Spray Theory and Applications
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Lecture – 26
Design of pressure swirl atomizer – 1

Welcome. We will continue our discussion on sprays, but we will switch gears slightly. We will start look at the insides of some of these atomizers and nozzles.

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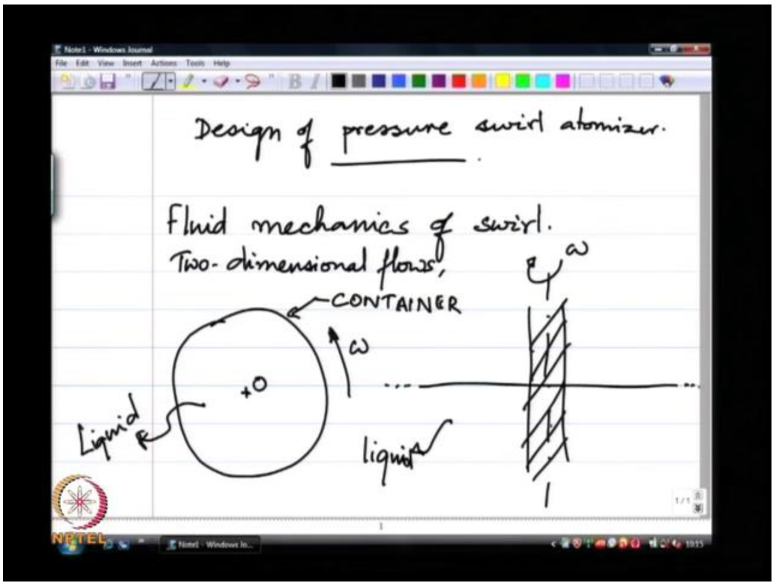

We are going for the next few lectures, we will talk of design of atomizers and we will begin with design of a pressure swirl atomizer. But before that I want to provide a quick primer on the fluid mechanics of swirl. We will see how swirl in general works. I will take 2 kinds of swirls and we will see what it looks like, if I take a container of liquid and spin it at some angular velocity omega, about a point o.

So, we are only going to look at two-dimensional examples to start with that is, sufficient to understand how things work. This is a container that is spinning and it has got some liquid in it. This liquid at least at the wall, is going to spin with the container and towards the middle, it is not going spins. This is one kind of swirl. The other is where I take, let us say an infinitely big bath of liquid and insert a rod into this and spin this rod at some velocity at some angular velocity omega. In one case, I have liquid inside a container and the container is spinning.

In the other case, I have an infinite bath of liquid. And the infinite bath of liquid; I emerge the rod into that bath of liquid and spin the rod, in some angular velocity omega. I want to understand, what the fluid mechanics is like in these 2 situations?

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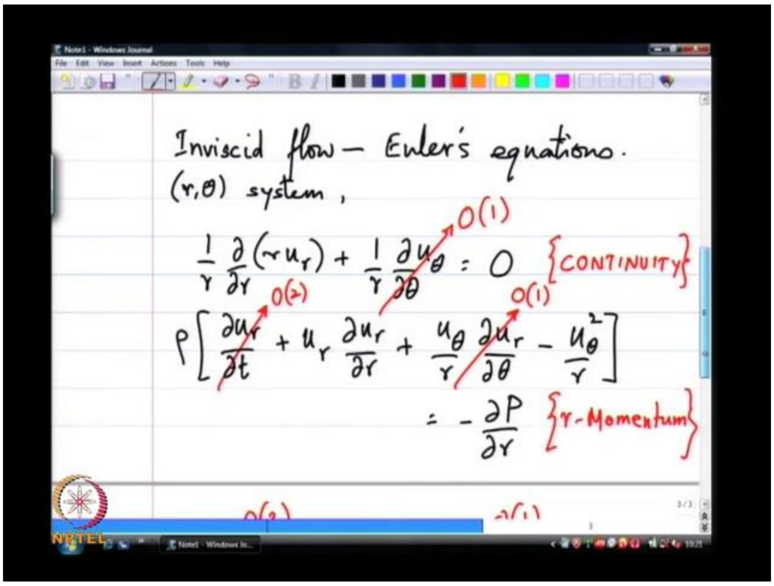

I am going to look at inviscid flow for now, governed by Euler's equations. If I write the Euler's equation in cylindrical polar coordinates, I will write the whole thing. This is our fame continuity equation.

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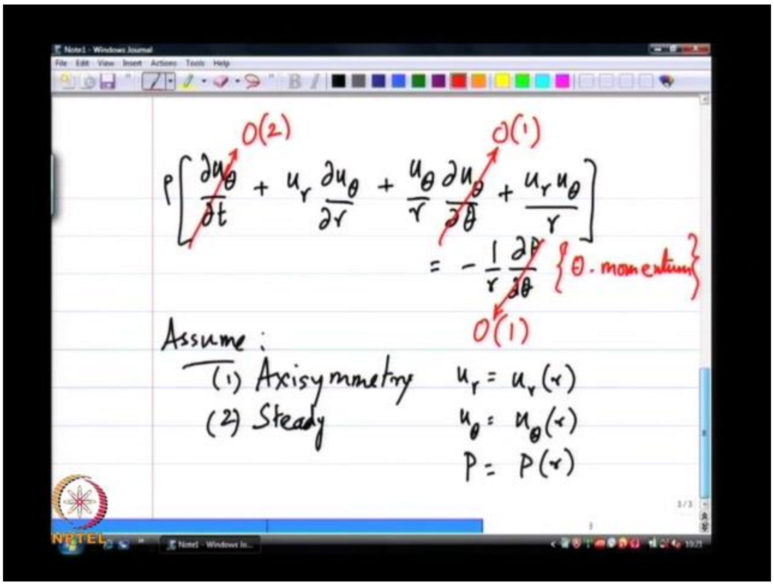

This is my theta momentum equation. What we find here is that I have 3 quantities u r, which is the radial velocity u theta, which is the angular velocity in the theta direction and P which is the pressure the fluid dynamic pressure in the hydro static pressure in the fluid.

And that hydro static pressure is represented by capital P, could be a function of r n theta. Now I am going to simplify these equations from a, some very simply making some assumptions, for assume Axisymmetry. Now the 2 kind of flows, I am interested in are these. In both these instances like, in the first instance o is the axis, in the second instance this happens to the o and the axis passes through o perpendicular to the pointing along the direction of the angular velocity vector in both these instances. Now if I assume Axisymmetry, what do you, I have? You say u r is only a function of r u theta is only a function of r; I will use the same symbols as is there P of r. If I use these Axisymmetry assumptions and scratch out terms that is not important, if u r is only a function of r and I am also going to assume steady flow.

Things are spinning and we are well passed, the spin up phase that is the phase where the fluid was adjusting to a rotating container or a rotating rod and we have well passed all of those transients. We are in a steady mode of operation. If I now go through scratch out terms and put in parenthesis, why that term is being scratched out? I will start with the first 1; this is scratched out because of Axisymmetry. This is scratched out because of the transient nature not being present, again this is 0 to Axisymmetry, and this is due to the transient nature not being present. This is due to Axisymmetry.

As well, this is due to Axisymmetry. I am going to take this simplified version of the continuity equation and see where it will take us.

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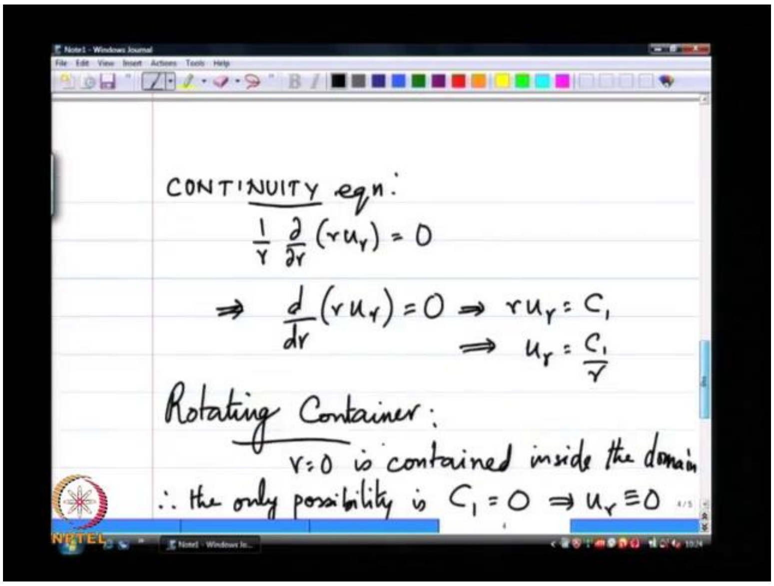

Say is 1 over r d d r of r u r equal to 0. Now if I simplify this, I can first of all replace the partial derivatives with total derivatives because u r is only a function of r u r, can only the of the form C 1 over r. If there is a radial velocity to this flow, it can only the way of the form C 1 over r C 1 is sum arbitrary constant that is yet to be determine.

Now, if I go to the theta momentum equation and see what that will tell us, now before I go much further, I am going to invoke, let us look at each case separately. If I take the solid body rotation case or if I take the, I call it the container case, then r equal to 0 is contained inside the domain because of which the, if u r is of the form C 1 over r then and this is the only at admissible form for u r if r equal to 0 is contain inside the domain that amounts to an infinitely large radial velocity at the centerline.

Therefore, the only admissible solution the only admissible solution C 1 equal to 0 which also implies that u r is identically 0 everywhere.

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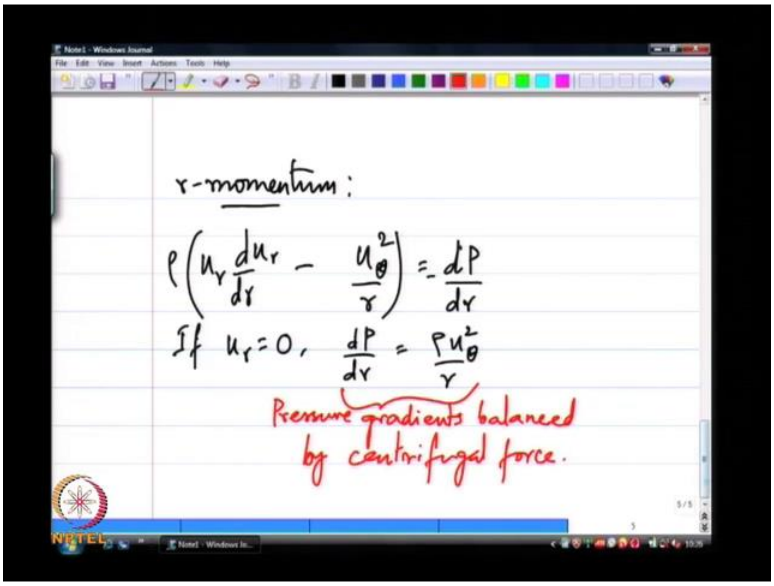

Now if I go to the r momentum equation, what I have? These are the only 2 surviving terms. So, if u r is equal to 0 then what I have is this further simplifies to rho u theta squared over r d p d r is equal to rho u theta squared over r. For the case where the axis is contained inside the container, this is the only possibility and this should look physically familiar that this basically says pressure gradients balance by centrifugal force.

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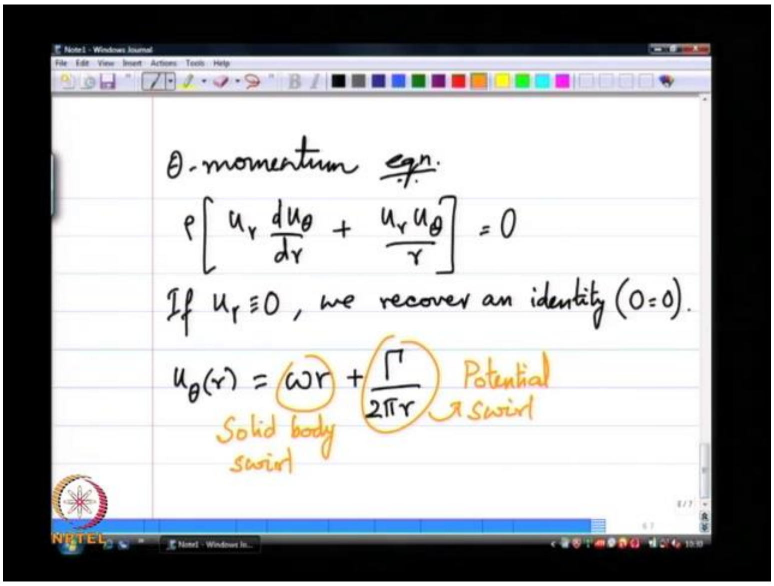

Now, if I take on the theta momentum and take the terms that I still have remaining u r d u theta d r plus u r u theta over r equal to 0, again if u r is equal to 0 like for the container all you get is an identity. As far as the Euler's equations are concerned, you really cannot solve for u theta and P. You essentially have one equation which says d P d r is rho u theta squared over r and from there that is the only equation that you have and theta and p r both separate are 2 unknowns. In the inviscid case; in inviscid swirling flow you end up with 2 with one equation in 2 variables that you really cannot a completely solve.

There are two forms of whatever be the form of u theta over r, you will recover the pressure distribution associated with that that is set off by that centrifugal action that is essentially what this thing what that equation says. Any form of u theta is a function of r that you impose that is that you will impose on the flow fluid would give you the corresponding pressure field. If I look at what are all the possible physical forms of u theta over r and this is not coming from inviscid theory, but actually from viscous theory.

But we will use it freely here essentially what you find is that u theta over r is of the form omega times r plus some gamma divided by 2 phi r this is called solid body swirl and this is called potential swirl. If you look at any kind of a swirling flow, remember that we are not restricted to one of these 2 forms except in the case of a viscose flow, we look at that later on, but as far as inviscid theory is concerned the only thing that you are allowed to conclude is that if you tell me what V theta is a function of r s there is a corresponding pressure field associated with that that main that can be calculated from the radial momentum equation.

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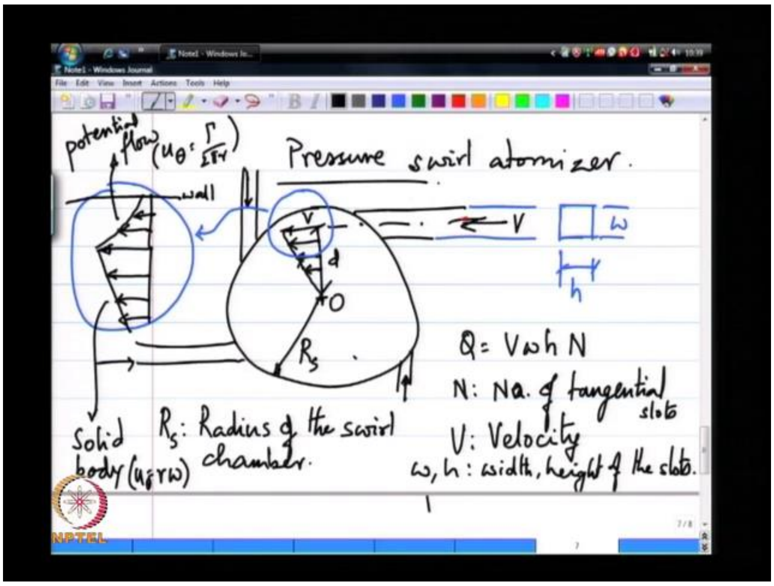

Now let us go back to a pressure swirl atomizer and see what the basic construction of a pressure swirl atomizer looks like and see where this theory would come in handing. A typical pressures will atomizer is composed of let us say some n number of tangential inlets that bring fluid n the tangential inlets may be located at distance d away from this center of this swirl chamber. I will just show one of the tangential inlets because the others are sort of hidden. This is a swirl chamber.

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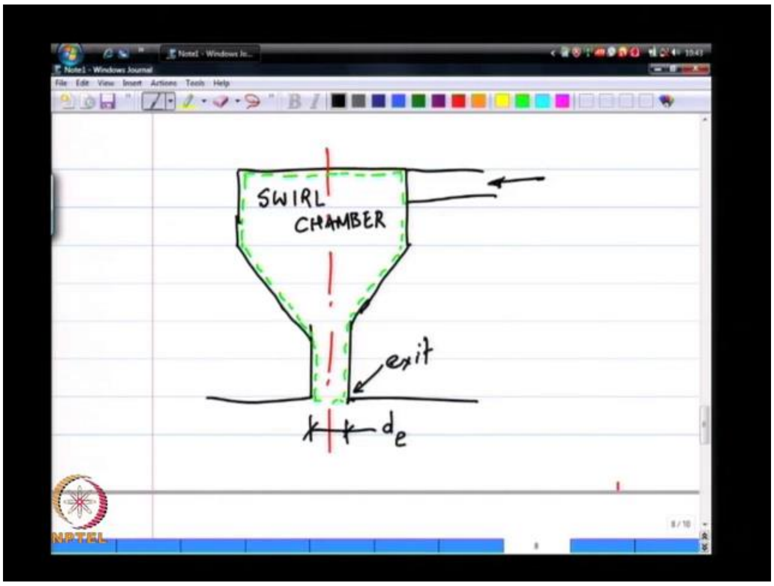

This usually a contraction passage, all of this the rest of this nozzle is Axisymmetric. This is a simplest kind of a pressure swirl atomizer that you will find.

If I go back to this case and let us say if I was to characterize this is the cross sectional area this is some width w and some height h. If the depth of the depth of the slot is some h and w is the height w is the width of this passage and v is the velocity coming in then the total volume flow rate is v times w times h times n, n is the number of tangential slots v is the velocity coming in w and h are the width and height of the slots. So, if I look at what the possible velocity field here would look like inside and I am going to include this dimension r sub s as the radius of the swirl chamber.

Now if v is the velocity here one could imagine invisible container between this point and the center line that is pinning at a linear velocity v or an angular velocity v over d. This part would have a velocity profile that looks like that, now if I look at what is happening outside here and if I draw zoomed out picture passed this point we essentially have to go all the way to the wall. This is the wall of the swirl chamber now from this point where you have this tangential slot coming in to there it is like a you can imagine an invisible rode that is spinning at an angular velocity v over d.

As far as this part of the fluid is concerned it is like it is being driven by a rode of diameter 2 d or radius d that is spinning with an angular velocity v over d. This part of the velocity profile is going to look like a potential swirl. I have 2 parts; you have this solid body rotation on the inside and a potential swirl kind of a flow on the outside. This part looks like v equal to r omega; this part looks like or rather u theta equal to r omega; this part looks like u theta equal to gamma over 2 pi r. See in reality, as you go close to the wall, you also have like a boundary layer that is due to again viscous affects. We will sort of try to stay away from that for now; we look at viscous affects in an empirical fashion little later on.

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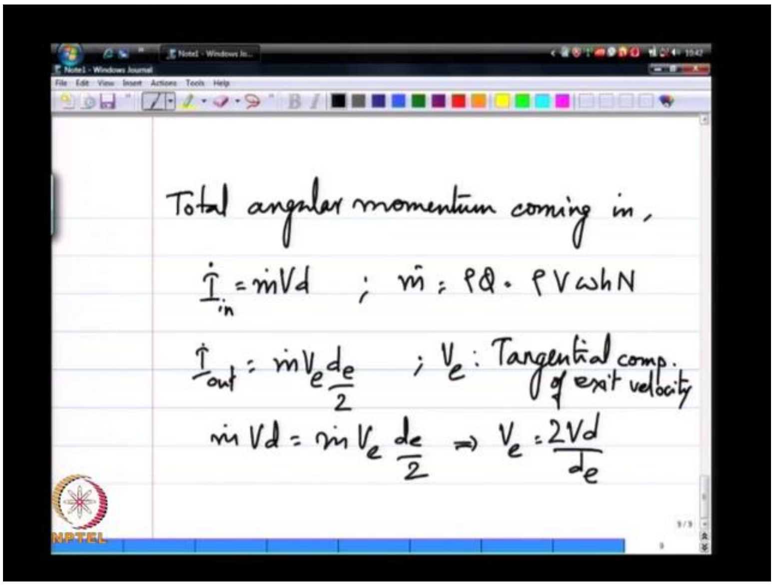

Now if you look at what this flow is doing total angular momentum coming in is m dot times v times d m dot equal to rho times q, that is equal to rho times v times w times h times n the number of slots rho is a density of the fluid. This is how much angular momentum is being input in to the swirl chamber. I were to draw a control volume around this part if I do an angular momentum balance on this because we are looking at in inviscid flow the walls are not exerting any kind of a torque on the fluid.

Whatever angular momentum comes in is exiting from the bottom. So, if the total angular momentum coming in is of this form we call this I dot in, I dot out is equal to m dot times v at exit times d at exit over 2 if d at exit is that whatever is the angular velocity of the fluid coming in has to equal the angular momentum of the fluid coming in has to equal the angular momentum of the fluid exiting.

If I equate the 2 this is only really true for a inviscid flow, this is just tangential component of the exit velocity now if because the fluid is coming in tangentially in to the swirl chamber it contains no axial momentum in the direction of the axis, but for the fluid exit out of this exit cross section it has to have an axial momentum.

Again if we do a simple control volume analysis, one would have to imagine that, that this part of the wall is essentially exerting a down word force on the fluid causing it generate an axial momentum. This axial momentum is now going to depend on the exit cross the mass flow rate divided by the exit cross sectional area divided by the density.

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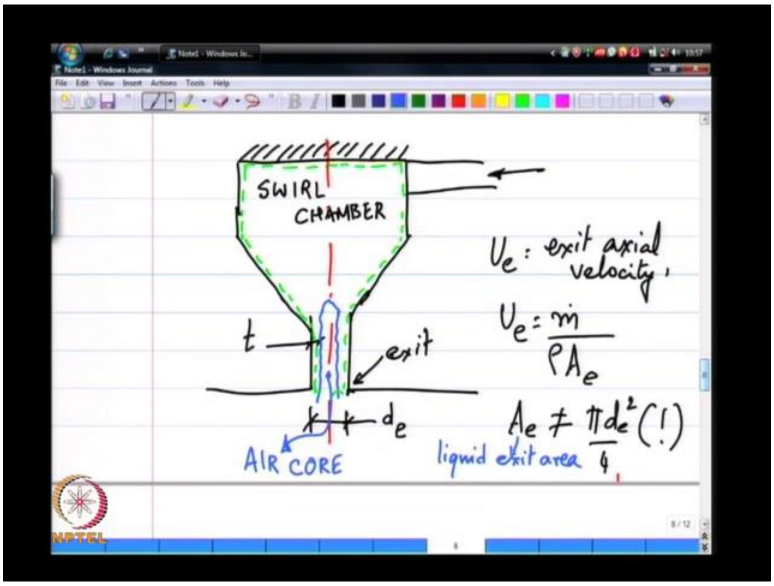

If I look at the axial velocity, I will call this u at exit this is m dot divided by rho A e the only problem here is A e is not equal to pi d e squared over four this is where the design process gets a little complicated A e is the exit area available for the fluid flow and because you have a swirling flow there is no guaranty that the entire cross sectional area is flooded with fluid fuel flooded with the liquid. In order to understand this we will go back to our container rotating problem and see if we can make sense of it.

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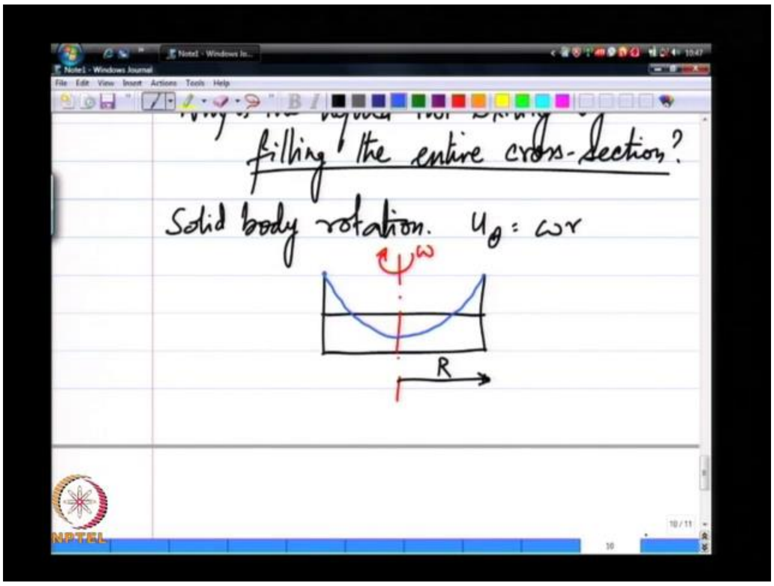

Why is, you will go back to the solid body rotation. If I take a container filled with a fluid up to some height h and I spin this about an axis, it is we have observed this in the past, if not we can easily make this observations that this fluid meniscus is going to become deformed in the form of a parabola this is our static steady condition.

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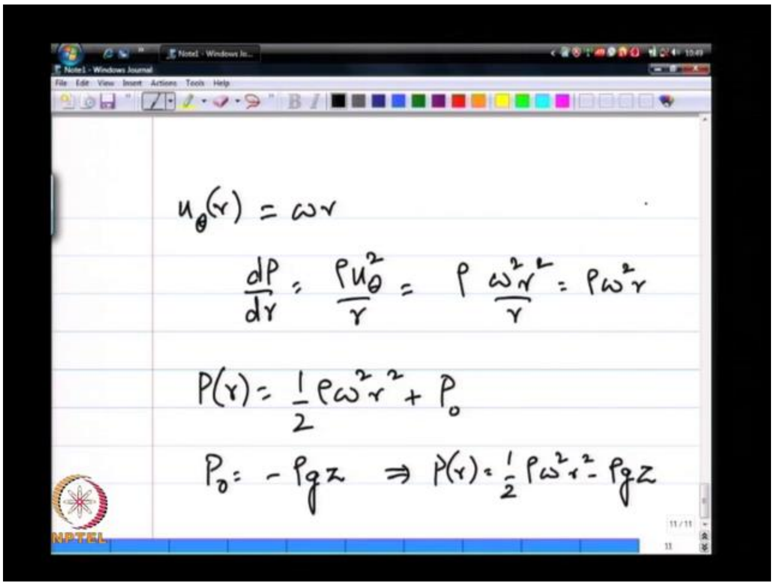

Where does this come from this come from say first of all u theta being equal to omega times r u theta of r being equal to and, if once I have this I have d P d r equal to rho u theta squared over r that comes from our radial momentum conservation.

This rho omega squared r squared over r which is rho omega squared r. If I integrate this P of r equal to half rho omega squared r squared plus a constant I will call P 0. Now this is the case with no gravity included in this equation if I include gravity then this P 0 is minus rho g z and when I use that this I mean I can show this to come from the z momentum equation if we want, but for now will just assume it is it is actually equal to P 0.

But it is I mean P 0 can be a function of z if I have gravity included which is acting along the, is that direction from here I can calculate the, an equation of an isobaric surface.

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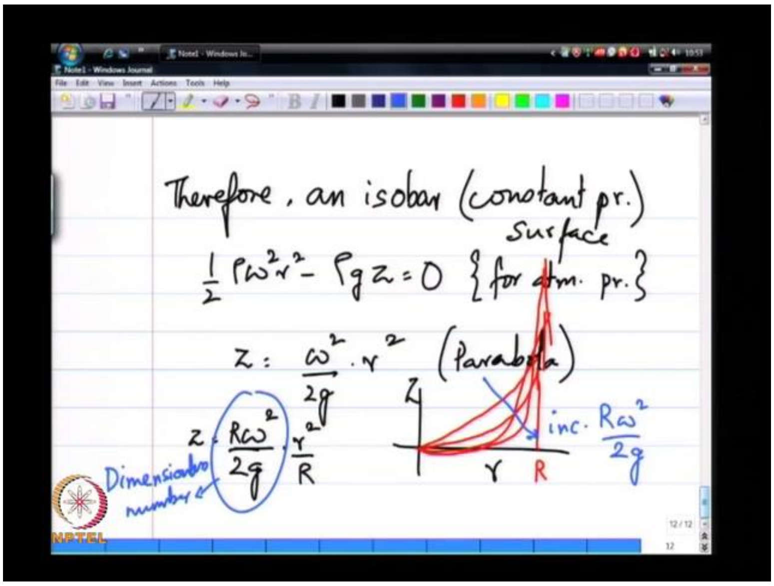

An isobar which is a constant pressure surface is given by half rho omega squared r squared minus rho g z equal to some pressure value and for atmospheric pressure I am going said that equal to 0. If I simplify this, what I have this is the equation of my parabola, if you take a set of parametric curves.

Now, if I take shapes of these z as a function of r for different values of this number omega squared over 2 g, it is like a length scale right or I will write this. This now happens to be a dimensionless number which describes the competition between the rotating velocity omega rotational velocity omega and gravity g. As this dimensionless number becomes larger and larger what do this curves look like essentially if in a let us say that is equal to one you get a curve that looks like this going up to R equal to R.

As this number becomes larger and larger, I am going to have to draw this one more time I will erase this part. As this number increases the fluid is going to be pushed more and more towards the wall the meniscus shape is going to look like a nearly flat meniscus at the bottom and it is going to rise up sharp linear the wall for a pressure swirl atomizer this is a gravity really has no role to play.

In other words omega squared or R omega squared in relation to 2 g is such a large number that g is like practically being equal to 0. Your essentially in other words the spray cone the spray properties of this nozzle is independent of whether you are looking at the spray horizontally vertically horizontally vertically up words does not matter, what the component of g s you are going to get essentially what looks like the same spray.

For that to happen this r omega squared over 2 g is a very large number. What looks like a parabola in our regular containers spinning is now going to look like a thin film that is sticking to the walls of the container walls of this exit orifice? The liquid exiting through the pressure chamber here through the swirl chamber is going to be in that form and t is some film thickness. Now what is it that is inside here the same stuff that is inside this parabola when it is spinning right at this point when I started this spin up process before I started the spin up process there was pure liquid.

Now, as I span the container up to some finite angular velocity omega the air which was outside here got dragged in here because of the pressure gradients and the net result of that is exactly the same as in this case excepts. Since there is no stabilizing force like gravity the air is dragged essentially all the way in to the swirl chamber and this is called the air core. If I look at where this comes from it is essentially coming from this isobaric surface. Now being equal to r equal to some constant value r naught which is less than the diameter of the pipe itself; this part of the flow this part of the flow looks like pipe a constant diameter pipe in which you have an inviscid swirling flow and that inviscid swirling flow is exiting in to atmosphere and that atmospheric air is dragged in to this in to the nozzle to create an air flow air core. This air core is primarily responsible for the cross sectional area a e not being equal to pi d e squared by four a e is the liquid exit area.

This liquid exit area is now being is being less than the pi d e squared over four is what is responsible for which is actually e is what is responsible for the diameters for these for this relation in reality though this is a this is the reason pressures swirl atomizers are very attractive because I can have a very large exit orifice d e and by appropriately changing the swirl geometry upstream I can cause the film thickness here to become thicker or thinner.

If you want to go back by varying this number d, d is the tangential of set of these slot by varying the tangential of set I can essentially create a higher amount of angular momentum entering the swirl chamber or lower as a consequence of that the swirl chamber the swirl chamber geometry then essentially dictates the value of a e the exit area available for the fluid flow. Just to complete this discussion, if I now take an isobaric surface including gravity or including some constant pressure p naught at the center or at the wall.

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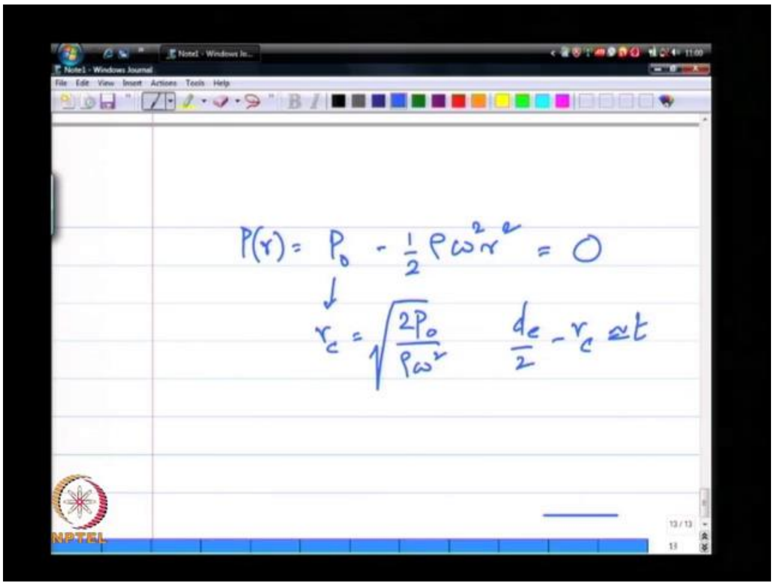

Let us say if I know the pressure at the wall then p of r is some p naught minus half omega squared r squared. If P naught is the absolute pressure absolute atmospheric pressure, the pressure as a function of r is of this form. I can find a particular value of r where this is equal to 0. If the pressure at the center line given, I can find a radial cut of point at which the pressure is equal to the atmospheric pressure.

Essentially it is using statics to understand a dynamic system which is not always the right thing to do, but it gives us the concept of the radius of the air core as being that point at which the flow field the centrifugal pressure due to the flow field is balanced by the atmospheric pressure. You could sort of find that value at which it is like a cut of point at which inside which the air core is stable and that is going to dictate the difference between these two is approximately equal to film thickness.

We will stop we will continue this discussion in the next class.