exit orifice.

Now I could also have let us say, this angle here alpha the angle of convergence these are all parameters that are important. Typical constraints that we have to meet, are supply pressure is usually specified, flow rate that is you want the nozzle to deliver is usually also specified, so that is Q we will say that is in meter cube per second. This is in pascals or bar typically, let us the more engineering rather than a SI unit will say this is specified in bar. This may be specified in liters per hour or liters per second.

I have a certain cone angle that I require, say for example in this schematic it could be that angle there of the liquid sheet exiting the nozzle. And in more resent instances you may have some sort of an indication of some sort of a requirement coming from drop size. I will use a script d for this although we will very quickly see that it is going to only be known after the fact not really controlled during the design process.

If I now go back to my free vortex that I have set up by this tangential slots w times r is w i r i which is, coming from angular momentum conservation. Now, mind you the moment I say angular momentum conservation I have already made a couple of simplifying assumption that we are only dealing with inviscid analysis. Actually, angular momentum conservation alone does not require inviscid see it could be true even in a viscous flow, but we are only going to look at an inviscid analysis; as we will see a little later on.

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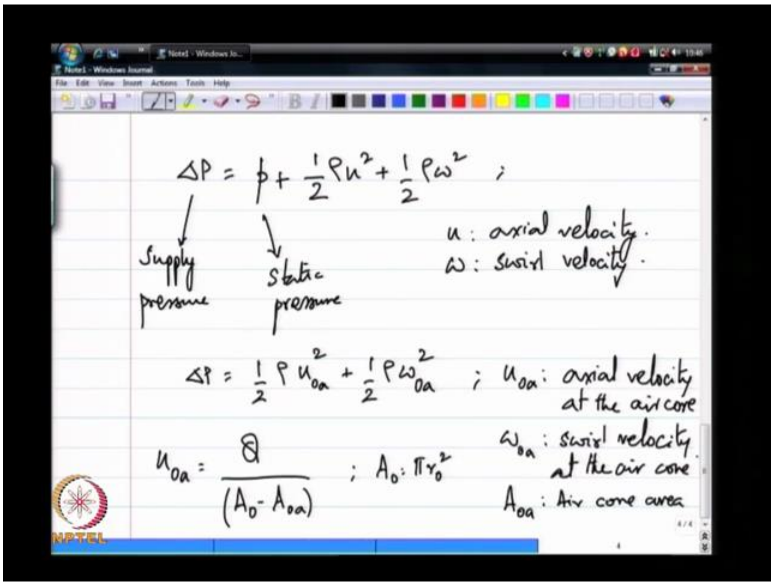

Now, since we have made the simplifying assumption of the flow being inviscid I can now use Bernoulli's equation. At any if delta P is the supply pressure P is the static pressure at any point in the chamber, u is the axial velocity, and w is the swirl velocity.

Now, if I go back to this schematic we talked about this in some detail the one of the main advantages of using a pressure swirl atomizer is the fact that it entrance an air core. So, your orifice size is no longer the determinant as far as the lengths scale of the liquid coming out of the nozzle. The lengths scale associated with the liquid can be much smaller than the orifice size itself and that is one of the advantages of having the pressure swirl like design.

So, let us see what that means here, what that means, what I have shown here in blue is the air core itself. And I am going to call the air core radius as r o a, will see that it does have a significant bearing to the performance of this nozzle. So, if I have r o a as being the radius of the of the air core I can apply the Bernoulli's equation from between the two points, if I take a point way upstream here where the pressure is delta P and if I apply it to a point on the air core here. So, I am applying Bernoulli's equation between this point way upstream of the atomizer where the fluid kinetic energy is very small and all of the and basically u and w are 0 and to a point just on the liquid in the liquid, but on the air core periphery. So, if apply Bernoulli's equation between these two points, what do I get I find the delta P is equal to half rho u 0a squared plus half rho w o a squared where, u 0a; is the axial velocity at the air core and w 0a, is the swirl velocity at the air core.

I think at this point it is good to take a small detour down the lane of undergraduate fluid mechanics again, remember Bernoulli's equation is only applicable on a streamline now if I take a streamline coming to this air core all the way extended into the tank. So, if I can draw a streamline going from this point here all the way into that tank which is at some high elevated pressure delta P. We are applying this Bernoulli's equation on that streamline. I can also apply it from that same tank to another point let us say, here or here does not matter, but if all the streamline are originating from the same tank I can apply this Bernoulli's equation between any point inside the swirl chamber and the tank.

Which also means, since the tank is at a constant supply pressure delta P it also means I can essentially apply Bernoulli's equation between any two points inside the swirl chamber, even if they are not on the same streamline, because I can apply Bernoulli's equation on two with on two points on a given streamline and if all the streamlines are originating from the same stagnation pressure delta P, I can essentially apply Bernoulli's equation even across two streamlines at two different points because there all amounting to the same stagnation pressure we will take advantage of this later on.

So, now back to, we have a delta P which is half rho u 0a squared plus half rho w 0a squared. Now what is u 0a, that is the axial velocity in this region and w 0a; happens to be the swirl velocity. We will write some simplifying expressions for that u 0a, would then have to be Q divided by A 0 minus A 0a.

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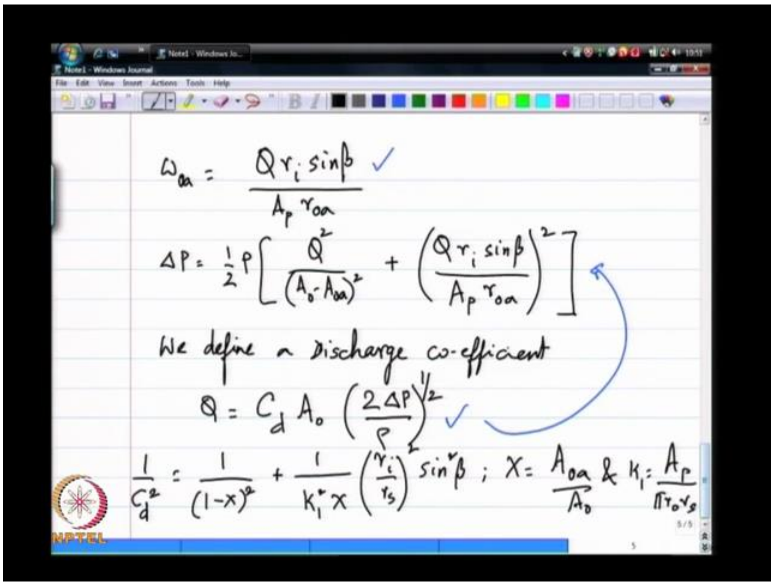

A 0a is the area of cross section of the air core A 0 is of course, pi r 0 squared, w 0a, I can get from our angular momentum conservation as that Q r i over A p is the linear velocity or speed of the liquid coming in, let us make some notes here this velocity vector as a magnitude of Q over A p and the component that is, in this plane here is, Q A p sin beta and that times r i is the angular momentum essentially times rho. Of course, but we are to instead of writing rho v r, but this going to write v r because rho is constant everywhere.

So, I can now substitute these two equations into the Bernoulli's equation at the air core what do I have delta P would then become half rho now, we define a discharge coefficient just like we would define let us say for flow in a pipe or flow faster flow through an exit orifice or sorts we were actually introduce to this earlier on, we use the terminology flow number and we showed how that flow number is going to be defended on some system of units that you have. We are now going to define, if I figure out a way by which we can make it relatively unit independent. So, I can now define a discharge coefficient as follows, Q by A naught is the, A naught is like the total cross sectional area available for the flow, so we are defining this discharge coefficient as though we have no swirl it is just flow through an orifice. When I substitute Q from this expression into the equation for delta P after some just a couple of simplifying steps we find this I have introduced the couple of new terms new symbols here, let me go head define them.

X is A 0a divided by A 0 which is like the fraction of the cross sectional area, that is occupied by the air core. X is physically the fraction of the cross sectional area that is occupied by the air core, K 1 is this dimensionless group A p over pi r 0 r s you can clearly see K 1 is dimensionless, so is x. So, I now have I have an equation relating C d 2 X and I also have K 1. Now K 1 is what I am after K 1 is something I need to get a value for before I can design C d is something that I may actually know apart from A 0. So, if I assume an A 0 if you look at this expression here one more time, if I know A 0 I know C d because Q and delta P are given to me and the form of constraints have a supply pressure and I have a flow rate I need to deliver.

So, for a given A 0 I can find the C d, if I have a C d I still have only one expression down here coming from a Bernoulli’s equation, which relates K 1 and X X is still something we have know, handle over that is determined by the fluid mechanics inside the atomizer. Apart of course, r i over r s and sin beta these are all also design variables. Design variables degrees of freedoms that are avail degrees of freedom that are available to us and we have only one expression. So, here in comes a simple principle that is often in worked to fix the value of X.

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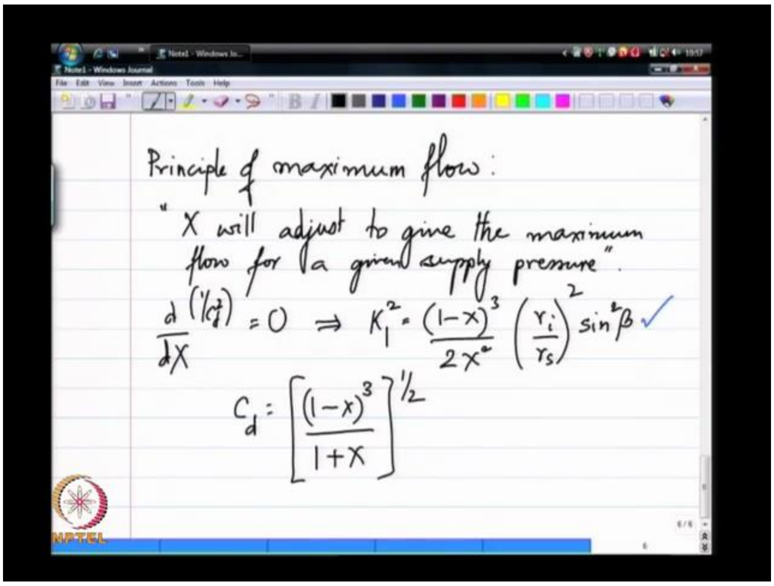

This is called the principle of maximum flow which says that X X will adjust to give the maximum flow for a given supply pressure. What does this actually, what how does this work physically just to sort of force some arguments if I have an orifice and let us say, X is the cross sectional area that is obscured by the air core. If X where large then you can sort of imagine that the back pressure behind inside the swirl chamber would be larger.

Essentially, that force would cause more liquid to flow causing the air core to shrink if the air core was too small then, the pressure there would be low that the pressure outside the ambient pressure would push their core out to a larger diameter. So, these two forces would balance themselves where the air core as such a diameter that the flow through is at a maximum this is not really something that can be proved other than through like posing simple arguments, but it is often invoked in fluid mechanic systems to give as one more equation.

Let us see what that does. We have an expression here relating 1 over C d squared 2 X and one way of employing this is to set d dX of 1 over C d squared equal to 0. So, essentially d C d dX is equal to 0 is the same as writing this expression and when I do that what do I have I will find K 1 squared. So, I have a value for K 1, K 1 related to X, now this is coming from this principle of maximum flow if I substitute this back into my expression for C d we end up getting this equation C d. So, if I give you a C d here is a nice expression that gives you the value of X value of the air core diameter. Essentially, if I know C d I can solve this into a cubic equation, cubic polynomial equation for X that usually has two complex roots and only one real root. So, you do you are guarantee to get a value for X that is real and usually between 0 and 1 that is what you want.

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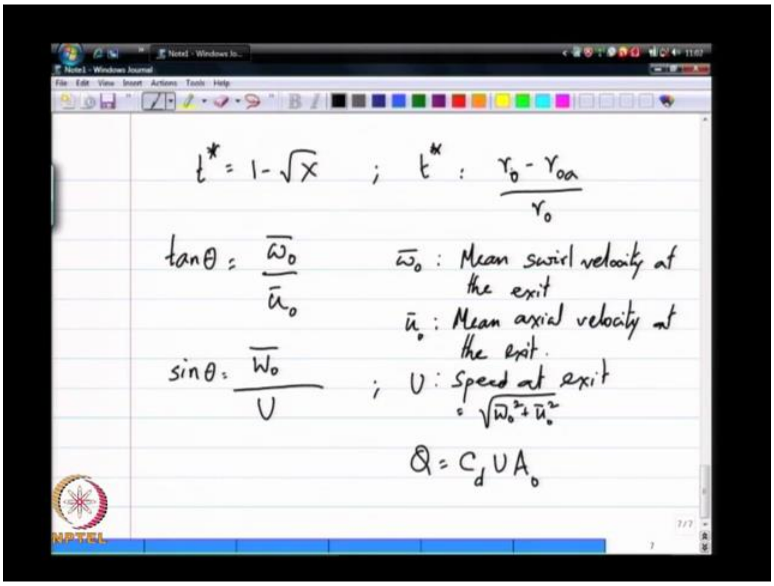

And now, if I from this definition of X I can define a non-dimensional film thickness if I define a t star to be r 0 minus r 0a divided by r 0 you can quickly see how that definition is the same as 1 minus square root of X. X is a 0a divided by a 0, square root of X is the same as r 0a divided by r 0. So, what have we done here essentially, if you tell me that is go back to the list of constraints that we have these are what we want to meet. So, if you give me a delta P and Q I am going to assume an r 0, I am only going to assume an exit orifice is diameter and for that exit orifice diameter Q and delta P I can find C d from this expression that is the basic definition of what a discharge coefficient is once I know a value for C d I can come back here and get a value for X.

Once I know a value of X, I know the non-dimensional film thickness coming out of the nozzle now why is the non-dimensional film thickness important because that as a bearing on the drop size. If I know delta P Q and A 0 I have a way of estimating what the drop size would be. So, if I know t star and if I know the fluid mechanic properties of the film coming out let us say the swirl velocity the axial velocity the air density I can use linear instability calculations to find the maximum the most unstable wave length on that film and from the most unstable wave length I can estimate a drop size that gives me an estimate for what the mean drop size would be coming out of the nozzle. There is also another side to this which is coming back from the cone angle a simple trigonometric calculation would tell us that, the cone angle is that. So, where w 0 bar is the mean swirl velocity at the exit and u 0 bar is the mean axial velocity at the exit we can also rewrite this where u is the speed at exit given by root of w 0 bar squared plus u 0 bar squared.

And I can write a simple expression as u times A 0 this is the basic definition of any discharge coefficient u times A 0 is the theoretical flow rate Q the actual flow rate divided by u times A 0 the theoretical flow rate is the discharge coefficient. Now I need an estimate for w 0 bar before I can tell you what the sub what sin theta is going to be, so for that we are going to use the idea that we have a free vortex inside the swirl chamber. So, free vortex as a 1 over r dependence of their, of the swirl velocity with the radial coordinate. So, we will take advantage of that.

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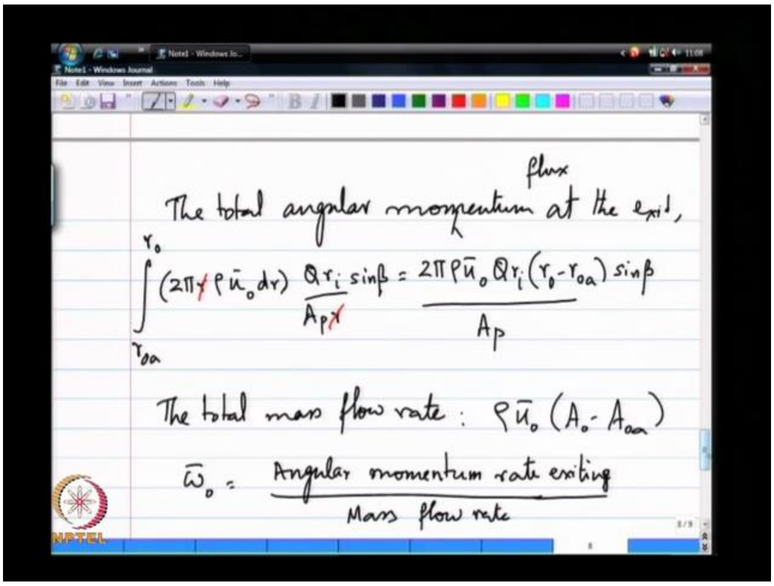

And once I do that, the total angular momentum at the exit is given by integral going from r 0a to r 0 of 2 pi r this is actually better stated as angular momentum flux at the exit. It is like how much angular momentum is exiting the swirl chamber at this point. So, the exit is essentially the mass flow rate times the angular velocity associated with the mass flow rate. So, I have 2 pi r rho u 0 bar d r u 0 bar is the mean axial velocity that axial velocity is actually, what is carrying the mass out of the nozzle the swirl velocity is not carrying mass out of the nozzle because b dot d a which we are use to in control volume analysis for the swirl would be 0.

So, the mass is being brought out of the nozzle by the axial velocity and this rho u 0 bar is the mass flow rate is rho u 0 bar times 2 pi r p r is like a differential cross sectional area at a radius r this is the actual differential mass flow rate coming out of thin slice d r width at some radial location r this times q r i divided by a p r sin beta. So, q r i sin beta divided by a p is the swirl velocity at the inlet is the angular momentum at the inlet, that divided by r remember our very first expression said at any point I have this w at any point inside the flow inside the swirl chamber times the radial location of that point is equal to w i r i and w i is given by q sin beta divided by A p, A p is the total area of all the slots as a reminder. So, this if I do the integration I can cancel out r here and essentially, what do I have this gives me is essentially have a constant the only and integral d r going r 0a to r 0 gives me just the difference between the (Refer Time: 42:17) the total mass flow rate exiting the nozzle is rho u 0 bar times A 0 minus A 0a. This is just a cross sectional area available to the fluid flow A 0 minus A 0a times u 0 it is like an, it is like the basic definition of mean axial velocity.

Therefore, if this is the total angular momentum that is being carried out by this mass flow rate the angular momentum divided by the mass flow rate is like the mean swirl velocity the angular momentum flux see the word I am not being very carefully here flux is actually, a word use to indicate per unit area. So, this is actually the total angular momentum flow it is actually total angular momentum rate and, if I take the mass flow rate in the denominator and take the angular momentum rate exiting the nozzle and take the mass flow rate exiting the nozzle these ratio is essentially what the average swirl velocity would be. So, if I do that what do I have this is a simple division of these two quantities we identified, w 0 would then become 2 pi q r i r 0 minus r 0a.

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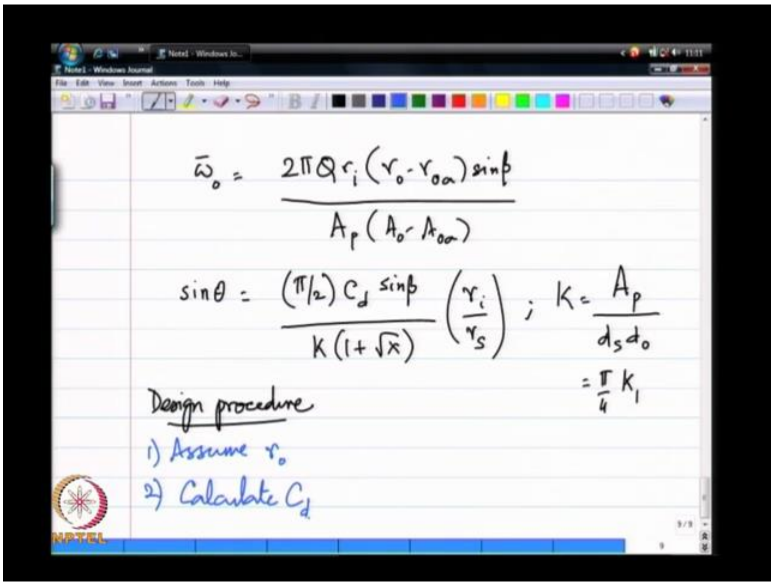

So, if I invoke the definition of sin theta that we put up there will find that is equal to pi over 2 C d sin beta times r i over r s in here I have added another K here which is essentially A p divided by d s d 0 it is simply pi over 4 times the K 1 that we had defined earlier we are defined a K 1 as A p divided by pi r s r 0 k is just pi over A K 1. Let us quickly recap and write down the design procedure. So, first calculate C d, the first step starts by assuming r 0 and then you calculate C d.

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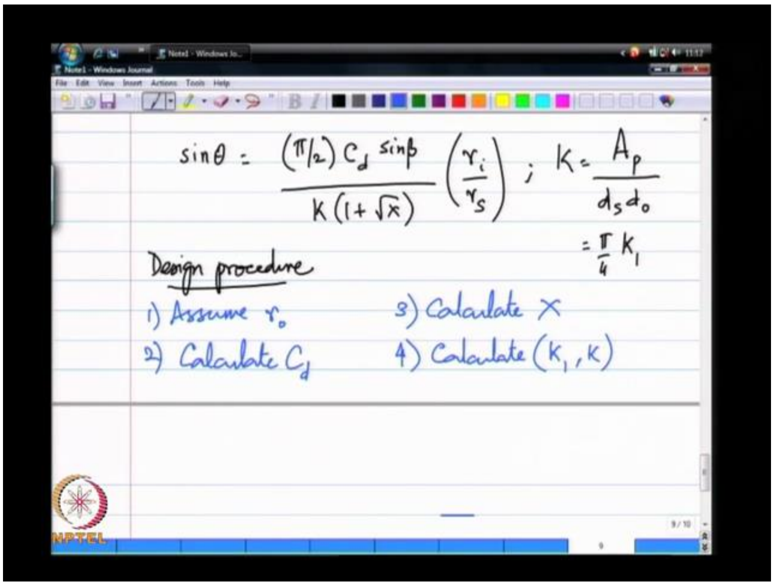

And from the calculated C d you get calculate X and then from there you can calculate K and K 1 K is a dimensionless group that involves some of your design some of your degrees of freedom areas of the pods these d s d 0 etcetera. So, I now have a way of going from a flow rate two by assuming r 0 which is one of my degrees of freedom, I can get some constraints on the other degrees of freedom. Design is always an open ended problem that there are multiple solutions to a given set of constraints if you go back to the list we clearly had more in our degrees of freedom list then, we had in our constraints and that is always going to be the case irrespective of where you go that is the way good design process always works.

So, we are still left with some degrees of freedom that we have to use to our advantage to achieve other constraints that earned explicitly laid out in quantitative terms like for example, you may have constraints coming from uniformity of this spray you could quantify it, you could have constraints coming from manufacturability tolerancing you could have constraints come from other performance issues such as drop size we have not talked about that.

So, there we look at a couple of examples in the next class where we will attempt to apply this design procedure and reach the set of values for our design for our degrees of freedom.