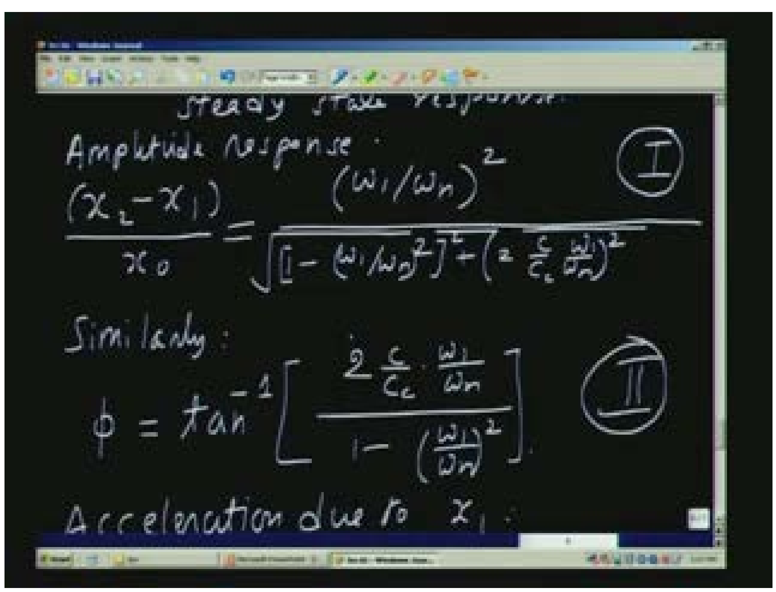
**Mechanical Measurements and Metrology**
**Prof. S. P. Venkateshan**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Madras**
**Module - 4**
**Lecture - 47**
# Vibration and Acceleration Measurement

We will move on to lecture 47 on our series of Mechanical Measurements. In the last lecture, we were discussing about vibration measurement and we were looking at a second order system, which basically represents a vibration measuring device and we were trying to understand the basic principles. Therefore what we will do in the present lecture is to recapitulate what we did in the last lecture and carry on from there.

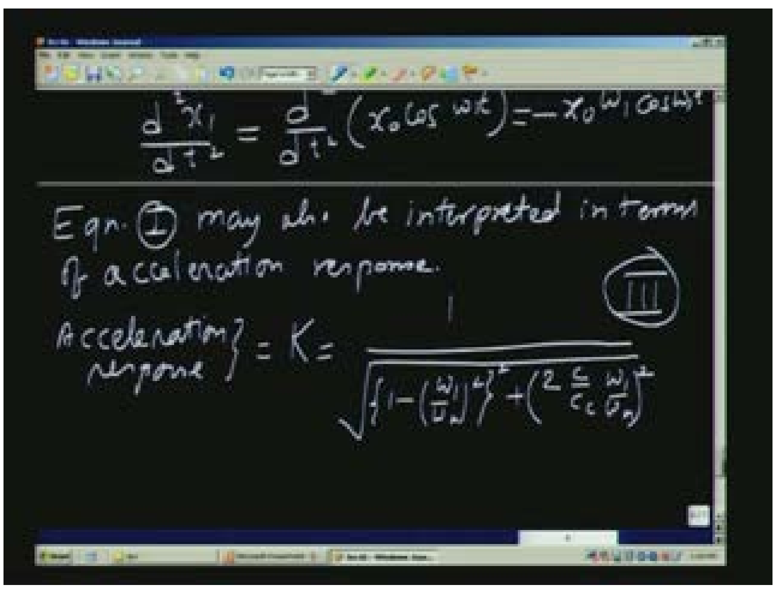
We will discuss the principles of operation of a vibration measuring system then we will subsequently look at the two examples 53 and 54, which will deal with the application of these principles, and then I will discuss one kind of accelerometer called the piezoelectric accelerometer. It is used very commonly in practice. Subsequently, we will look at a laser Doppler Accelerometer which is more recent in origin, and we will look at some of the principles involved in these. So if we go back to the slide, the equations which we have derived in the last lecture were labeled as 1 and 2 here.

(Refer Slide Time: 02:26)



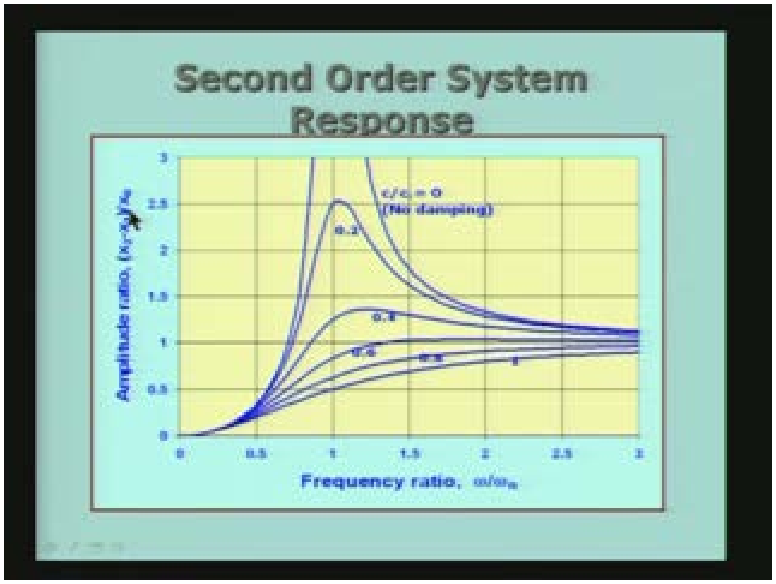
So, the amplitude response of a second order system which consists of a mass has a movement against the spring and there is a damping. The damping can be due to viscous effect, and the response is written in terms of a response to a sinusoidal input. That means that the vibration which is imposed on the system is having a frequency of $\omega_1$ and the natural frequency of the system is $\omega_n$. So what we are talking about is the response of the system to the input, input has amplitude $x_0$ and that is given by expression 1. So it is given by ($\omega_1$ by $\omega_n$) whole square by square root of 1 minus ($\omega_1$ by $\omega_n$) whole square plus there is a 2 into c by $C_c$, this is the ratio of the viscous damping to that in the case of critically damped second order system, multiplied by ($\omega_1$ by $\omega_n$) whole square, and there is also an accompanying phase lag which is given by the tan inverse 2 into c by $C_c$ $\omega_1$ by $\omega_n$ by 1 minus ($\omega_1$ by $\omega_n$) whole square. Similarly, we can also look at the acceleration response for which we know that the acceleration due to the variation of $x_1$ in a sinusoidal, or in a periodic fashion is given by this formula, minus $x_0$ $\omega_1$ square (cos $\omega_1$t).So equation 1 can be modified to give the acceleration response as given by,

(Refer Slide Time: 04:19)




K is equal to 1 by square root of 1 minus ($\omega_1$ by $\omega_n$) whole square plus 2 into c by $C_c$ ($\omega_1$ by $\omega_n$) whole square. So these are three equations which we have derived and these are the basic things which are going to govern the behavior of accelerometer. We will look at some of these, by looking at the response plotted in a particular way. First we will look at amplitude ratio, which is given by $x_2$ minus $x_1$ by $x_0$ plotted as a function of frequency ratio omega by omega$_n$, and what we have done is, we have given the response as a function for different values of the damping parameter.

(Refer Slide Time: 05:02)

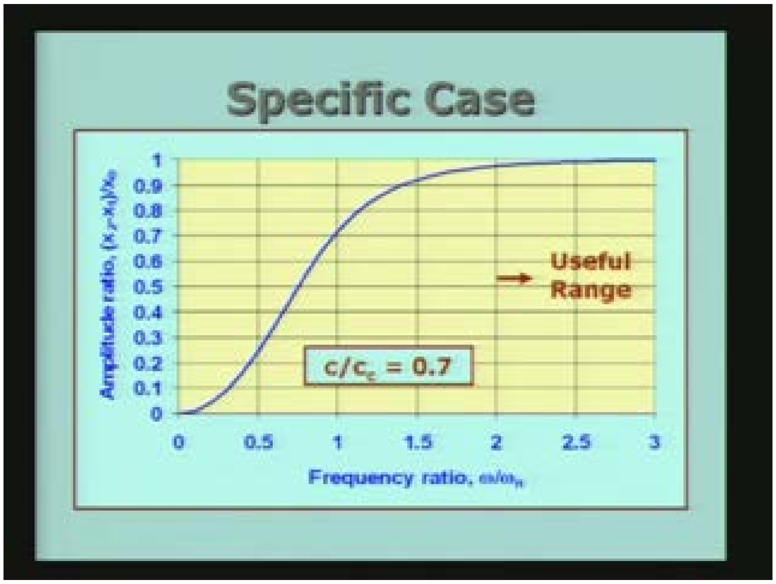



So, if you do not have any damping at all, at omega$_1$ is equal to omega$_n$ that is the input frequency equal to the natural frequency of the system in principle, the amplitude is going to become infinite. Of course, in practice, there is no system with zero damping, and therefore, this is only an idealization. In other words, if the damping is very small, the amplitude can become very large at the condition omega by omega$_n$ is equal to 1 that is what it means. As the damping ratio is increased, so I have plotted for 0.2, 0.4, 0.6, 0.8 and 1 you see that the amplitude becomes more manageable.

For example, if the value of the damping ratio c by $C_c$ is greater than 0.6, the response is never greater than 1, it is always below 1. That means that the amplitude ratio is reduced to a value less than or equal to 1. You will also notice that, the amplitude ratio varies quite significantly, for omega lesser than omega$_n$ and actually it grows in size, and for large values of omega by omega$_n$ that is for large values of the input frequency compared to the natural frequency of the system, the value more or less tends to 1, at least it is close to 1 here. Here, if you take the look at a particular case with c by $C_c$

is equal to 0.7, the damping ratio is equal to 0.7, you see that the amplitude ratio if the value is more than about 2 if omega$_n$ is greater than about 2, you see that it is very close to 1.

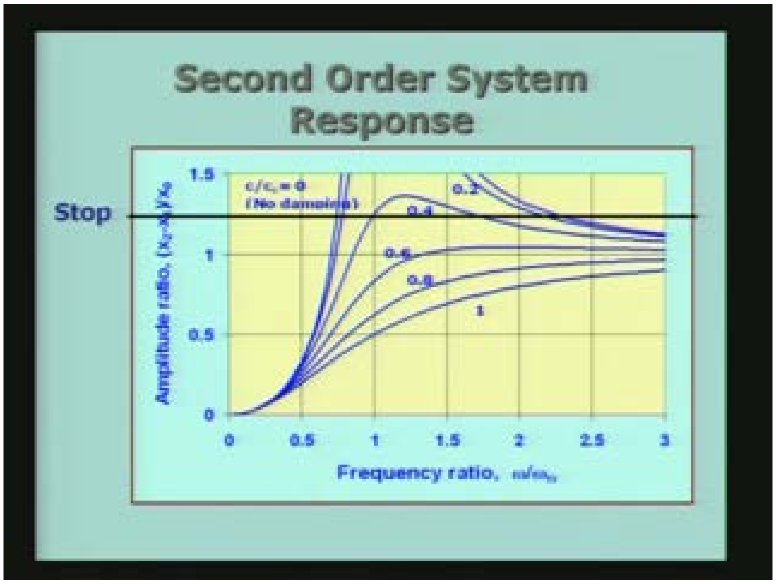
(Refer Slide Time: 07:12)




So we say that any value omega by omega$_n$ greater than 2 is a useful range for measurement of the amplitude of the vibration. So this is for the measurement of amplitude of the vibration. The second point, I would like to mention here is that, if you remember, in the previous lecture we indicated that we can limit the amplitude of the vibrometer or vibration sensing device by putting stops. So, if I put a stop such that it is not more than 1.25, then for different damping ratios, you see the kind of behavior you are going to get.

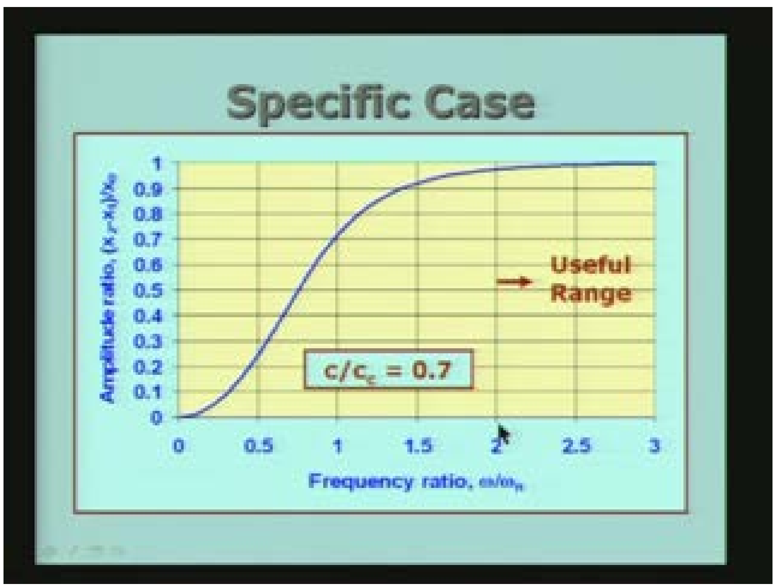
For example, if I do the experiment with c by $C_c$ is equal to 0.4, it will go up to this and when it hits the stop, it is not going to change. So it is going to stay constant, and then it is going to come back like that. So one way of preventing the run away of the transducer is to put stops so that the amplitude is limited to a certain value, in this case I have taken 1.25 you can put even less than 1 or more than 1 as the case may be and that will prevent the vibration sensing device from executing too large an amplitude and probably wrecking the sensor itself.

(Refer Slide Time: 07:51)



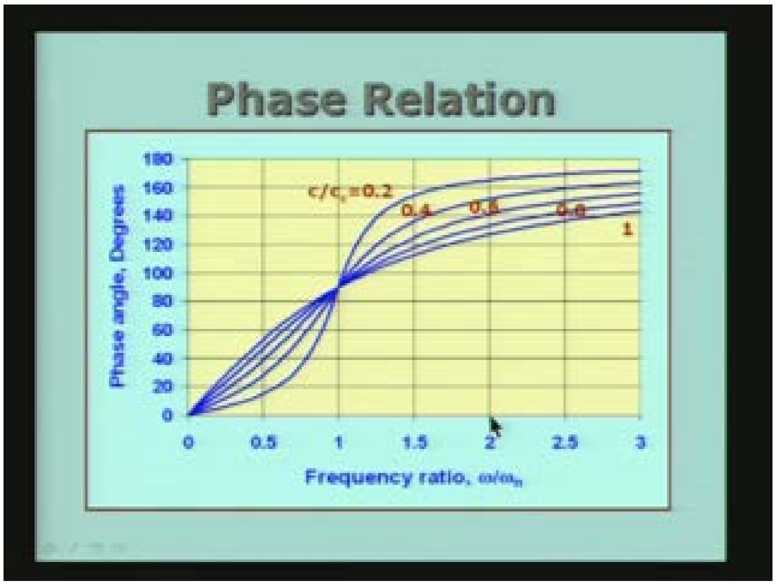

So the thing we notice is that, for measuring the amplitude of the vibration, the useful range of the instrument is such that, you must have the input frequency greater than about two times the natural frequency of the system.

(Refer Slide Time: 09:01)




So you can immediately see that, if we are going to measure the amplitude or vibration of a system varying at a fairly low frequency, then omega$_n$ must be even lower than that. For example, if I have a frequency of 1 Hz, this factor 2 here means that, the natural frequency must be even less than that half of the impressed frequency, therefore your natural frequency must be chosen even smaller than that. The second relation we had was with respect to phase relationship, and the phase angle in degrees is plotted here again with respect to frequency ratio omega by omega$_n$.

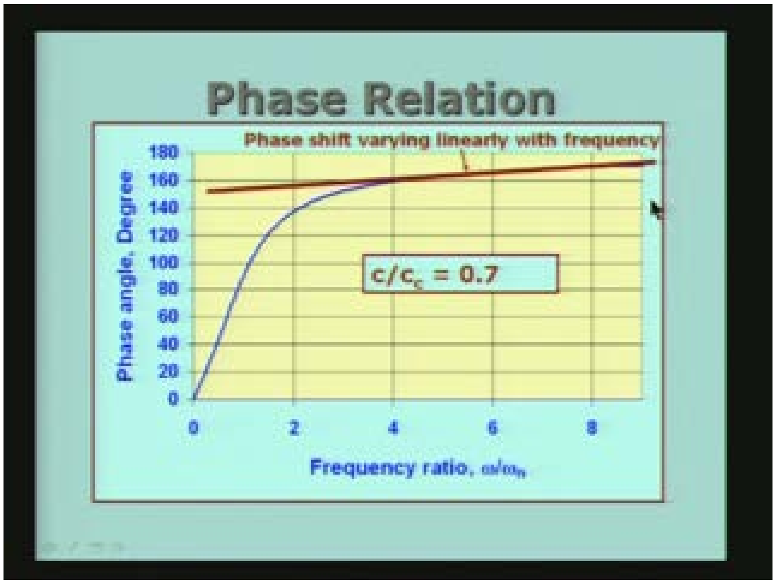
(Refer Slide Time: 10:03)




And you see here that the phase changes from 0 that means that, when the frequency ratio is very small, the movement or the motion of the transducer is in phase with the input disturbance. But as you see when the frequency becomes almost equal to the natural frequency, actually lag is there of 90 degrees there is a phase angle of 90 degrees difference between the input and output.

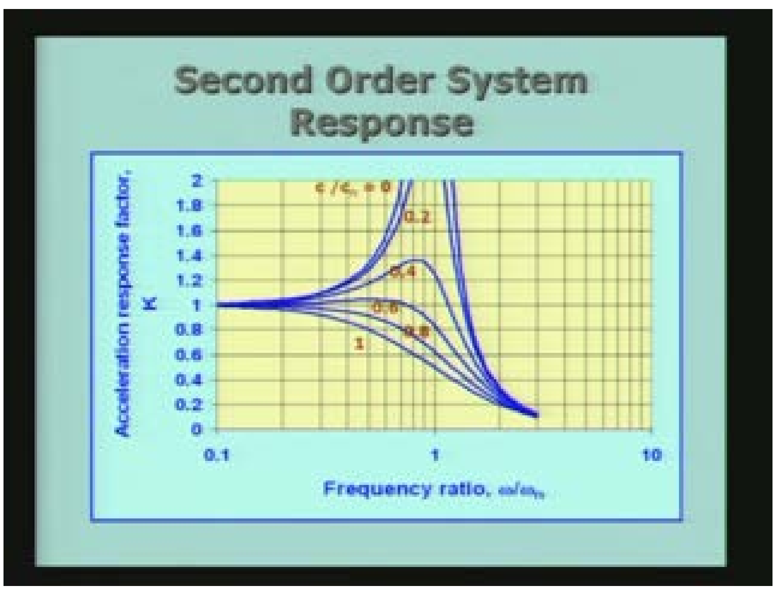
And for omega by omega$_n$ greater than 1, you see that the phase is actually becoming larger phase difference and in the limit as omega by omega$_n$ tends to infinity, the phase difference goes to 180 degrees. The phase relationship is very important. So how does the phase relationship become important? We will be actually taking a look at a particular example, which will make it clear as to what the simplification of this is. For example, if I look at the special case of c by $C_c$ is equal to 0.7, that means that the damping ratio is 0.7 times the critical damping ratio, then you see that the frequency versus phase shows this kind of a change and you see that for a value greater than about 4, there is almost a linear phase shift with respect to frequency.

(Refer Slide Time: 11:18)




So, higher the frequency, higher the phase shift, and the phase shift is linearly varying with respect to frequency. This is the second observation we are going to make. Let us see how these are going to be useful in practice.

(Refer Slide Time: 12:08)

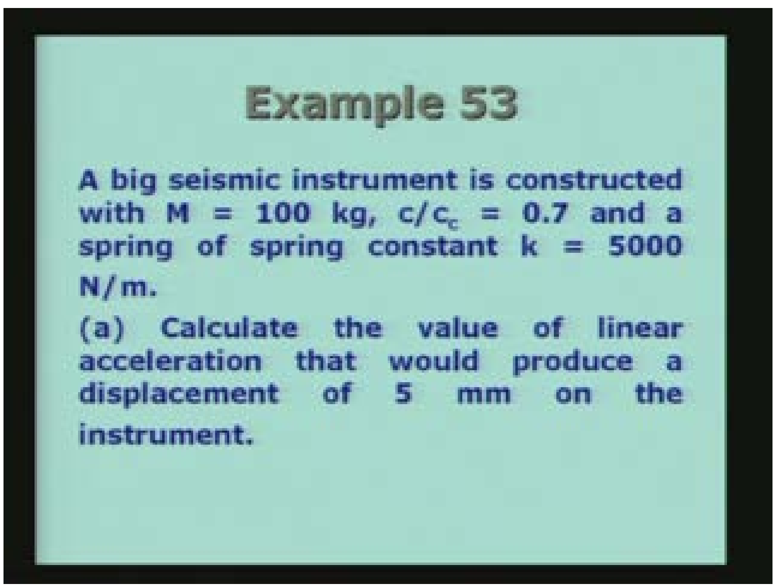



The third one is the acceleration response factor K which is expression 3. Here I have plotted the acceleration response factor K versus the frequency ratio, but I have used a logarithmic scale on the x axis, so that the figure looks slightly different from what it was earlier, and you will see the important thing here. The acceleration response is almost 100%, or close to 1, for omega by omega$_n$ much less than 1. If the impressed frequency is much less than the natural frequency of the system, the acceleration response is much better.

If you remember previously what we discussed the amplitude response is better, and it is exactly the opposite omega by omega$_n$ must be larger than 2 or so what we saw in that case. Therefore, omega much greater than omega$_n$ you have a good amplitude response, and if omega is very small, compared to omega$_n$ you have a better acceleration response. That is, if you are interested in measuring the acceleration due to the vibratory motion we would have to choose the natural frequency of the transducer to be much larger than the actual frequency at which we are going to measure the acceleration.

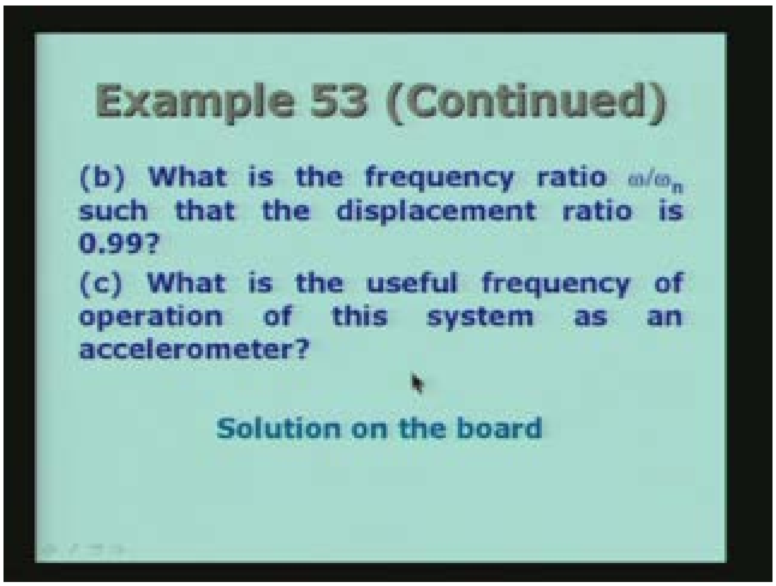
This is the major thing which comes out from this particular theoretical analysis. So with this back ground, let us look at two cases. Here are the details about the example. Let us workout the solution. What we have here is a big seismic instrument. Seismic instrument is used in measuring mostly the earthquake forces generated during earthquake.

(Refer Slide Time: 13:50)



It is constructed with a mass of 100 kg a fairly large mass c by $C_c$ has been adjusted to 0.7 by suitable means, and we have a spring of spring constant is equal to 5000 N by m which is a fairly strong spring with a large spring constant of 5000 N by m. So we want to calculate the linear acceleration that would produce a displacement of 5 mm on the instrument, this is part (a). And part (b) we want to find out what is the frequency ratio omega by $omega_n$ such that the displacement ratio is 0.99.

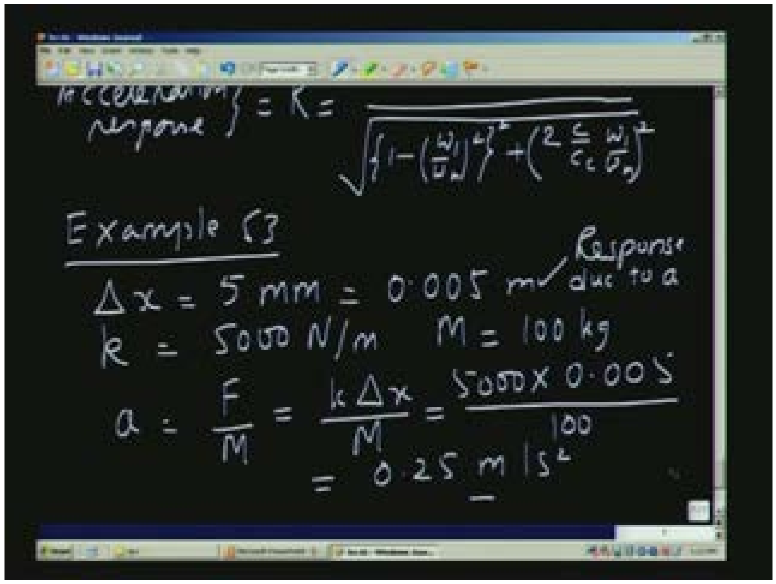
(Refer Slide Time: 14:40)




And thirdly, we would like to find out the useful frequency of operation of this system as an accelerometer. So, three parts of the question are given here. What we will do is, we will try to work it out by calculating the various things. All it requires is the use of the equations which you already derived. Therefore let us just find out how to go about it.

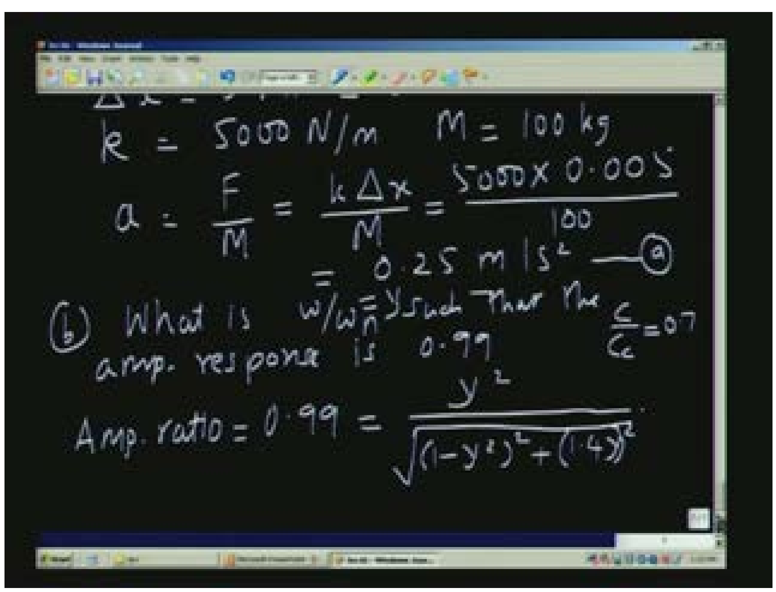
In example 53, the displacement, delta x is given 5 mm is equal to 0.005 m, and we know that the spring constant of the system k is given as 5000 N by m and the mass or the seismic mass, M is 100 kg. So, if the system is subjected to a linear acceleration we would like to find out what would be the response. So this is the response, due to acceleration a. It is a very simple thing to find out. The acceleration is nothing but, mass times acceleration is equal to the force, and force is proportional which is equal to K into delta x therefore, all I have to do is, to obtain the acceleration as a ratio of force to the mass, and the force itself is given by k into delta x by M which is very straight forward.

(Refer Slide Time: 17:25)




This will be 5000 into 0.005 by 100 kg and that gives you a value of 0.25 m by s square. So a linear acceleration of this mass, by 0.25 m by s square will give rise to a displacement of 0.005 or 5 mm, it is a very sluggish system. So this is the part (a) of the answer. For part (b),it is as follows:

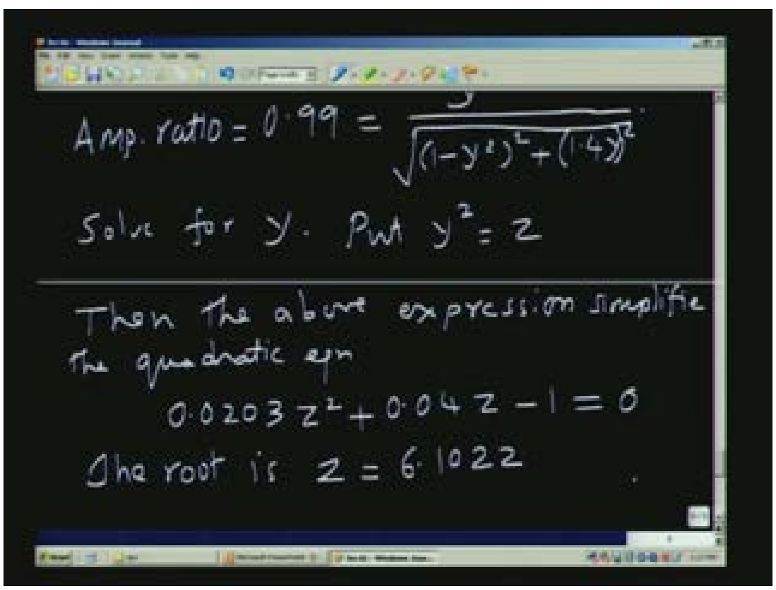
(Refer Slide Time: 20:04)



The reason why I am working out this example because it is going to actually tell us about the principles involved in acceleration measurement or in vibration measurement. What we want to do is to find out the value of omega by omega$_n$ such that the amplitude response is 0.99, it is 99%. Of course, acceleration is linear, that means it is a constant value as in this case. Linear acceleration means the accelerometer is simply accelerated in a particular direction with a constant acceleration. That means there is no periodicity it is a constant value. So when you have a constant value for the acceleration it gives you a constant amplitude output, and it is same as the actual amplitude. Now, once we start varying it sinusoidally, you see that the response becomes less than 1. It will not be equal to 1, but it will be less than 1, and we want to find out when it becomes 0.99 assuming that up to about 0.99, we can use the instrument treating it as a useful range.

Therefore, all I have to do is, to use expression 1 which we had earlier. So I will say amplitude ratio is equal to 0.99, I will consider omega by omega$_n$, I will write it as y, so that it is easier to write down the expression. So, that becomes, y power 2 by square root of (1 minus y square) whole square plus 2 c by $C_c$ and c by $C_c$ is equal to 0.7. So we are going to assume c by $C_c$ is equal to 0.7 so that becomes (1.4 y) whole square. Therefore, this is the equation: y power 2 by square root of (1 minus y square) whole square plus (1.4 y) whole square actually we have to solve for y.

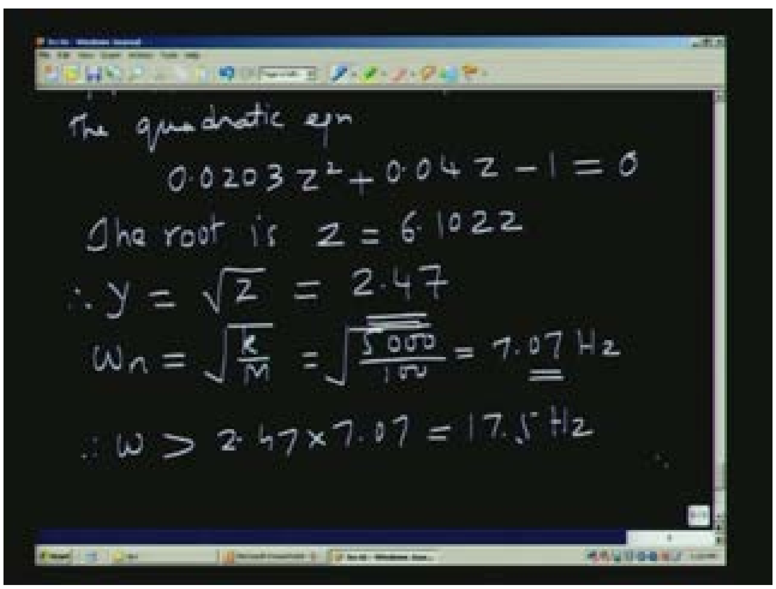
(Refer Slide Time: 21:38)



We can square both the sides and then simplify it. So, if I do that put y square is equal to z, then the above expression will become like this. You can work out and get this result. The above expression simplifies to the following quadratic equation.

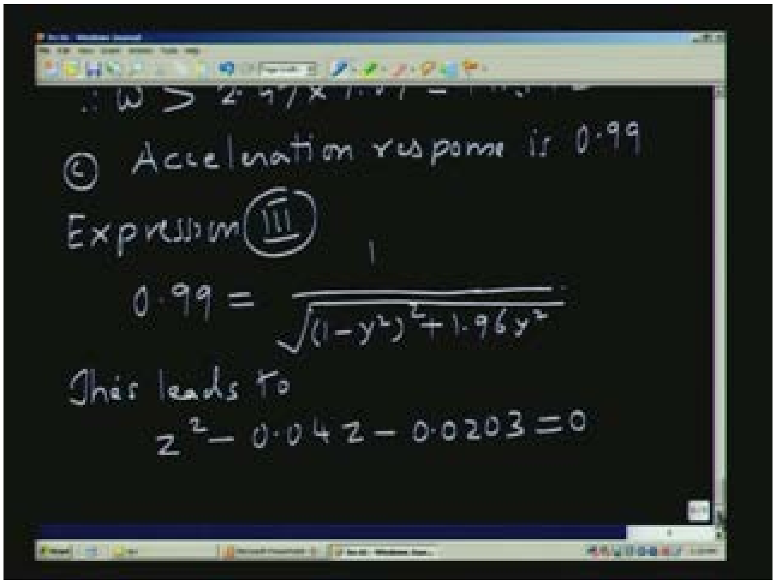
The quadratic equation will be in z. 0.0203 z square plus 0.04 z minus 1 is equal to 0, that is the equation I am going to get. So this has got two roots, one root will not make sense, one root will probably make sense, so the root which makes sense is, z is equal to 6.1022, but z itself is equal to y square so we can immediately say that y is equal to square root of z is equal to square root of 6.1022 and that will give you 2.47. By looking at the graph, above a value of omega by omega$_n$ is equal to 2 is as good as a sensor of amplitude but here we see that, 2.47 is the proper value taken. So if you have a omega by omega$_n$ greater than 2.47 the amplitude will be faithfully recorded. What does it mean? So omega$_n$ is equal to if you calculate for this system we can do that it is simply given by 2pi square root of K by M, is equal to square root of 5000 by 100 kg this is your natural frequency it is about is equal to 7.07 Hz.

(Refer Slide Time: 23:18)




Therefore, omega should be greater than 2.47 into 7.07 is equal to 17.5 Hz. That is the answer to the second part of the question. The third part of the question or part c wants to look at the acceleration response. Again I am looking at the acceleration response being equal to 0.9 as the limiting value for the utility of the sensor.

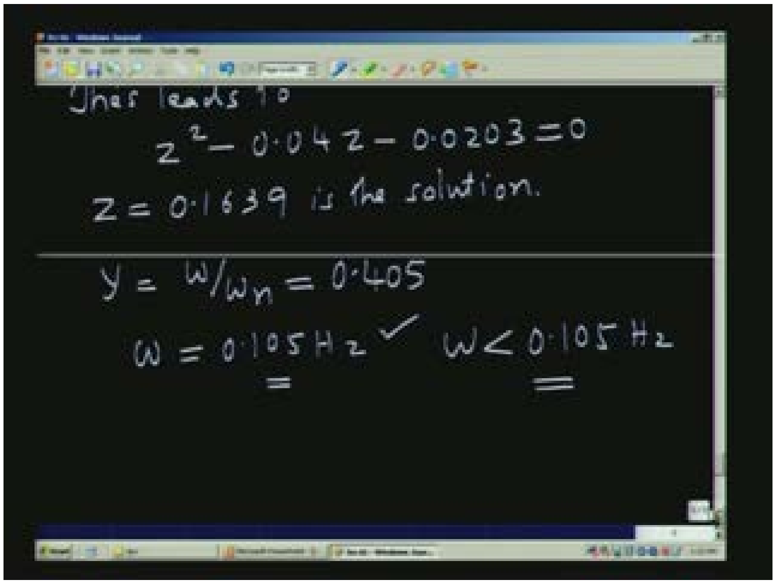
(Refer Slide Time: 24:48)




So again I go back to the expression 3, which was given earlier. This is the starting point, so again I will put it equal to 0.99. So in this case again, I use the same symbol y is equal to omega by omega$_n$ this becomes this expression (1 minus y square) whole square plus 1.96 y square where 1.96 is nothing but that (1.4) whole square. So again I will be able to simplify this expression, and we should be able to get the following equation: z square minus 0.04z minus 0.0203 is equal to 0, so you see that we have a different equation this time.

We can actually compare this equation with what we had earlier. In the earlier equation, 0.0203 was here and 0.04 and 1 and you see that the numbers have somewhat interchanged, 0.0203 has gone to this place and the coefficient has become 1 for z. This equation has a solution given by 0.1639. That means that is small, that is exactly what we mentioned earlier. That for small values of omega by omega$_n$, the response to acceleration is better and for large values of omega by omega$_n$, the response to amplitude is better. So it is corroborated by this particular discussion. So z is equal to 0.1639, and this corresponds to y is equal to omega by omega$_n$ is equal to square root of 0.405.

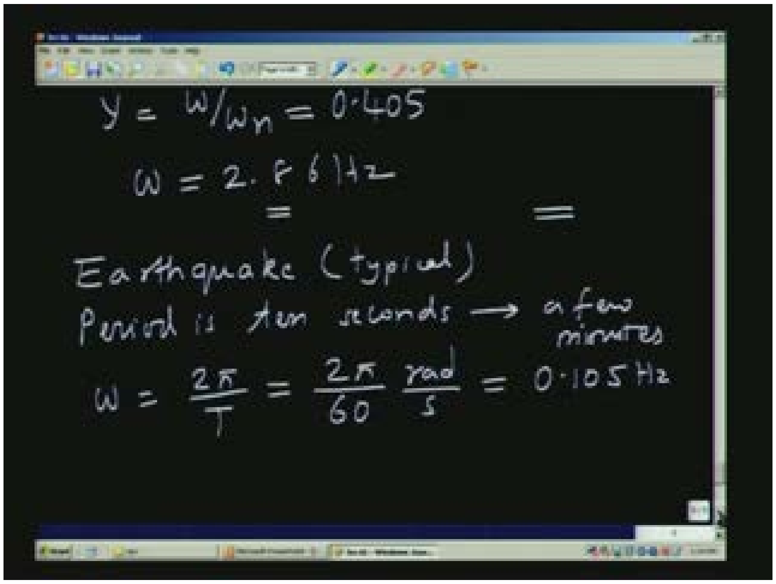
(Refer Slide Time: 26:18)



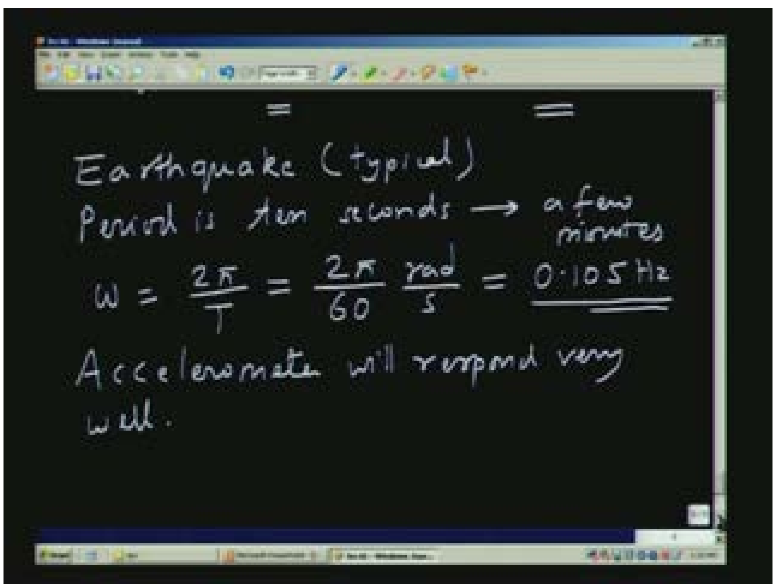

And if you multiply by the value of frequency if you remember what we did earlier, $\text{omega}_n$ we have already found out so we can find out the value of omega, this comes to about 0.105 Hz, may be, we have made a small mistake let me go back and omega will come out to be 0.105 Hz, which is the value of the omega below which so omega should be less than 0.105 Hz. So very small omegas only it will respond to. So if I look at a typical earthquake, the period of the waves is something like tenths of seconds is ten seconds to about a few minutes.

The waves generated by the earthquake are periods in this particular range, ten seconds to about a few minutes. And if you calculate the corresponding frequency, 2 pi by the period so if I take 1 minute it becomes sixty seconds so this will be so many radians per second, and that comes to about 0.105 Hz. **The above result is incorrect**. This will be 2.86 Hz. And here that if you have 1 minute as the period of the earthquake generated wave you get a very fairly low input frequency, and the accelerometer will certainly respond to this very well.

(Refer Slide Time: 29:00)




(Refer Slide Time: 29:56)

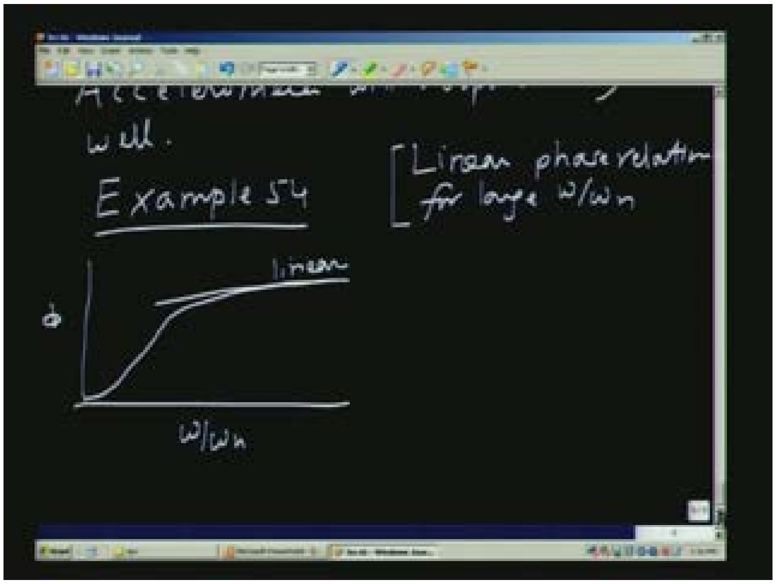



So an accelerometer must have a large mass, and a large spring constant as in this case, and it must have a natural frequency which is much larger than the frequency which is going to be measured, and that is what we have seen in this example. Let us look at another example.

We already discussed about the linearity of the phase difference, the linear phase relationship for large omega by omega$_n$. We had something like this,

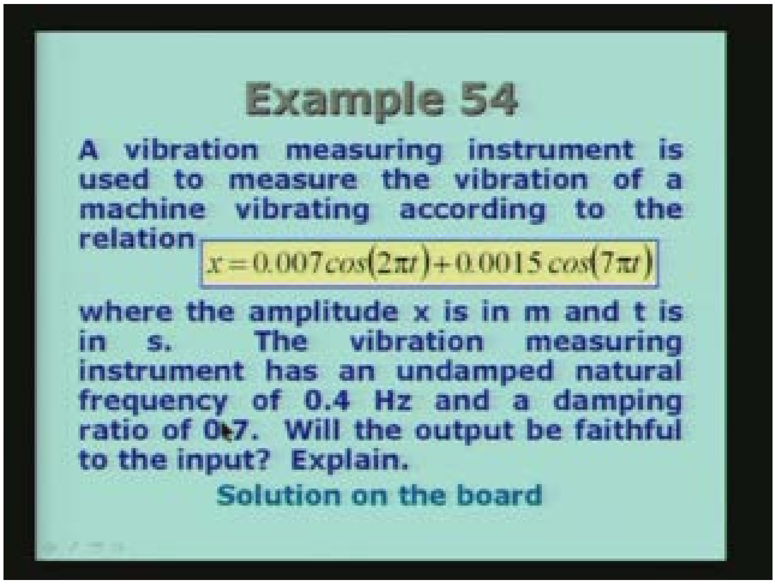
this is the phase versus omega by $omega_n$ it looked like this, it was almost linear here. So what is the consequence of this?

(Refer Slide Time: 30:46)




Suppose we have the following problem; the vibration measuring instrument is being used to measure the vibration of a machine, vibrating according to the relationship which is given, x is made up of two components, that means that the input vibration is made up of two components, one given by 0.007 cos (2pi t) where it is all in meters. So 0.007 means 7 mm is the amplitude of this particular component and cos (2pi t) is the first wave or the first component, any periodic wave can be made up of a set of sinusoidal or co-sinusoidal by using the Fourier series concept. This particular wave is made only of two components with significant energy and others are not important. So the second component is 0.0015 is 1.5 mm amplitude but with a higher frequency of cos (7pi t) so I have cos (2pi t) so two different frequency components are present in the input.

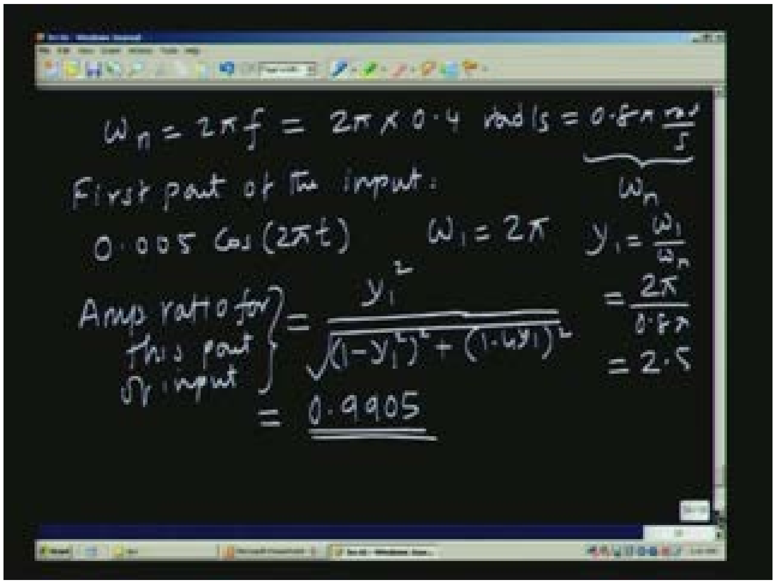
(Refer Slide Time: 30:54)




The vibration instrument has an undamped natural frequency of 0.4 Hz and a damping ratio equal to 0.7, so we would like to find out whether the response of the vibration measuring instrument will be faithful to the input. That means two things we can say. If it is faithful the amplitude will be more or less the same as the amplitudes which are given here, and secondly both these waves must be combined without any distortion. So we will look at the consequence of this particular wave.

We will look at the consequence in terms of the linear phase relationship for large omega by omega$_n$. What we can do is we can treat the input as two separate components, and find out what is the response of the instrument to these two components and then we will add the two responses that should be the response of the instrument. So the vibration of the instrument is given by, omega$_n$ is equal to 2pi (0.4 Hz) the frequency. This will be in so many radians per second or it is equal to 0.8pi. Therefore the first part of the input is given by 0.005 cos (2pi t) so omega$_1$ is equal to 2pi and this is omega$_n$. So all I have to do is, substitute these values into the expression or amplitude ratio which we also had in the previous example.

(Refer Slide Time: 36:02)

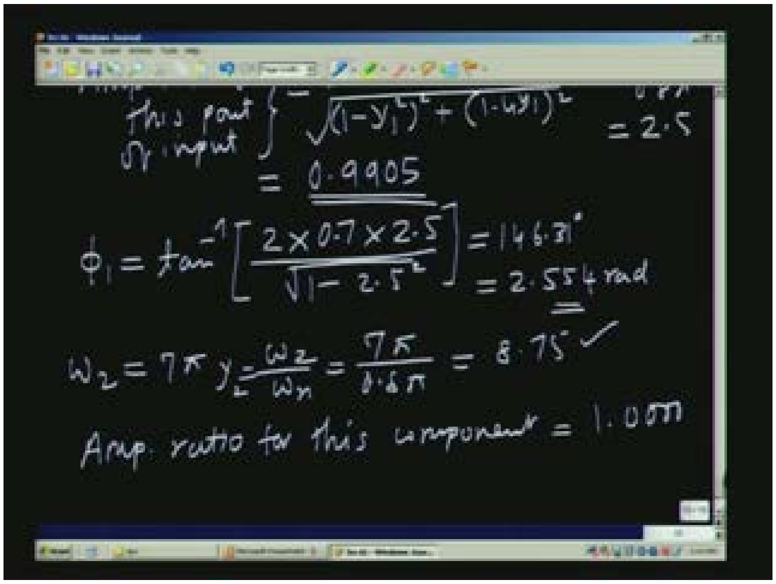



So amplitude ratio for this part of input all I have to do is to substitute omega by omega$_n$ into that formula so it will be $y_1$ square by square root of (1 minus $y_1$ square) whole square plus 1.4 $y_1$ square. So $y_1$ is nothing but omega$_1$ by omega$_n$ is equal to 2pi by .08pi that ratio is 2.5. So I have to put $y_1$ is equal to 2.5 here and work out the value of this and it comes to 0.9905 so it is better than 99%.I can easily calculate the amplitude which is 0.9905 into 0.005 so this is the amplitude response. Let us also look at the phase response.

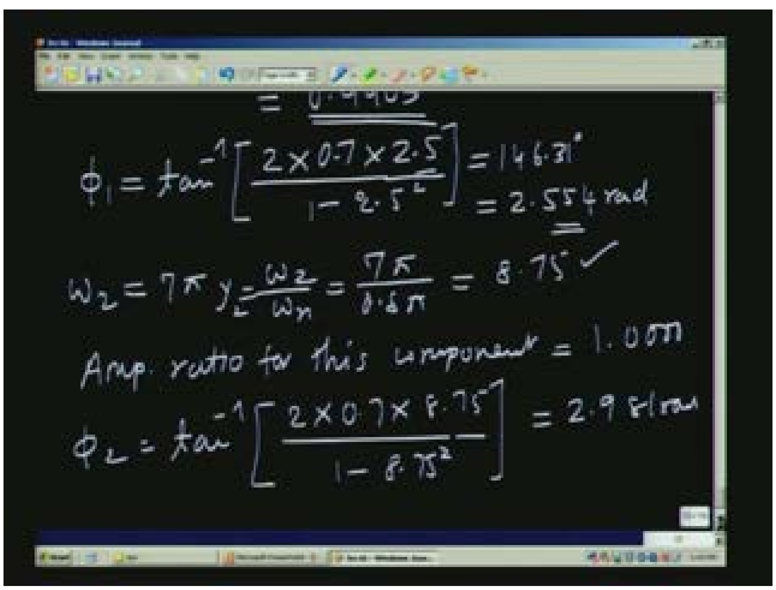
Phase is given by phi$_1$ is equal to $\tan^{-1}$[ (2 into 0.7 into 2.5) by (square root of 1 minus $2.5^2$], is equal to 146.31 degree or in terms of radians, it gives you 2.554 radians. We have worked out for the first part of the input. The second part of the input, has a higher frequency, and we know omega$_2$ is equal to 7pi and therefore if you take omega$_2$ by omega$_n$ we call it $y_2$ which is 7pi by 0.8pi is equal to 8.75. So, for this value of $y_2$, I can again work out and we will see that, the amplitude response is almost unity.

So, amplitude ratio for this component is equal to 1 with up to four decimal places, and we will also find out what is the phase for this case phi$_2$ is again the same formula, but using the 2 into 0.7 into 8.75 by …….. and in fact, I should have put a negative value here, but because of the phase lag, I am just showing the magnitude otherwise it is somewhat confusing if we put that value also. So the value is 2 into 0.7 into 8.75 by 1 minus 8.75 square is equal to 2.981 radians the value of phi which you get.

(Refer Slide Time: 37:45)

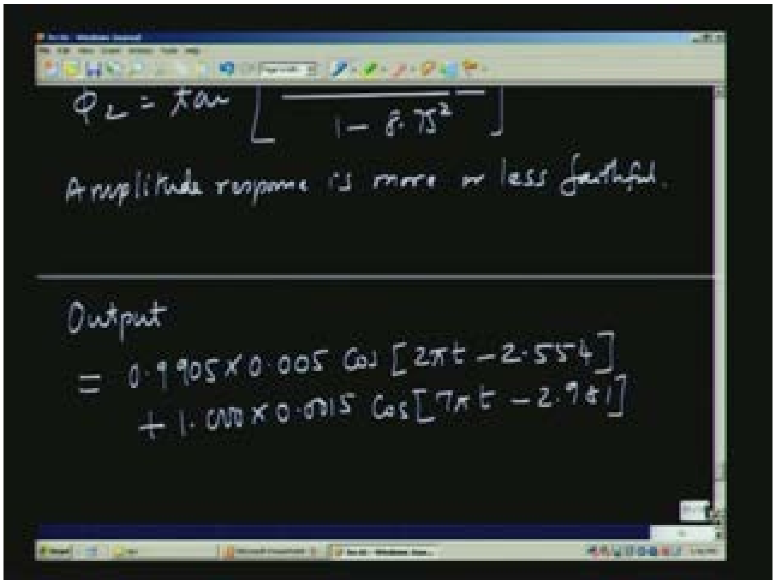



(Refer Slide Time: 39:16)



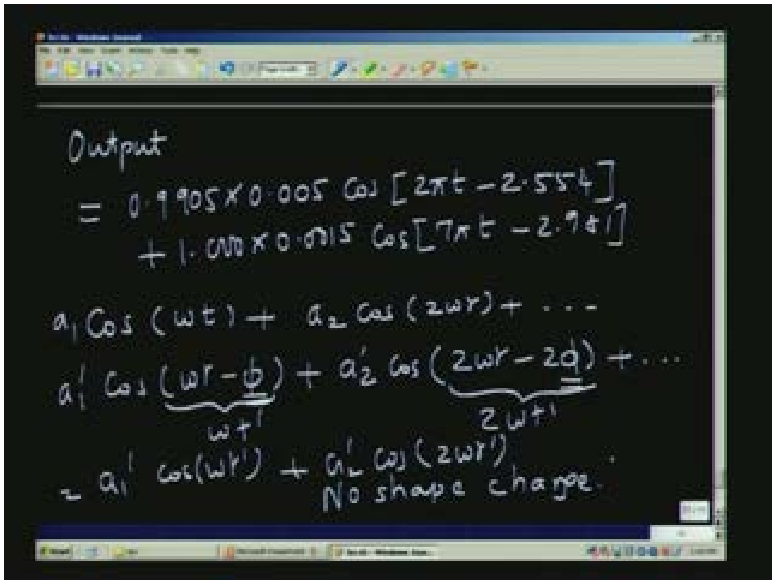

So we have the amplitude ratio for the first case which is 0.9905, for the second wave is 1, therefore, there is a very small amount of distortion, because the first part is going to be one percent less than the input amplitude. To that extent there is a small impact on the value. So amplitude wise amplitude response is more or less faithful. That means very small change in the first component the second component is more or less preserved.

(Refer Slide Time: 40:49)




However, if I look at the phase relationship, the output phase of the first component is different from the output phase of the second component. Now, we can write the output in the following form; it will be 0.9905 into 0.005 cosine (2pi into t) this is the input wave, there is a phase lag of 2.554 of the first component plus 1 into 0.0015 into cosine [7(pi t) minus 2.981]. If the phase response should be preserved, if the shape of the output must follow the shape of the input wave the following should hold. Suppose, I have the input $a_1$ cos (omega t) plus $a_2$ cos (2omega t) etc.

(Refer Slide Time: 43:18)

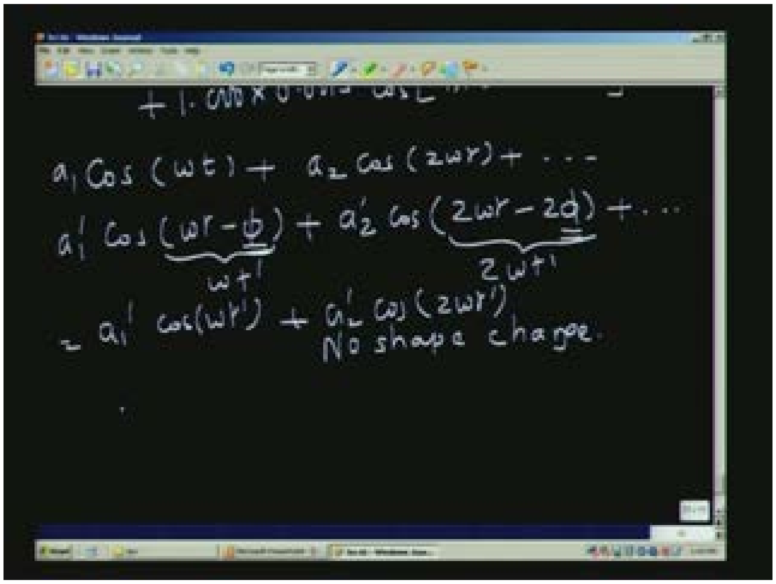



Suppose the response is the following this becomes $a_1^1$cos(omega t minus phi), suppose it becomes $a_2^1$cos(2 omega t minus 2 phi) that means this is phi and this is 2phi so the phase lag for the first component the phase lag for the second component is just twice that value. This means it is linearly increasing. Then if I put (omega t minus phi) is equal to (omega $t^1$) this becomes, (2 omega $t^1$) and therefore this becomes, [($a_1^1$ cos(omega $t^1$) plus $a_2^1$ cos(2 omega $t^1$)] so the shape is preserved because the two are the same.

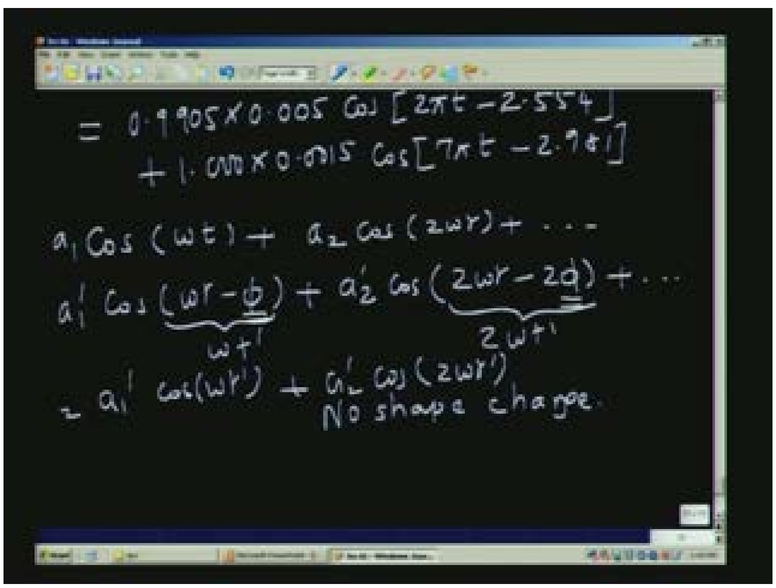
If you look at these two the only difference is there is a small change in the amplitude. Of course, if I choose the omega by omega$_n$ large enough then this $a_1^1$ will be very close to $a_1$, $a_2^1$ will be very close to $a_2$, and you will also see that the phase is changing by exactly the same ratio, as the frequency of these two components. So omega t and omega t will become 2 omega t will become minus 2 phi so this is an ideal case, there is no shape change and no distortion.

That means we can say that the response is faithful. Now what I would like to do is, to find out whether what we have got here for these two components is either in agreement with this or not. Let us look at (2pi t) minus 2.554, this is 7(pi t) minus 2.981. So what is the ratio of the frequencies, between these two this is 2.775 it is a factor of 3.5.

(Refer Slide Time: 43:38)

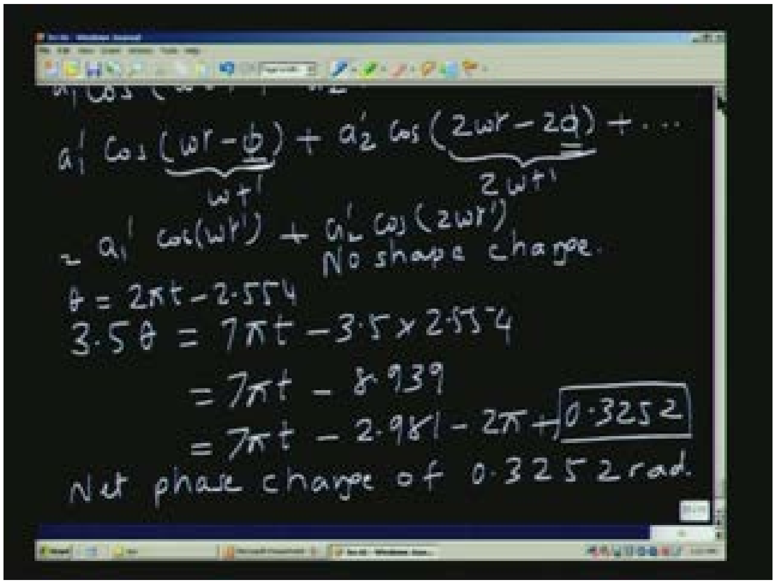



(Refer Slide Time: 44:11)



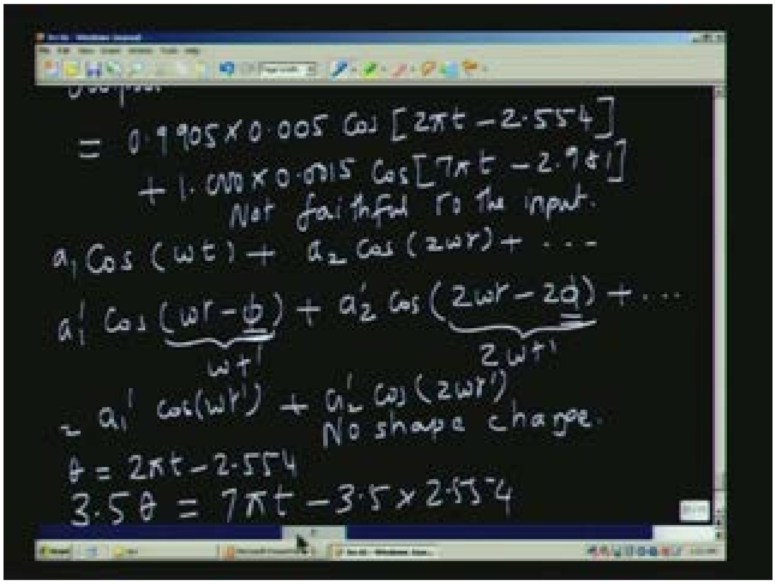

So, if this 2.554 changes by a factor of the same factor then we would have no change of shape. Of course 2.554 and this is 2.981, I can always add a certain number of pi to that. That means suppose I take 3.5 theta, where theta is equal to omega t minus 5 that means (2pi t) minus 2.554. If I take as omega as theta like t prime instead of that omega t is equal to 3.5 theta is what I am writing because this will become three times this portion.

(Refer Slide Time: 46:04)



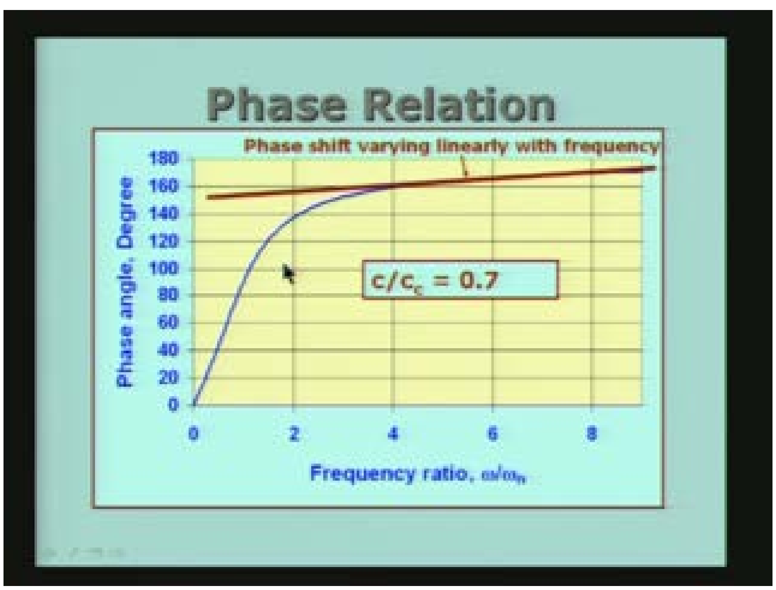

So omega $t^1$ will be here this will become 3.5(omega $t^1$). So 3.5 theta will be nothing but (7pi t) because that will give you (7pi t) for the next component minus 3.5 into 2.554 is the value, this will be (7pi t) minus 8.939, so I can write it as (7pi t) minus 2.981 minus 2pi plus 0.3252. So I am rewriting in this particular form. So, if certain multiples of pi are added here it is not going to change the value because every time it comes back to the same value. Therefore essentially what is happening is there is a net phase change of 0.3252 radians between the two components. Therefore we can conclude in this example that this is not faithful to the input. Now I will go back to the response curve, and immediately you will see that it is in agreement with what we have already seen.

(Refer Slide Time: 47:29)



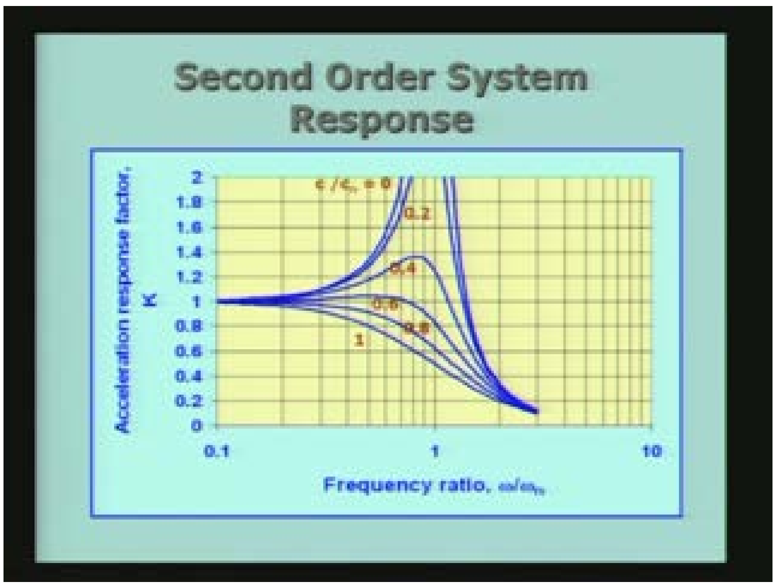
So, if I look at the response curve there is a factor of 3.5 and omega by omega$_n$ is equal to 2.5 which is somewhere here. And the second one is somewhere here, therefore you see that now it is quite departing from that, this is the difference we are talking about. If I had a case, where it was 4 and let us have 3.5 into 4 is equal to 13.5 you will see that it would have followed faithfully because it is in the linear part of the curve.

(Refer Slide Time: 46:29)



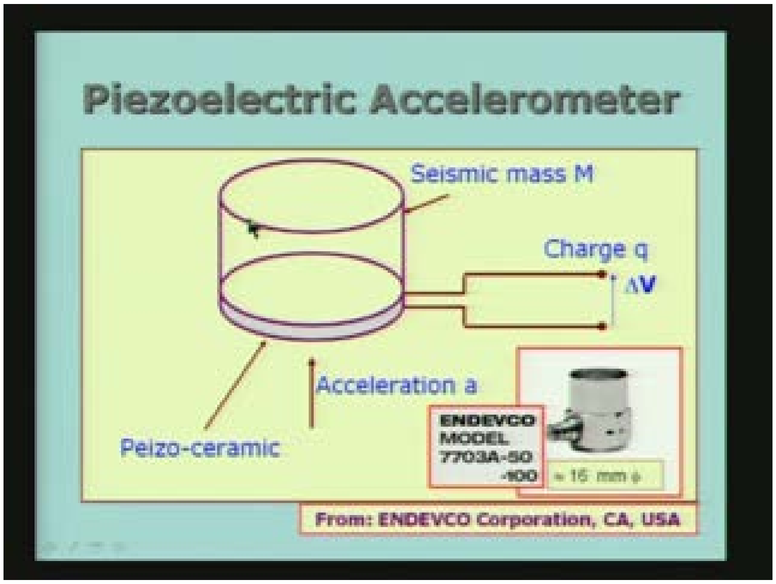
So in this case, because the first point is actually in the non linear part of the curve here there is a net phase difference, this is the phase difference we are talking about that will correspond to 0.3252 radians, it will be a plus 0.3252 radians in the present case. So you see that the two things which we should do for vibration response that is amplitude response is to have omega by $omega_n$ large enough so that the amplitude is more or less equal to 1.

(Refer Slide Time: 47:26)




Secondly, we should choose that part of the curve, where the phase difference between the input and output is proportional to the frequency. That means, I must be on the linear part of the curve to get a faithful response for the vibration measurement system. For the acceleration measurement system it should be in this part of the curve so that you have a good faithful response with respect to the acceleration response. Let us look at one more vibration transducer which is very popular, it is called the piezoelectric accelerometer and the principles are exactly the same as we discussed. It consists of a seismic mass or a mass which is attached to the piezoceramic material usually quartz, and this is a thin layer of quartz material. And if the quartz material is subjected to acceleration it develops a charge across that.

(Refer Slide Time: 48:21)

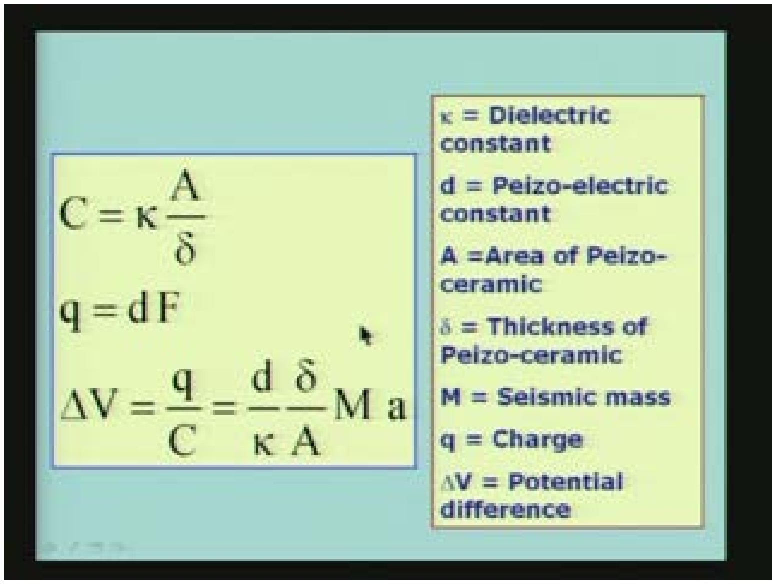



So there is a charge developed across the terminals. The terminal is at the top another terminal at the bottom and you subject this piezoceramic material to a compression, or stress due to acceleration a, then it will immediately give you a charge and this charge will be appearing as a voltage. Actually you can look at the following way. This piezoceramic is like a capacitance, the positive charge or negative charge will appear on one electrode and on the other electrode there will be a net charge of the capacitor and therefore this deltaV is proportional to the charge which is generated, and this charge is proportional to the acceleration. Therefore you can see that, we are going to have a nice piezoelectric accelerometer having a good acceleration response.

Again the same principles which we have just now enunciated are going to hold for this case also, omega by $omega_n$ large values is going to give you a good amplitude response and small values are going to give good acceleration response and so on. And a typical shape of this transducer is shown here in the inset which is roughly 16 mm diameter, this is the model made by ENDEVCO and the model reference number is given here.

The ENDEVCO Corporation makes these transducers, and you can see what happens, here you take the electrical output out, inside this you have the seismic mass as well as the piezoelectric ceramic, and this is attached to the body whose vibration we want to study by using a suitable arrangement.

(Refer Slide Time: 50:55)

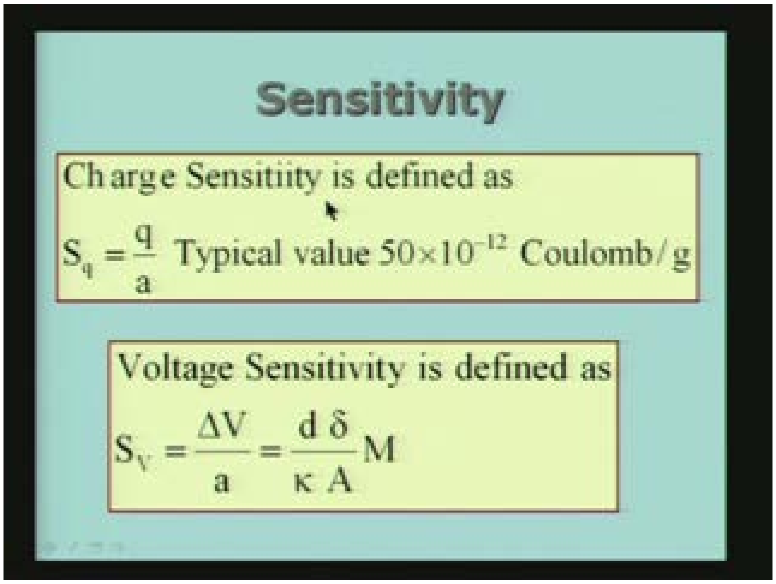



Principle of this sensor: We know that the capacitance is the equal to a constant K into (A by delta), where, A is the cross section of the area and delta is the thickness of the piezoelectric, because it is just a dielectric constant, piezoelectric constant. So q is equal to d into F, where F is the force and d is the dielectric constant for that particular material.

Therefore you can see that, deltaV is equal to q by C, the charge divided by C, q is equal to d into F and F is equal to M into a, so I have taken F outside d into M into a is equal to d F that kappa A by delta has come here and you can see that deltaV is proportional to acceleration and (d delta M) by kappa A is the sensitivity of the instrument. So it is given by the seismic mass, the piezoelectric constant, and the thickness and the area of the material, and the dielectric constant of this thing. So the potential difference is proportional to the acceleration.

So the charge sensitivity is q by a, where q is the charge divided by the acceleration and typical value is about 50 into 10 to the power minus 12 by g where g is the acceleration due to gravity the standard value, and voltage sensitivity is defined as deltaV by a, and that is given by delta by kappa A M. These are the two things which we usually talk about.

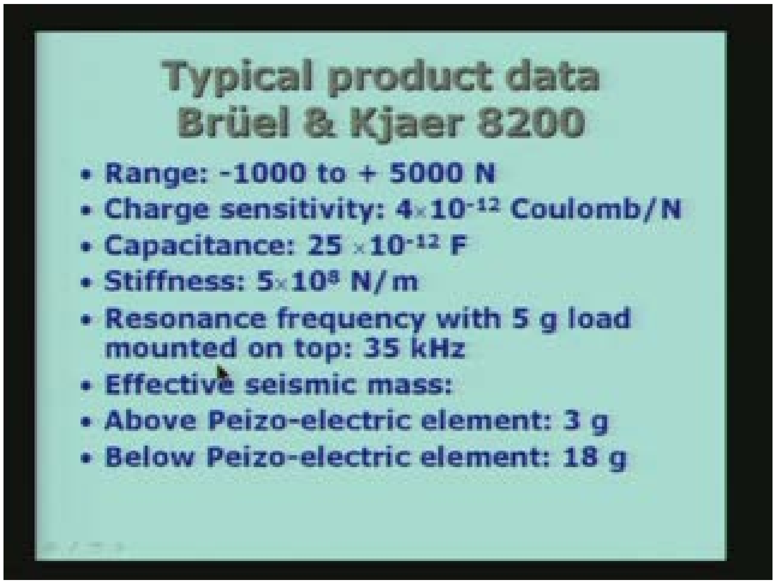
(Refer Slide Time: 52:09)



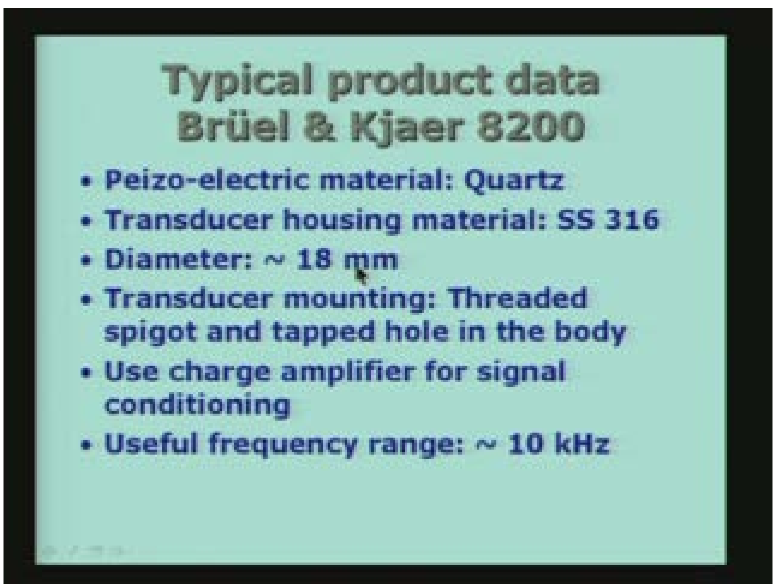

For example, if I take a typical sensor made by Brüel & Kjær 8200 this is the name of the product, it is useful for measuring forces between minus 1000 that it is compressive to tensile 5000 N, charge sensitivity is 4 into 10 power minus 12 coulomb by N, capacitance is about 25 into 10 power minus 12 F and stiffness is 5 into 10 to the power 8 N by m so the material itself has got a stiffness which is the spring built into the system.

Resonance frequency with 5g load mounted on top is 35 kHz so see the advantage of this particular instrument, the frequency is very high, and therefore it is very useful for measuring even high frequency accelerations in this particular case. Effective seismic mass above the piezoelectric element up to 3 g and below 18 g and material of the piezoelectric material is quartz, transducer is housed in a SS 316 stainless steel diameter which is about 18 mm.

(Refer Slide Time: 52:36)

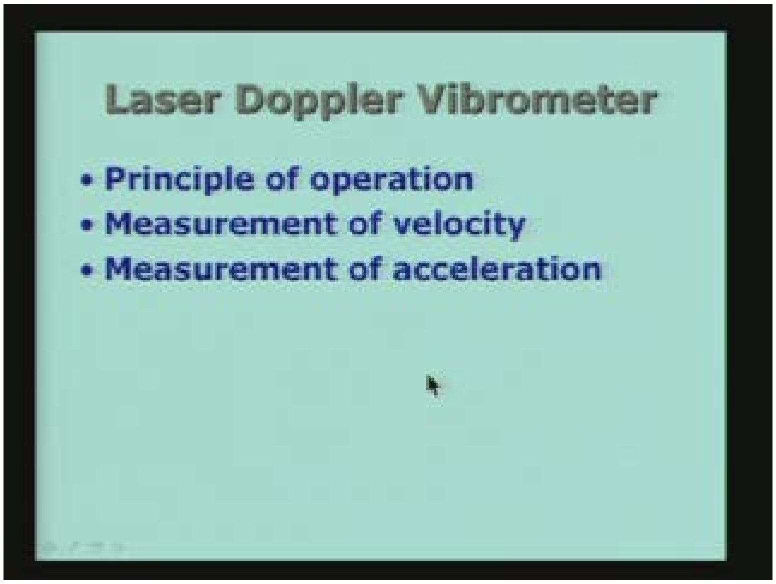



(Refer Slide Time: 53:40)




The transducer mounting is done by threaded spigot and tapped hole in the body. So the use is made of a charge amplifier. Useful frequency range is up to 10 kHz so this is a very small instrument, capable of being used for very high frequencies. Now we are going to do is, look at some of the recent developments in this particular field. If you remember when we were talking about measurement of velocity of gases and so on we were talking about new non invasive method of measurement, we also talked about ultrasonic and then the laser Doppler instruments. So the same principles are also used in the measurement of velocity, measurement of acceleration, measurement of displacement in a vibration system.

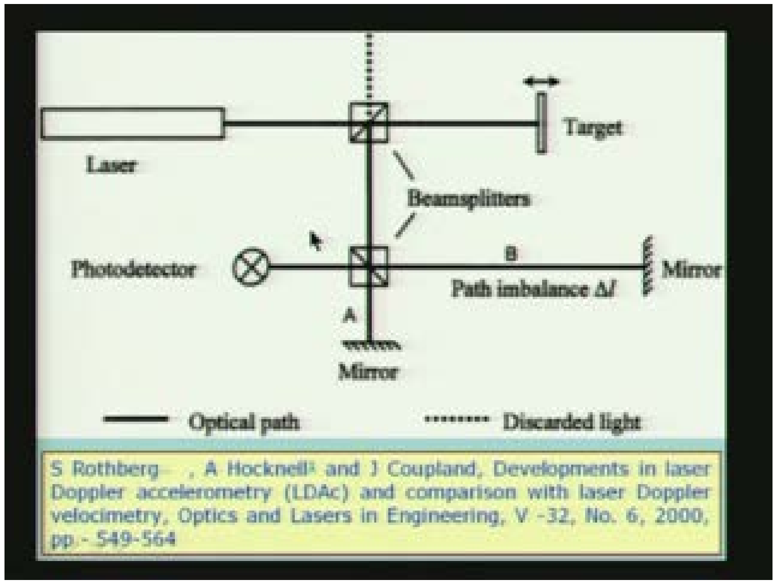
(Refer Slide Time: 54:12)




So the laser Doppler vibrometer is used in the measurement of either velocity or acceleration as the case may be in the case of vibrating system. So we will look at the principle of operation of such. This has been taken from a recent paper S Rothberg and it is from Optics and Lasers in Engineering, it talks about the development of a Doppler accelerometer, and of course it compares with laser Doppler velocimetry. So we have a laser source then we have a beam splitter, part of the beam is sacrificed it goes away from the system, and part of it goes and hits the target. The target is one which is going to be vibrating.

So we want to measure the vibration, or either the amplitude or the velocity, or the acceleration of the target, and this beam is going to hit the target and it is reflected back retro reflected and then the beam splitter will sent part of it at 90 degree to that, and it will be incident on a second beam splitter, and it will be reflected and it will go and fall on a mirror, and we have deliberately a long path which we call as path imbalance deltal.

(Refer Slide Time: 55:06)




So the reflected light from here, and the light which goes through comes back here and these two are going to combine together and are going to be falling on the photo detector. So we have a reference beam, and in fact there are two beams now. Both the beams have undergone some change because of the target. The target is moving and these two are combined at the photo detector and the information we get at the photo detector will have some information about the acceleration of the target. We will look at the principle of operation by actually looking at how these two are going to combine here and what is that information which we are going to get which is going to be proportional to the acceleration of the target. Thank you.