Principles of Mechanical Measurements

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Lecture No. # 20

We have seen last time an instrument which measures the force under closed loop condition with high accuracy. Now we see another principle, this is unique in the sense we are measuring the force by digital means because when we want to give that force measured signal to computer, now we have to put an analog to digital converter. Then the analog signal is converted to digital signal and then it can be fed into the computer. So that additional converter is required whereas if we can sense the force by digital means, directly by digital means then the signal can be fed without the analog to digital converter to the computer that is the advantage. The principle is digital the sensing the force itself by digital means. A typical example is the veena instrument when we apply different forces to the string that it's naturally frequency and the tone is varied or sound emanating from the string is varied. That means when we are tightening the string in a veena instrument what we are doing is we are changing tension in the wire by that the natural frequency is changed; when the natural frequency is changed the tone is changed. That is the principle what is made use of in this force measurement.

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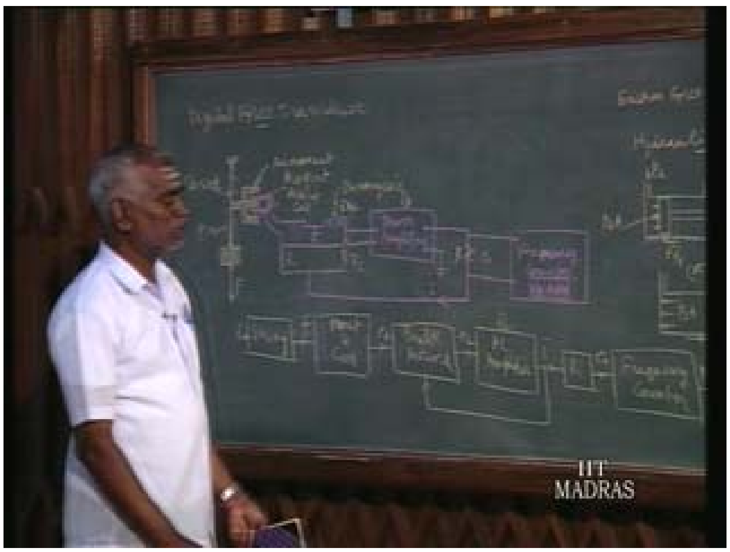

Suppose force F is applied here, this is the string. So when force is applied, different forces induces different natural frequency for the string, the natural frequency of the string is varied for different forces. Suppose we apply a force F and there is little disturbance in the string then it vibrates, so that is what is shown by dotted line, it is vibrating at its natural frequency. So for a given force it vibrates at one frequency, now this is to be picked up and then it should be read somehow that is done like this.

You fix here a permanent magnet to the string, of course should not be very a large weight, a small weight permanent magnet we fix it. So that the amplitude of vibrations of the string is not altered. So when this magnet moves to and fro we have got a coil fixed coil, an inductive coil. So the magnet moves, the coil is fixed the magnetic lines move cutting the windings. So a voltage is induced in the coil and this coil is forming one or the adjacent arms. One of the forearms of an AC bridge and adjacent arms here it is no here I have got twice times (Refer Slide Time: 04:04). Already earthed is shown there so here one terminal is earthed for excitation, other terminal you have got voltage. Then the other two common points it is something like I mean diamond construction but it is done in different way that's all. This is one pair of output signal for a bridge, the other pair of output terminals for the bridge, four terminals are there. Against two terminals we give excitation and the other two terminals we take the output signal.

So if there is any excitation in the bridge then you will find the excitation voltage will be available in the dummy coil. That is adjacent coil it's called dummy coil here. In this active coil, this we will say active coil. In active coil you have got two voltages; one is applied voltage excitation voltage plus the induced voltage due to the motion of the permanent magnet. So here only applied voltage so what is the bridge output? We have learnt in bridge output when we saw under signal conditioners bridge output is equal to the voltage difference between two adjacent coils. Now here you have got the excitation voltage, this is excitation voltage and here excitation plus induced voltage. So difference between these two excitation, excitation will cancel. So induced voltage is taken out as output of the bridge network, this is the principle. So excitation voltage you will have the same frequency of the string, string vibration is the induced frequency. It's an excitation plus induced voltage, so excitation cancel so induced voltage comes out.

So induced voltage is due to the amount of the permanent magnet that is having at the same frequency of the string. So the voltage output to this power amplifier, voltage input to the power amplifier will be having a frequency of the natural frequency of the string. So it is being amplified power amplified and that current is used to excite the bridge again at the same frequency. That's how once it starts vibrating then you will find the excitation or the vibration of the string is maintained because of the excitation. We have got external power through this at the same frequency the vibration is maintained but we allow the current I to flow through a constant resistor R and a voltage drop e_o that is this voltage with its frequency natural frequency of the string is given to a frequency counter where you can read the frequency here.

Frequency can be stepped down or stepped up by a suitable electronic circuit and the final number what you get will be same as say force in Newton or force in kilogram force whatever it is. So for different force and different frequency that is picked by the coil and it is read there and the signal flow diagram is drawn here. By using string we convert that force into frequency, frequency convert into a voltage signal this is mechanical frequency. This force to frequency is mechanical frequency that is put in voltage that frequency voltage at this one and this induced voltage comes out as bridge output here as e_2 and then that is amplified and that current flowing through R gives a voltage drop with the frequency, natural frequency of the string and it is fed in the frequency counter as a reading N there which you can adjust to be same as the force number, whatever number. That number you can get it in the frequency counter by suitable electronic circuit. So this is the digital force transducer, all other transducers what you have learnt are analog transducers. This is the digital transducer.

Now the friction force which we commonly encounter in mechanical engineering. When friction will be available? When there is relative motion between two bodies so the chalk is moving over the black board, so when it moves it moves against a friction force between the chalk and the board. So whenever there is relative motion between two bodies there is always friction and that measurement of friction is often done in mechanical engineering and friction is a force so we will see one method by which friction is measured.

In many instances in mechanical engineering you will find friction is always combined with other forces. So when you want to measure the friction force alone you have to separate that force from other forces and then you can measure. So the main problem in designing a setup for measuring friction is you should design in a way that friction force is separated from other forces. Later on you can use any one of the methods for measuring force.

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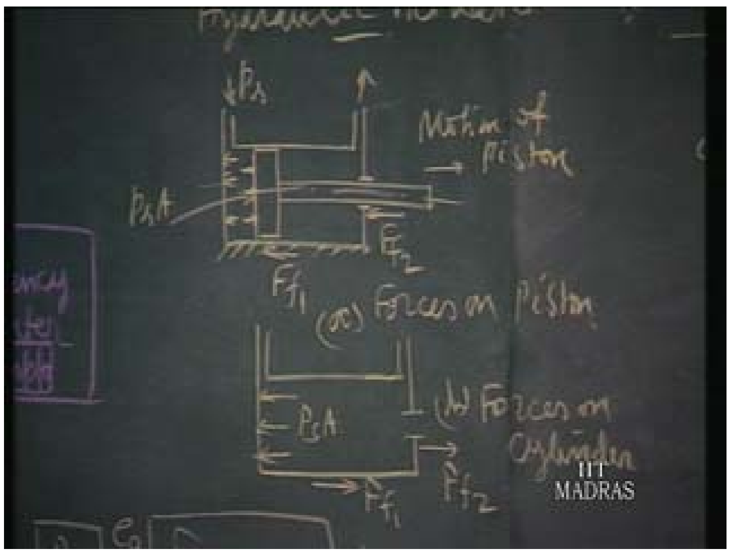

For example here you are going to see a hydraulic actuator where friction presents between the piston and the cylinder. So here is the assembly, the cylinder is normally fixed to the ground and the piston is moved rightwards, when the supply is given to the port at the left hand side and this is the return line for the oil because oil will be here, that will be taken out. So the supply is interchanged I mean changed from here to here by having a so called directional control valve but that we don’t see. As far as actuator is concerned, we see the piston moves from left to right under action of the force obtained from the supply pressure Ps, Ps multiplied by the area, this is the area A of the piston will give the actuating force.

So this pushes the piston but since the piston moves inside the cylinder you have got friction force F_{f1} between the piston and the cylinder and also we find the piston rod is supported at the other cap or the other end of the cylinder. So there is a friction force between the piston rod and the end cover. So that is friction force F_2. Now if you see the free body of these pistons alone, you find one force is pushing from left to right and two forces, friction forces one at the cylinder surface other at the cap who opposes the motion of the piston. So two forces are acted from right to left and the actuating force from left to right. So there are three forces on this and separating the friction forces is impossible because it is on the same mass and acting simultaneously, it is not easy to separate friction force in the piston itself.

So now we analyze what are the forces in the cylinder that is shown here. Now when you supply pressure oil, hydraulic oil then this end cap takes up the pressure and a force developed Ps into A, that force is pushing the cylinder towards left but that is normally taken up by fixing the cylinder to the ground. So that force will go to ground normally but anyhow that is under the force of Ps into A, same force which actuates the piston which moves the piston from left to right that is one force. Then the other two friction forces will be felt by the cylinder in the opposite direction. That is when the piston moves from left to right the cylinder also will be dragged to move in the direction of the motion of the piston.

So we have got the dragging force on the cylinder F_{f1} and F_{f2}, these are the two friction forces which was felt by the piston in opposite way because for the moving body, friction opposes the motion. For a stationary body friction drags the stationary body towards the moving member so it is being dragged. Now we find there are three forces again in the cylinder, somehow if we can absorb this end force by some other means then the cylinder will be left with two friction forces which you can measure. That is what is done in the setup; this is a corresponding setup for measuring the friction in a hydraulic actuator.

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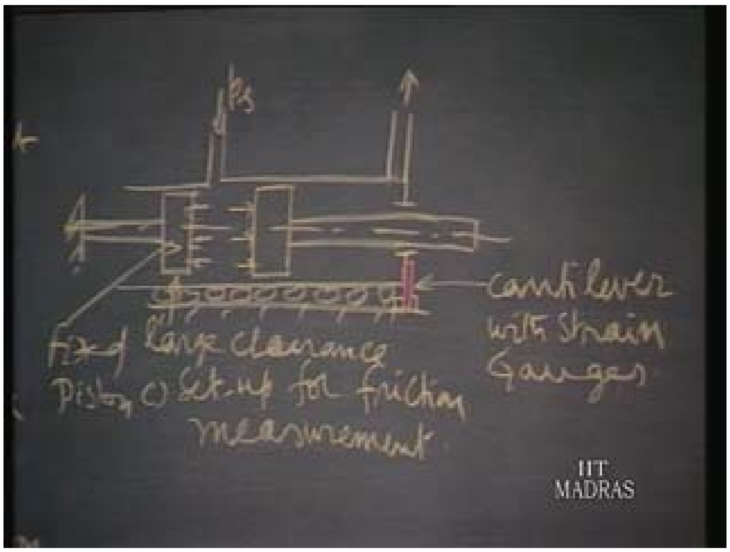

So this end cover is obtained by having a fixed piston, this is a fixed piston. So now the cylinder is open, we have a fixed piston so this reaction force on the cylinder is taken up by a fixed piston and sent to the ground. Now you find the whole cylinder is just floating or it is supported on precision ball bearings with a very negligible friction and also we find large clearance. We put large clearance between the fixed piston and the cylinder so that there is very less viscous friction. So we find now when the piston moves from left to right and we find in the cylinder, only dragging force is there that is F_{f1} and F_{f2} the reaction force is being taken by the fixed piston and now this due to drag force it will try to move in this direction which we can restrict it by having a cantilever.

This is a cantilever and you can put strain gauges near about the fixed end of the cantilever and you will get a signal proportional to the friction force. That is how the friction force is measured in such setups. That is now we have separated the reaction force and the cylinder from the friction forces so net friction forces are measured.

That is separating the forces, friction force from other existing force. Once you achieve it later on measuring the friction is simple affair. Now we work out three problems.

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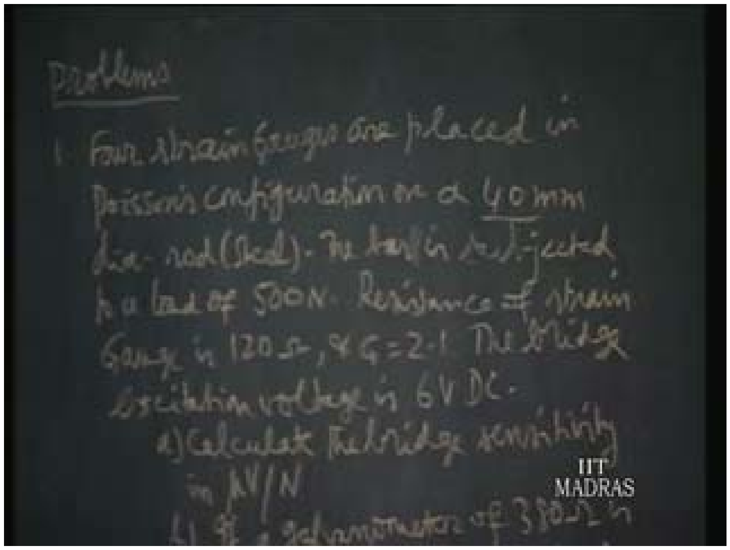

First problem is 4 strain gauges are placed in a Poisson's configuration on a 40 mm diameter rod made of steel. The bar is subjected to a load of 500 Newton, resistance of strain gauge is 120 ohm and the gauge factor is 2.1. The bridge excitation voltage is 6 volts DC, now calculate the bridge sensitivity in microvolt per Newton of this force and if a galvanometer of 380 ohm is connected to the bridge output terminals, calculate the sensitivity in terms of micro amp per Newton. Assume for steel 2 into 10 to the power of 11 Pascal and Poisson's ratio as 0.3. So this is the problem.

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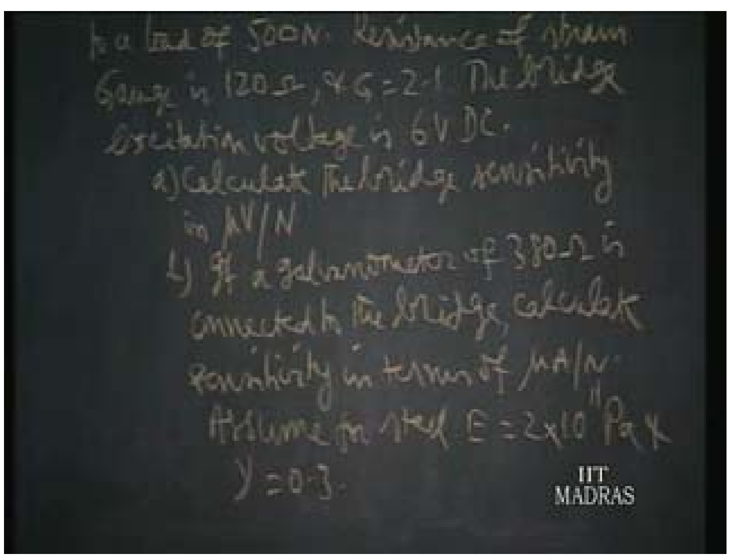

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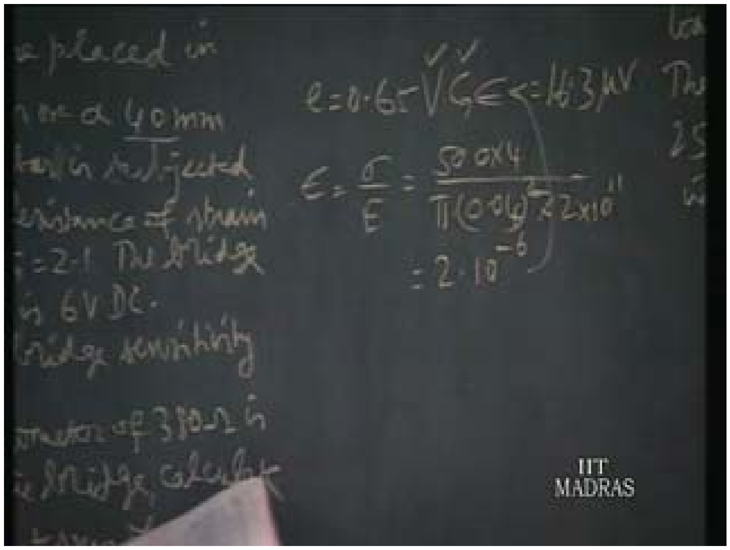

Now we know this is a Poisson’s configuration, voltage output in a Poisson’s configuration we know e is the output 0.65 time. Suppose excitation voltage V, VG into epsilon. This is the bridge output for a Poisson’s configurations, strain gauges connected in Poisson’s configuration. What is Poisson's configuration? Two strain gauges in the axial direction of the bar and two strain gauges in the perpendicular direction to pick up the Poisson’s strain. For such a strain gauge bridge will give so much output. Now the excitation voltage is given, gauge factor is given and epsilon alone is to be found out. For that 500 Newton is the load and the diameter is 40 mm diameter. So Young’s modulus is given, epsilon can be found out.

Epsilon is equal to sigma by E, stress by Young’s modulus so stress by strain is equal to Young’s modulus. So from that we get this equation. So stress equal to load we have got 500 Newton by the area pi D square that is 40 millimeter. We convert in terms of the meter so 0.04 pi D squared over 4 that is the area and divided by the Young’s modulus 2 into 10 to the power of 11. So this gives rise to 2 microns, 2 into 10 to the power of minus 6. That is what is produced, so now substitute this is E epsilon here so you will get an an output voltage of 16.3 microvolt.

So the sensitivity is equal to 16.3 divided by 500 so it's equal to that is the microvolt per Newton, 16.3 by that is 0.033 microvolt per Newton that is this is in 10 to the power of minus 6. It is so many microvolts per Newton. Now the bridge network, this is your bridge network. This is our 6 volts DC and here you have got the output. Now this is connected to a galvanometer having 380 ohm. So each one is 120 ohm. So you have got the voltage source and equivalent circuit is, of the bridge is 120 ohm as it is seen from the terminals 120 ohm and for this we connect your voltmeter, this is galvanometer that is the 380 ohm. So this is equivalent circuit for a bridge connected with the galvanometer.

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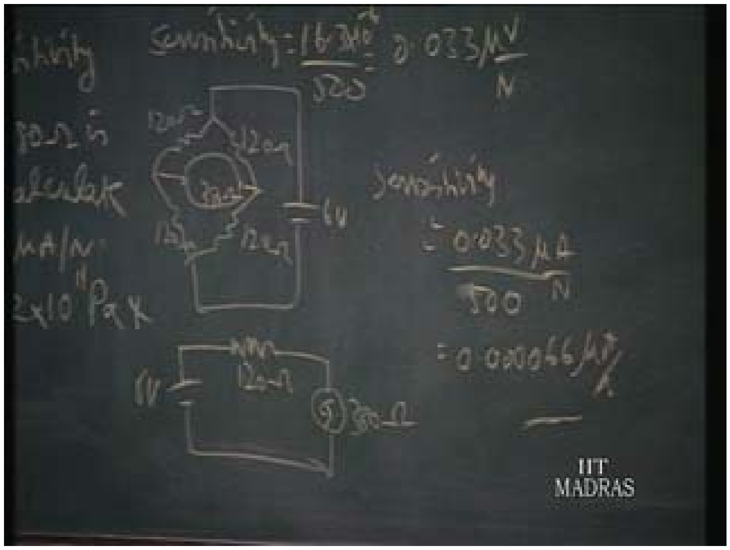

So now the current flow will be, that is it is coming here and through this it is connected. So now it is 6 volts is there, so current flow will be is equal to 6 volt divided by 120 plus 380 so 6 volt by 500 that is the current. So sensitivity in terms of that is we know already the microvolt per Newton, that the microvolt divided by the total resistance, will give that microampere per Newton. So this is equal to, that is sensitivity again, sensitivity is equal to 0.033 divided by 500, it is 120 plus 380, 500 that is the microvolt that is micro amp per Newton. So this comes as equal to 0.000066 micro amps per Newton so this is the solution for this problem.

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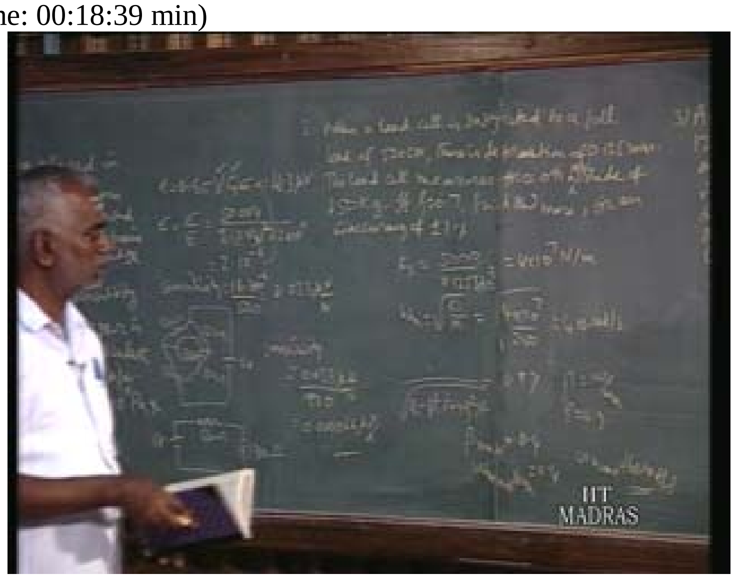

Then second problem, when a load cell is subjected to a full load capacity of 5000 Newton there is deformation of 120 millimeter. That means now we can find out the spring constant of the load cell that is 5000 divided by the 120.125 will give the so many Newton's per millimeter or we can convert Newton per meter also.

The load cell measures the force on a machine slide of 250 kilogram and if the damping ratio is 0.7, find out the frequency of the load which is coming on the instrument. That is omega_{max} for an inaccuracy of plus or minus 1%. So now we have got the spring constant K_s is equal to the full load 5000 Newton by 0.125. So we can convert in terms of the meter, so in 10 to the power of minus 3 that gives a spring constant of 4 into 10 to the power of 7 Newton per meter. So we have got the natural frequency, root of K_s by M. We know that for the natural frequency you have to consider the mass of the force transmitting member that is our 250 kilogram whereas inside the instrument that sensing number may be of negligible weight when we compare with 250 kilogram.

So we can take 250 kilogram alone here to decide the natural frequency that is the important thing in deciding the natural frequency of the instrument of the load cell you have to consider the load transmissibility member and that varies from application to application. That is why natural frequency of any load cell will not be given by the manufacturer. So here we know the application where 250 kilogram is what the force transmitting member so now 4 into 10 to the power of 7 divided by 250. So root of this will give the natural frequency that is 400 radian per second. So now we know omega_n, now for the magnitude ratio 1% error so 0.7. The magnitude ratio will be 0.99 and we know the magnitude ratio is a root of 1 minus beta squared, whole squared plus 4 psi squared beta square.

This is the magnitude ratio this equal to 0.99 and now in this beta is equal to omega by omega_n, psi is given as 0.7 psi is equal to 0.7. So when you substitute 0.7 and solve for the beta. That is our beta maximum, beta maximum comes about 0.4 and that is equal to omega maximum by omega_n. So this is actually omega maximum by omega_n is equal to 0.4 and omega_n we know already, so omega maximum comes about omega maximum for this 1% accuracy it is 160 radian per second. So that is the answer for this problem, up to 160 radian per second we can measure with 1% accuracy.

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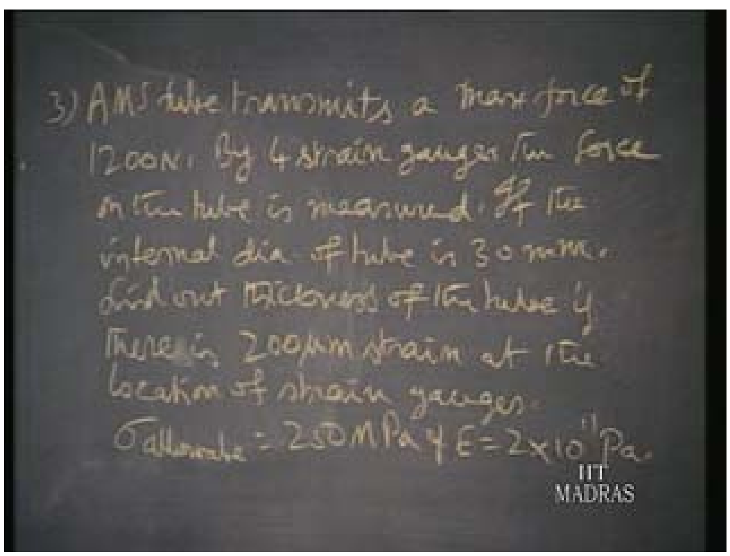

Then third problem, a mild steel tube transmits a maximum force of 1200 Newton. Actually this is a problem in automobile applications. So that is the tie rod from the steering wheel, the force is taken to the wheel to turn the vehicle.

So for rough road probably there may be 1200 Newton in the tie rod and by 4 strain gauges the force transmitted by the tube is measured. If the internal diameter of the tube is 30 mm and we have to fix the strain gauges there. So we have to get a strain of 200 micro strain. So find out thickness of the tube if there is 200 micro strain at the location of strain gauge because earlier rod is designed for strength, there we cannot paste the strain gauge. So we have to weaken it, so that sufficient strain is produced. So for that strain what should be the outside diameter of the tube? If sigma allowable, allowable stress in this mild steel is 250 mega Pascal and Young's modulus is so much so it's a practical problem which we solve for an industry. So here you can find out for this strain of 200 micro strain, what is the corresponding stress.

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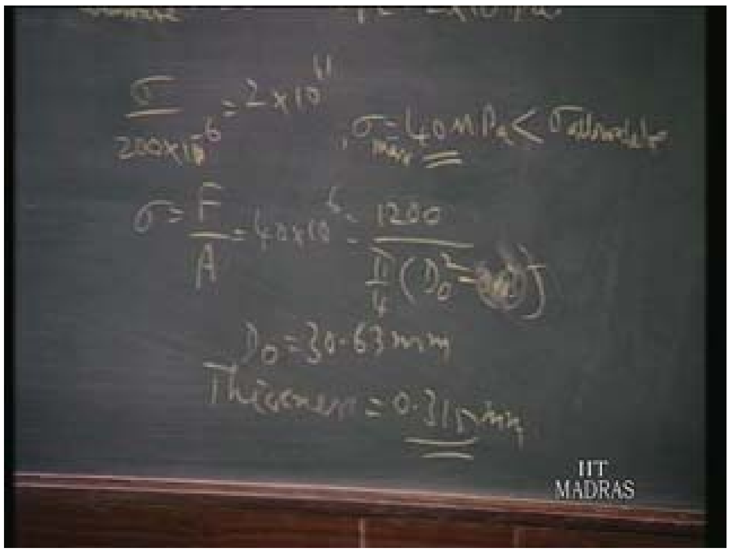

Sigma by stress by strain where strain is 200 micron, 10 to the power of minus 6 is equal to Young's modulus 2 into 10 to the power of 11 for 200 micron. So sigma is equal to that is sigma is equal to 40 mega Pascal. So sigma allowable that is sigma maximum in this application of 1000 Newton, sigma maximum is this much. So this is less than naturally sigma allowable, so it is permitted so it is below the allowable. So it is it is below the yield strength so the rod will be safe. Now produce this much of strain, this much of stress 40 mega Pascal that will give rise to 200 micro strain. What should be the cross section of this tube? That is a simple one. So we find stress equal to force by area. Now it is 40 mega Pascal, 40 into 10 to the power of 6 is equal to force is given as 1200 Newton and area is we know pi by 4 D_o square minus D_i square, D_o square output diameter minus D_i squared given is 30, it comes to 30 millimeter square. We can have everything in millimeters, so this is mega Pascal.

So in terms of meter we can have it, 0.03 square in terms of meter, D_o will be in terms of meter. So we can solve this equation and get D_o as, D_o is equal to 30.63 millimeter that is thickness equal to 0.3 millimeter, so 0.315 millimeter. So that should be the thickness of the tube where we are going to paste the strain gauge, otherwise 200 micro strain will not come and your instrumentation will not have any sensitivity.