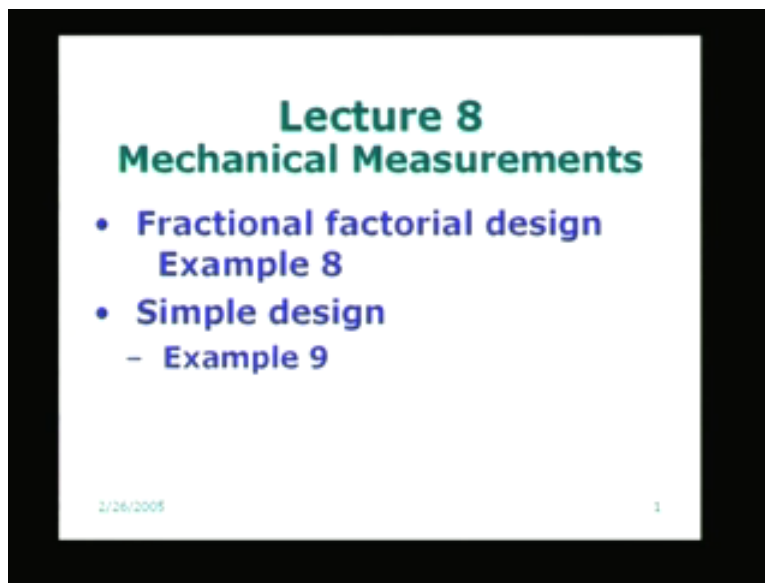


Mechanical Measurements and Metrology
Prof. S. P. Venkateshan
Department of Mechanical Engineering
Indian Institute of Technology, Madras
Module - 1
Lecture - 8
Design of Measurements (Continued)

So this will be lecture number 8 in our ongoing series on Mechanical Measurements. Towards the end of the last lecture, we were talking about fractional factorial design. We just started talking about it but before that we had solved simple problems, which involved factorial design, a 2-level, 2-factor experiment which involved machining process, and we were describing the results. In those results, there were one or two small mistakes. I would like to correct them before we proceed to the current lecture.

(Refer Slide Time 1:42)



If you remember, we had this earlier and in this we had written p_0 here and cancel this p_0 . Therefore there is no p_0 here. In the earlier lecture p_0 was shown here; that was a mistake.

(Refer Slide Time 1:52)

$$\begin{aligned}
 (y_i - \bar{y}) &= \mu_0 + \mu_A + \mu_B + \mu_{AB} - \mu_0 \\
 &= (\mu_A + \mu_B + \mu_{AB}) \\
 \sum_{i=1}^4 (y_i - \bar{y})^2 &= \frac{4\mu_A^2 + 4\mu_B^2 + 4\mu_{AB}^2}{3}
 \end{aligned}$$

The second mistake was that there was 8 here instead of 1; and it was a minor error but we have to make the correction so that the student does not get confused.

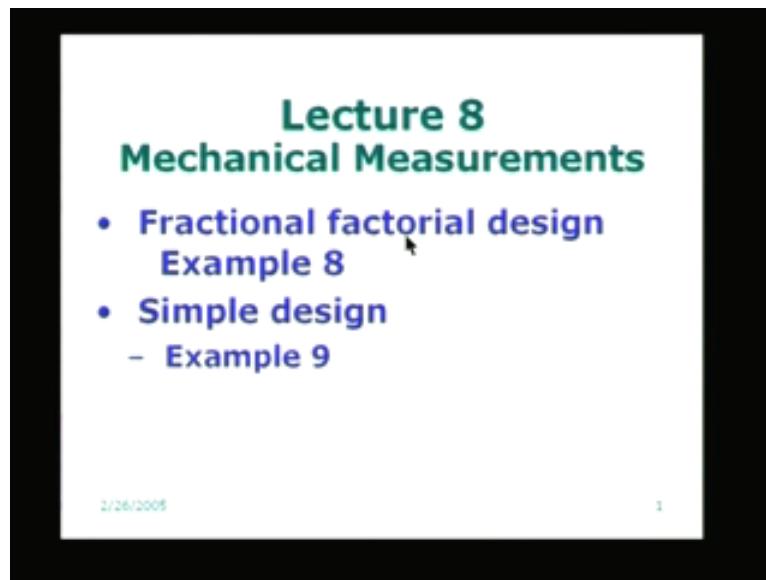
(Refer Slide Time 2:10)

$$\begin{aligned}
 \sigma_y^2 &= 5.60 + 2.25 + 1.47 \\
 &= 9.32
 \end{aligned}$$

	% Effect	
A	60.1	MAIN
B	24.1	
AB	15.8	Interaction
SUM	100.	

Otherwise the three terms will add up to more than 100.

(Refer Slide Time 2:35)



With these two corrections, I will go back to the slides pertinent to the current lecture, lecture number 8. Initially I am going to talk about factorial design, which we have already covered in the previous lecture. Then I will talk about fractional factorial design, which will also let you know why we do that. In fact, towards the end of the last lecture itself, we mentioned about the fact that the number of experiments will become very large. If the number of factors is very large even with 2 levels for each factor, the number of experiments that need to be conducted will include all the effects.

When I say effects, it means the effects caused by individual factors; then the effects caused by the product of 2 factors taken at a time and so on and so forth. If there are a large number of factors, you will get factors like main factor, then you will have product of 2, product of 3 and so on and so forth. So the factorial design, which we talked about earlier, will of course faithfully give information about all the effects including the main effects and the factors, the product of the factors and so on. But the number of experiments will be very large. So the main reason we want go for fractional factorial design is to reduce the number of experiments and it can be done in a sensible fashion. If we are able to get some information either from past experience or by study of similar processes, some interactions may not be important.

In general, it is reasonable to assume that very large number of factors coming together and affecting the outcome of the experiment—the probability of that is small. Of course it has to be very fine if one wants to really verify their hypotheses. But many times it is alright to do that. Supposing there are n factors; all n factors coming in the formal product and affecting the results will certainly be much smaller than the main effects, the main factors affecting the outcome of the experiment. Therefore the fractional factorial design takes advantage of the fact that some interactions may be unimportant. Therefore we can reduce the number of experiments.

(Refer Slide Time 5:38)

3 Factor, 2 Level Design			
<ul style="list-style-type: none">• Example 8• Full Factorial Design needs $2^3=8$ experiments	A	B	C
	+	+	+
	+	+	-
	+	-	+
	+	-	-
	-	-	+
	-	+	+
	-	+	-
	-	-	-

Then I will, towards the latter part of the lecture, take you back to the simple design. Taking an example from fluid mechanics where I will show that simple design is necessary for various reasons. I will give basic reasons and work out using 1 simple example to illustrate the use of simple design. Instead of the 2-factor, 2-level experiments we had in the previous lecture, I have a 3-factor, 2-level design; I will treat this as example 8. If we go for a full factorial design, we will see that we require 2^3 is equal to 8 experiments in all. So what are these 8 experiments? I have A, B, C. This is actually x_A , x_B , x_C . Instead of it I have written A, B, C.

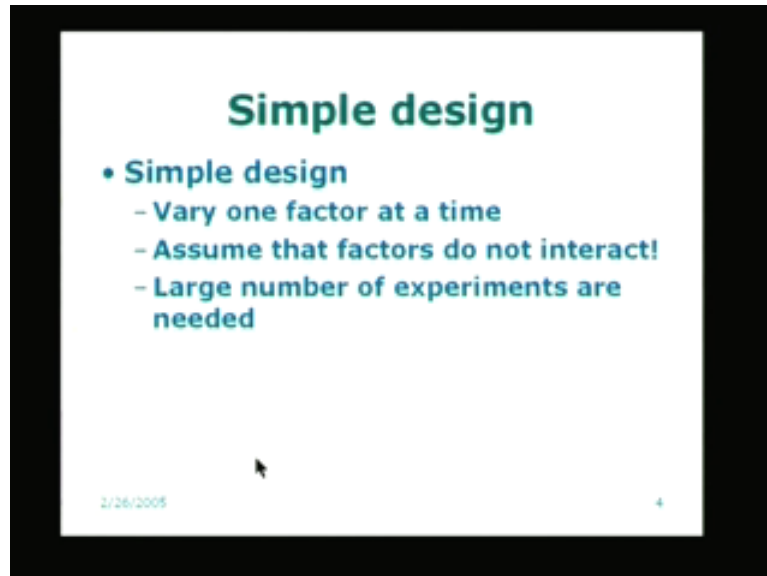
The values are at 2 levels and I will choose positive here, positive here, positive here for this one combination. All of them are high, A,B,C. The two of them, high and 1 low, and so on and so forth. We can see that we can create 3 columns or 3 vectors and these vectors are of course linearly independent. That means they are orthogonal vectors. These three are the possible arrangements, and if you count the number of experiments, you will see 1, 2, 3, 4, 5, 6, 7 and 8. There are 8 experiments in this particular 3-factor, 2-level design. If you want to capture all the effects, we need to do these 8 experiments. In fact I have indicated the column vectors. What are all the column vectors which will be important? There will be the main effects x_A , x_B , x_C . These are the same as the ones shown here, A,B,C. These are the same as shown there. The first 3 columns are nothing but the 3 sets of the values for x_A , x_B and x_C . Then we can have interactions between 2 parameters, two taken at a time you see there are 3 parameters and I can form 3 such

(Refer Slide Time 6.49)

Column vectors are all independent						
x_A	x_B	x_C	$x_A x_B$	$x_B x_C$	$x_A x_C$	$x_A x_B x_C$
+	+	+	+	+	+	+
+	+	-	+	-	-	-
+	-	+	-	-	+	-
+	-	-	-	+	-	+
-	-	+	+	-	-	+
-	+	+	-	+	-	-
-	+	-	-	-	+	+
-	-	-	+	+	+	-

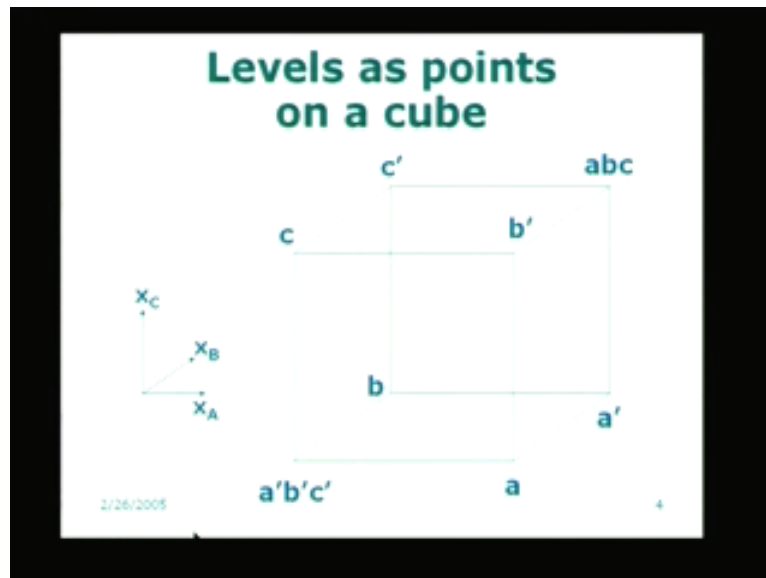
products $x_A x_B$, $x_B x_C$, and $x_A x_C$. Therefore $x_A x_B$, $x_B x_C$, and $x_A x_C$ are the 3 parameters, 3 products, 2 taken at a time. You can see that I can create this column by simply using this plus into plus is plus and again plus into plus is plus, plus into minus is minus and so on and so forth. So these are products of 2 columns written here and similarly $x_B x_C$ is the product of these 2 columns. Similarly, $x_A x_C$ is the product of these 2 columns and I can also form a product of all 3 taken at the same time— $x_A x_B x_C$. That will be the product of these 3 numbers, that is plus and so on. So, if you carefully observe this table, you will see that all the columns, all the vectors represented by these columns are orthogonal to each other. That means the product of 2 column vectors turns out to be 0. That means they are independent; that means we have 8 different parameters; 7 here and of course, there is 1 more, which corresponds to the intercept parameter, which I have not included here. Therefore after 8 experiments, I will have enough number of equations to look for solutions to all the parameters that characterize the relationship between y and x_A , x_B and x_C .

(Refer Slide Time: 7:14)



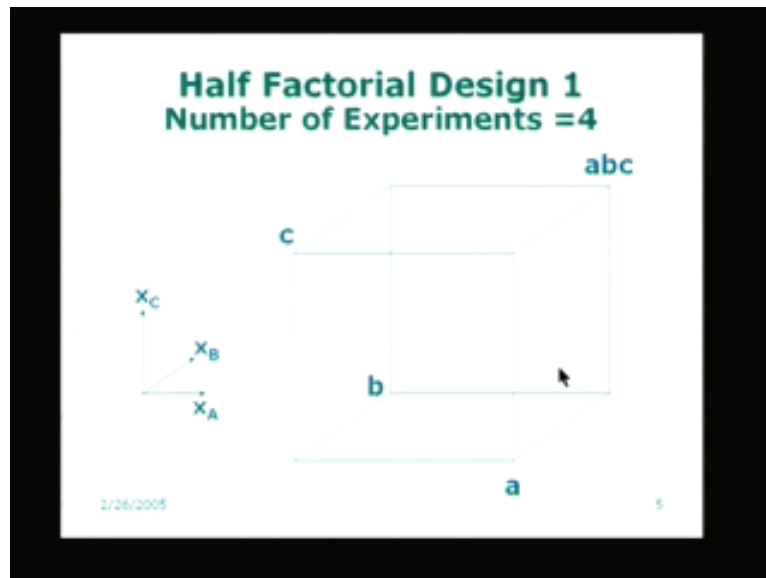
If you remember in the last lecture, we had indicated the values of the factors as the corners of the square. In fact, we showed a point in the middle and said it represents mean. If we have 3 factors, we require a cube to represent what is going on. So this is similar to the square we have drawn earlier when the case was for 2 parameters. So the cube has got 8 corners; and I have deliberately given labels a , b and c for these 3 corners, abc is this corner and abc is not a product. Similarly, I have got a' , b' , c' , and $a'b'c'$. Why do I do this? Suppose you were to look at this cube from the right-hand side, that is perpendicular to this surface. You will see a square and you will see the corners b' , abc , a' , and a . Now that is not I want to do. What I want to do now is to do a slightly different thing and that is shown in the next sketch.

(Refer Slide Time 9.20)



I am only taking 4 corners as shown here. I have a corner abc. I have a corner a here, a corner b here and a corner c here. This is different from the previous sketch—I have got 8 corners and I am suppressing the ones with the primes: a' , b' , c' and $a'b'c'$. I am removing those corners. I am ignoring the presence of those corners. Then you will get abc, c, b and a. Now you project it on to this plane. What you will see is c, b, a will look from this side and abc will appear from here. You are going to see c, abc as though it is projected here, b at this corner, a as though it is projected to this corner. Therefore when you see from one side you are going to see a 2-factor experiment. In the case of 3-factor experiment with 2 levels, I have got 8 corners, I am going to look at only 4 corners at a time and what I am going to see is when I see from this edge or any other edge, I will see the 4 corners of a square. That means I am reducing the number of points, number of corners where I am going to take the parameters to 4. That means I am reducing the number of factors, number of experiments by a factor of 2. So this is called half factorial design.

(Refer Slide Time 10.30)



The first half factorial design I get is by taking the values a, b, c, abc. Of course, I will get a second one by taking b', a', a'b'c', and c'. Therefore I can have 2 half-factorial designs. That means the number of experiments I am going to perform is reduced by a factor of 2. Of course if you reduce the number of experiments, some information must be lost. So exactly what is going to happen is seen from the half-factorial design number 1, which I have shown here. So I have put point a, point b, point c and point abc here and the values of x_A , x_B , x_C . We will come to this later on; we will see that a represents high value, b represents low value, c also represents low value, but abc represents high value.

All you have to do is go back to the sketch and see that a represents x_A , which is measured in this direction; x_B in this direction; x_C in this. a is this corner here. We are going to the right-hand side; this has high value. This also has high value, and so does this. But this is of a low value. So you have what I have shown here is high value for a, low value for b, low value for c, and high value for abc. So I can now construct 3 columns and we are going to have 4 experiments corresponding to this. x_A will take plus minus minus. This will be minus plus minus; this will be minus minus plus and plus plus plus. So these are 3 independent columns. We can see that if we take a product of any 2 of this column, it's going to become 0. However, if I now multiply x_A and x_B , I am getting the column here, plus into minus is minus, minus into plus is minus, minus into minus is plus, and plus into plus is plus. You see that this column is no different from this column.

(Refer Slide Time 13.30)

Half Factorial Design 1 Number of Experiments =4							
	x_A	x_B	x_C	$x_A x_B$	$x_B x_C$	$x_A x_C$	$x_A x_B x_C$
a	+	-	-	-	+	-	+
b	-	+	-	-	-	+	+
c	-	-	+	+	-	-	+
abc	+	+	+	+	+	+	+

That means that x_C , $x_A x_B$ product look similar. They look alike. That means we say that these 2 are aliased; that means this is like an alias of this and we say that these 2 effects cannot be resolved. x_C , x_A and x_B are together; therefore we cannot resolve these 2 effects separately and we say that these 2 effects are confounded. In fact you can do the analysis and see that column x_B and $x_A x_C$ are also confounded. $x_A x_B x_C$ is also confounded and in fact the product $x_A x_B x_C$ which is positive is confounded with the intercept parameter because it is also going to be plus plus plus. Therefore in the half-design experiment when the number of experiments is equal to 4, I can only determine 4 parameters, p_0 , p_A , p_B and p_C because p_A is also containing information about $x_B x_C$ product; p_B also contains the information about $x_A x_C$ and similarly $x_C p_C$ contains information about this.

Of course, p_0 will also contain this. Therefore the price we pay for doing a half factorial experiment is to reduce the number of parameters we can represent, and that is the main problem. In fact we can similarly analyze design number 2. You will see that the resulting thing is like this. The only thing is x_A into x_B , these 2 products will be minus x_A and x_B . In the other case it would have been plus x_A and x_C plus here. It is just going to be a change of sign. Actually $x_A x_B x_C$ product is equal to minus 1. In the previous case, the product of these 3 is equal to plus 1. That is the characteristic of these 2 designs.

(Refer Slide Time 16.06)

Half Factorial Design 2 Number of Experiments =4							
	X_A	X_B	X_C	$X_A X_B$	$X_B X_C$	$X_A X_C$	$X_A X_B X_C$
a'	+	+	-	+	-	-	-
b'	+	-	+	-	-	+	-
c'	-	+	+	-	+	-	-
a'b'c'	-	-	-	+	+	+	-

So we have 2 half-factorial designs and the advantage of half-factorial design is that only 4 experiments are needed to be performed. But we will not have the facility of representing all the 8 parameters so I have further explained this aliasing and confounding. Suppose I write y is equal to p_0 plus $p_A X_A$ plus $p_B X_B$ plus $p_C X_C$. These three terms are going to be called the main factors and then $p_{AB} X_A X_B$, $p_{BC} X_B X_C$, and $p_{AC} X_A X_C$. These are the products of 2 things taken at a time. There are 3 of them and finally p_{ABC} is multiplied by $X_A X_B X_C$. If you look carefully it is p_0 p_A p_B p_C p_{AB} p_{BC} p_{AC} p_{ABC} . Therefore we have 1 plus 1 3 such 4 3 such 7 8. There are 8 constants and it is not possible to resolve all of them because I have also shown p_0 with 2 bars on the top and $p_{ABC} X_A X_B X_C$ with 2 bars on the top. That means these two are confounded.

(Refer Slide Time 17.00)

Aliasing and Confounding

$$y = p_0 + \overline{p_A x_A} + \overline{p_B x_B} + \overline{p_C x_C} + \overline{p_{AB} x_A x_B} + \overline{p_{BC} x_B x_C} + \overline{p_{AC} x_A x_C} + \overline{p_{ABC} x_A x_B x_C}$$

- Only main effects are resolved. The terms that are confounded are indicated with bars!

2/26/2005 9

We cannot resolve these two separately. The effects of these two are going to be intertwined. Similarly, $p_A x_A$ and $p_{BC} x_B x_C$ are confounded and so on and so forth. Therefore what it means is that only the main effects are resolved and the terms that are confounded, which are indicated with the bars here, are not going to be resolved separately; so this is further brought out by seeing that the model, which we can have for this particular case is a linear model, y is equal to p_0 plus $p_A x_A$ plus $p_B x_B$ plus $p_C x_C$.

(Refer Slide Time 18.20)

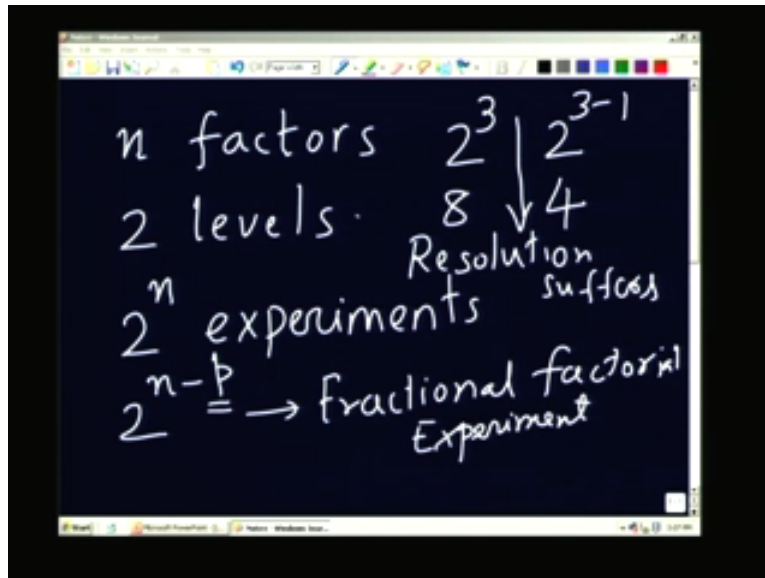
A linear model!

$$y = p_0 + \overline{p_A x_A} + \overline{p_B x_B} + \overline{p_C x_C}$$

- What is possible is shown above

2/26/2005 10

(Refer Slide Time: 20:10)



What is possible is all this; that is, I can only have a 4-parameter model and I can have linear relationship between y and x_A , x_B and x_C . Now the question is, what should we do when we have done 4 experiments? We get only the 4 constants and some information. Suppose we want to further explore, I can add one more experiment. I can add one more experiment and find out whether that experiment is also going to follow this relationship. If these discrepancies are large, that means whatever effects we left out must be important. That means the results are going to be dependent on the terms we had ignored earlier.

Therefore, I have to do one more experiment or maybe two more experiments. That means the fractional factorial experiment is a prelude to a larger experimental set. So you would like to first of all get an idea of what factors are important and what factors are not important and their relative magnitude and so on. For that this could be a preliminary design and once the preliminary design gives me some information regarding what is happening may be at that time we can go and do more experiments, plan a bigger experiment of certain form. This is one aspect of the problem. Suppose in the 2-level design we are taking just 2 levels.

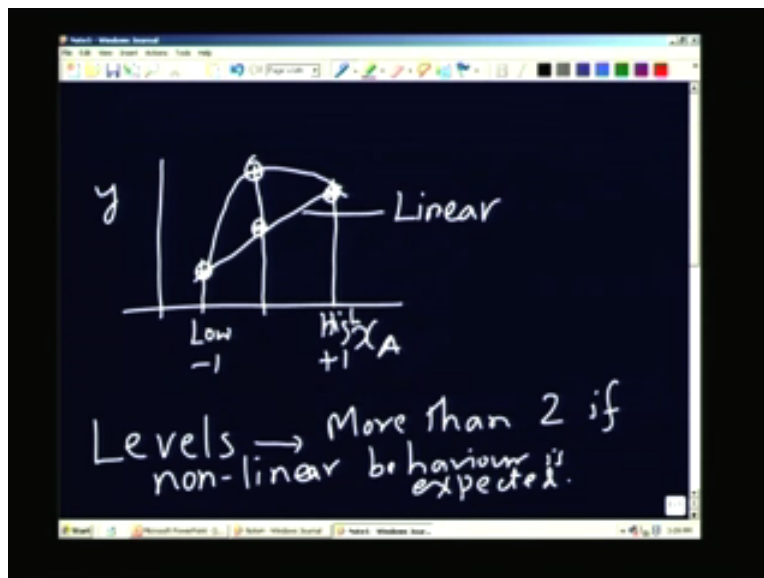
Let me explain the 2 levels. What I am trying to do is going to the board and give some information. So we have n factors and we have two levels and we know that we require 2^n to the power of n experiments to resolve all the interactions. This is a factorial experiment. But what we were doing is 2^{n-p} to the power of n minus p . This is the fractional factorial experiment. So in the case of fractional factorial experiment, I will not be able to resolve all the factors, the main factors, then the product and so on and so forth. That means this is going to reduce the number of experiments; at the same time it also resolves less number of factors. So we say that the p here will reduce. For example, we had 2^3 and then we had 2^{3-p} .

1 here. How many factors are there? There are 8 factors that are constant, x_A , x_B , x_C ; then the products of x_A , x_B etc; then $x_A x_B x_C$. So what we resolve here is only 4 factors. We are able to resolve only 4 factors. So that means the number of factors that can be resolved is reduced. The resolution suffers. This is one aspect of the problem. The second aspect, however, is also equally important. Let us look at that one.

Let me just draw an impulse cage I will say x_A . Let us have 1-factor experiment. So suppose I do at a 1 low value high value plus 1. I get 2 points. All I can do is join by a straight line so this assumes a linear relationship. Suppose the relationship is nonlinear and we want to find out. You must do at least one more experiment somewhere in between. For example if I do an experiment at some intermediate value, I may get a value here. That means now you expect the relation should be like that. Therefore the factorial design, which uses only 2 levels, can be an exploratory design which gives some information about what is happening. But to resolve things like whether it's a linear relationship or a nonlinear relationship, at least one more experiment has to be done.

If I do one more experiment and suppose it came very close to here, then I am confident that may be the linear relationship is not bad. Therefore you will say that the levels are more than 2 if nonlinear behavior is expected. Only then you will be able to find out whether it is nonlinear or not. You can resolve only by doing at least one more experiment at least at 3 levels. So when you go from 2-level experiment to a larger number of levels, obviously the number of experiments will become excessively large. The second point with respect to these multiple level experiments is that some of the factors may have linear relationship. Some factors may have nonlinear relationship. If you recognize which are linear which are nonlinear, we may still be able to bring down the number of experiments to some extent.

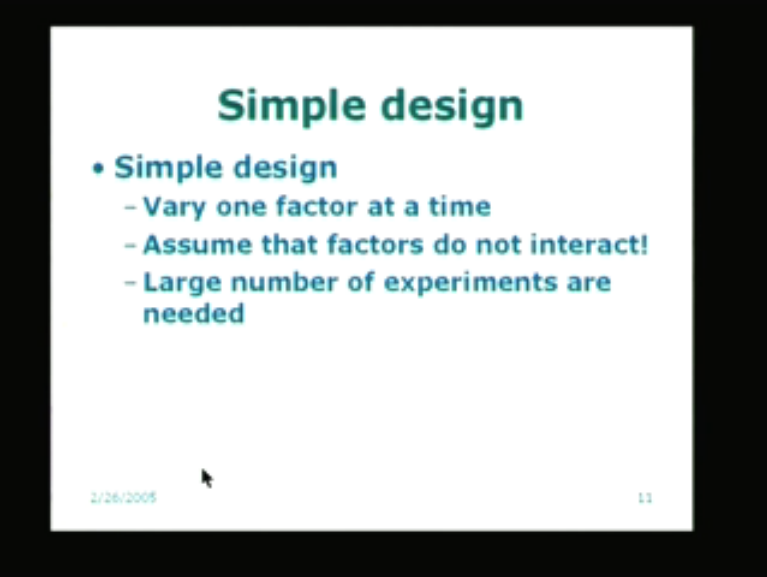
(Refer Slide Time 25.06)



For example, for those which are nonlinear I will have to do more experiments to resolve the nonlinearity in the dependents. In the case of linear factors, for the factors which are linearly dependent, that is the experimental outcome the linearity is dependent on the factor. Then I can do a fewer number, even 2 levels may be able to resolve that. So the number of levels is something which is to be determined by looking at the behavior, whether linear or nonlinear. These are some of the ideas which one must keep in mind before jumping into any experimental activity.

Do some explorative experiments, find out whether linear or nonlinear, what are the factors that need to be resolved and once you have the information, you can go for a more complete experimental procedure, experimental program and complete the experimental activity which you have in mind. That will give us the maximum amount of information or the amount of information which is required to be obtained. So with this background let me now go back to a simple design. In fact I mentioned earlier on that we will first consider the factorial design which is usually useful for process modeling and so on, where the outcome is one particular quantity we have in mind—either the quality or some such attribute of the process. But in the case of simple design, the goal is slightly different. We are interested in finding out what is the relationship between the outcome of the experiment and either one factor or several factors. This is the more ambitious program; we are not interested in one particular operating point of the design.

(Refer Slide Time 26.28)



Simple design

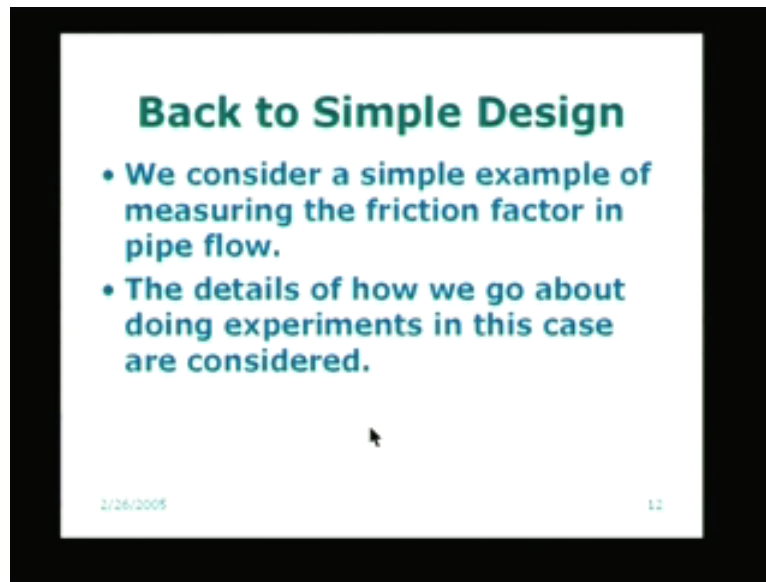
- **Simple design**
 - Vary one factor at a time
 - Assume that factors do not interact!
 - Large number of experiments are needed

2/26/2005 11

We are also interested in knowing what is happening for different values of the input. The number of input factors should be quite large. In case the number of input factors is large and the outcome is dependent on many factors, of course, this is more cumbersome work, but there are

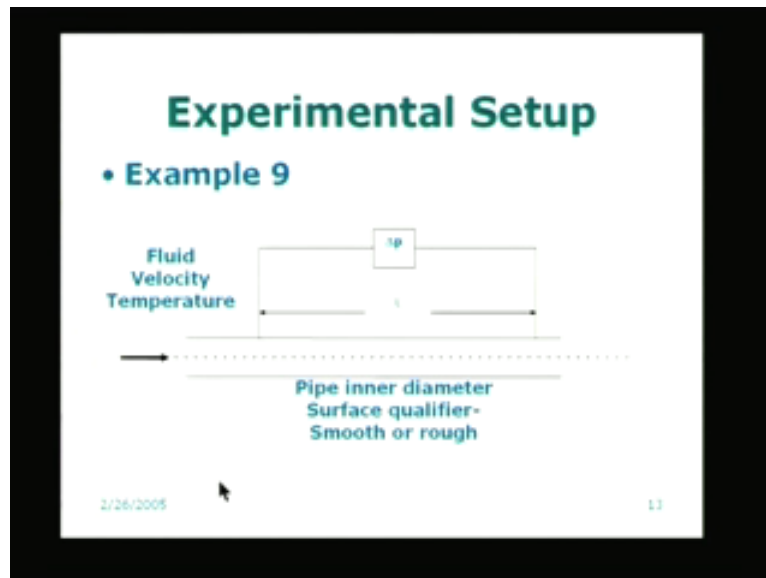
some redeeming features which I am going to talk about. So the simple design is characterized by varying one factor at a time. Suppose there are 3 variables, which are going to be affecting the outcome, you vary each one at a time, keep other two constant. Of course this will increase the number of experiments required. We can find out which ones are dominant or which are less dominant and reduce the number of levels as the case may be. Sometimes what we do is we simply assume that the factors do not interact or that they interact in a particular way and then we simplify or reduce the number of experiments.

(Refer Slide Time 28.18)



In general, the simple design requires a large number of experiments and that cannot be avoided. But let me just take a simple example, which is shown and which is going to be discussed in some detail for the rest of the lecture. So we consider a simple example of measuring the friction factor in pipe flow. Most engineering students who have taken fluid mechanics course would be able to understand what we are going to talk about in this particular experiment. So the details of how we go about doing experiments in this case are going to be considered and let me just indicate the type of experimental setup one requires for this.

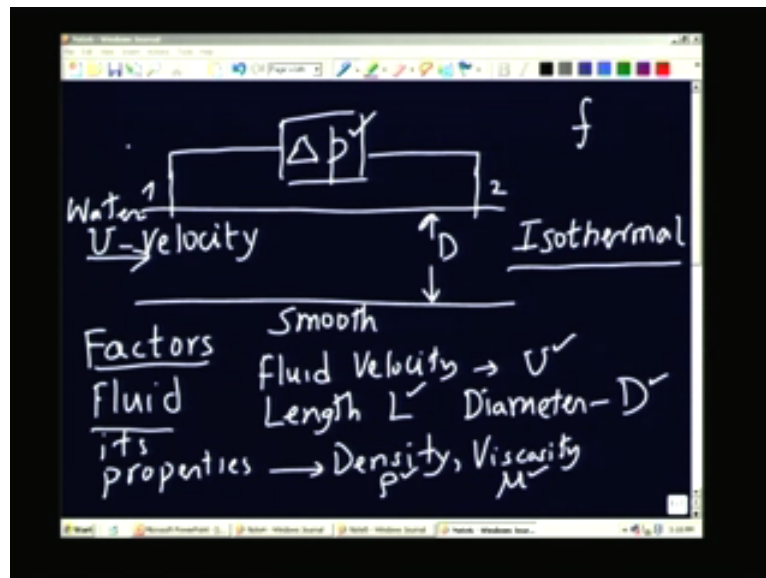
(Refer Slide Time 28.46)



So what is given is I have got a tube which is shown here, and we can take a circular tube for the present. There is a fluid that is entering the tube from the left and it moves along down the axis of the tube. The fluid is specified; maybe it is water or air or it is any other fluid. Its velocity, as it enters, is given. Actually it has to be measured. That's something which we will talk about later. Temperature may be specified, it may either be cold or room temperature experiment, where the temperature is not going to change. Or it may be temperature that is held at higher value by heating the fluid, in which case, it is a high temperature experiment.

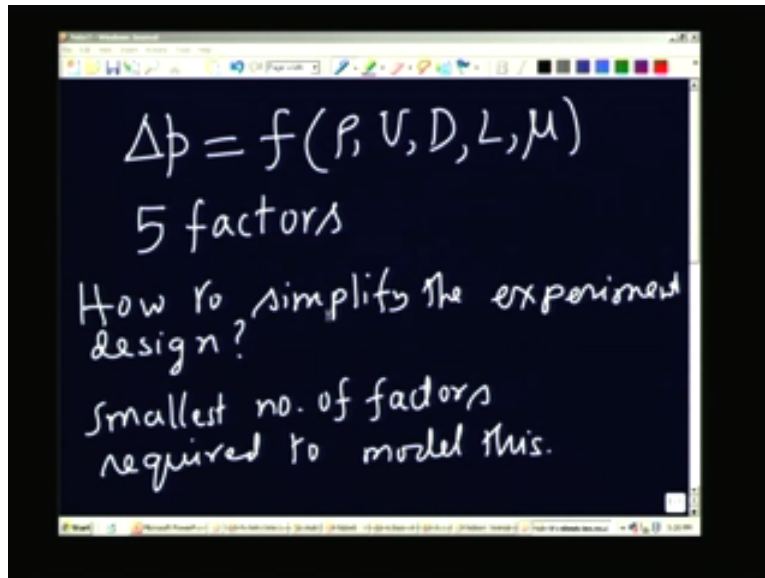
But in general, when you are measuring the friction factor, you would like to have isothermal flow conditions. That means, the flow is at one single temperature. We identify certain length of the pipe of the tube as is shown here and that is length l and I am going to put a pressure measuring or differential pressure transducer or differential pressure measuring instrument. I will get this Δp ; this is the outcome of the experiment and of course, the inner diameter of the pipe is specified. Its surface qualification, whether it is smooth or rough, is also given. For the present it is sufficient to look at how we go about doing an experiment and this is example 9 for our purpose now. So what I will do is I will go to the board and look at this in some detail and will see how to simplify the experimental design.

(Refer Slide Time 34.06)



So I will make a simple sketch here. This is the tube and the fluid is entering here. I have identified two stations; suppose I call them 1 and 2. I am going to measure the pressure difference between them and the diameter of the tube is D . As far as the fluid is concerned, we know what the fluid is. For example, it may be water and velocity is given as U . I will assume that is the mean velocity, and we assume that the entire experiment is isothermal. That means the temperature is going to be constant throughout the experiment. What I am going to do is to look at what the factors are? The factors are nothing but those quantities, which are going to affect the outcome of the experiment. The outcome of the experiment is this: Δp is measured. That is the outcome of the experiment; of course, we will later modify these definitions slightly. What we are going to look for is the friction factor f . I am going to look for f and we will define it slightly later. So what are the factors in this experiment? We will immediately say that it is the fluid, that means, its properties. Fluid is characterized by certain properties and there may be the density and in the case of pressure drop we expect viscosity to play a role.

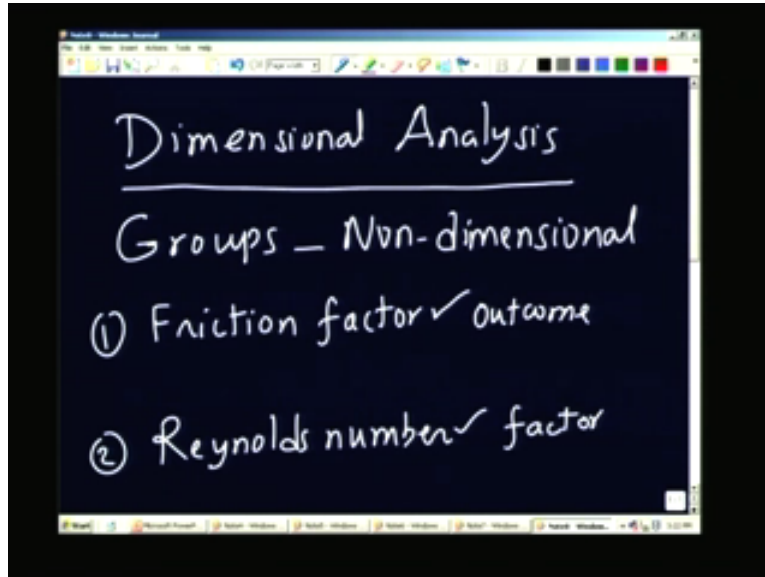
(Refer Slide Time 36.06)



So suppose I indicate density by the symbol ρ and viscosity by the symbol μ , these two are going to be important. So this is fluid and its properties. The velocity, the fluid velocity is U , the length of the pipe and the diameter of the pipe D . So the number of factors is 5, assuming that it is a smooth pipe. We will say this is the smooth pipe to reduce the complexity. So we have 5 factors and this is the quantity, which depends on the 5 factors. So let us look at what is going to happen. So we can say that Δp is the function. This is not friction factor, but the function symbol. So we have ρ , U , D , L and μ . You can write in the any particular order. Therefore there are 5 factors and you will see that if I were to do a factorial design, there will be 2 to the power 5 experiments to resolve all the interactions. So we require 2 to the power 5 experiments; that will be 32 experiments that are required in this case. Therefore that is one way of looking at it. The second way is how to simplify the experiment process or experimental design. That is the question I am asking. For this, what we will do is we will look at a different approach altogether to this problem. I am going to find out the smallest number of factors that can describe this experiment.

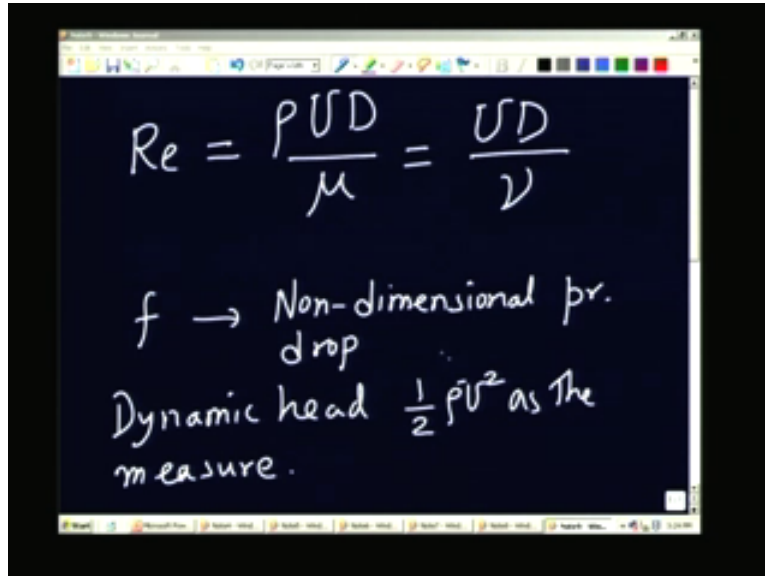
We are looking for the smallest number of factors required to model this. For this we take recourse to dimensional analysis; dimensional analysis for this particular problem has been done long ago by many people in this area it is nothing new for the first-time students. Of course it's a new activity and I am not going to indicate how dimensional analysis is done. This is taught in the course of fluid mechanics and I will simply assume that such an analysis can be done. The basic idea of dimension analysis is to go for what are called groups, non-dimensional groups. In fact in the present problem, only 2 groups will be involved: one is the friction factor and the second one is called the Reynolds number. That means I can look upon the friction factor as the

outcome and this as the factor. I call the friction factor as friction factor but it's actually the outcome of the experiment.
(Refer Slide Time 37.36)



What is the thing which I am going to vary? What is the factor which influences the friction factor? In this case of a smooth pipe, it is only the Reynolds number that is going to affect the friction factor. This is already a large simplification. We had 5 different factors, which were shown earlier. Now I have come to the conclusion that there is only one factor. So the reduction of factors here has taken place by knowledge of the concerned field; in this case, fluid mechanics. From the basic ideas of nondimensional or dimensional groups or dimensional analysis, we are able to reduce the number of groups to just 2. So let's look at the definitions of these two, so that we will be able to appreciate what is going on. The Reynolds number, which is named after the physicist Reynolds, is given by ρ , U and D by μ . Of course μ by ρ is also given by the dynamic viscosity. If we take μ by ρ , in kinematics viscosity it is usually given by the symbol ν . So this also becomes UD by ν . The friction factor is actually a nondimensional pressure drop parameter and what we do normally is to use the dynamic head, which is given by half ρU^2 as the measure.

(Refer Slide Time 39.45)



Handwritten notes on a digital blackboard:

$$Re = \frac{\rho U D}{\mu} = \frac{U D}{\nu}$$

$f \rightarrow$ Non-dimensional pr. drop

Dynamic head $\frac{1}{2} \rho U^2$ as the measure.

So with is as the basic idea, I will say that Δp is equal to $(f L \rho U^2)$ divided by $2D$ or I can write the friction factor as f is equal to $2D$ by $L \rho U^2$ into Δp . Notice that this is measurable; L and D are the length and diameter of the pipe, which are also measurable quantities; U is also measurable. Therefore, we have to measure these in this particular experiment. Of course we are measuring large number of quantities. But we are able to express everything in the form of a simple expression involving only the relationship in the form of two groups. That is the simplification we have been able to bring out.

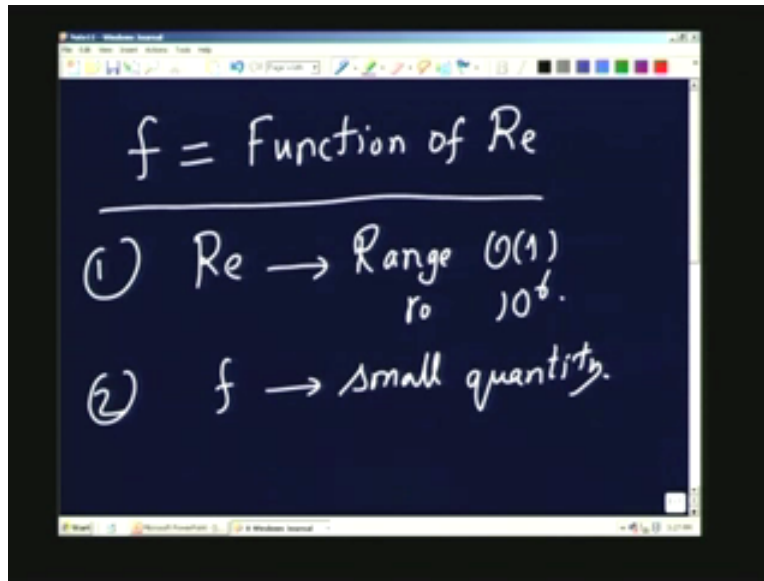
(Refer Slide Time 40.38)

$$\Delta p = \frac{f L \rho U^2}{2 D}$$

$$\text{or } f = \left[\frac{2 D}{L \rho U^2} \Delta p \right]$$

So in the experimental design, never mind the things we are going to measure. I am going to look at only one outcome, which is the friction factor and 1 factor which is Reynolds number. So the simplification is already apparent. Therefore it is not necessary to go about thinking in terms of a factorial design and so on, which in principle, one can do if one didn't know anything about the process. In this case the knowledge in the field has helped us in looking for the right kind of relationship. Therefore I will summarize thus. We are going to say that f , the friction factor, is a function of Reynolds number and of course I don't know whether it is a linear function or nonlinear function. We don't know anything about it; that's what we have to find out and from experience, we know that this relationship is not linear and there are certain subtle features. What are the subtle features? (1) Reynolds number is an unbounded quantity; that means it can be any value it can be without flow; if flow velocity is 0 the Reynolds number is 0. If the Reynolds number is high, the flow velocity is also high. In another words, Reynolds number range is very large. Let us put it as order 1 to may be 10 to the power of 6 or even higher. It depends on the particular occasion; of course the friction factor is normally a small quantity.

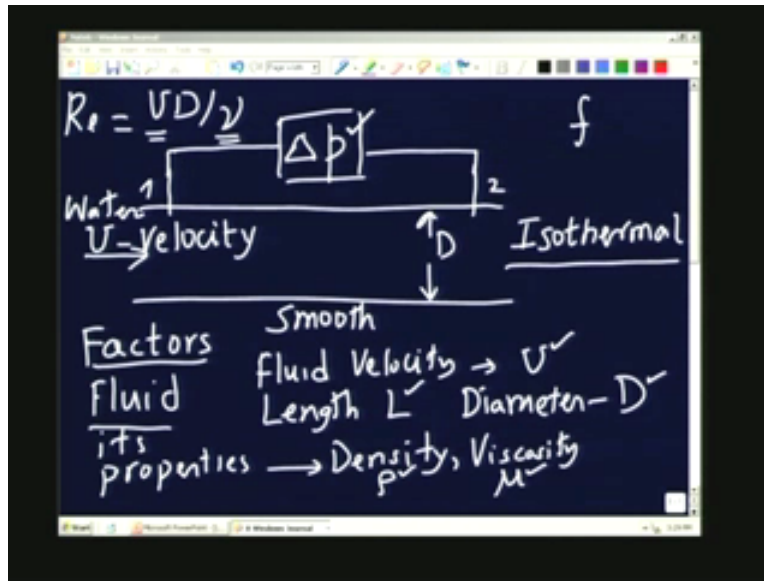
(Refer Slide Time 43.04)



So if the range of the particular variable is very large, as in this case, it means that it may be difficult to get or cover the entire range by using a single fluid or single diameter. So that is where the experimental design has to concentrate. Let me just briefly explain that. I will go back to the first slide, where I introduced the idea of a certain flowing tube. I will write here that the Reynolds number definition is UD by μ . If I fix the experiment to be the isothermal experiment, the viscosity does not change. Because we know that the viscosity is of what temperature and if the temperature is fixed, it is going to be fixed. Therefore I cannot vary this μ .

Once I have chosen the particular fluid value of μ gets fixed. I have however control over the diameter of the pipe and the U . Of course, I cannot change the diameter of the pipe at will. I may be able to get few different diameters, which are manufactured and available in the market. Therefore I can choose a few levels, may be 2 or 3 diameters I can think of. So I will write these later on; so I can think in terms of few levels for D , and I can vary the large range of U if I have a good enough pump capable of pumping the fluid through the circuit. I can do it with low velocity by putting a valve. By opening the valve slowly, I can increase the flow rate and I can increase the velocity U . Therefore U can be controlled or kept at any specified value over a fairly large range.

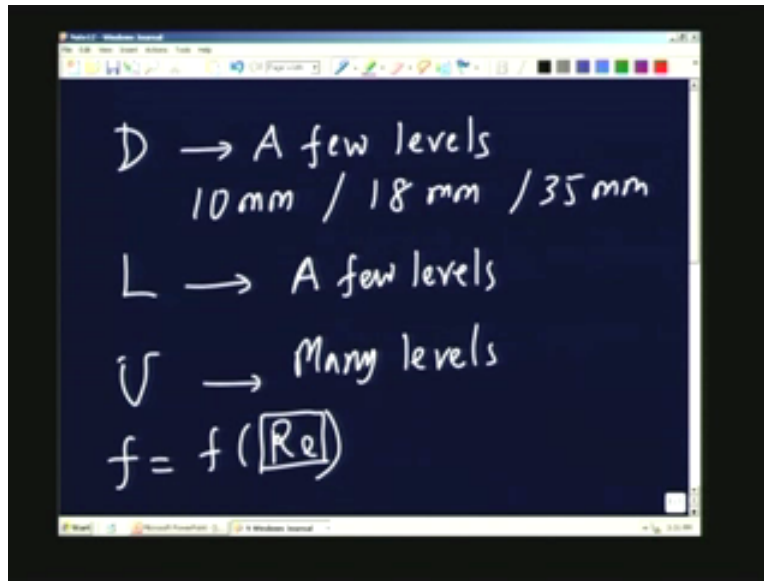
(Refer Slide Time 44.49)



Therefore if I look at the possibilities of D , a few levels are possible. For example, I can take 10 mm diameter; I can take 18 mm diameter; I can take probably 35 mm diameter. It has just 3 numbers just to indicate that few levels are possible. Even though, in principle, D can be varied continuously, it is not possible in practice because tubes available come in certain diameters. In fact the ones I have given may not be the correct ones. I am just trying to say that there are only few diameters possible.

Of course the length can be of any value; you can have different lengths L . You can again have a few levels; the maximum length possible, which is available in the market, is about 6 or 7 meters and you can cut it shorter for a smaller length. If you want to do that, a few levels are possible. For U many levels are possible; so the point is, if we look at the relationship we had got earlier, f is equal to a function of a Reynolds number. The fact that there are only a few levels for D and many levels for U is not going to be a major problem at all. In fact I can do all the experiments I want with the single diameter pipe and by varying the velocity alone. Actually I am, in other words, varying one factor.

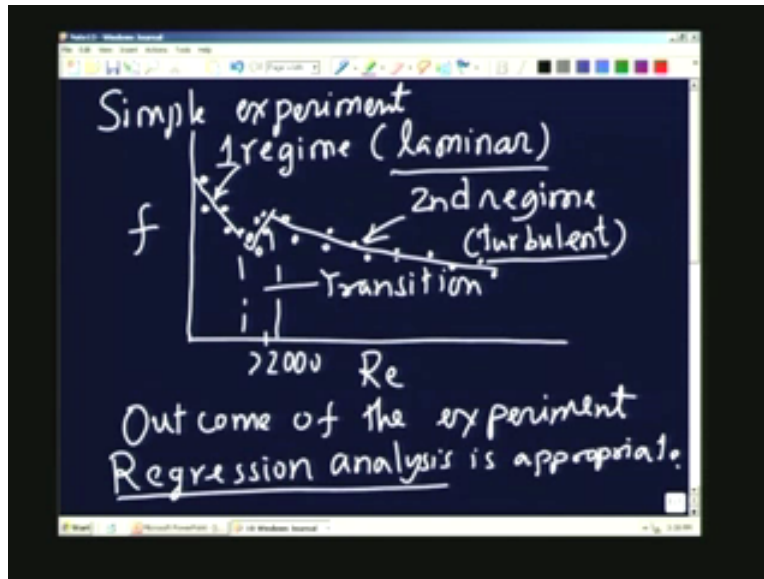
(Refer Slide Time 47:01)



In the factorial experiment we are talking about the main effects and interaction effects and so on. Here we have only the interaction effects; there is no main effect. The effects are only through the product of various quantities. Therefore Reynolds number involves the fluid property, the diameter and the velocity. So, all these are coming together to give me the effects on the measured quantity. Therefore the interacting effect we measured, the main effects, are not resolved or need not be resolved because there is no mean effect in that sense of the term. So that is the main difference between what we did in the earlier lecture regarding factorial experiments in the case of machining process.

Here we are talking about the fluid mechanics problem and the difference between the two. So essentially, whether I vary the D and L or only vary the U, it doesn't matter in principle. So if we are able to cover the range of Reynolds number required in an experiment by varying only 1 of them, it is sufficient. That is the great advantage in this particular thing. What I do normally is choose a diameter as a representative diameter; take a suitable length so the delta p is not very small. I can measure it easily; then I vary the velocity over a range such that I cover the entire range of Reynolds number. By doing an experiment of that type we get a plot of the following type. It's just qualitative; I am not going to give any numbers here, for small values of Reynolds numbers, it more or less goes linearly. At about greater than 2000, suddenly there's a change in the value; it goes up and goes like that. So we say that this is one regime we call this a laminar regime and we call this second regime the turbulent regime. Of course there is a confusion here; this we call transition.

(Refer Slide Time 53.44)



Let me briefly touch upon how we are going to perform the analysis. I may get data points here like this. I may get confusing here; we don't know. Then I may get some points here like this and therefore by looking at the data, after making a plot—of course this is a logarithmic graph—I have not mentioned that. If I make a graph suitably, I will see that the data points are going to indicate some kind of a relationship or some kind of a regression model. So the regression model is a more useful thing to do in the case of fluid flow experiments rather than look for linear models like we did earlier. And the big difference between the 2 models is that there the number of experiments and the number of parameters were identical if we wanted to resolve all the effects. In this case the number of experiments is quite large and we cannot resolve; we are not going to get too many parameters. We want to actually put it all in terms of as few parameters as possible.

If you look at the friction factor data which is given in any book on fluid mechanics, f is equal to constant divided by Reynolds number in the laminar range, and in the case of turbulent range, it will be some constant divided by Reynolds number to the power of some value. So it is a simple model we are talking about. So we will say regression analysis is appropriate for this kind of experiment. So the number of experiments must be so chosen that we are able to resolve the regime. We know that there is a laminar regime and there's a turbulent regime—we must take enough number of data so that we can get some points here and some points of the second region. Only then will we be able to resolve all these features.

Of course when we talk about the regression analysis I am not looking for a uniform or one regression equation valid for the entire range. I will divide the range into several regimes or several regions. In this case I can do 3 regions: the first one is laminar; second one is transition; and third one is the fully turbulent regime or range and then obtain 3 formulae, which relate f to the Reynolds number in that fashion. Therefore the experiment design is a simple experiment, not a factorial experiment like we had earlier, because some parameters can be taken only with

few levels. You can take some parameters with large number of levels and in fact many times what we do is we cover the entire range, we may even change a fluid from one which has lower viscosity to higher viscosity and so on and so forth. So the Reynolds number, being a single factor, can be obtained by the various combinations and we look for these combinations, which give a particular range of values. Therefore we will be able to cover the entire range. So with this, we have more or less come to an end of the experimental design.

Before we wind up this lecture, let us just recapitulate what we have done in the last 8 lectures, which forms module 1. In these 8 lectures, we had started with general discussion on measurements; why we do measurements; what is the goal of the experiments; and so on. Then we follow with some ideas with errors in measurements and their ubiquitous nature; that is, the error is always present. There are two types of errors: the bias as well as the random errors. We discussed at quite a length the nature of the random errors, how to characterize them, and so on. And then we have looked at the regression analysis, which is a very important tool for representing the outcome of the experiments.

And then we talked about design of experiments in process kind of applications, where we have a certain number of factors and levels, and we have rounded off with a set of simple experimental procedures, where the number of factors is small. Even though the number of original factors is very large, we were able to reduce the number of factors considerably to a small number and therefore a simple design is adequate for this kind of experiment. In fact the simple design with enough number of experiments was done to resolve some of the features in the result like we talked about, the laminar, turbulent and transition regimes. That is what one should look for. Therefore experimental work involves some prior understanding or knowledge of the field. Then you must also have all the knowledge about different instruments, which you can use for performing the measurements.

You must have the important tool called regression analysis to interpret the data in a proper fashion; so this is called a good presentation of data. When you are able to perform regression analysis, put everything in terms of a few parameters and simple formulae, you have been able to communicate your work to the wide audience in a proper fashion. We will stop here at this time and from the next lecture we are going to start module 2, which will look at measurements of different quantities of interest to us as mechanical engineers. Thank you.