Mechanical Measurements and Metrology
Prof. S. P. Venkateshan
Department of Mechanical Engineering
Indian Institute of Technology, Madras
Module -1
Lecture -7
Design of Experiments

So towards the end of the last lecture, we were talking about design of experiments and in lecture 7 on Mechanical Measurements, which is the current lecture, we will also look at the design of experiments somewhat greater detail. There are some basic ideas which we would like to understand and also basic terminology which is used. It should be made clear before we proceed with some of the details.

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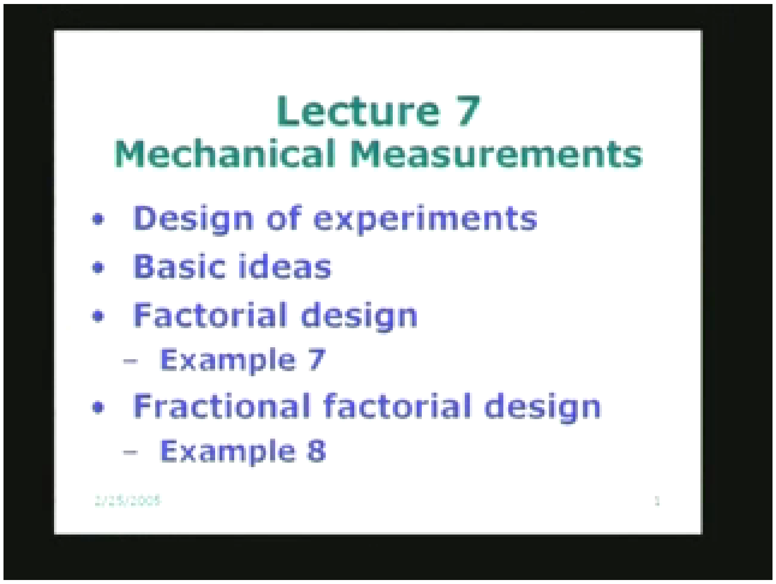

If we look at the first slide what we are going to cover in this lecture is to talk in general about design of experiments: some basic ideas involved in the design of experiments, which of course, depends on the type of experiments we are interested in, what is the end purpose of the measurements that to a great extent determines what are the details we have to understand. Then we will talk about a class of measurements, which is termed as factorial design. We will explain all the terms which are used with respect to factorial design and then we will follow it up with a simple example, which I call example 7, and then I will move on to fractional factorial design. The difference between factorial and fractional factorial design is to reduce the number of experiments that is to be conducted and this is to be done by using some uniform basic principles and that is what is going to be covered in this lecture. So the, first thing we are going to do, if to look at the definition for various quantities which automatically become important when we decide to discuss design of experiments. What are these?

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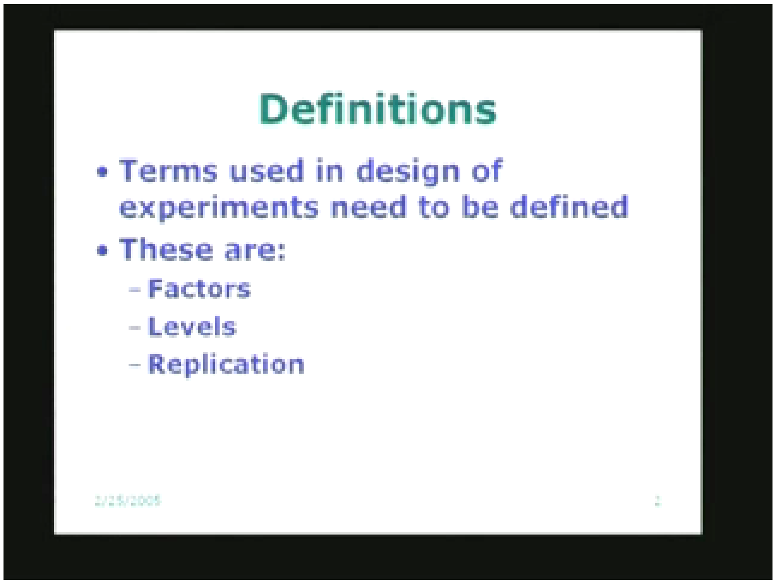

So the terms which we will meet with, we will call it factors. Then we talk about levels and then we will talk about replications. These are the three important things and let us see what they mean.

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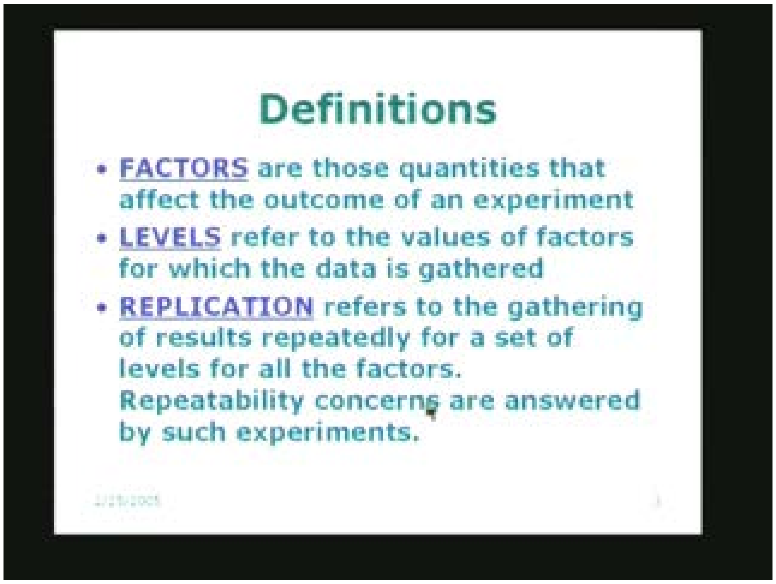

Factors are those quantities that are going to be important because they are going to affect the outcome of an experiment. Whatever may be the nature of the experiment there will be some quantities or parameters or some variables which are going to play a role in determining the outcome of an experiment. So what are these factors, we call them factors, and of course all factors need not be equally important. Some factors may have a dominant effect, some of them may not have that much of an effect and so on. So in order to recognize the relative importance of various factors, we have to conduct an experiment. So, that will become clear as we proceed with the discussion on design of experiments.

We talked about the factors; there may be N number of factors. In the case of the example I took in the last lecture, we have a machining process and, the speed of a machine and the depth of cut were the 2 factors. They will be recognized as the 2 factors and the outcome of that experiment will of course be the surface finish of the product, which can be given in terms of some numerical index. So the levels we are talking about here, the levels are the values the factors will take in the experiment. I may have a lower limit for the factor and I may have a higher limit for the factor and in conducting experiments, I may set these at different values. So each value which I am going to use between the maximum and the minimum becomes a level.

For example, the speed may be varied from a lower value of, let us say, 1000 rpm to a higher value of 3000 rpm in terms of revolution per minute. I may choose 1 or 2 values in between these 2 limits. Therefore the number of levels should automatically be equal to the number of values I am going to choose; the lowest, may be 2 or 3 values in between,and the highest. So you can have for example 5 levels. In another experiment, I may have only 2 values: 1 is the smallest value 1 is the highest value. I just want to see how sensitive the process is to the level of this variable or factor. So the levels refer to the values or the factors given in conducting a certain experiment. So these 2 are very important and in the design of experiment we will see that these are going to a play a significant role.

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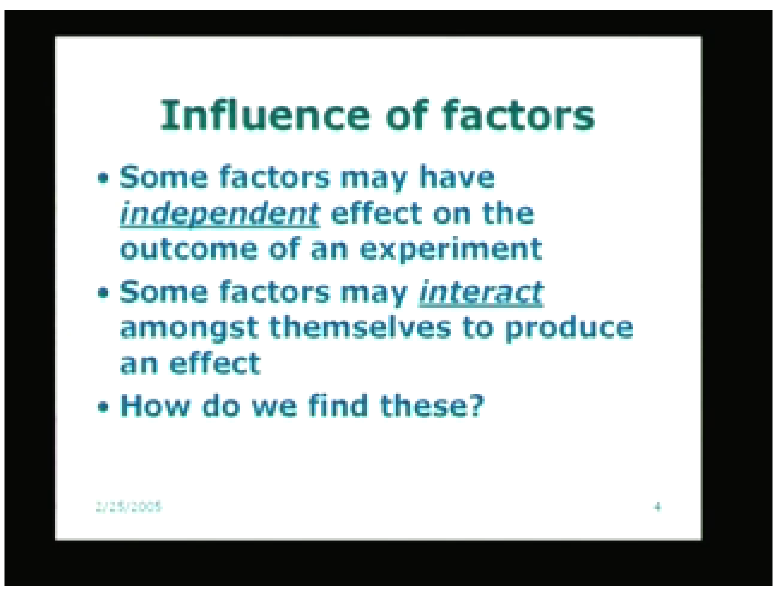

The third aspect is replication or repeating the measurement. For example, here I have written replication refers to the gathering of results repeatedly for a fixed set of levels for all the factors. In fact, I talked about the 2 factors here: the speed and the depth of cut. With a given speed, and given depth of cut, I will repeat the experiment again and again with different stock or different material or same material, I may have different specimens and I am going to vary the speed and the depth of cut, but in particular experiment, I am going to keep these 2 values at fixed levels and at those fixed levels, I am going to repeat the experiment again and again. The replication is necessary to address the concerns regarding repeatability and also the spread of the variation of the outcome of the experiment. So this replication is to basically get statistical information about the process that is being modeled or being studied. When we have identified a certain process or a certain experiment we have in mind and before we conduct the experiments, we must find out what are the factors which play a role and, in fact, some of these factors may be important in their own right; they may play a role independently. That means some factors may have independent effect on the outcome of the experiment. Some of them may also interact amongst each other, for example, the depth of cut and speed may interact. That means the two values may decide the outcome of the experiment not only independently but their interplay. That means we can, in other words, think in terms of some independent factors; that means the outcome of the experiment is directly dependent only on that effect.

For example, if I were to conduct an experiment with different values of one factor while fixing the other factor at a fixed value, if the effect is independent of the factor which is fixed, then I should get the same value if the independent effect should be only manifested because of the variation in the value of the level of the factor, which I am varying. However, sometimes more than 1 factor will interact with each other, in other way with different types of interactions. We will look at this later, these interaction effects are more difficult to bring out and the design of experiments basically is to find out what independent affects are, what the interacting affects are and how we can quantify these effects on the outcome of the experiments.

So the question rightly will be, how do we find this, whether the factor is independent or there is an interaction. In fact, we have sometimes also called the independent factor the important factors as main factors and the interactions sometimes may be classified as less important factors. The more important factor is direct involvement of the independent factor or the main factor, and the interactions may play a lower role. In fact we expect in many cases, purely from the knowledge of what is going on, that the interacting effects may be less significant than the main effects. So there will be a hierarchy of effects, the independent effects or the main effects and then the interacting effects, which will have lesser influence on the outcome of the experiment.

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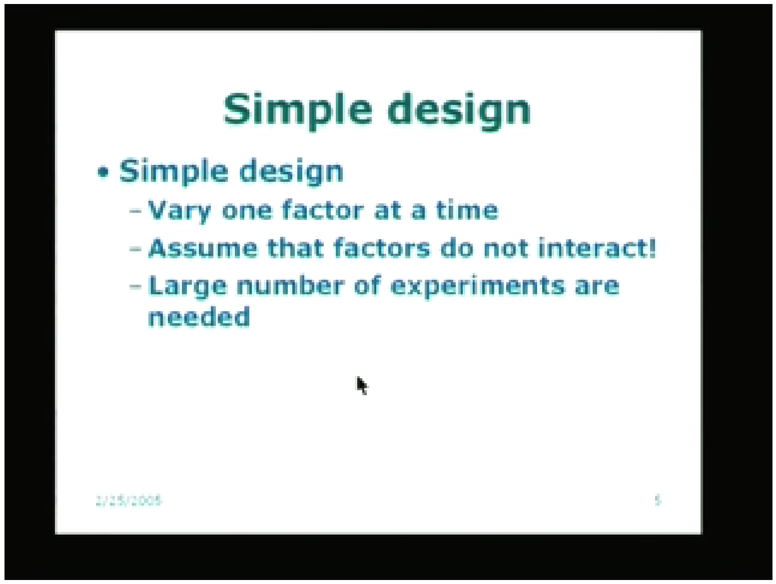

There are several ways of designing an experiment or a set of experiments. One is, of course, a very simple design. It is simply called as a simple design in which what I do is, if I have several factors which can vary in an experiment, I will vary one at a time, only one factor at a time; keep all others fixed at some values. I will come back to this simple design later when we talk about some difficult experiments where it may be necessary to do that. So vary one factor at a time, and many times when we do this, we assume that factors do not interact. This information about whether they interact or not has to come from independent experiment or some independent argument or some theoretical considerations. The only problem with this simple design is that a very large number of experiments are needed to get any significant amount of information about what is happening. So the simple design is avoided most of the time, but there are cases were there is no alternative but to go for a simple design. So we will come back to this simple design a little later on. Let us look at the next way of doing it.

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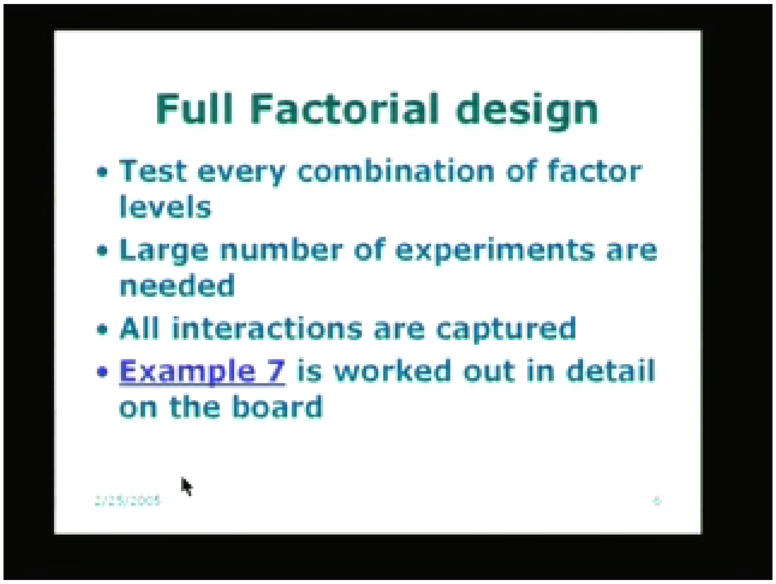

We want to study the effect of each of the factors on the outcome of the experiment and we can use what is called the full factorial design. Factorial design stands for all factors that are being taken into account in running the experiments and then trying to get the information regarding the effect these various factors have on the outcome of the experiment. So if I look at the full factorial design and what it tries to do, it should test every combination of factor levels. Suppose I have 5 levels for a factor, I will conduct the experiments for each value of these on all the 5 levels. I will also have all the levels for all the other factors that are going to be playing a role in the particular experiment. The disadvantage of full factorial design is, of course, that large number of experiments are needed again, but the advantage of this is that all interactions are going to be captured. We will explain this of course with a simple example. I am going work out example 7 on the board. It will become clear to us as to what we are talking about. So the advantage of a full factorial design is that it is going to give you information about all the parameters or all the factors, which are going to decide the outcome of the experiment.

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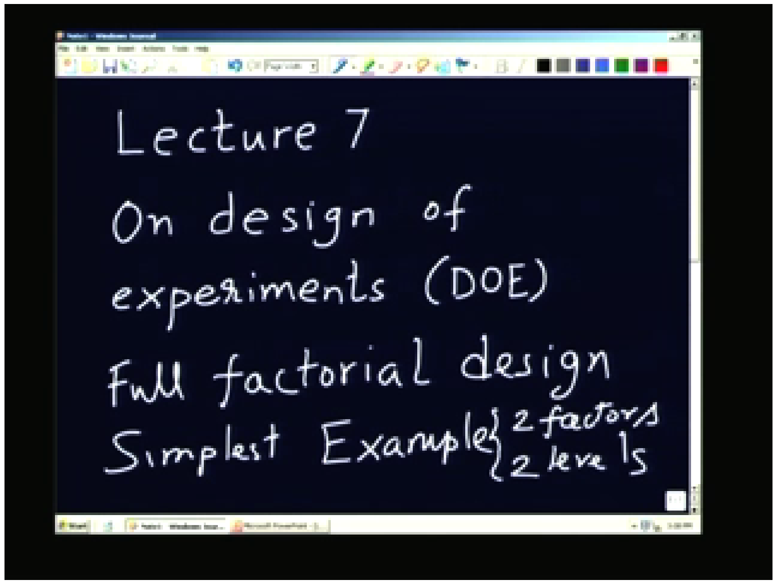

So let us get on with the idea of full factorial design and let us explain the basic things about full factorial design. I will work out the simplest example, which will be consisting of 2 factors and 2 levels; but this is like a representative case. Once we understand how 2 factors and 2 levels of experiment work it is easily possible to extend the arguments to generalize for any number of factors and any number of levels. Therefore, whatever basic information is required for design of experiment is already present totally in a 2 factor, 2 level experiment. So let us look at the way we are going to go about.

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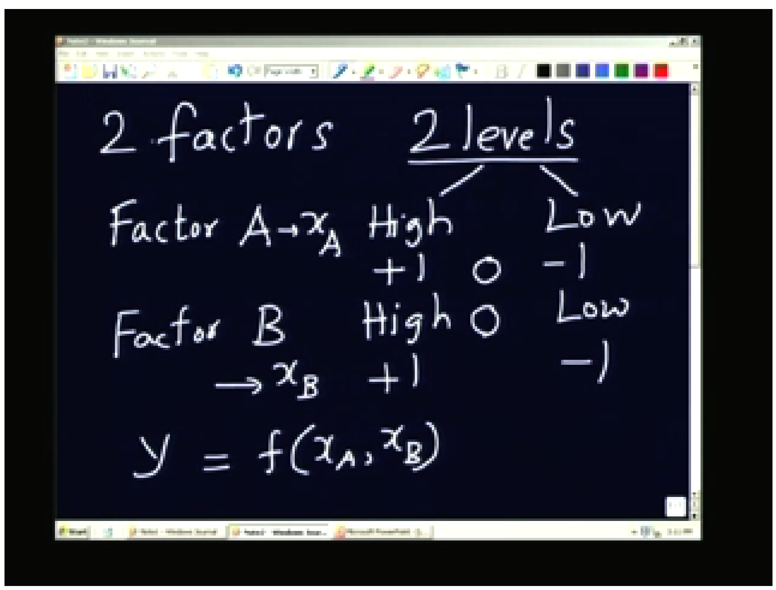

So we have 2 factors: this is the example I am taking; I will take a specific example where I will indicate what the factors are as we go along. We have 2 levels, so factor 1 or factor A would be the speed of the machine, factor B would be the depth of the cut in the experiment in which machining was the process which was being modeled. I can have a high value and a low value because there are only 2 values; it is allowed to take only 2 values as far as this particular design is concerned; again, a high value here, a low value here. Normally what we do is we take the high value as plus 1, low value as minus 1. Similarly, here plus 1 and minus 1; it is very convenient to work with such numbers. In fact, you can also see the mean of these 2 is actually 0, as it is written 1 is on the plus 1 side as the higher value, the lower value is minus 1 and in between, we have the 0 value and we are not going conduct the experiment here.

What we are doing is only 2 levels: the high level and the low level of the 2 factors. In fact I can represent factor A by x_A and factor B as x_B and in general, what we are trying to do is y is the outcome of the experiment; is the function of x_A, x_B. This is the general expression. What kind of functional relationship are we going to have? How to do that? We'll take it up as we go along. So if we look at the levels I got a plus value and a minus value and what I will do is I will indicate it in the next slide in the form of a matrix, which is easy to manipulate.

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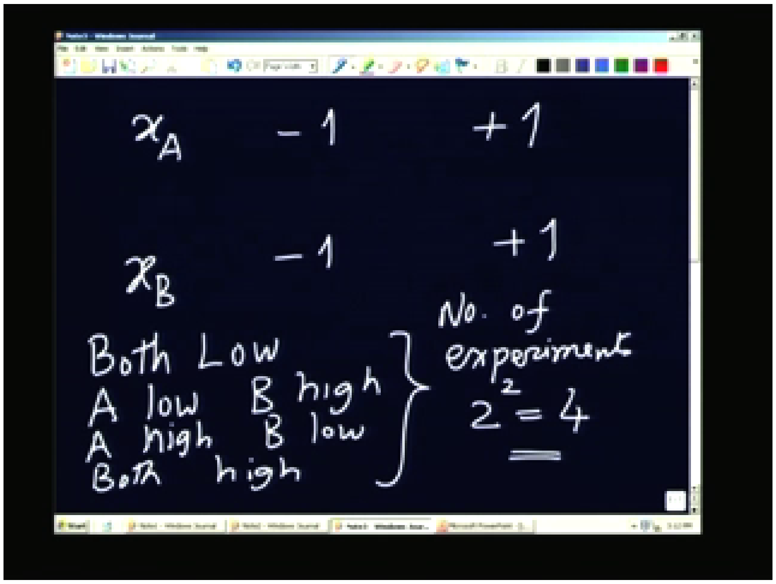

So I will say x_A, I am going to have minus 1,plus 1; for x_B I am going to have minus 1,plus 1. Now you can immediately see that the number of experiments they can perform is, I can have minus 1 and plus 1.Minus 1 here, and plus 1 for x_B. I can have minus 1 and minus 1: both low values for x_A and x_B. So we will just write it in words: both low, then we can have A low, B high, and then we can have A high, B low, and both high. So the number of experiments is easily seen to be 2^2 is equal to 4. So we are now able to understand what is going on. We are conducting an experiment and if I go back to the previous page (Refer the previous Slide 16:42) I have 2 factors, 2 levels. I am taking the 2 levels as the highest and lowest values, high or the low value in the experiment so factor A, I will call it x_A is the numerical value or the identifying value for that high and low. I am going to use the values plus 1 and minus 1, factor B high value and low value again plus 1 and minus 1 and we are looking at the outcome of the experiment, which is y and it depends on the both x_A and x_B. And as I was explaining earlier, it may be directly x_A, x_B that are called main effects or it could be through cross effects means the product of x_A and x_B can also influence the outcome of the experiment. Therefore we have 3: x_A, x_B and the product x_Ax_B. These are the 3 effects we are going to be interested in.

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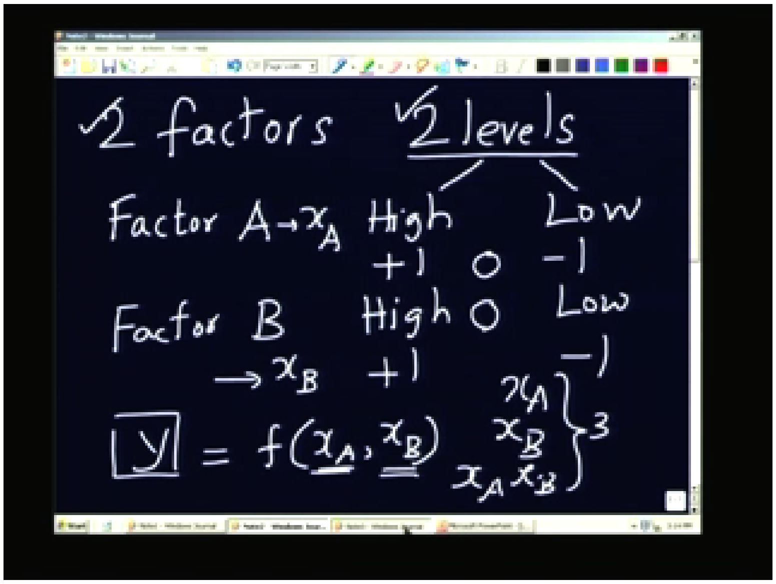

So let us look at the matrix we have written here. So I said that we can have minus 1 plus 1 for x_A and minus 1 and plus 1 for x_B. Therefore if I do the experiments, I can have both low, one of them low and the other high and vice versa, and then both high and that represents the number of experiments equal to 4.

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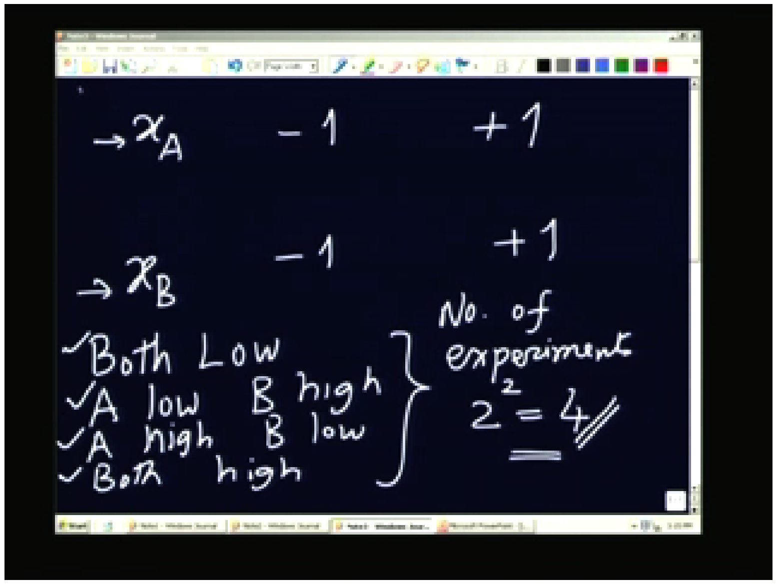

So let us look at this in slightly more detail. So I will just write down all the experiments we are going conduct in the form of another matrix. So I will write here, the experiments and the factor level combinations. I will put it in the form of columns in a matrix again; so I will say $x_{A,}$ x_B. I will also include the product x_Ax_B here,because we will see later that is also important as far as the effect on the outcome of the experiment is concerned. If we remember, we had plus 1 and minus 1 are the high and low values I simply put them as plus and minus; the numerical value 1 is there. It is plus or minus 1 and therefore, to be economical in writing because we understand what we are doing now, I will just say plus and minus 1.

For example; I can take x_A negative, x_B positive; that means this is minus 1 and plus 1: this is low value and this is high value. I can take plus here minus here, I can take minus here, minus here, I can take both positive. So x_A x_B will be simply the product of these 2, minus into plus is minus, plus into minus is also minus, minus into minus is plus, plus into plus is plus. So what do we gather from this representation? If we look at these, we can look at each column as a vector, we have minus plus minus plus, when this is minus this is plus , this is plus this is minus, this minus, these 2 vectors are linearly independent. They are independent of each other because if I take the product of these 2 which is what x_A x_B is, if I take the product of these 2 columns, I am going to take x_A, x_B and I am going to multiply the particular component and these are the other 4 components of the vectors.

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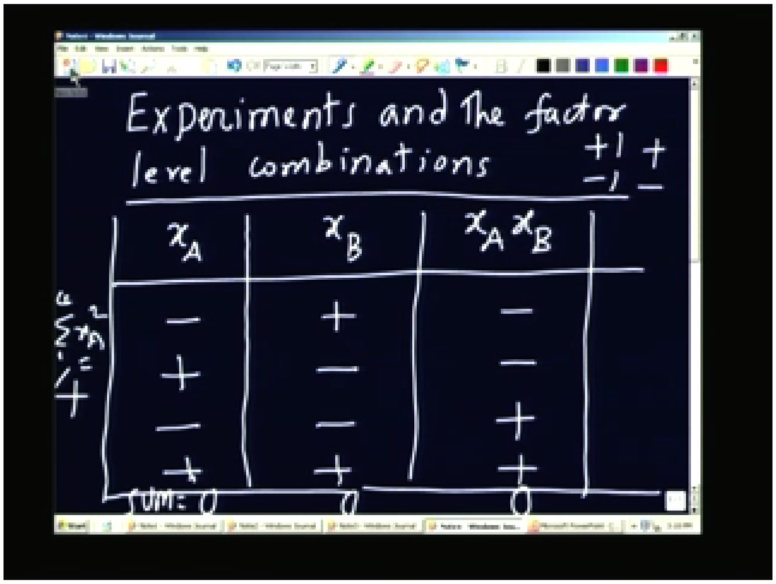

I am going to just multiply the components and put it here you see that if I add these columns, it is going to give 0. If I write sum equal to 0 for this column, in fact it is also 0 and this is also 0 here, so the property of this particular matrix is that the column sums are all equal to 0. However, if I were to square each one of these components so you take the square of this, square of this, square of this and square of this and add, you will get a sum equal to 4. So, Σx_A square 1 to 4 is equal to 4. So, sum of the squares of these is equal to 4. I think I will write it in the next pane. So what I am doing is Σx_A 1 to 4 is equal to Σx_B equal to $\Sigma x_A x_B$ is equal to 0. The column sums are going to be 0 because for an equal number of times they are positive and for an equal number of times, negative; so they just add up to this.

However, if I take Σx_A square this will be equal to 4 Σx_B square. So just to explain what I am trying to do, I am just trying to understand the idea of or understanding the basic properties of these coefficients, which I am using for the measurement process because these are going to play an important role in the way we are going to interpret the result of the experiment. So now let us look at the relationship between y and x_A and x_B and may be the product $x_A x_B$. This can have up to 4 parameters. Why is it so? Because I am conducting 4 experiments, I have got 4 y values and the corresponding values of the x_A, x_B and the product $x_A x_B$. Therefore I will be able to fit a curve or find a relationship, which can support only 4 parameters. Therefore I will think in terms of what is that function which can have 4 parameters.

I will take the simplest I can think of; it will be a linear relationship so I will say y equal to the first one will be of course a constant because the value of y can have a value independent of these parameters. In fact we will show that there is a mean value p_0 plus p_1 or $p_A x_A$ plus $p_B x_B$ plus, I will say, $p_{AB} x_A x_B$. Each of these is the relationship between y and the parameters x_A,x_B and the interaction parameters x_A x_B is nothing but the interaction between the two factors. That means when I have only 4 experiments, 2 parameters and 2 levels, this is all I can do. I cannot have any other relationship between y and the quantities x_A, x_B and product of x_A x_B. So how do I find out these values? I find out the outcome of the experiments—with the 4 different experiments I am conducting—in each experiment I know the outcome of the experiment y. So now I will have 4 values and I will substitute those 4 values here and I will find out the corresponding values that are in excess on the right-hand side and then I will get 4 equations; so we have 4 equations, 4 unknown p_0, p_A, p_B, p_{AB}.

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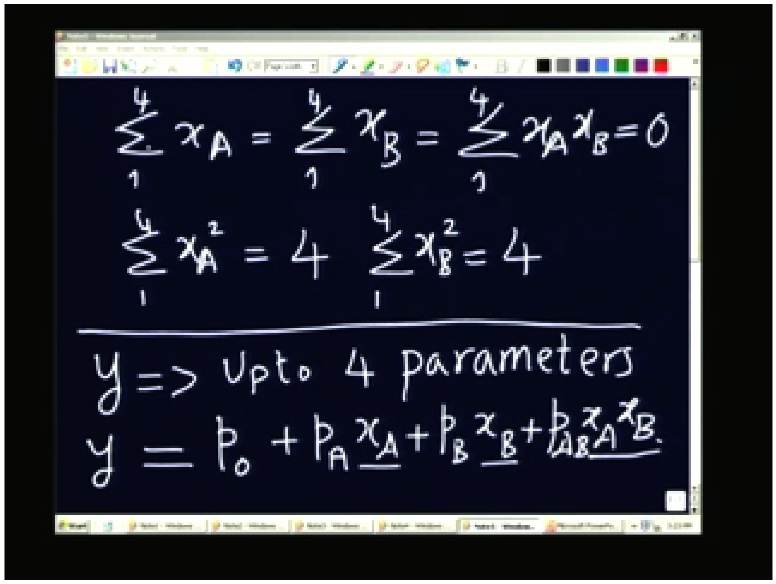

Therefore I will be able to determine the 4 constants from these 4 experiments. For doing that, I will take a simple example, which will clarify because I am going to use some numbers. Therefore let me take an example which was also mentioned in the last lecture, the same machining example of the surface finish. I am just taking one example, which is familiar to us. As mechanical engineers we can take any number of examples: surface finish will be represented by y. I will represent speed of the machine by x_A and the depth of cut by x_B. In real machining process, there may be many other parameters which may play a role, but we have taken a very simple one, where we have ignored any other parameters or factors coming into the picture. We have only 2 factors so these are the 2 factors: speed of the machine and the depth of cut, and the outcome is the surface finish. That means the assumption is that if I vary the speed of the machine or depth of cut, the surface finish must depend on the 2 values, which I choose for these.

Again I can identify high value and low value; for example, the speed of the machine may be 200 rpm, 2000 rpm or 4000 rpm arbitrarily. So one of them will be minus 1 and the other one will be plus 1. So we don't have to worry about the actual number, one of them given is minus 1 the other one is plus 1. You can always transform the coordinates at these 2 values. You get depth of cut; it may be a tenth of a millimeter, it may be .25 millimeters, so you can have two: upper limit and lower limit and those will become minus 1 and plus 1 respectively. I am going to measure the surface finish using some instrument. This is measured after the machining process is done; we are going to conduct the experiment by machining the part. After the machining I am going to take it out and measure the surface finish using some instrument and I want to link the value of y to the value of x_A and x_B, which were used in the experiment. So the measured values of surface finish may be given some values. For example I have taken something like 1 to 10 so this is 1 to

10; in that scale I am going to fit in the value at some place by some formula. I don't have to worry about the details now, they just indicate how the example is worked out; as in example 7.

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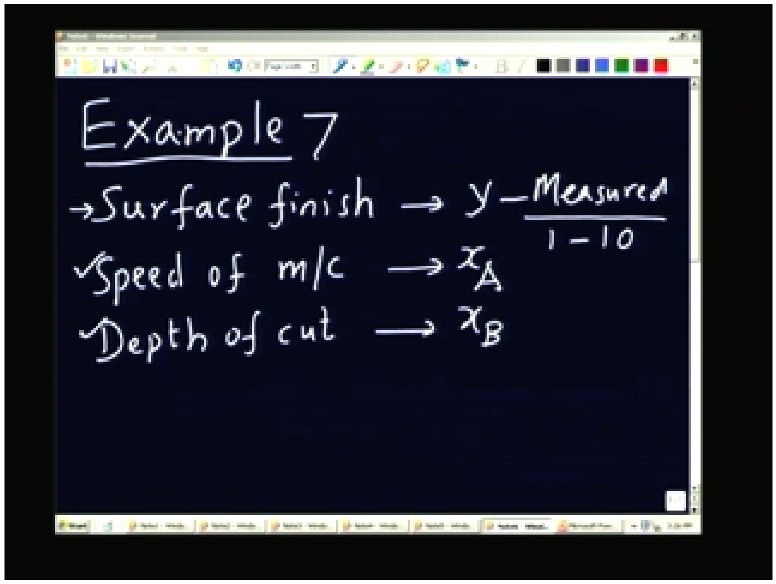

So I am going to look at the surface finish, speed of machine, depth of cut and make a tabulation of the results of the experiment. So the results of the experiment are tabulated in the following way. So I have minus 1 and plus 1 or in the terminology I am going to just say minus and plus for speed of the machine it is x_A. I am going to write x_B here minus and plus and the surface finish obtained is given in the form of a table, with 4 entries of the values I have obtained: 2, 8.2, 1.5 and 3.5 in this. This is the actual example; this is the outcome of the 4 experiments, of the 2 squared experiments. So the outcome of the 2 squared experiments or 4 experiments minus plus , minus plus : these are values of the factors. These are the levels and these are the values: 2, 8.2, 1.5, 3.5; of course, this table itself immediately tells me that somewhere here I have a likelihood of getting good finish because the outcome of the experiment is surface finish and in my terminology 10 would be a very good value for the surface finish 1 would be a very poor value for the surface finish. So I am talking in terms of some relationship between the numerical value and surface finish or the quality.

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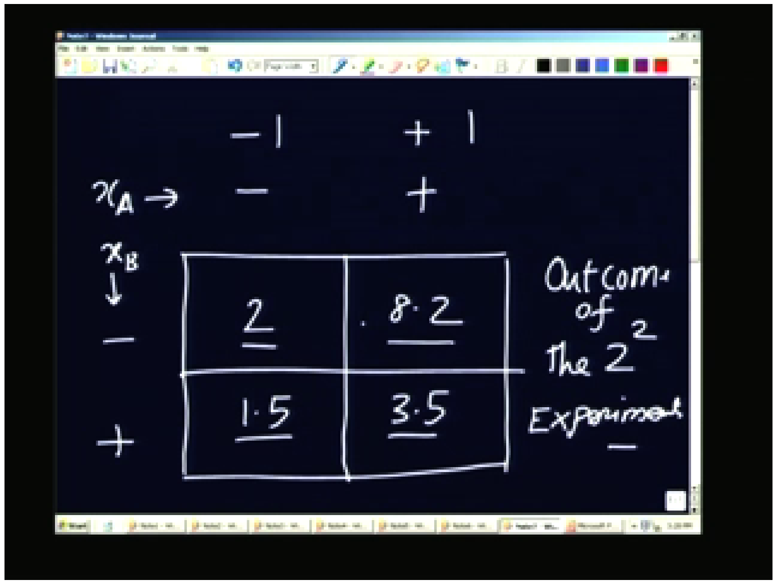

So now I will write the 4 equations using the model which we have proposed. I will just write the model here,y equal to p_0 plus $p_A x_A$ plus $p_B x_B$ plus $p_{AB} x_A x_B$. There are 4 values in the table and for each one of them I can write the corresponding equation here, so if I want to refer here, I can just write here in capital form, I have the 4 values: minus, plus, minus, plus. These are the 4 values: 2, 8.2, I think 1.5 and 3.5. This value is, let us say 2. So 2 equal to p_0; both are minus so minus p_A minus means minus 1 minus p_B, $x_A x_B$ is 1 therefore this becomes plus p_{AB}. This corresponds to the first entry in the table. The next one, 1.5: p_0 doesn't change; it is constant. The value is minus p_A plus p_B, this becomes minus p_{AB} because this is a product of x_A and x_B. Now minus into plus is minus is minus, that is why it is equal to minus 1. I am going to have 8.2 as the third one. I want to write little bigger so that you can easily see what is happening. 8.2 equal to p_0; that corresponds to this where the x_A is plus 1, x_B is minus 1. Therefore this becomes plus p_A minus p_B again minus p_{AB}. So lastly 3.5 is equal to p_0. Both are high, so plus p_A plus p_B plus p_{AB}: these are the 4 equations available to me to determine the 4 coefficients. What are these coefficients? These are nothing but the coefficients, which represent the effect of each one of the parameters: the larger the coefficient value, the larger the influence of the particular factor.

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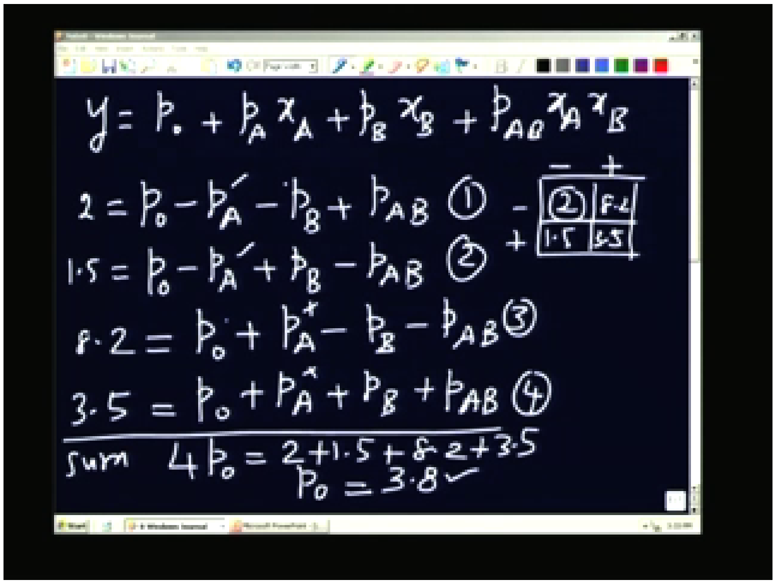

So, if p_A is dominant, p_A is much larger than the p_B or p_A is greater than p_B, that means the parameter x_A has an important role to play in a machining process than the parameter x_B. Suppose p_{AB} is very small; then we will say that the interaction between x_A and x_B is not very important. Some of the conclusions, which we are going to draw regarding the nature of the effect of the factors, are coming from these 4 equations.

We will also notice that if I simply add these equations all the other quantities will become 0 because p_A minus p_A minus p_A then there is a plus p_A and plus p_A. These 2 are going to cancel off with this, minus p_B plus p_B minus p_B plus p_B: they will cancel, therefore if we simply add these equations suppose I call it 1, 2, 3, 4; so the sum of the equations gives you the $4p_0$ is equal to 2 plus 1.5 plus 8.2 plus 3.5 and this gives a value of p_0 is equal to 3.8, that simply means that suppose the value of x_A and x_B were taken to be the mean values for the high speed and low speed and will take the increment value similarly depth of cut, high depth of cut and low depth of cut: if I take the mean of these values, this is what I am going to get.

Therefore let us look at the interpretation of this result graphically in the next tablet here. So I am going to draw a sketch of what is going on so I will say x_A and x_B, like they are coordinates along the 2 directions. This corresponds to plus plus because both x_A is equal to plus 1 and x_B is equal to plus 1. So there is double plus or plus plus. If you want it, put a comma here. This corresponds to plus minus, this corresponds to minus minus, and this corresponds to minus plus and this is the center. So what we are trying to do is, suppose because of some reason I had assumed that this was the reference set, I have a reference set for x_A x_B, then what I am trying to do is to find out what happens if I go to a value to the right or to the left to the top to down. So I am trying to study the effect of the parameter on how the reference value, which is nothing but p_0, is going to change.

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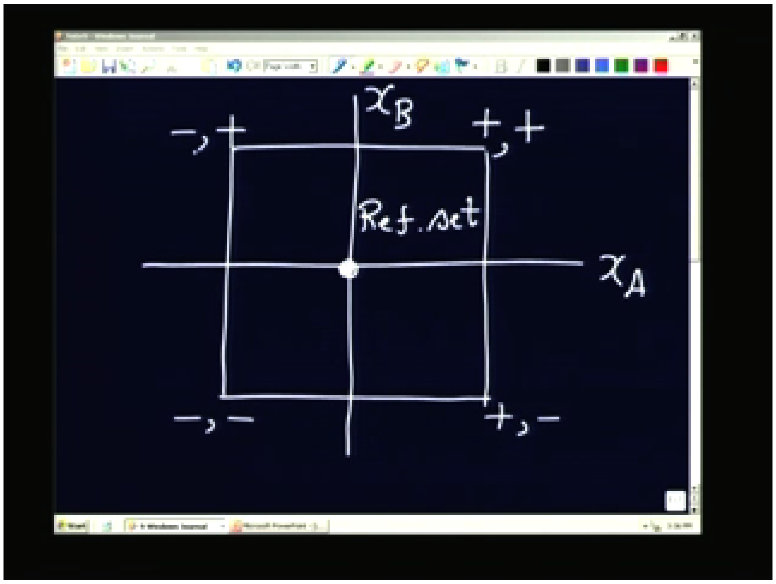

So the experiment can be looked upon as a study of the process and the variation when, in the process outcome, we vary the values of the two factors away from the mean value. So it is like trying to understand what is going on in the process. So this is a very physical way of explaining what is happening and what I did in the previous tablet was this: give the same in the form of mathematical expressions. So p_0 I have already obtained 3.8 and the student can correctly obtain the other values and I will give the values in the subsequent tablet here. So before we do that let me do this: indicate how a solution has to look. I think some details can be mentioned here. So, by adding 1, 2, 3, 4, the 4 equations, I was able to get the average.

Now you notice that suppose I want to find the value of p_A I have to make this plus so I would take minus of equation 1 so it is minus 1, minus 2, minus 3, minus 4. What will happen if I do that? If I multiply by minus, it will become minus 2. It will become minus p_0, this becomes plus p_A, this becomes plus p_B, this becomes minus p_{AB}. Then I multiply this by minus this becomes minus 1.5, this becomes minus p_0, this becomes plus p_A, this becomes minus p_B, this becomes plus p_{AB} and if you look at column sums, excepting this column, all other columns are also to be 0 when I do this, when I add these things. In fact minus 1 is 3 minus 1, this is like 3 minus 1, 4 minus 2, equation 3 minus equation 1 will involve the difference between these 2 and similarly differences between 4 and 2 are referred to as contrasts. So we take minus 1 minus 2 plus 3 plus 4 add to get p_A. Then you can do the same thing with the next one, so I have to make this plus and also this plus so minus 2 minus 1 plus 2 minus 3 plus 4. I get p_B. So you will see that by very simple arithmetic operations, I am able to get all the values of the parameters; so let me just give the parameters in the next tablet.

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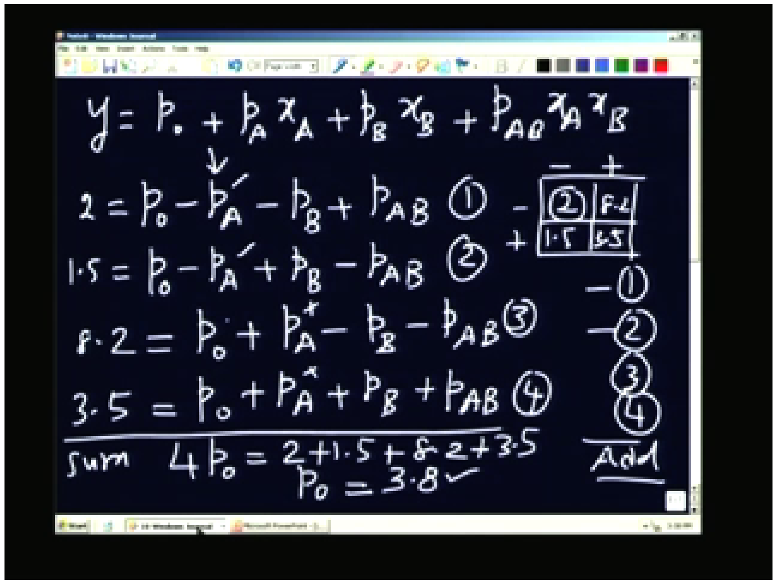

So the solution using the properties of the matrix is given by p_0. Of course, I have already given 3.8, so p_A works out to be 2.05. The student can verify these values by doing what I indicated minus 1.3 and p_{AB}. The interaction parameter is given by minus 1.05. Now I am going to ask myself the question, how do I interpret these results in terms of identifying what was the relative importance of different factors on the outcome of the experiment. So the question is, what is the relative importance of each factor in terms of the main effect due to p_A and p_B and the interaction effect due to p_{AB}. So I have got 4 coefficients; 1 is the mean and you see here by having nonzero values for x_A and x_B I am going to perturb this value from p_0 equal to 3.8. So the mean value is this. So I can use the variance with respect to the mean as a measure of this effect.

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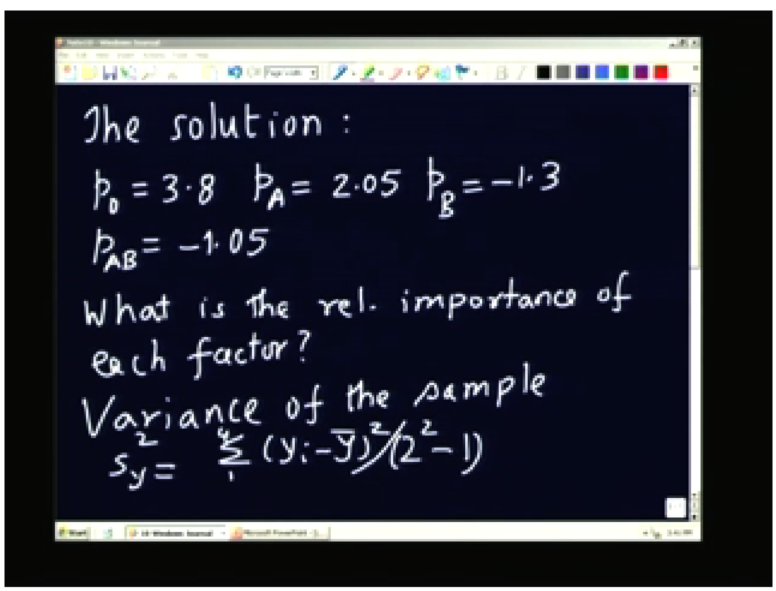

I will tell you how to calculate the variance of the sample. It will be calculated as follows. I will call it as s_y square. The sample variance is given by Σ 1 to 4 y minus ybar whole square divided by n minus 1; if you remember we have 2 square minus 1 because we have already got p_0 from these measurements, therefore I have one less value and therefore I will use the formula n minus 1 in the denominator n is 2 square, 2 square minus 1 is nothing but one third.

Let us look at Σ y_i minus ybar whole square. I can take one of them and see what is happening. So y_i minus ybar is nothing but p_0; for example, if I take plus plus that means both high values, it will be p_0 plus p_A plus p_B plus p_{AB} minus p_0 because the mean value is nothing but p_0 plus p_A plus p_B. I am going to square this, because I need the square of this quantity. So you can see from the property we talked about earlier, the sum of the squares of the column is going to be 4; that is going to be important.

In fact if you do this we will see that it is nothing but the square of this piece. They will add up to 4; therefore it is easy to find out what is going on. So p_0 is simply a coefficient p_0 multiplied by the column sums of the squares of the column, ok, and sums of the square of the columns are going to be 4, which we already showed how it happens. I am going to leave it as an exercise of the student to figure out; divided by 3 it is nothing but 4 p_A square plus p_{AB}; so 4 p_A square plus 4 p_B square plus 4 p_{AB} square divided by 3. So the variance of the sample or Σ y square contribution is 4 by 3 p_A square, 4 by 3 p_B square, and 4 by 3 p_{AB} square; therefore coefficients are going to be giving an idea as to what is their role in the outcome of the experiment.

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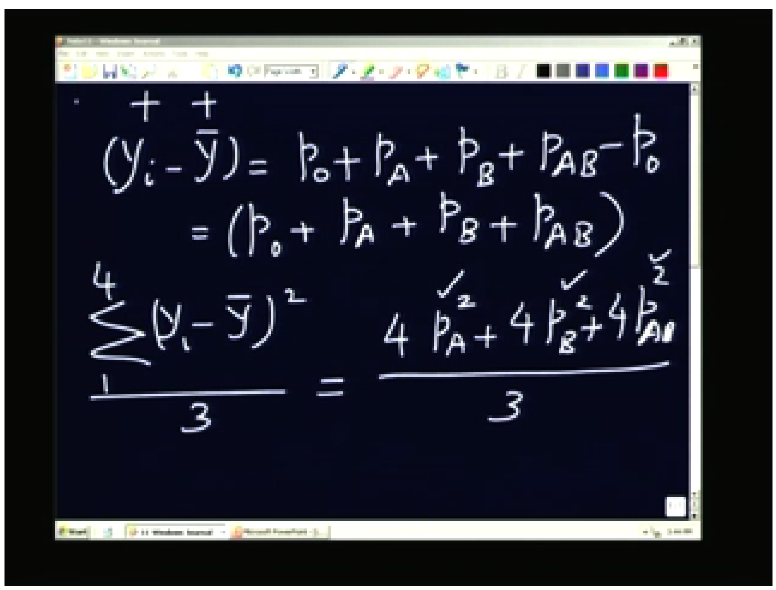

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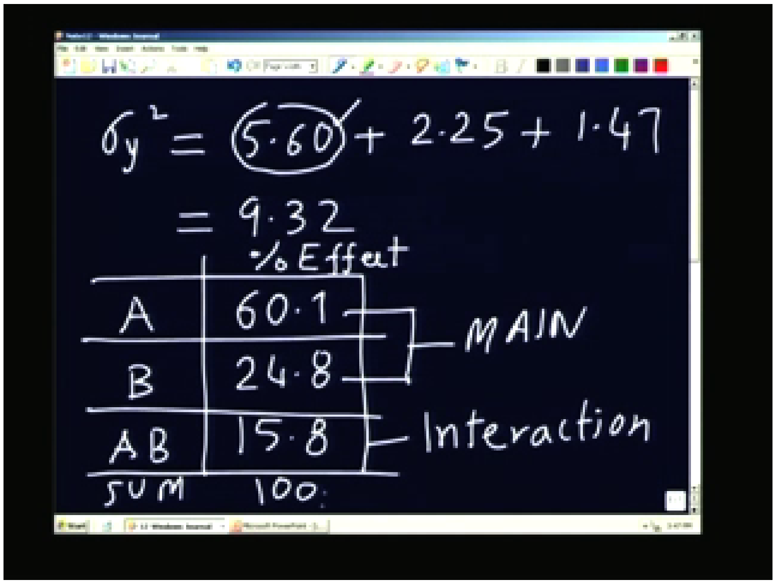

Now I can put the values p_A, p_B and p_{AB}, which was determined in the last slide. So what I can do is I can introduce this and just give the values so we can show that Σ y square is equal to 5.60 plus 2.25 plus 1.47 and if we add all these things, we get 9.32. So this is going to be 9.32. So what I can do now is take the ratio of each one of these with respect to the total. That will give the percentage effect of the particular factor on the outcome of the experiment. Therefore I can say A, B, AB—another way of writing the effect, here I will say percentage effect, the effect is measured by the contribution to the variation. Variance is the variation which is the effect of a particular factor. So we are looking at the contribution of the each one of these, so I will say 5.6 by 9.32 into 100. I will put this as percentage. This will be 60.1, next one is 24.8 and the third one is 15.8. If we add all of them, we should of course get a sum equal to 100. That means 60% of the variation in the outcome of the experiment is because of the speed of the machine. Therefore speed has got a more important role to play in the process, which we are talking about. The depth of cut has got an effect but it is less than the effect of the speed and the interaction between the 2 is not unimportant. It is important in this case but it is much lower than the effect of each one of the individual factors. So we will say these are the main effects: 60 plus 24, 84.8% and the interaction effect is about 15% so this not necessarily ignorable or negligible in this particular case. Therefore in this particular experiment or in this particular process, the speed, the depth of cut and the product of these 2 are all playing significant roles. However, the relative significance of each is determined by the relative magnitudes of the variances due to each one of these and that's what we are trying to do.

Let us get back to the slide which we were looking at before we went to the tablet pc to write out the things. You will now appreciate that by doing that full factorial experiment, I am able to calculate all the interactions. In this case the interaction is only one interaction—x_A into x_B was the only interaction. If number of factors becomes larger the number of interactions also becomes larger as we will see in a little while from now. So let us look at what might happen if I do an experiment with a full factorial design. What will happen is that the number of parameters or number of factors is large, the number of experiments needed to capture all the effects. In the previous case I had only one interaction effect x_A product x_B.

In the case of many number of parameters, I will have many such combinations. I may have combinations like x_Ax_B, x_Bx_C and so on; I may also have $x_Ax_Bx_C$ product of 3 terms at a time. I may have $x_Ax_Bx_Cx_D$ for 4 things taken at a time. So there are a lot of such effects that may be important or we may have to study them. Therefore if we want to capture all these effects, the number of experiments to be performed is large. For example, a 10-factor, 2-level experiment requires 2 to the power 10 is equal to 1024 runs to bring out all the interactions. In a laboratory experiment where we want to conduct these experiments, 1024 experiments are going to take months and months. If each experiment takes several levels, it is going to take a large amount of time and effort on the part of the experimenter. The question now is, is it possible now to reduce the number without losing too much of the information?

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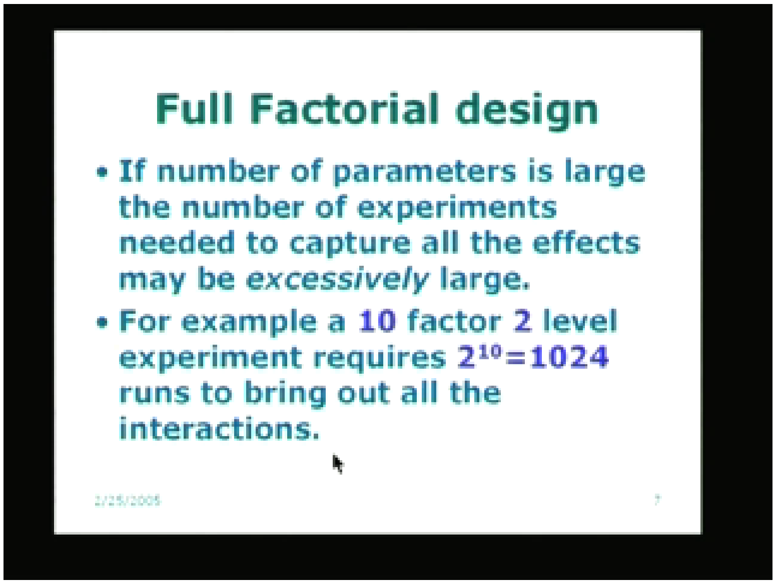

That means, we go for what is called a factorial fractional design. The idea of this fractional factorial design is to use not 2 to the power of 10 as in the previous case. Instead of that I will take 2 to the power 10 minus 4; that means I am not going to use all that. I will choose from 1024 experiments, which are possible, I will choose the one in a very careful manner so that it will bring out the most important effects I mean to study because as you saw in the previous case x_A, x_B are having more important role than the product x_Ax_B.

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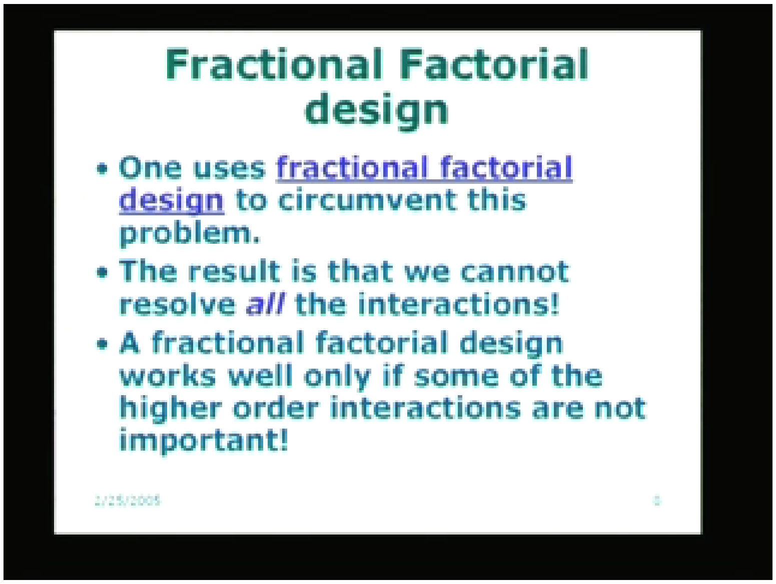

So in general, if the number of parameters is large, the main effects will be due to each one of these factors and the product of 3 quantities taken at a time, or 3 factors taken at a time. So they may become less and less important if it is found from some other study or somebody else studying the problem before or if they have some independent reason to believe that only certain interactions are important. Why do we have to do all the experiments? You do only a certain number of experiments to bring out the important effects, which we want to study and that's what the fractional factorial design does. The result is that we cannot resolve all the interactions very clearly, but some of the interactions may be unimportant. Therefore what we will do is try to find out is whether we can run a fewer number of experiments and get whatever information we want. So we will discuss this idea of fractional factorial design in the next lecture and also discuss simple design, which we talked about in a very small, meaner detail in this lecture. We will try to continue with that in the next one. Thank you.