Mechanical Measurements and Metrology
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Module - 4
Lecture - 46
Force Measurement

So this will be lecture 46 in our ongoing set of lectures on Mechanical Measurements. Towards the end of the last lecture, we were actually looking at the measurement of force and we will resume from there. In fact we were discussing different ways of measuring force and one of the ways of measuring is to use a spring element.

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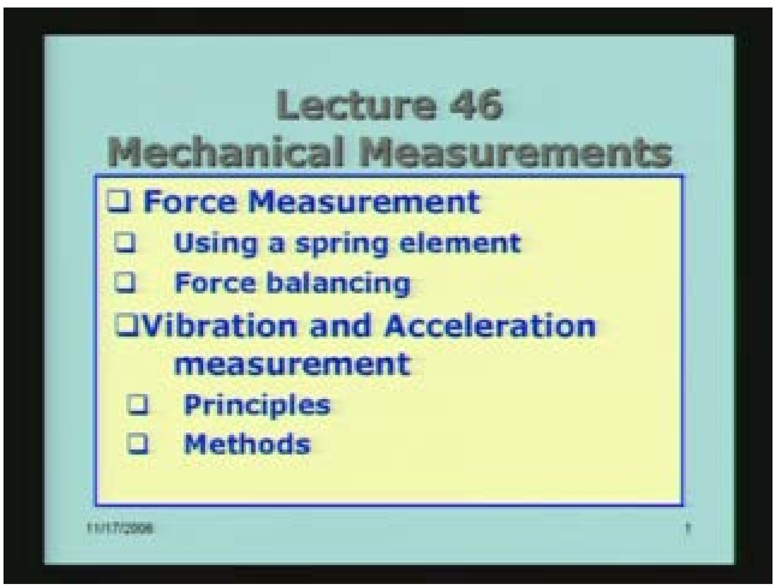

We discussed a few difficult cases, where this force is estimated by looking at a displacement in a spring element. We will also look at force balancing or using balance of forces to measure the force. So these are the two things we are going to look at in this lecture. And then, we will look at the very important application of force measurement to the study of vibration and measurement acceleration. So we will try to set forth the principles of operation of vibration and acceleration measurement devices and then, we will discuss a few methods which can be used for that. However, to start with let us look at force measurement. It simply says that one of the simplest methods is to use a transducer that transforms force into displacement.

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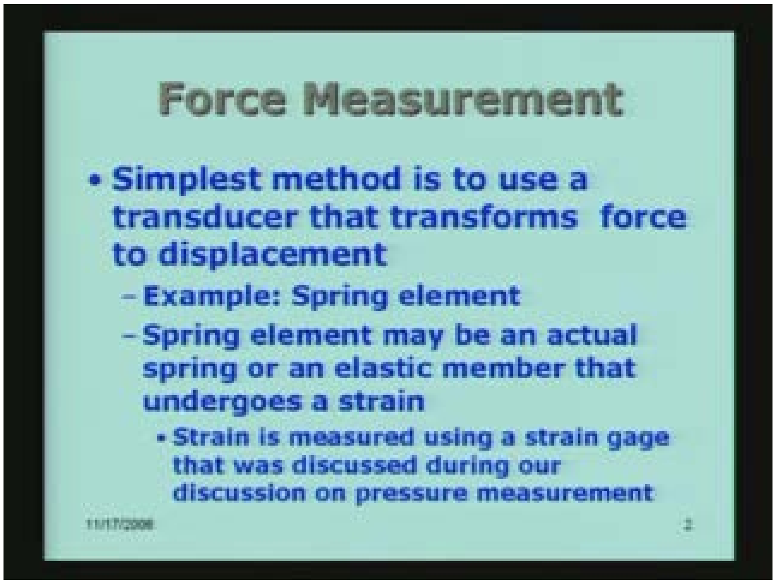

So, displacement is a measured quantity, and the force is the inferred quantity or the estimated quantity. A spring element may actually be a spring like in the spring balance, or it could be an elastic member which undergoes a strain under the action of the force, and the strain is measured using for example, a strain gage. We talked about this when we discussed pressure measurement. Let us look at an example. So I take the case of a cantilever beam which was actually given as an example, in the last lecture. It is made of spring steel whose Young's modulus is 200 GigaPascals. GPa stands for Gigapascals.

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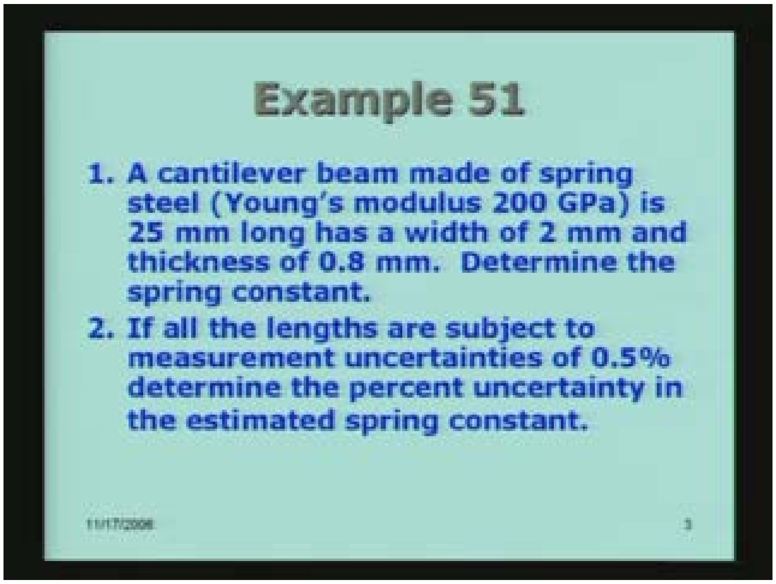

That is, 200 into 10 to the power 9 Pascals. It is 25 mm long and the width of 2 mm. It is in the form of beam and thickness of 0.8 mm. So we want to determine the spring constant for this particular case and the second part of the question says that if all the lengths are subjected to measurement uncertainties of 0.5%, we want to determine the percentage uncertainty in the estimated spring constant.

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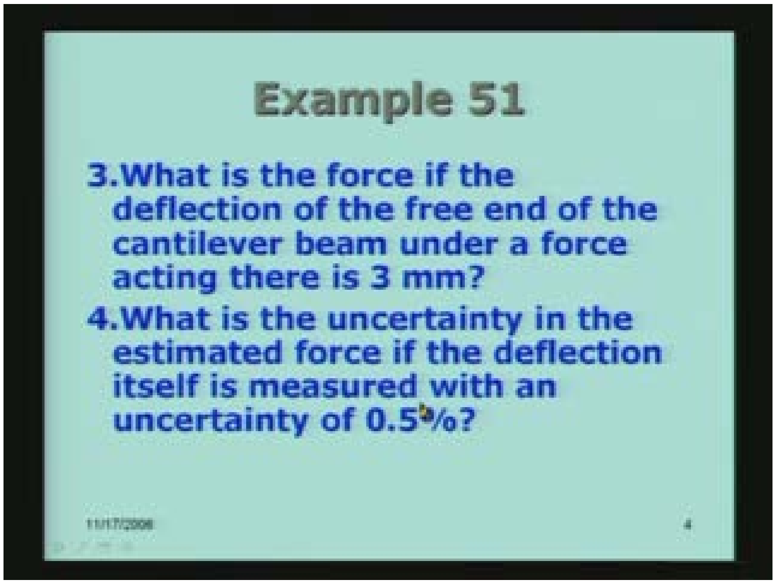

Thirdly, if the displacement which is measured under a force acting at the tip of the cantilever beam is 3 mm, what is the force corresponding to that displacement? We would also like to know, because all the measurements are susceptible to error. What is the uncertainty in the estimated force if the deflection itself is measured with an uncertainty of 0.5%?

So let us look at the following sketch which shows the type of thing which is happening. You have a beam in the form of a small strip, or spring steel, this end is fixed, and we are applying the load at the other extreme, and the consequence of that is that, this end is going to deflect a certain amount in the downward direction so that displacement is actually measured. In fact, in this case we can even use a simple Vernier scale, and measure the deflection easily.

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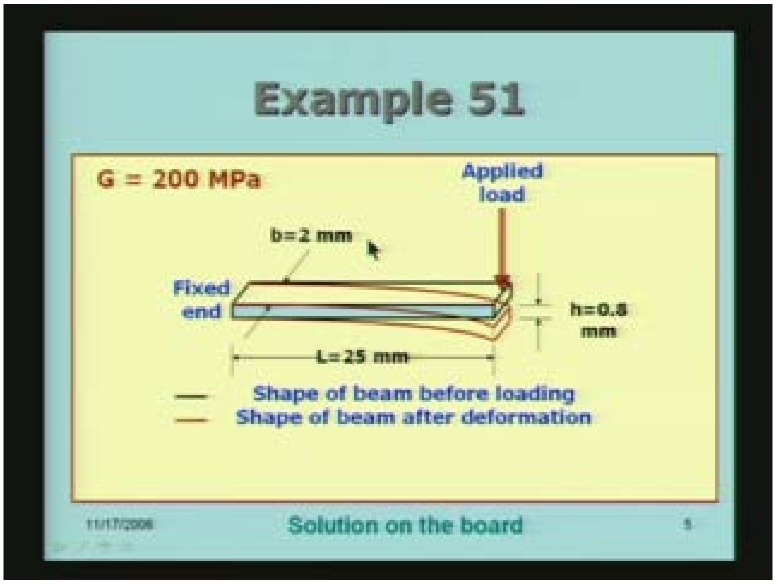

So the breadth of the beam is b is equal to 2 mm, the length is 25 mm and the height h of the beam which is the same as this, is 0.8 mm. So what I have shown here is the shape of the beam before loading, that is the straight thing here, and the shape of the beam after deformation, it has been bent like this. So essentially the element is a beam element, and the beam undergoes bending, and therefore this displacement of the free end is actually given by, a well known formula from Strength of Materials.

So let us look at the solution. We are given the material property as E is equal to 200 Gigapascal which will be 200 into 10 to the power 9 Pascals. Then we have, the width is b is 2 mm. I will convert everything into SI units, so this becomes 0.002m and the thickness of the beam h is 0.8 mm, which will be 0.0008m. The length of the beam is given as 25 mm which is 0.025m. So we can calculate the moment of inertia of the section, moment of inertia of the cross section is given by (b into h cube) by 12, where b is the breadth, h is the thickness and (b into h cube) by 12 is the moment of inertia which is given by 0.002 into 0.0008 cube by 12, and this works out to 8.5333 into 10 to the power minus 14 m^4.

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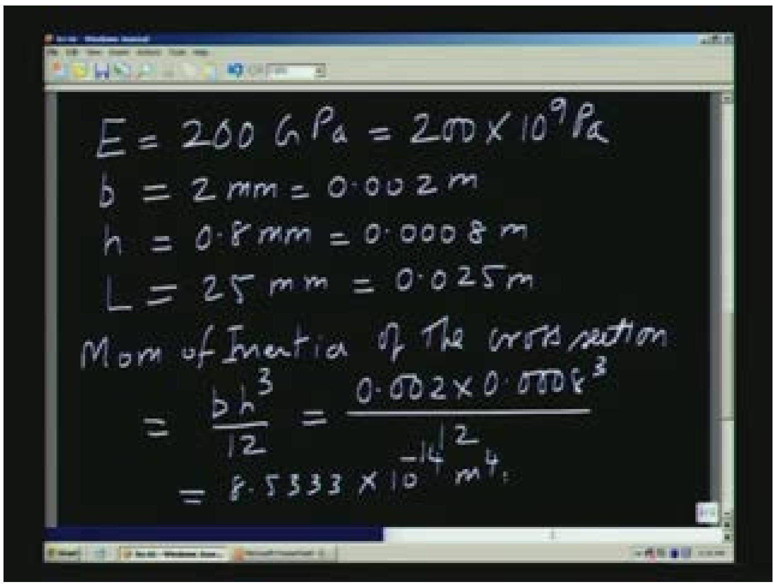

So we have the moment of inertia, because the spring constant is actually given by a simple formula. The spring constant that we are trying to find out is given by, let us use the symbol K is nothing but 3EI by L power 3 and this will be 3 into E is equal to 200, into 10 to the power 9 into I is this quantity 8.5333 into (10 to the power minus 14 by 0.025) whole cube and the unit of K will be N by m.

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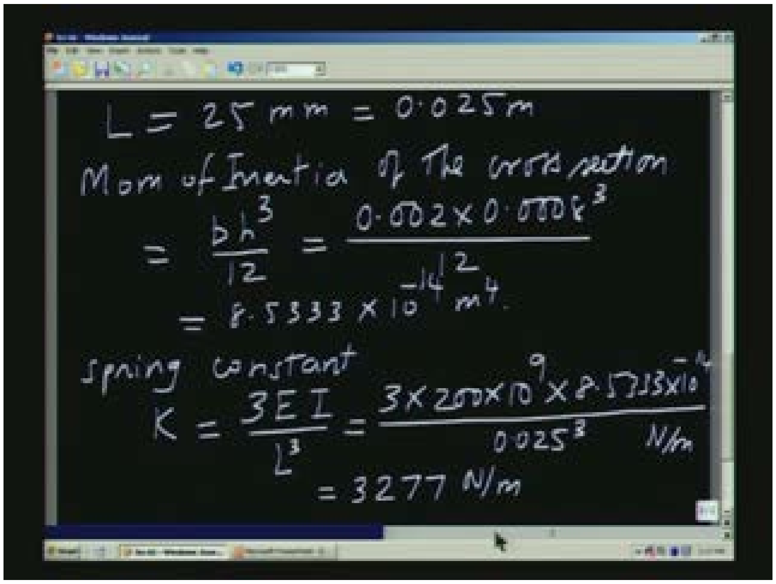

You can actually verify that EI by L power 3 this is Gigapascals or Pascals, this is in m power 4 and the m cube and then you can show that this is equal to N by m which is going to be 3277 N by m. So that is the first quantity of interest to us. We want to calculate the spring constant for this particular application, where the load is applied at the free end of the cantilever beam made of spring steel of the given dimensions.

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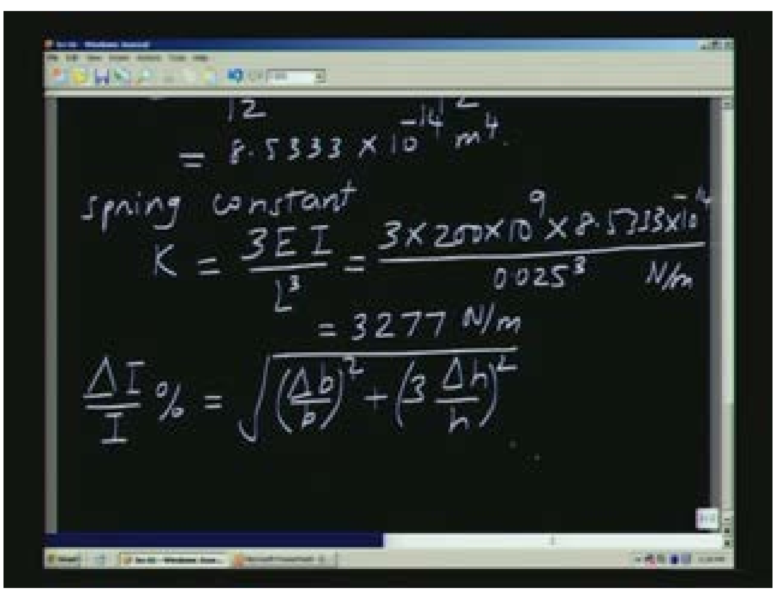

Now we have to calculate the error in the value of K as given by this formula, because all the lengths measured in the problem are susceptible to errors. And because the formulae involve mostly products and products of powers we can use the logarithm differentiation which we are familiar with and, therefore directly I can work with the percentage error. For example, the error in percent of the moment of inertia is given by bd cube by 12 so (delta b by b) in percentage whole squared, this is in percentage plus three times (delta h by h) both in percentages whole squared because of the reason that I is equal to bh cube by 12 is in the product of quantities and raised to the power, therefore, I can use logarithmic differentiation, and obtain this one. So (delta I by I) % is equal to this, and therefore (delta b by b) is simply point 5%.

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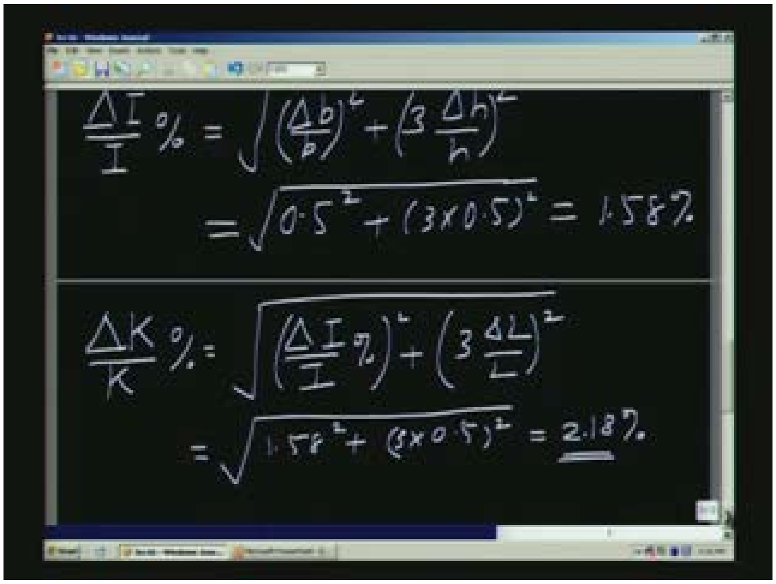

I can directly put this percent here so square root of 0.5 square minus (3 into 0.5) whole square is equal to 1.58%. So the moment of inertia is in error by 1.58%. And if you remember, the formula K is equal to 3EI by L power 3, then I is now in error of 1.58%, L is also in error by the same percentage, therefore now I can use a similar formula, and assume that E itself has no error specified on it, therefore we will assume that the error is only due to measurement of various length. Therefore now it is delta K by K in percentage and this will be simply given by it is 3EI by 3 is simply a constant, E is also now assumed to be having no error so this becomes (delta I by I%) whole square plus L^3, that will give you (3 delta L by L) whole square that will give you again square root of 1.58 square plus (3 into 0.5) whole square is equal to 2.18%. This is in percentage so this will give you 2.18%. So the error in the value of the estimated value of the spring constant is about 2%.

That is the first answer we require. Therefore we can say, K is equal to 3277 plus or minus 71.4 N by m which is 2.18% of that value. This is the answer to the first part of the problem. Now we know that the deflection under the load, if I call it is 3 mm is equal to 0.003 m deflection under the load. Therefore the force which gave rise to this particular thing is KY and by definition of spring constant that will be 3277 into 0.003 N is equal to 9.83 N.

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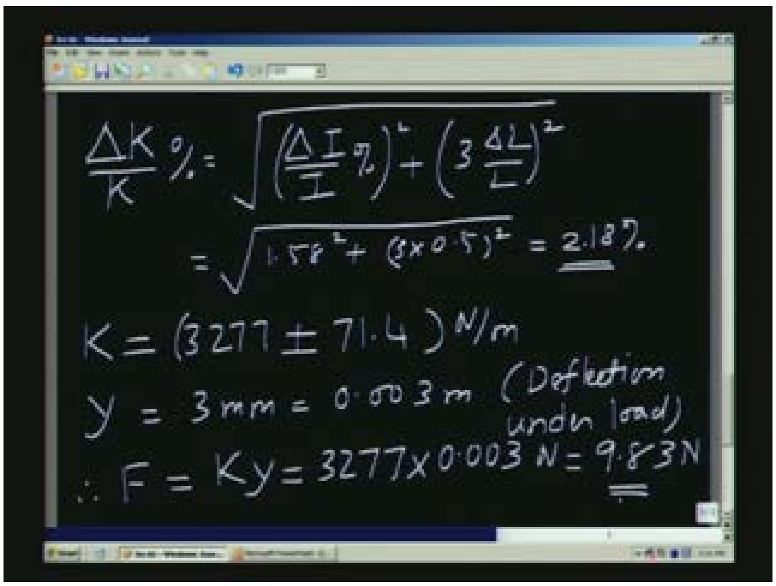

This is just the nominal value. So that is the force that has been applied and that gave rise to a displacement of 3 mm under the load. And again, I can use the same K which is susceptible to error, we already know Y is also susceptible to error, therefore I can also calculate delta F by F% is equal to square root of (delta K by K%) whole square divided by (delta Y by Y%) whole square is equal to square root of $(2.18)^2$ plus $(0.5)^2$ is equal to plus or minus 2.24%, which is the percentage error in the force measured, and where 2.18 is the value of deltaK by K%. So the error in the measured value of the force is given by 2.24% ,and therefore I can actually calculate what the force is which has been estimated, what is the error in the force and therefore, I can also say F is equal to 9.83 that is the value we got plus or minus 2.24% of that comes to 0.22 N, this is the answer to the problem.

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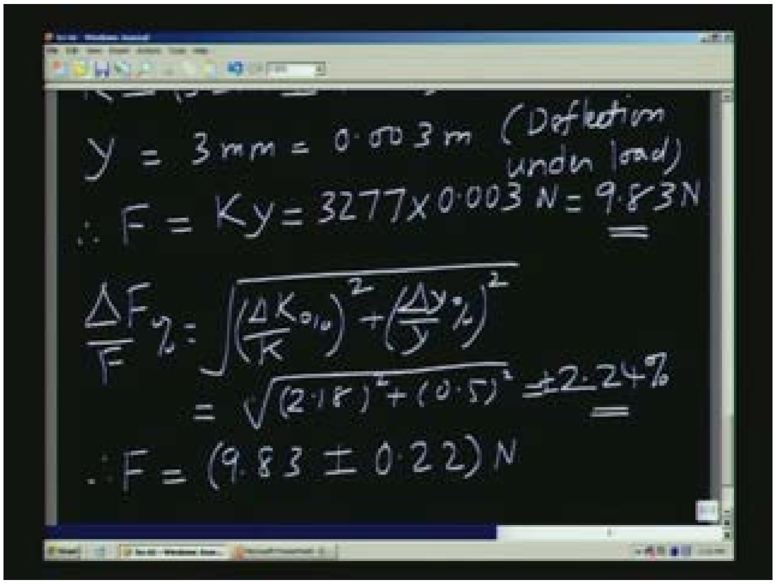

So you see that in this example, the elastic member that undergoes a deflection which is in the form of a cantilever beam made of spring steel, can measure a force of about 10 N which is 9.83 which is very close to 10 N for a deflection of about 3 mm under the load. This is just an example, to show what kind of numbers you expect in a problem like this.

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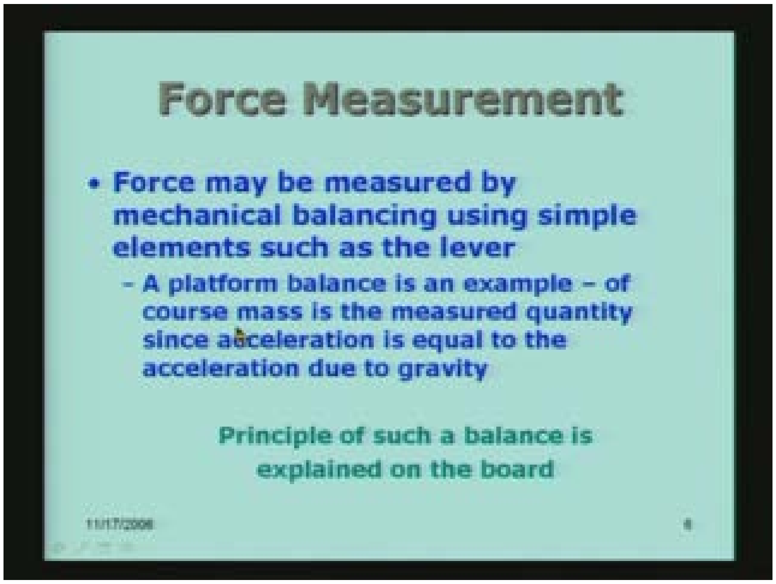

Now let us look at force measurement by other methods. For example, we can measure force, by mechanical balancing using simple elements such as the lever. Platform balance is an example, where the force measurement by balancing is used for determining the mass of an object. In fact, you can take the case of a simple balance. In the simple balance, we have horizontal member which is balanced in the center of the member, and on the two sides there are two pans and on one pan you put the weight, and on the other pan you put the mass the object whose mass you want to measure and then you find out when there is balance that means the horizontal arm of the balance must be perfectly horizontal and that is usually indicated by a needle attached at the center whose verticality you note against a scale.

What we are doing here is, actually the mass is measured by balancing a known weight or weight of an object whose weight has to be determined with respect to a known weight which is calibrated whose weight is known and by balancing you are able to find out what is the mass. Because both the pans are subjected to the same gravitational acceleration, same gravity is acting on both the sides and therefore it is actually the force balance, it is actually the moment balance, the moment of one force on one of the pans is the same as the moment, because of the other force in the other pan of the balance.

Similar thing can be done by using what is called, a platform balance which is used usually for measurement of very large masses, or if you want to call it as very large weight. Here is a simple schematic through which you can learn the principle of such a balance. Let us see what it consists of, this is the platform, what you see here is the platform and the weight can be put anywhere on the platform, that is the whole point of this platform balance.

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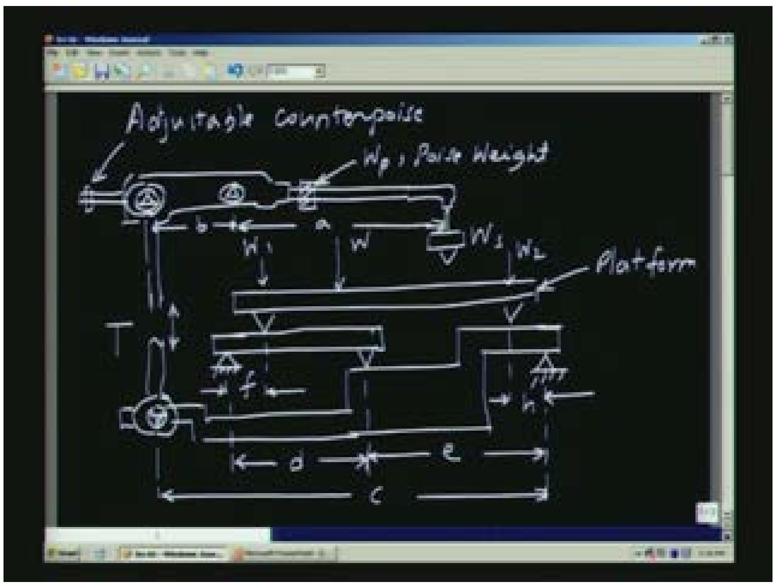

It can be placed any where on the platform. There are two knife edges which rest here and one of the knife edges rest on this beam and the other one rests on this beam. There are three beams as you can see; one here, two here and the third one here. So this is resting here and here and the force is applied here and you can see what is going to happen. This force is distributed here and here. These two forces W_1 and W_2 are due to W, which may be placed anywhere on the balance Of course, W_1 plus W_2 is equal to W. W_1 and W_2 itself is determined by the position of W depending on where it is put W_1 and W_2 will be in different amounts, that is the whole point here. So this rests on a knife edge here, and you can that this beam also rests on the third one.

The third member has got W_2 coming here at a distance of h, from this knife edge, and you can see that this is resting here therefore, there is another force coming at a distance, equal to e, and then you see that this is attached to a vertical rod, this rod is going to be either pulled down or pushed up depending on whether there is enough balance or depending on which way the balance is. Hence, there is a certain force in this member, and this is applied to a knife edge here, and this is a knife edge, one force is applied here, and there is a counter poise and this is the fulcrum and this is another lever here. And the weight W_s is going to be put here at a distance equal to a from this fulcrum. And the force T which is in the member is acting at a distance b from this place. We can analyze this very easily by going through part by part. Let us look at what equations we are going to get. It is just balance of moment.

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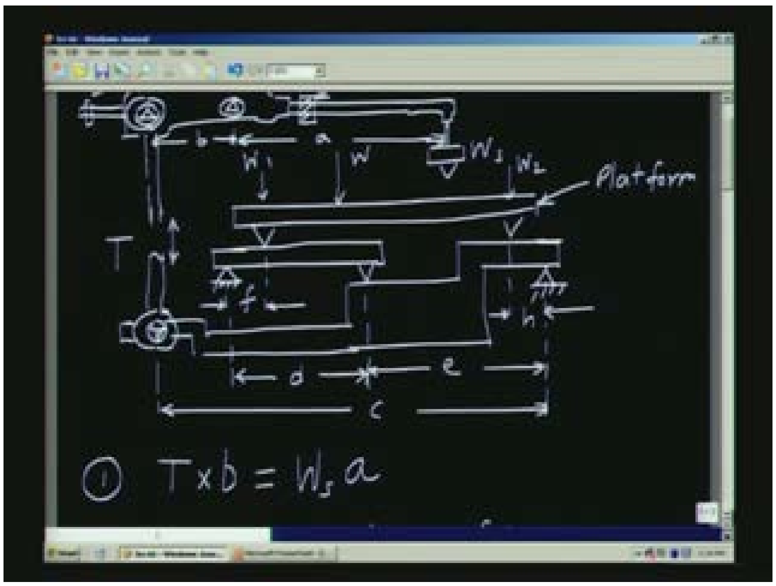

So you can see here that, if you apply it to this member here, W_s into a is the moment acting in the clockwise direction must be equal to T into b which is acting in the counter clockwise direction, if there is a balance. Hence it is W_s into a is equal to T into b. So, balance is usually indicated by a needle here whose position is going to indicate whether it is balanced or not. So T into b is equal to W_s into a where W_s is the weight which is going to be the known weight we know what this W_s is.

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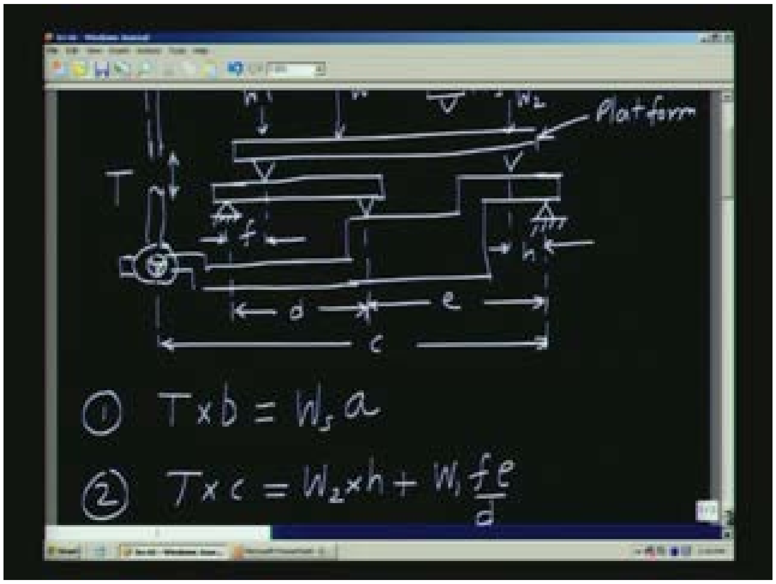

The next equation is, if you look at this member, there are several moments on that, and you can take moments at this particular point, so the sum of the moments must be equal to zero for the balance of that. So T into c where T is this force here multiplied by c is the force acting on it, and, W_2 is acting here at a distance h, W_2 into h plus W_2 is acting here, and there is another lever here therefore there is a factor of (f into e) by d where e is this distance this f divided by this f by d comes from this lever. Therefore what we have is T into c is equal to W_2 into h plus W_2 fe by d.

I can rearrange this equation, by writing for T is equal to e by c and I will take it outside here so this will be e by c into W_2 into h by e plus W_2 into f by d. suppose we arrange h by e is equal to f by d because this is something which we can choose, h by e is equal to f by d, if we choose these two factors are the same.

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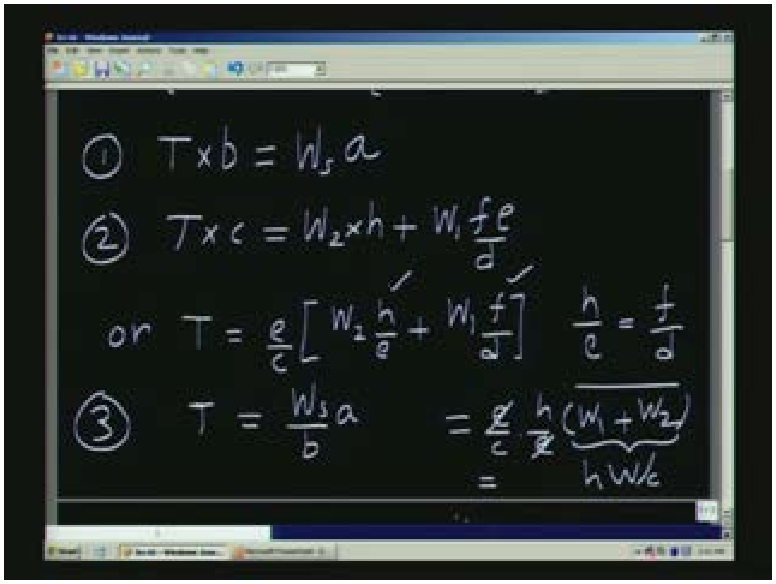

So, you see that if I take h by e outside this becomes e by c into h by e into W_1 plus W_2 which is nothing but W. So e will cancel off, h into W by c and is equal to T. So I can write here T is equal to W_sa by b is equal to h this part. So these two are equal, and therefore I can calculate W.

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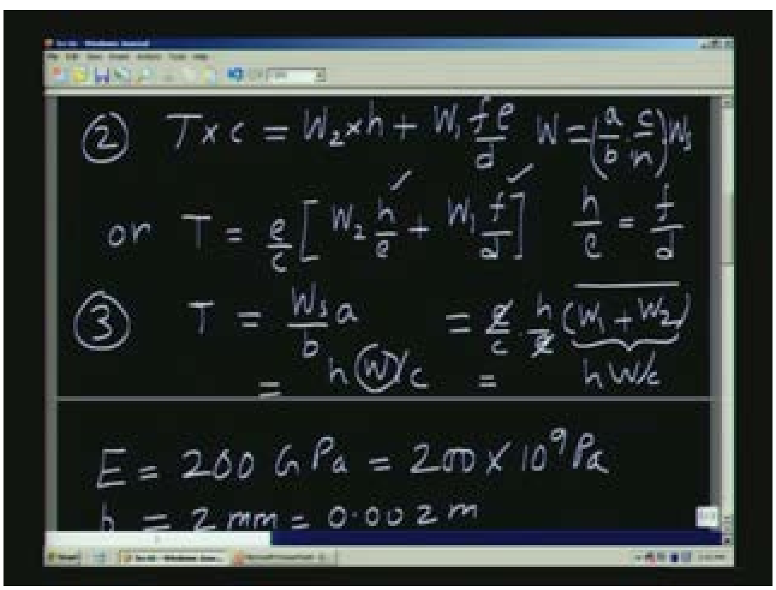

So W is equal to (a by b) into (c by h) into W_s. So, W which is to be measured, can be placed anywhere on the platform is equal to the product of (a by b) and (c by h) and these are all constants,and this, (a into c) by (b into h) is the gage constant. Therefore the value of W_s itself directly gives you W by multiplying it by a product which is a gage constant, chosen once we have all the lengths a, b, c, h. Once they are known, we can calculate the gage constant directly. This is how you get the weight. Weight is obtained by in terms of the added weight in the pan.

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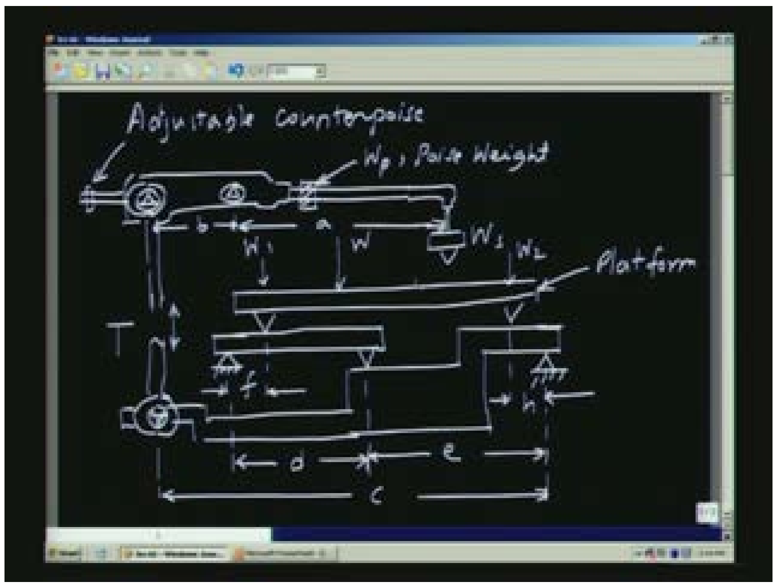

Apart from this, we also have a small weight which can be moved here on this arm, this can be called the poised weight. We also have what is called a counter poise weight which is on this side. This counter poise is actually used to balance the balance when there is no load on it we want to balance it, and that can be done by the adjustable counter poise. Now you can see what happens, if I put a poise weight here it is like putting a smaller weight here because this is W_p the moment arm is only this portion, whereas the moment arm for this is much larger. Therefore, it is going to give me a control. For example, W_s is 10 kg, and I want an improvement in the measurement, and 100 g is what I want to measure using this.

I can use a weight here at a smaller distance, so that it gives you a smaller least count of the instrument goes up, or this can be moved on a scale and therefore I can find out the scale reading here will correspond to a smaller weight and that is the way it works. So, the platform balance is simply an instrument where force is measured, the force in this case, happens to be the weight of the material which I am going to put here is measured in terms of balancing of the various moments acting on members, and what we have seen is that the position of W on this platform is immaterial. Wherever you place it on the platform it is going to read the same value. This is an example, which is indicative of another way of measuring the force.

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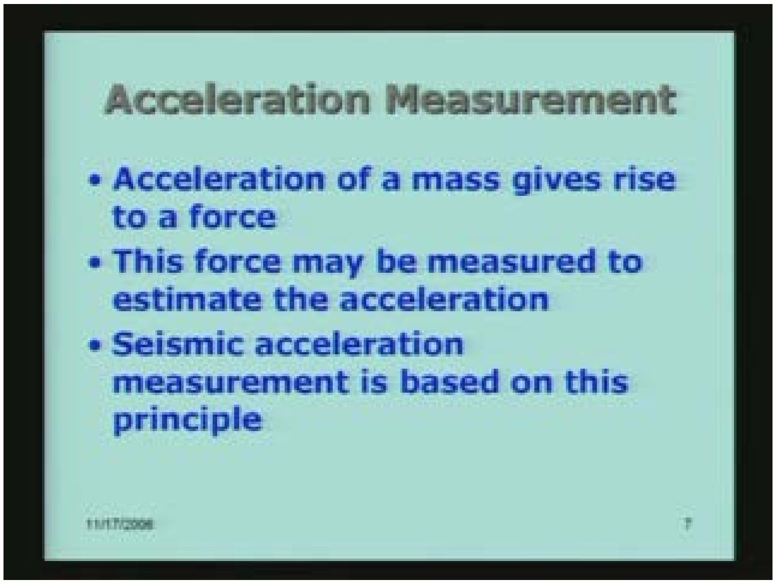

Now, we are interested in measuring acceleration. And an accelerometer or a meter measuring the acceleration of a member or a mass is called the accelerometer. The acceleration may be due to vibration of a machine for example, and we would like to measure the acceleration because of the vibration. Or, if you have an instrument you would like to find out how much vibration it can withstand, and I would like to find out what is the acceleration it is subjected to during these vibrations.

The measurement of acceleration is one measurement which we do to study the vibrational response of a system. We may also be interested in measuring the force which the member experiences, because of vibration. We may be also interested in the displacement, because of the vibration. There are many aspects which need to be measured. If I have mass and acceleration is imposed on it, then it means there is force which it gives rise to. So the force is nothing but mass into acceleration and acceleration imposed on a mass gives rise to a force.

And in fact, this force may be measured to estimate the acceleration. Because acceleration is not directly measured the force can be measured by using any of the methods available. For example, a spring can deflect and the force can be measured with the spring and from that I can find out what the corresponding acceleration is if I know the mass which is undergoing the acceleration and that is how you work it out. For example, a seismic acceleration measurement or seismic accelerometer is based on this principle.

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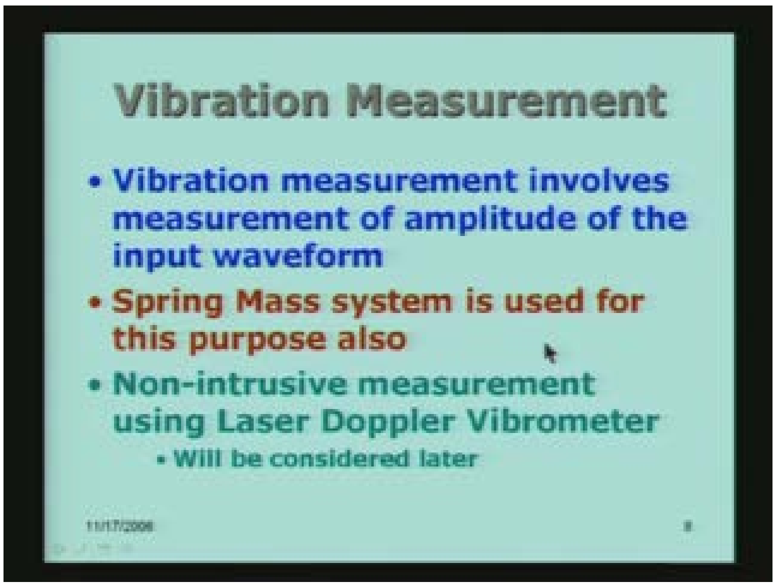

Secondly, as I talked about vibration measurement it involves measurement of amplitude of the waveform and the input waveform may be any waveform it need not be sinusoidal it could be non sinusoidal. That is one complexity that is going to come when you want to measure the vibration amplitude. For this purpose also I can use a spring mass system or in fact you can use a non intrusive measurement using laser Doppler vibrometer. This laser Doppler vibrometer is just like the laser Doppler velocity meter which was used in the measurement of velocity of a fluid. So similar principles are going to be involved but we can use a laser Doppler vibrometer to measure vibration. Let us look at some of the basic principles involved especially in the case of the spring mass system.

A typical spring mass system is shown here. This is the mass and there is a spring attached to the mass, and there is a base plate which is in the form of a bent plate like this and the spring is attached between this and this **[30:09].** And what I am going to do is, I am going to subject this base here to the acceleration.

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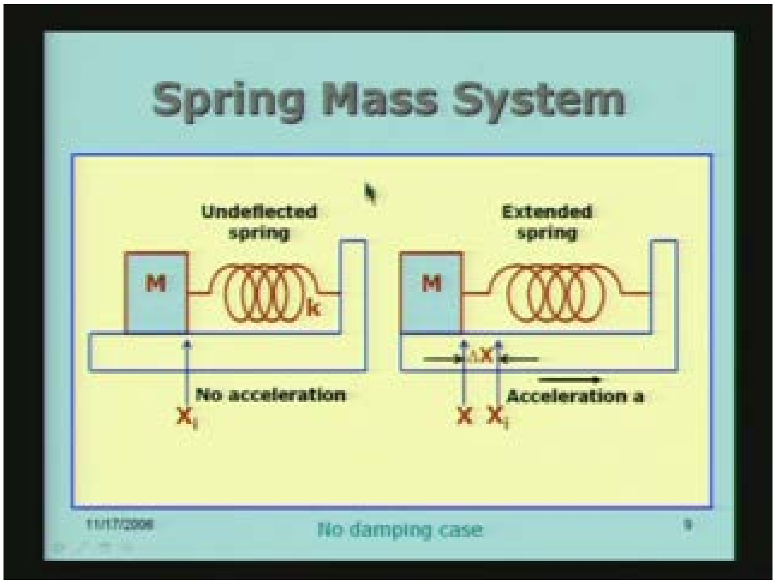

If you want you can say that this is the table and the table is the one which is going to be accelerated. This is like a table on which I have got a mass and for the moment we will assume that there is no friction between the M, and this, and the spring is having a spring constant of k. Undeflected spring means, there is no force, there is no acceleration so this will be undeflected so M is sitting with stress there. Therefore the length of the spring will be the undeflected length of the spring. If the table accelerates to the right with an acceleration given by a, and because the mass has inertia, the mass will almost stay where it was, and with respect to the mass or with respect to the table, the mass looks like it has gone back.

That means because of the extension of the spring the force is now felt by the mass, or because the mass is staying where it is and the ideal case is that the mass should not move but only the table should move in which case the total force will be simply given by the extension of the spring multiplied by the spring constant which will directly give the mass times acceleration. Therefore acceleration is obtained by simply taking the spring deflection multiplied by k and divided by the mass.

So the displacement of the spring is a measure of the acceleration as you can see. Now I call it no damping case because I do not have any damping in this. Here we have a spring mass system and if you have a spring mass system without damping, it is having a natural frequency, and suppose I oscillate or vibrate this table at a frequency equal to the natural frequency, the displacement of the mass will be enormous.

In principle, it will go into resonance and the displacement will be too large, and we do not want that to happen. So sometimes what we do is we provide two stops. If you have a vibrating table you place the vibration measurement device on top of that.

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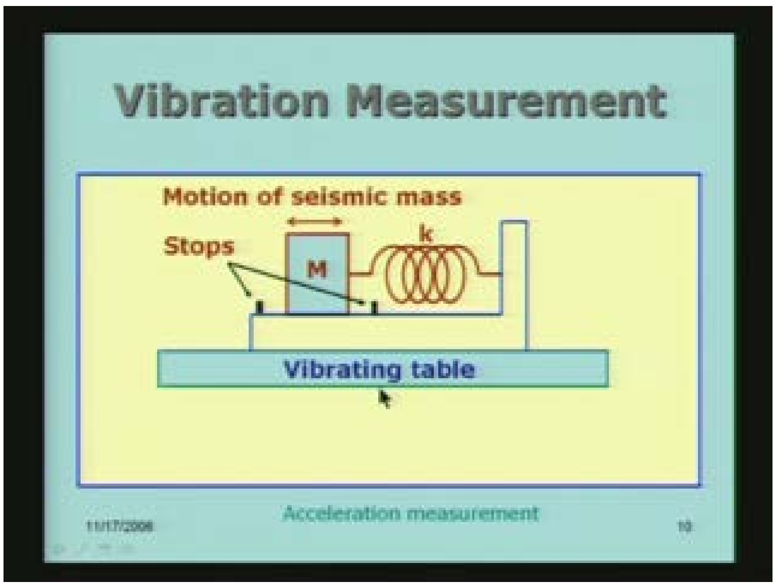

So what I will do is that I will put two stops here so that the spring is not allowed to extend beyond some value, it will be stopped here, or it will be stopped here on the compression side. This is called the seismic mass, so the motion of the seismic mass is a mass which is more or less isolated from this vibration source so it more or less stays in the same position. It is best if it can stay in the same position.

Then you have the k the motion is stopped by the stops which I have kept here, so that you are preventing the damage to the instrument when the natural frequency of this spring mass system coincides with this impressed vibration frequency. If the same frequency is there, then it will go into resonance. We have an accelerometer which has a seismic mass of mass equal to 0.02 kg or 20 gm and a spring of spring constant equal to 2000 N by m, and it is a very hard spring.

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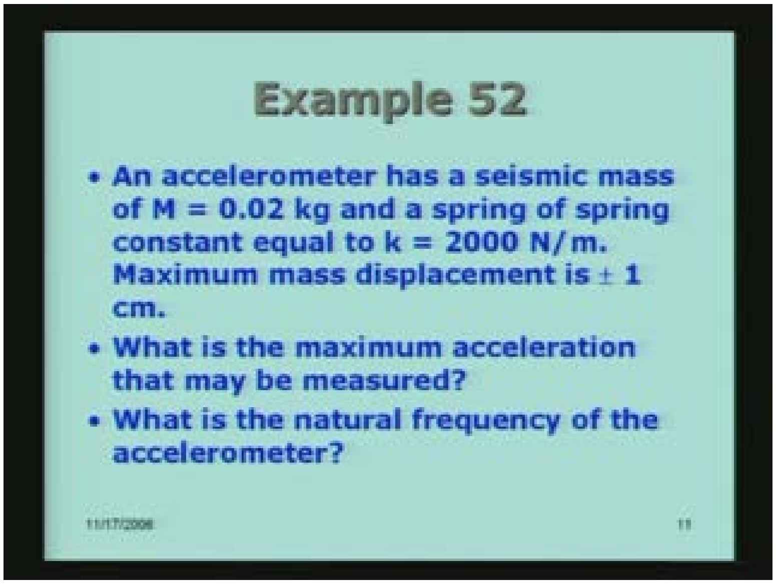

The maximum mass displacement is plus or minus 1 cm that means I have provided the stops so that it will not go beyond that. So what is the maximum acceleration that may be measured? What is the natural frequency of the accelerometer?
If I have a stop here, it means that the mass is more or less going to be on the same place, the mass is not going to move, but it is only the spring which is going to extended, and if it extends this mass will appear to reach this point. Actually the mass is there the spring extends such that this will come and hit it, and similarly the other side, this is ideal.

Here is a simple problem. This should be example, 52. The problem is very simple,because the maximum amplitude or maximum moment is given because of the stop. That is given by delta $x_{maximum}$ is equal to 1 cm. Actually you can put it as plus or minus 1cm, because it may be either compression or in other words the acceleration will be either to the right, or left, as the case may be, this will be plus or minus 0.01 m. Simple $a_{maximum}$ is given by (k by M) into delta x. (k into delta x) is the force, force divided by mass is the acceleration, it is a very simple formula. So k is equal to 2000 and delta x is equal to plus or minus (2000 by 0.02) into 0.01 m by s square so that works out to plus or minus 1000 m by s square.

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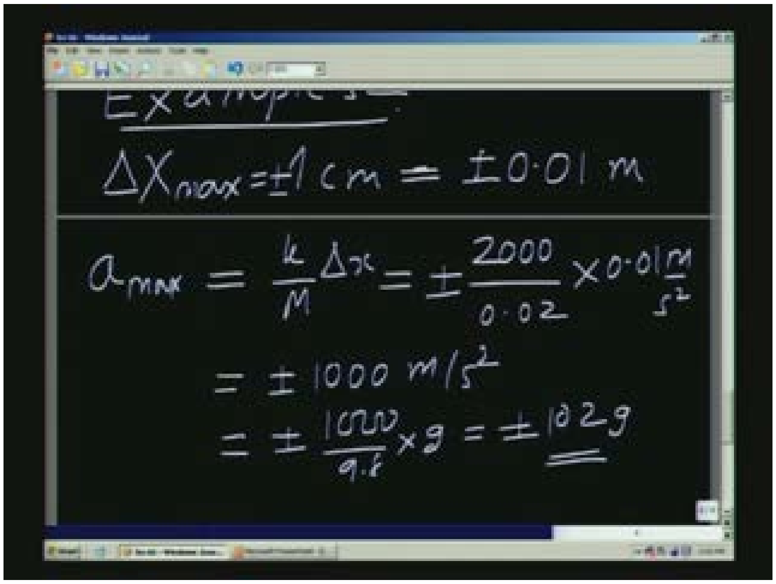

You can notice the magnitude here it is a very large acceleration. I actually divided it by the acceleration due to gravity, so (1000 by 9.8) into g, as 1g is equal to 9.8 we can see that this comes out to plus or minus 102g. So, if you allow the mass to move about or the spring to compress or expand by about 1 cm then we are talking about 100g appearing as the acceleration impressed which is a very large acceleration.

So natural frequency can be also be obtained. We know this from our study of mechanics. The natural frequency I will call it as f_n which is nothing but 1 by 2pi square root of k by m 2pi into fn is called the circular frequency omega$_n$. We are just using that formula. So this will be 1 by 2pi square root of 2000 by 0.02, this will be in so many hertz and it works out to about 50.33 Hz. So the natural frequency of the system is about 50 Hz.

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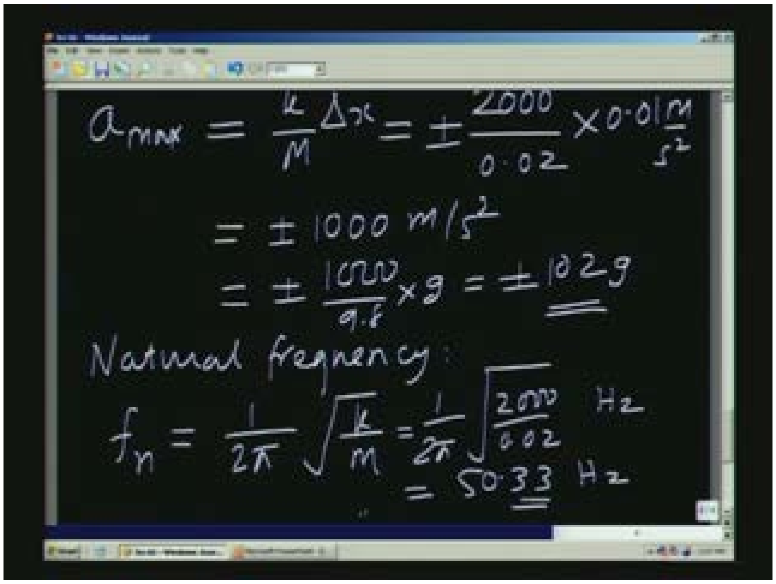

The natural frequency plays a very important role as we will see. Hence, let us look at a more tangible case, where you also have some damping, because it is very difficult to have the system without damping, and it is just not possible. So, if you have damping the nature of the response of the spring mass damped in the system is totally different from what we are talking about.

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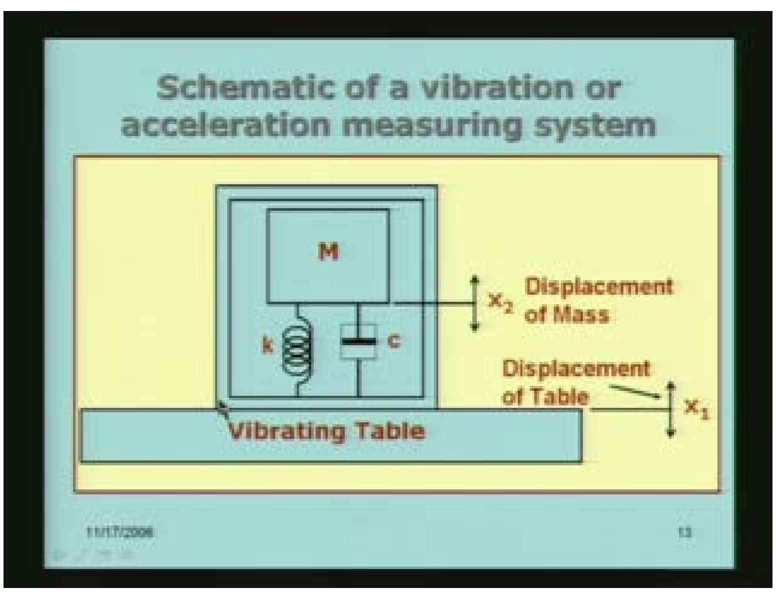

The slide here shows the schematic of a vibration or acceleration measuring system. For both cases we are going to use similar instrument the vibrometer or the accelerometer are one and the same but the various quantities, M, k, c, etc., that are shown are going to be different. Essentially it consists of a housing the accelerometer or vibrometer is the housing this housing has to be attached to the vibrating table or the source whose vibration we want to study.

It should be intimately connected with that,and it should not have any relative motion with respect to the vibrating table. So it is necessary to screw it down, or attach it in some way so that it is a part of the vibrating table. So what I am talking about is only the housing. Inside the housing I have a seismic mass M just like what we had in earlier case, I have a spring just as in the other case but I also have an additional thing shown here.

This is a damping arrangement which has got a damping coefficient or damping constant of c. So the difference between the case we considered earlier, and the case we are having now is that there is an additional item coming here. Suppose the table is vibrating up and down, if it is vibrating sideways, then nothing is going to happen, because the spring is able to go up and down, and not sideways so this will measure the vibration only in this up and down direction.

So accelerometer has directional response, it responds only in this direction. Let us assume that x_1 is the displacement of the table up and down, and here I have shown up and down arrow because it is vibrating. Sometimes it is up and sometimes it is down, sometimes it goes to one end goes to the other end and so on. As the vibration goes on, this x_1 will go through variations from 0 to a maximum then again to 0 back to minimum and so on.

So it will go up and down like that here this table movement is the same as the movement of the housing. The accelerometer housing undergoes the same moment in step with this x_1, and in principle M can go up and down by $x_{2.}$ This is the motion of the seismic mass. It may be ideal to have the mass x_2 to 0 that means it is standing still. If it is standing still you can see what is going to happen, the compression of the spring is simply due to the up and down motion of the vibration, and therefore directly, you can find out the acceleration or the force, because the spring deflection is what is measured.

The mass is undergoing a vibration motion x_2 as shown by x_2, and let us see what is going to happen schematically. Because of x_1, if this moves by x_2, the spring and the dash part or the spring and the damper are going to undergo a change x_2 minus x_1, x_2 minus x_1 is the displacement of the spring as well as this element here. So we have to see the force on the mass. Mass is of course moving up and above up and down by x_2. Therefore M into the acceleration of the mass which is given by d square x by dt square, which is the force experienced by the mass this must be equal to, the sum of the forces due to these two elements, because that is what is going to come on that. So if you recognize that you will be immediately able to write the appropriate equation. We can show that the spring mass system with damping is governed by a second order differential equation, because acceleration of the mass is going to give rise to a force, and this force is now shared between the spring and the damper.

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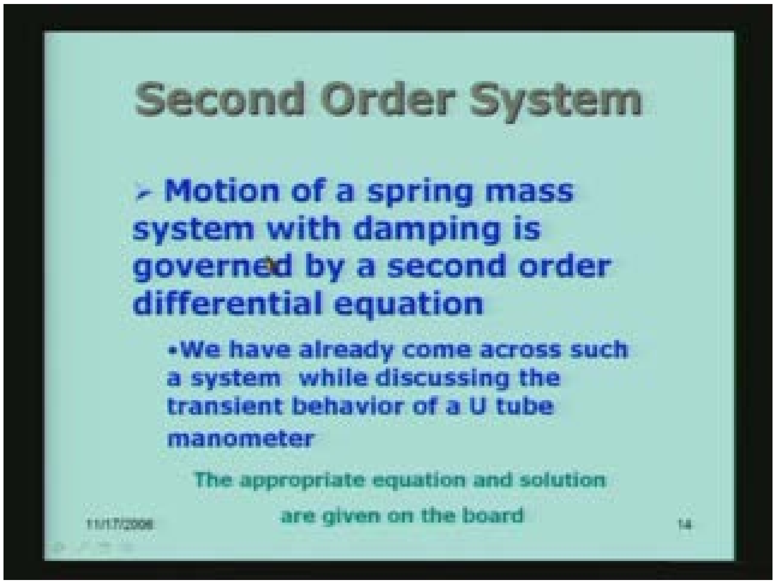

Now we are going to look at a second order system. In fact we have already come across such a system while discussing the transient behavior of a U-tube manometer when we were talking about pressure measurement. Therefore we are going back to that equation but we are going to interpret the solution in a slightly different way. Here is the sketch. This is the housing that is the seismic mass M, there is a spring, and there is a dashed pot here, and this is x_1, this is moving up and down x_2.

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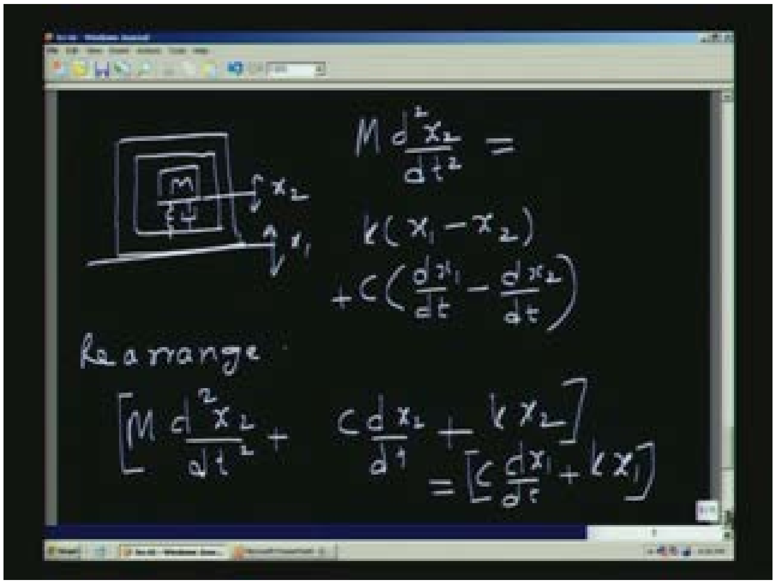

So, I want to find out what is the equation governing the change of x_2. For that we can immediately see that, force on the mass because of the acceleration is M d square x_2 by dt square. That is the force experienced because of the movement of the mass. The mass is going to have a force M d square x_2 by dt square.

The spring force is given by k into (x_1 minus x_2), because the difference between the two is the displacement of the spring, and similarly for the damper we assume that the damping force is proportional to the velocity. In fact we have done that earlier also (dx_1 by dt minus dx_2 by dt), so this equal to this plus this and that is what the problem is about. M into d square x_2 by dt square is equal to k into (x_1 minus x_2) plus C into (dx_1 by dt minus dx_2 by dt) and we can rearrange it in the form of a second order equation (M into d square x_2 by dt square) plus (C into dx_2 by dt) plus k into x_2 is equal to (C dx_1 by dt) plus k into x_1.

So the left hand side, I have got all the x_2 terms and in the right hand I have all the x_1 terms. And if you notice, x_1 is actually known to us or it is given as the input. So we can take x_1 for example, I can treat it as some x_0 cos omega$_1$t. This is assumed to be the input from the vibrating table. And for the moment, I am assuming a single component which is sinusoidal and has got a single frequency. Of course, we can generalize it to any type of input which may not be sinusoidal.

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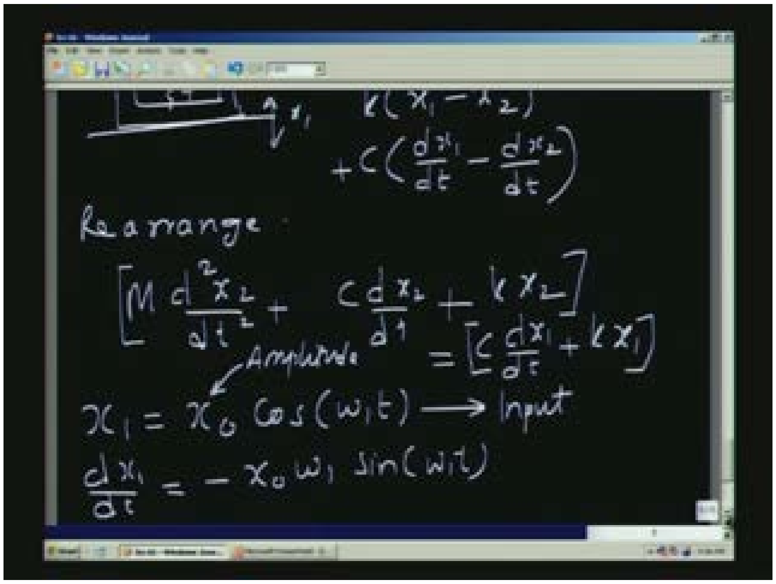

Now I am going to impress only one particular frequency if it is shaking with a single input frequency. Now I can obtain this dx_1 by dt from this, and that will be nothing but, minus x_0 omega$_1$ sine omega$_1$t. Therefore the right hand side can be written in terms of a given function of t and x_0 will be your amplitude of vibration. In a vibration measurement what I would like to measure is that x_0.

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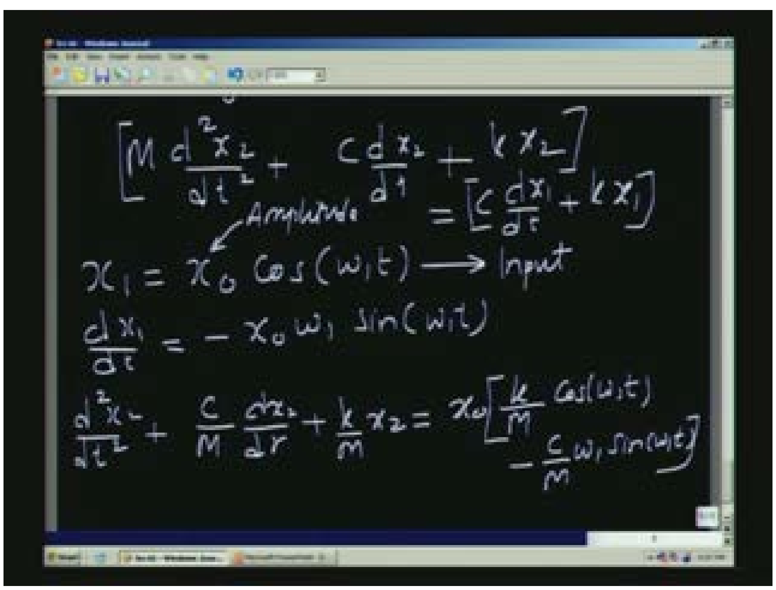

So I can write the equation now, I can also divide by M, so I will have d square x_2 by dt square plus (C by M) into dx_2 by dt plus kx_2 by M is equal to I am going to divide throughout by M; x_0 I will take out as a common factor (K by M) into cos $omega_1$t minus (C by M) into $omega_1$ into sine $omega_1$t. This is the equation whose solution is required, if you want to find out what is the response of the system. In fact the response, consists of two parts. As you already know any second order or any differential equation has got a complementary function and a particular integral. And the complementary function in this case, will give rise to a damped oscillation which is transient. The transient will go off after sometime, and only the steady state part of the solution is important to us, because we want to look at the steady state response of the system. Here are some of the quantities which will be useful.

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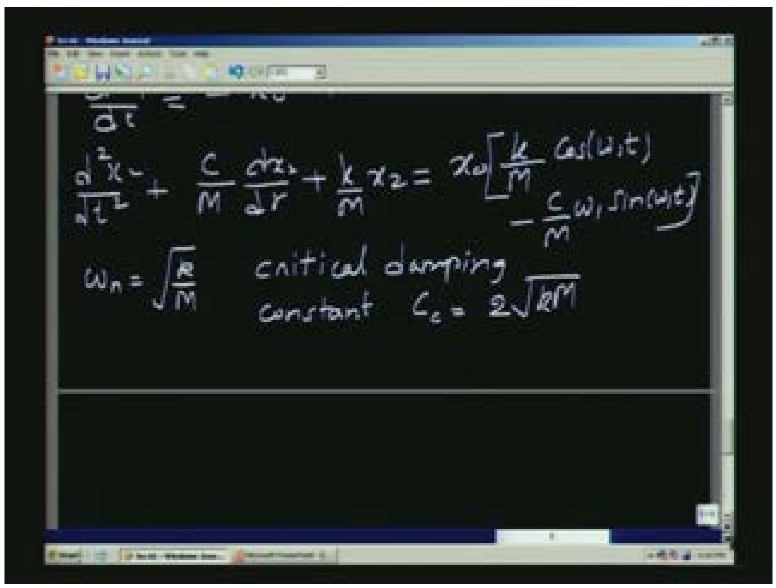

So the natural frequency $omega_n$ we already know is square root of (K by M) and we will also introduce what is called the critical damping coefficient or damping constant. The critical damping constant, C_c is 2 into square root of (K into M). So with these quantities, we can solve the equation very easily and the general solution I can give for x_2 minus x_1 the solution can be shown by e power minus (c by 2m) into t into (A cos omega t plus B sine omega t) and you notice that the e power minus c by 2m into t, this is the

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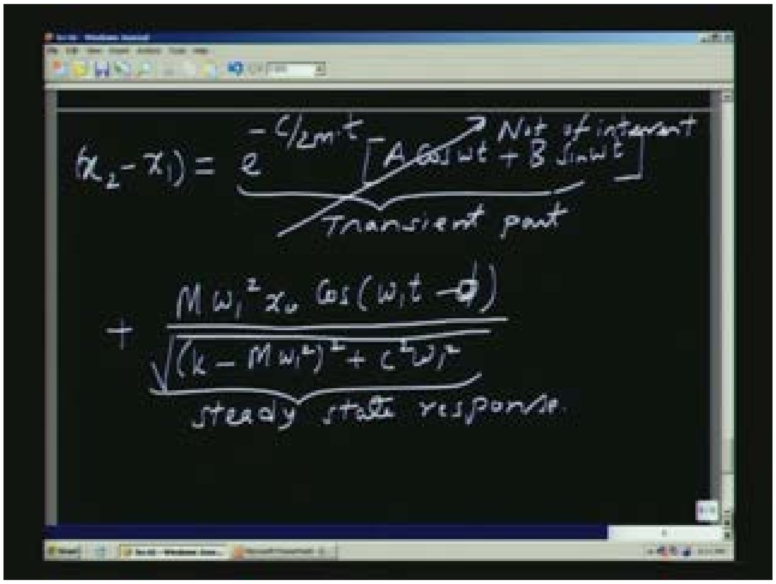

damping part and it is going to reduce with respect to time, and as t increases this is going to become smaller and smaller and this is actually the transient part, which will go to 0 for long time. Here this part is a more important one. This is given by M omega$_1$ square x_0 cos omega$_1$ t minus phi, where phi is the phase angle by square root of (k minus M omega$_1$) square plus (C square omega$_1$ square), so this we call as the steady state response, and that is what of interest to us. In fact, what we can do is, in this equation which we have written down, the steady state response I can divide x_2 minus x_1 by x_0, x_0 is the amplitude of the input thing and I can remove cos omega t minus phi, this is just a time factor. So what is important is x_2 minus x_1 by x_0 equal to this factor which is nothing but the response of the system.

So the response of the system in terms of the amplitude x_2 minus x_1, x_2 minus x_1 is equal to x_0 we have solved the problem. So we will say that amplitude response is simply given by x_2 minus x_1 by x_0 this is the maximum value of x_2 minus x_1 with respect to x_0. I will also introduce the omega$_n$ square that was the natural frequency, and then we also had the critical value, C_c which was introduced earlier using that and then re arranging the equation we can show that this is equal to square root of (omega$_1$ by omega$_n$) whole square by 1 minus (omega$_1$ by omega$_n$) whole square plus 2 into C by C_c into (omega$_1$ by omega$_n$) whole square, that is your response function we will call that as equation 1, this is very important so let us use this as equation 1. That gives you the amplitude response of the system, and the amplitude response you can see is dependent on the omega it depends on the ratio of C and C_c and also it depends on the natural frequency.

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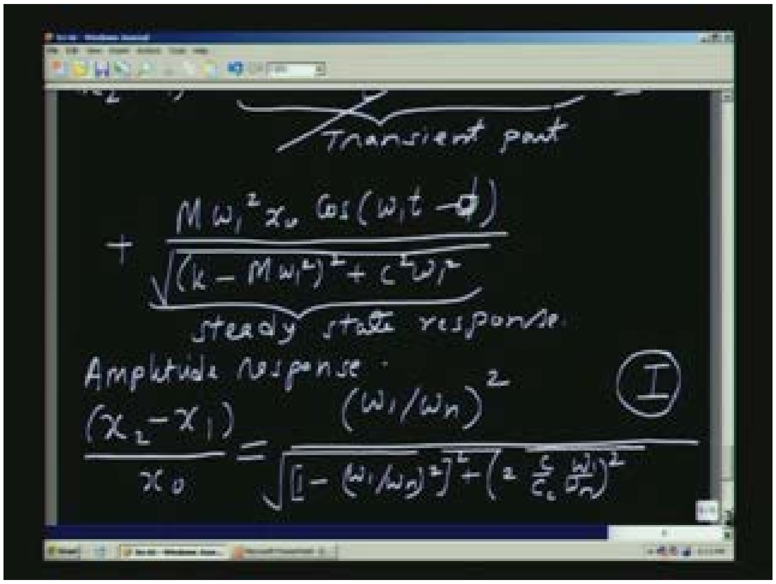

So, for a given system, that means that the c is known, k is known and M is known for such a system, I know all these quantities omega_n is known, C by C_c is known, and with different values of omega_1, I would like to find out what is the amplitude response. If you look at the phase information, we can similarly show that, I can write phi in the same fashion as we did here, it will tan inverse of the following quantity: 2 into C by C_c into omega_1 by omega_n by 1 minus (omega_1 by omega_n) whole square.

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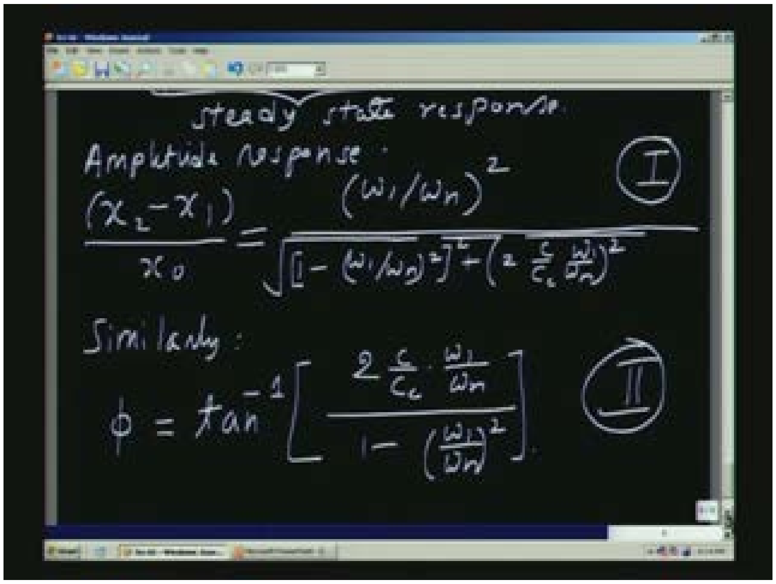

This we can call as 2. And in fact I am going to write another equation, which is also useful. And if you go back, and see the acceleration due to x_1 is nothing but d square x_1 by dt square that is the acceleration of the table at any given instant of time, because of the vibratory motion of that. So this can be written as d square by dt square of x_0 cosine omega$_1$t and this will be nothing but minus x_0 omega$_1$ square cos omegat. That means the acceleration, because of the vibratory motion is proportional to the amplitude of the motion multiplied by the square of the frequency of the change, omega$_1^2$ is the frequency change.

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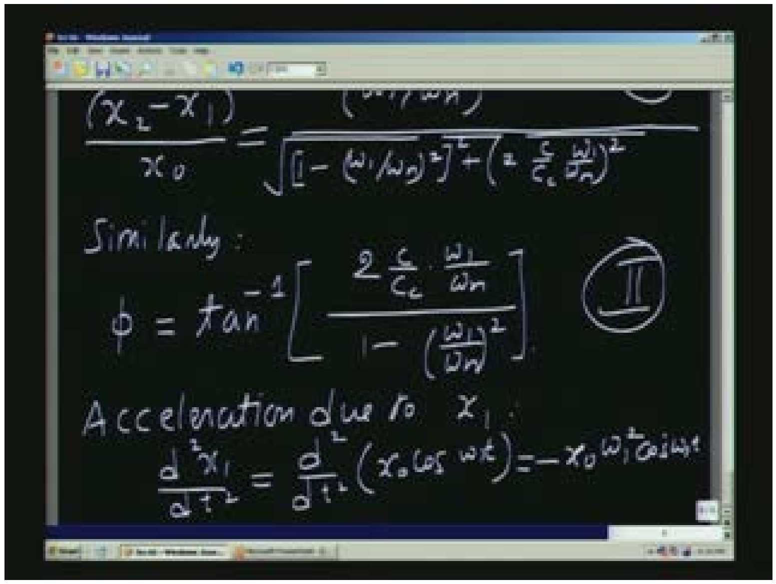

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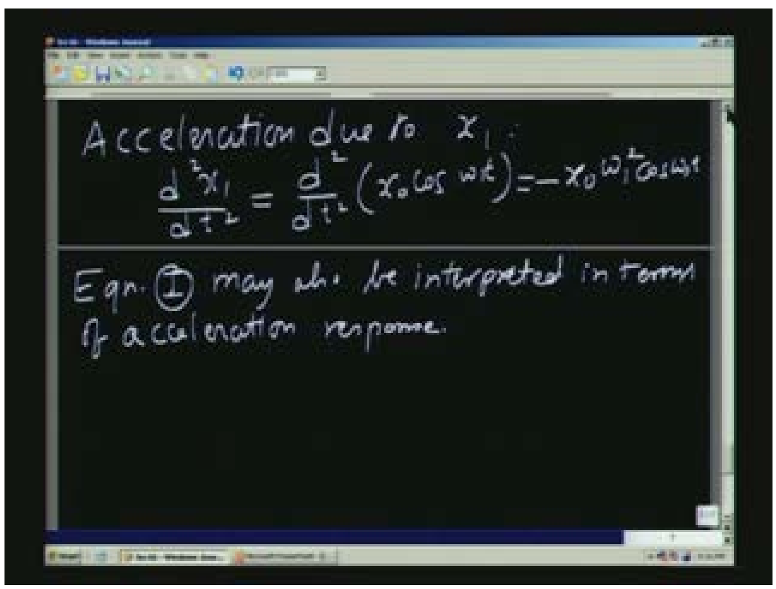

So I can interpret equation 1, in terms of acceleration response. Let us look at the equation and see how this interpretation is done.

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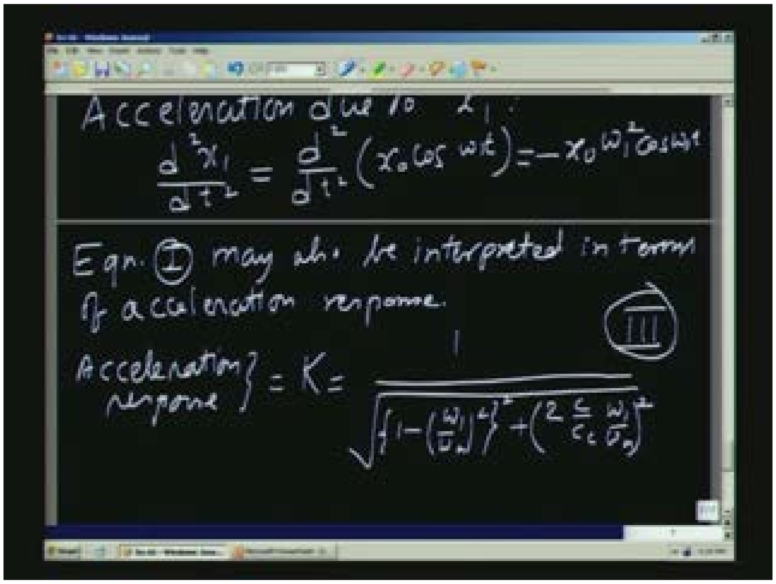

There is already omega$_1$ square here, so all I have to do is, take this omega$_1$ square out, because that is proportional so x_2 minus x_1 by x_0 if I take out this omega$_1$ square, because that is the acceleration you can see that 1 by omega$_n$ square multiplied by this whole thing is going to be response due to acceleration. Therefore equation 3, I can write down and because omega$_n$ is constant for any particular system, I can say that the acceleration response is given by the factor, where I use symbol K, for that factor I am removing that omega$_n$ square from that expression and I am going to say that, this is equal to [1 by square root of 1 minus (omega by 1 omega$_n$) whole square] to the whole square plus 2 C by C_c (omega by 1 omega$_n$) whole square under the square root so we will call this as expression 3. Thank you.