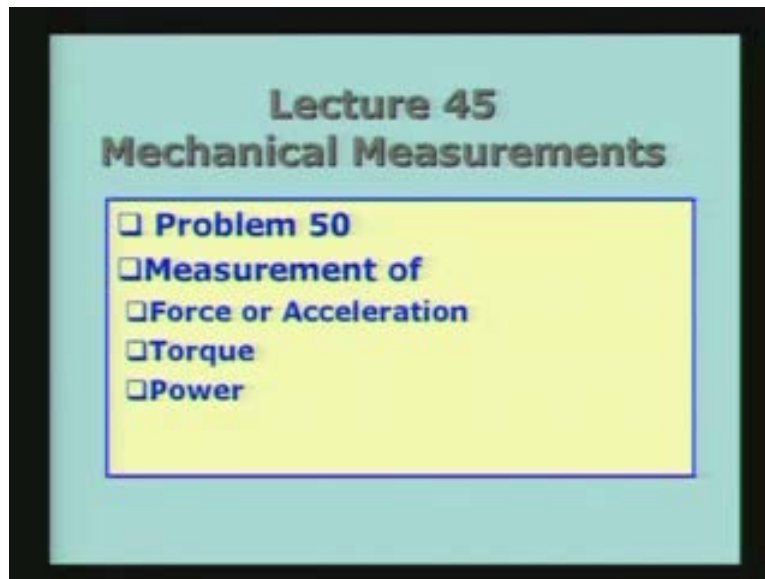


**Mechanical Measurements and Metrology**  
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**Module - 4**  
**Lecture - 45**  
**Measurement of Force**

This will be lecture 45 on our ongoing series on Mechanical Measurements. Towards the end of the last lecture, we looked at the problem of measurement of particulates in air and we were thinking of doing a problem. So we will start with that problem. In fact that will also mean we are going to complete our discussion on measurement of particulates and then move on to the next topic.

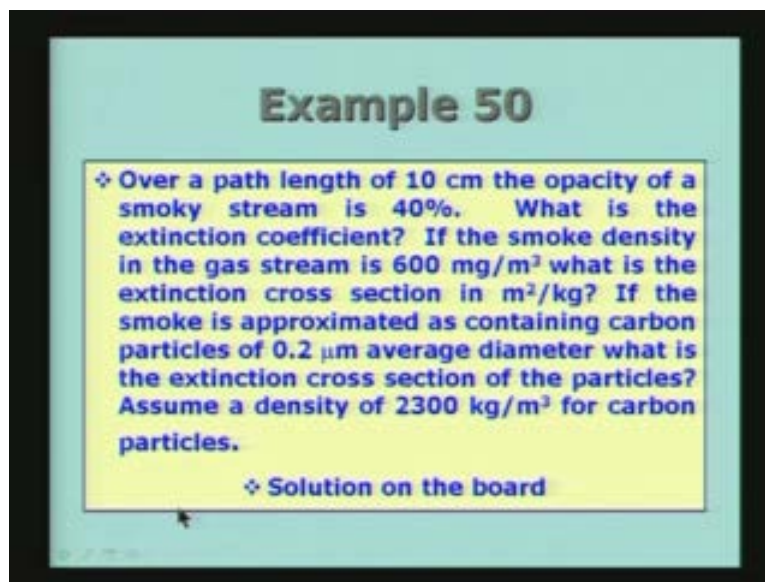
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In this lecture the problem 50, is the one which is going to come from the previous lecture whatever is left out. Then we will look at some interesting things about measurement of force or acceleration, torque, and power. The measurement of force and acceleration, torque, and power are very important from the point of view of mechanical engineering applications. For example, if you have an engine or a power producing device we would like to measure the power output, and the power output requires the measurement of torque, and the speed, of the engine or the machine.

In some other applications, for example, force is involved in the measurement of weight of an object, or force experienced by an object under acceleration, and therefore we may be interested in measuring force as well as acceleration, independently. For example, in the case of vibration problems, one would like to measure the forces experienced by an object when it is subjected to acceleration, because of the vibrating motion of the system. So, acceleration may also be involved in the measurement, for example, when an earthquake takes place there are some accelerations which are experienced by various objects on the ground and we would like to measure these accelerations, because these are going to determine whether those systems will survive or not in an earthquake. The problem 50 is specified in this particular slide. We have a path length of ten centimeters, and if you remember the opacity measurement

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**Example 50**

◇ Over a path length of 10 cm the opacity of a smoky stream is 40%. What is the extinction coefficient? If the smoke density in the gas stream is  $600 \text{ mg/m}^3$  what is the extinction cross section in  $\text{m}^2/\text{kg}$ ? If the smoke is approximated as containing carbon particles of  $0.2 \mu\text{m}$  average diameter what is the extinction cross section of the particles? Assume a density of  $2300 \text{ kg/m}^3$  for carbon particles.

◇ Solution on the board

We have a light emitting diode as source of light, and photo diode, or a photo detector as a detector for energy transmitted across the path length. And as you remember the radiant energy is going to be absorbed and possibly also scattered, because of the presence of the particulate matter. So we are interested in looking at what happens when you have such a situation. So we have a 10 cm path length and the opacity of a smoky stream is given to be 40% this is from measurement. So we would like to find out what is the extinction coefficient corresponding to this 40% opacity. Then, further we are given that the smoke density in the gas stream, after a small amount

of particulate matter is present in the gas as a mixture, and this 600 mg by m cube is the total content of that.

So 600 mg by m cube is the amount of particulate matter present in the gas. So we would like to find out what is the extinction cross section in m square by kg and if the smoke is approximated as containing carbon particles of average diameter of 0.2 micrometer, we would like to find out the absorption cross section, or extinction cross section for the particles, because extinction I am using a term which includes the both absorption and possibly scattering, and may be mostly scattering and to some extent absorption. So we assume a density of 2000 and 300 kg by m cube for the carbon particles.

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Example 50

$$\begin{array}{c} I_0 \\ \text{LED} \end{array} \xrightarrow{L=10\text{cm}} \begin{array}{c} I_t \\ \text{PD} \end{array}$$

Opacity = 0.40

Transmittivity

$$= 1 - 0.4 = 0.6$$

$$\frac{I_t}{I_0} = 0.6 = e^{-\beta L}$$

$L = 0.1\text{m}$   
?

Essentially we have a source which is an LED, and there is a photo detector. I will just use the symbol PD for that, and we have a radiation going across, and this is 10 cm, path length, so L is equal to 10 cm, and if you assume the intensity of radiation leaving the LED is  $I_0$  and what is measured is transmitted. So we know that in this case, the opacity is nothing but 0.4, means the transmitted amount or transmittivity is, 1 minus 0.4 which is 0.6, that means that this transmitted amount  $I_t$  by  $I_0$  is equal to 0.6, that is what it means.

We will write this as e to the power minus the extinction coefficient, beta into L. So 0.6 is equal to e power minus beta into L. So L is known, and this

is nothing but, 0.1 m, so I can work out the value of the beta. So all I have to do is take logarithms on the two sides with respect to natural logarithm then you see that minus beta L is equal to logarithm of 0.6, and I can put the value of 0.1 for L, and therefore beta will be given by minus of logarithm of 0.6 by 0.1. So this comes out to be, 5.1083 by m, because, I am using 0.1 is 10 cm is what is given and this is equal to 0.1 m.

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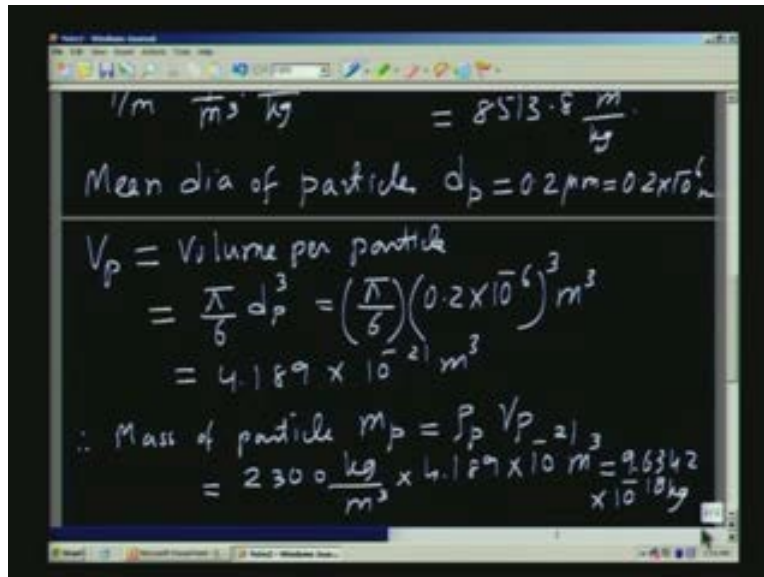
The image shows a blackboard with handwritten mathematical derivations. At the top, it starts with  $I_0$  and a question mark. The first equation is  $-\beta L = \ln(0.6)$ , which is rearranged to  $\beta = -\frac{\ln(0.6)}{0.1}$ . This is then calculated as  $\beta = 5.1083/m$ . Below this, the concentration  $P_s$  is given as  $600 \frac{mg}{m^3} = 600 \times 10^{-6} \frac{kg}{m^3}$ . The final equation shows  $\beta = P_s \sigma_e$ , where  $\sigma_e$  is the cross-section, and it is calculated as  $\sigma_e = \frac{\beta}{P_s} = \frac{5.1083}{600 \times 10^{-6}} = 8513.8 \frac{m^2}{kg}$ .

Here, everything is in SI units, so the extinction coefficient beta comes out to be 5.1083 m. This is per meter this is the first answer which is required. We also are given that the sample has got a concentration, so the concentration of the smoke, concentration is measured either in terms of the number of particles per cubic meter of the entire volume, or it could be amount of material per unit volume.

In this case what we have given is the amount of material in terms of mass per unit volume. So we will call it as concentration is measured in rho of the smoke, the density of the smoke 600 mg by m cube. We have a very small amount of the particulate material. This is actually 600 into 10 to the power minus 6 kg by m cube, because 1 mg is equal to 10 to the power minus 3. Because I am putting everything in terms of minus 6 kg by m cube what we have to do is, now we can define the beta which is the quantity we described earlier the extinction coefficient can be written as the density rho<sub>s</sub> times the cross section, we will call it the sigma<sub>e</sub>, the cross section of the particulates,

and therefore  $\sigma_e$ , actually if you remember beta is 1 by m, and this is in kg by m cube, and  $\sigma_e$  is in m square by kg so the kg will cancel it will give you 1 by m. So it is nothing but given by the density multiplied by the extinction cross section per unit mass,  $\sigma_e$  is extinction cross section per unit mass. So  $\sigma_e$  is beta by  $\rho_s$  so we can put it as 5.1083 by 600 into 10 to the power minus 6, and units will be m square by kg, then this comes out to 8513.8 m square divided by kg, that is the second answer we require. That is the per unit mass the extinction cross section. It is in the form of an area. Therefore, extinction cross section means that the amount of radiation removed is given by this particular cross section. It is as though there is so much of cross section per kilogram. Of course, the amount of material is only 600 mg by m cube, therefore it will be proportional to that. Now the mean diameter is given of the smoke particles, I will call it as  $d_p$  it is 0.2 micrometers, so that will be 0.2 into 10 to the power minus 6 m.

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$$\frac{1}{m} \frac{m^3}{kg} = 8513.8 \frac{m}{kg}$$

Mean dia of particle  $d_p = 0.2 \mu m = 0.2 \times 10^{-6} m$

$$V_p = \text{Volume per particle}$$

$$= \frac{\pi}{6} d_p^3 = \left(\frac{\pi}{6}\right) (0.2 \times 10^{-6})^3 m^3$$

$$= 4.189 \times 10^{-21} m^3$$

$$\therefore \text{Mass of particle } m_p = \rho_p V_p$$

$$= 2300 \frac{kg}{m^3} \times 4.189 \times 10^{-21} m^3 = 9.6342 \times 10^{-18} kg$$

So we can find out the volume per particle. We note the volume per particle, if you find out the volume per particle then, I can multiply by the density of the particle to get the mass per particle. Already I know the extinction cross section per kilogram of the material. So now, by knowing the volume per particle then, I can calculate the mass per particle therefore, I can find out what is the cross section per particle. That is the idea of this calculation.

So, volume per particle is given by  $(\pi \times 6)$  into  $(d_p \text{ cube})$  that will be  $\pi$  by 6 into  $d_p \text{ cube}$  is  $(0.2 \text{ into } 10 \text{ to the power minus } 6)$  whole cube so this will be in  $\text{m}^3$  and this works out to  $4.189 \text{ into } 10 \text{ to the power minus } 21 \text{ m}^3$ . We are working with very small volumes, and the corresponding mass of the particle,  $m_p$  is the  $\rho$  of the particle,  $\rho_p$  multiplied by  $v_p$  and  $\rho$  is given as  $2300 \text{ kg by m}^3$  for carbon multiplied by  $4.189 \text{ into } 10 \text{ to the power minus } 21$  this will be  $\text{m}^3$  so that will give you the mass of each particle which will be  $9.6342 \text{ into } 10 \text{ to the power minus } 18 \text{ kg}$ . So we are talking about very small mass per each particle. And we already know the extinction coefficient. In fact we can also find out the number density of the particles. Because, we know the amount of material present per unit volume this is given in  $600 \text{ mg by m}^3$  so I know the volume mass of each particle therefore, I can find out the number of particles.

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Handwritten calculations on a blackboard:

$$\therefore \text{Mass of particle } m_p = \rho_p V_p = 2300 \frac{\text{kg}}{\text{m}^3} \times 4.189 \times 10^{-21} \text{ m}^3 = 9.6342 \times 10^{-18} \text{ kg}$$

Number density of particles:

$$n_p = \frac{\rho_s}{m_p} = \frac{600 \times 10^{-6}}{9.6342 \times 10^{-18}} = 6.228 \times 10^{13} / \text{m}^3$$

Extinction cross section per particle:

$$\sigma_p = \beta m_p = 5.1083 \times 9.6342 \times 10^{-18} \text{ m}^2 = 4.92 \times 10^{-17} \text{ m}^2$$

So simply given by  $n_p$ , the number of particles per  $\text{m}^3$  the number density is given by the density of the smoke divided by the mass per particle. So this  $\rho_s$  is  $600 \text{ into } 10 \text{ to the power minus } 6$  by  $9.6342 \text{ into } 10 \text{ to the power minus } 18$ . This will give you a number density of  $6.228 \text{ into } 10 \text{ to the power } 13$  by  $\text{m}^3$ . So, number of particles per  $\text{m}^3$  is actually a large number like  $10 \text{ to the power } 13$  that is almost  $10 \text{ to the power } 14$ . So now we can calculate the extinction cross section on per particle basis, it is simply given by  $\sigma$  or extinction coefficient per particle, this  $\beta$  which is per  $\text{kg}$  multiplied by the mass of each particle. So you see that you will get



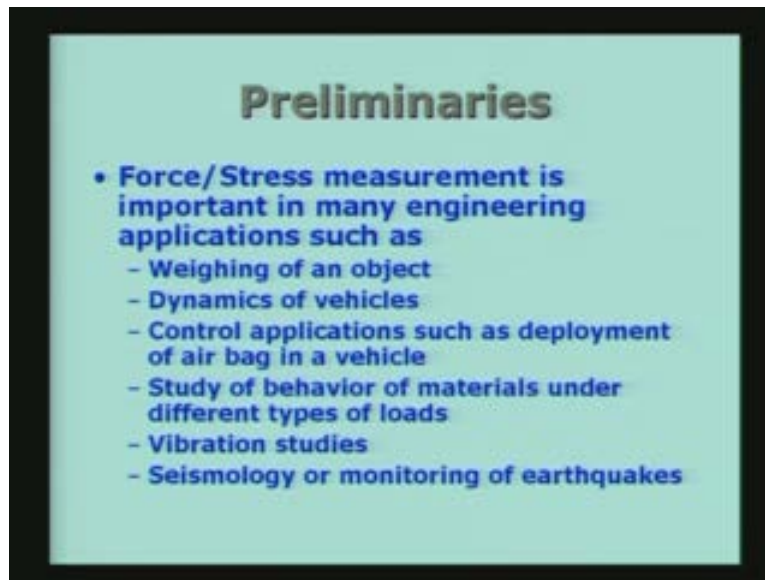
5.1083 multiplied by mass of each particle 9.6342 into 10 to the power minus 18 and this will be in m square by particle. So I need not mention per particle so beta square is the per particle basis and this gives you 4.92 into 10 to the power minus 17 m square.

The reason why I took up this problem is, just to give an idea as to the kind of numbers that describe the process we are talking about. When you have 40% opacity that means 60% is transmitted and if you assume the particles to be about 0.2 micrometer diameter the absorption cross section or extinction cross section per particle is only 10 to the power 5 into 10 to the power minus 17 m square. And because the number of particles is very large in m cube, you see that, the absorption is quite strong or the extinction is quite strong.

Now let us look at the measurement of various quantities like force, stress, torque, power and things like that. Let us look at the reason why we do that. There are many interesting important applications in engineering, where such measurements are required. Stress measurement is very important in the characterization of materials, studying the mechanical behavior of materials. Forces are involved in many mechanical systems, and for design or for characterization we require to measure forces. So let us look at the slide which shows some of the typical cases which we will meet with in practice.

Force or stress measurements are important in many applications such as weighing of an object. Just to weigh an object we require an instrument, a weighing machine, and it is usually a machine which actually uses force or balance of forces. The measurement of force and stress are also involved in dynamics of vehicles like four wheelers and two wheelers and so on and they are subject to lot of forces, because of the road surface the way the road surface varies, the shape and waviness and things like that. Therefore dynamics of vehicles means the study of forces which is subjected to because of various conditions prevailing when the vehicle is moving.

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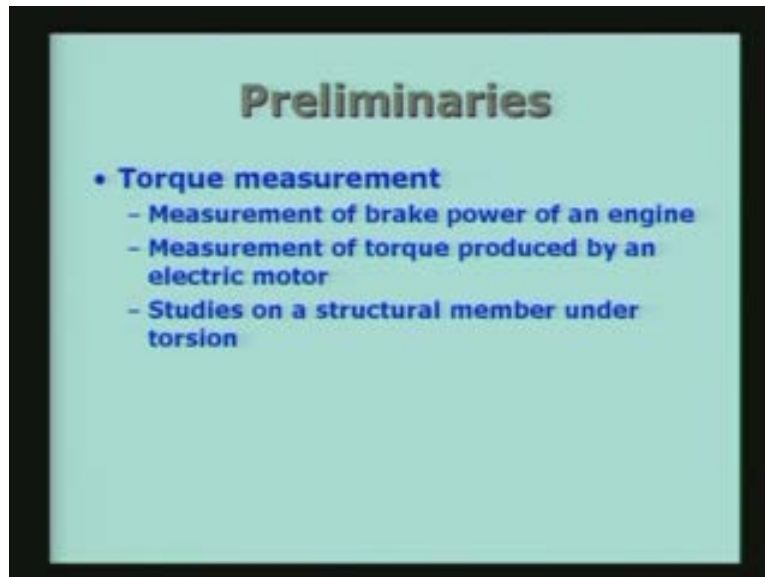
There may be control applications. For example, in the case of an automobile, for the safety of the passengers we have air bag system. Air bag is supposed to deploy as soon as there is an indication that there is likelihood or there is a collision which has taken place and the collision has to be actually sensed by a sensing element, and that sensing is done by measuring the force an element is subjected to, and if this force is more than certain value the air bag will deploy. This is the control application where the deployment of air bag takes place only if the force is more than certain value, and this is measured by a force transducer or a force measuring device. It is also an important application where it involves the safety of the passengers in the vehicle. Then we have study of behavior of materials under different types of loads.

For example, you may have a beam which is subjected to bending, you may have a ring element which is subjected to compression or expansion. For example, a pipe carrying a high pressure fluid inside will subject the wall of the pipe to stresses, and these stresses we need to measure or we need to find out what these stresses are, find out whether the safety is taken care of by seeing that the forces are less than, or the stresses are less than the allowable stresses for the material of the pipe and so on. Sometimes, we also have vibration because of time varying sourcing, and this vibration study involves the study of vibration forces in a particular system that is subjected to, and also the amplitude of these vibrations which the system responds to.



So we would like to measure both the vibration in terms of its frequency, in terms of forces it induces and also the accelerations because of that. In fact in seismology which is the study of earthquake to monitor the earthquake, we require a measurement instrument which will measure the acceleration or the forces which are going to be operative because of the earthquake. So, in all these cases, forces and stresses are going to be the measured quantities. Secondly, we are interested in torque measurement.

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And to just explain the measurement of torque let me just look at the definition of torque. If you have a rotating system, and if this is the radius, and if the rotation is due to some force so  $F$  into  $R$  is your torque. This is basically, the definition of the torque; the force applied on a rotating shaft for example, and if the force is applied with a momentum  $r$ . So the product of  $F$  and  $r$  is actually the torque. So we can see that we can measure the force and the radius separately and evaluate the torque or we can think of some arrangement by which the torque can be directly measured. So either measure independently or measure directly. Of course, direct measurement has some advantages.

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Handwritten notes on a blackboard:

$\sigma_p = m_p \cdot 9.6342 \times 10^{-18}$

Extinction cross section per particle:

$$\sigma_p = \beta m_p = 5.1063 \times 9.6342 \times 10^{-18} \text{ m}^2$$
$$= 4.92 \times 10^{-17} \text{ m}^2$$

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Torque

Measure directly

$$\vec{F} \times \vec{R} = \text{Torque}$$

↑ ↑ =

Independently measured

Diagram: A circle with a center point. A vector  $\vec{R}$  points from the center to the right. A vector  $\vec{F}$  points downwards from the right end of  $\vec{R}$ . A curved arrow indicates a counter-clockwise rotation.

Because the errors in measurement of  $F$  and  $R$  are going to translate into error in the measured quantity that can be avoided it by measuring the torque directly. So why do we measure the torque? If you look at the following if you have rotating machinery, the torque multiplied by the angular velocity actually gives the power. So the reason why we are interested in measuring the torque is, of course, because we would like to measure the power through the same.

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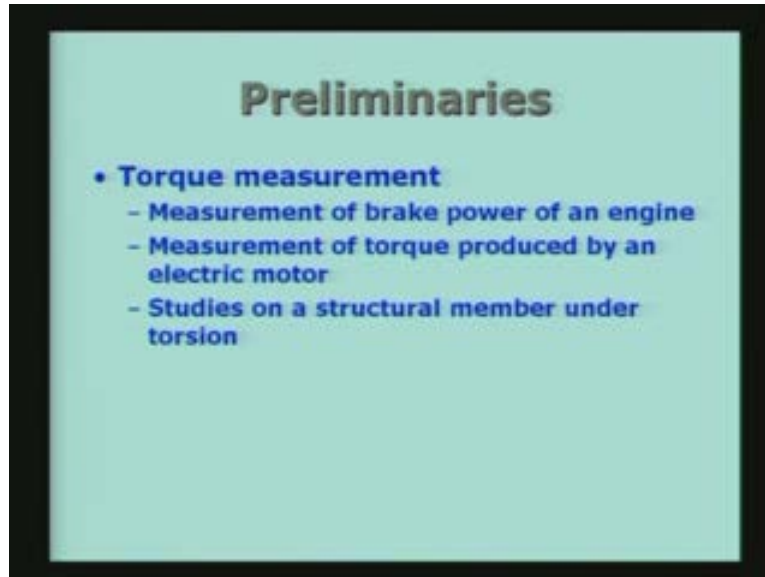
Handwritten notes on a blackboard:

Rotating m/c

$$\text{Torque} \times \text{angular velocity} = \text{Power}$$

The point is, the measurement of torque is related to the measurement of power and this could be in different cases.

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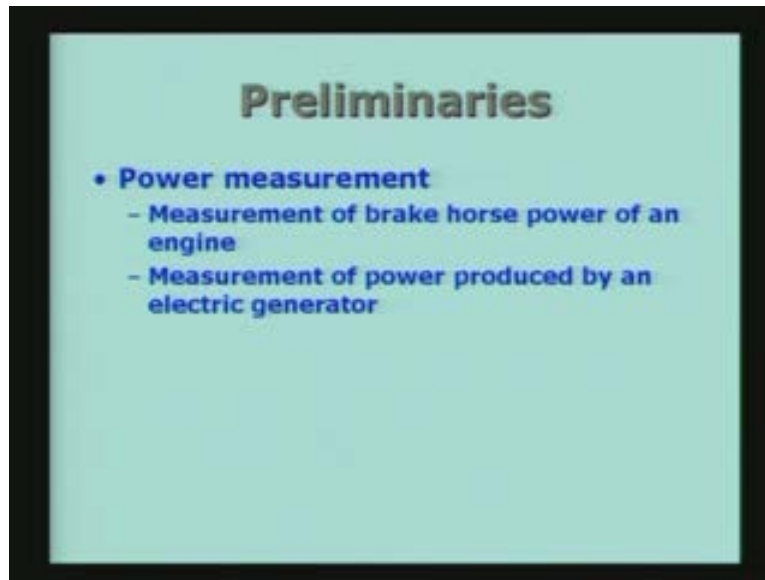


For example, measurement of brake horse power or brake power of an engine which is very common, or measurement of torque produced by an electric motor. Electric motor is used to run a pump or run some other apparatus, and you require knowing the torque produced by the electric motor, and we will have to measure it. It can even be studies on a structural member which undergo torsion. So torque measurement is required when you have a system where some structural member is under going torsion like for example, the shaft of an engine itself. The shaft of an engine is going to be transmitting power, and during this process it is going to under go torsion, and we would like to find out whether the torsional stresses set up in the shaft or below the allowable stresses for that material or how to characterize that. The next one we are going to be interested in is power measurement.

We have the important case of measuring of break horse power of an engine, and you know that always the specification of the engine is in terms of its rpm that is the highest speed it will be operated at, and also it will tell you what the horse power it will produce is. This is an important quantity which we have to measure. In a laboratory where engines are being tested one of the most important things we do is to test for the horse power or the power

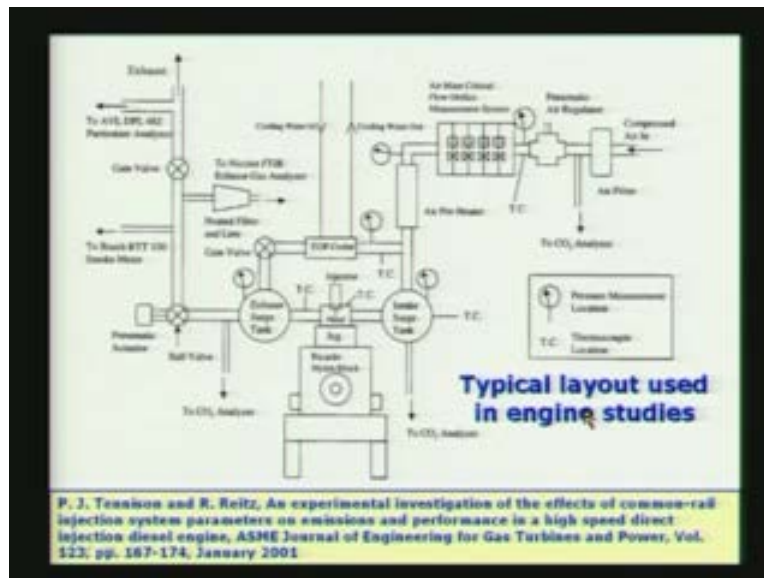
output of an engine and it is usually called the brake horse power. You may be interested in finding out what is the power generated or produced by an electrical generator for example. There are various ways for measuring these quantities.

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Here is an example. This is the typical layout used in engine studies. This is a paper taken from the ASME journal of engineering for gas turbines and power 2001 and the creator is Tennison and Reid. This is just an example of what are involved in a typical engine study. In the slide we show a typical layout used in engine studies. The study is basically on engine. Wherever TC is written, thermo couples are there to measure temperatures, and such a symbol here is a pressure gage. So, pressure measurement locations are also given. So you see that, large number of pressure measurements are involved, and large number of temperature measurements are involved, and apart from that a few other important things are present.

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For example, you have the smoke meter, exhaust is taken into a smoke meter where we are talking about the opacity measurement. You also have a particle analyzer to find out the diameter of particles the size distribution and the number and so on. The particle size measuring devices usually operate on the principle of scattering of radiation from the particles. And by looking at the pattern of the particles one can in fact get an idea of the diameter, distribution and so on. So this is the particulate analyzer, then we have an exhaust gas analyzer as you can see here this is an equally FDIR.

FDIR instrument is a Fourier transform infrared spectrometer. The spectrometer will give you a spectrum corresponding to the species present in it. And by looking at the magnitude or the size of these signals due to each one of the gases we can find out the concentrations. Therefore, you have a gas concentration measuring device here. And of course the cooling water is coming in and cooling water is going out, this is an exhaust gas recirculation cooler so some amount of gas is re-circulated. There are several processes that can be studied and one of them is to look at the EGR cooler or the Exhaust Gas Recirculation then they also have air pre heater, they have so many orifices, and so on to control or to measure the amount of various fuel and also the air used in running the engine and so on. Then we have the CO<sub>2</sub> analyzer both here as well as here.

So you see that, typically an engine laboratory a single engine with so many different instruments attached to it is a major item in any laboratory. It is a very expensive one, it contains a lot of instruments, and a lot of information can be obtained by doing the experiments with all these measurements taken simultaneously. For example, here there are so many pressure gages, pressure transducers, there are many thermo couples, and there are other devices to measure air mass, air flow and so on. All these require simultaneous measurement.

One cannot of course go on taking readings like in the olden times where we did that. One student will take the reading here, another student will take the reading here at the same time and so on and so forth. Now we do not do that. When we can afford to have a laboratory as good as this, we will have a data logger that means all the information from various instruments which are shown here are going to be locked in real time using a data logger. That means you have many channels and all these inputs go into the various channels of the data logger, and the output is recorded almost on a continuous basis with respect to time.

And the other advantage of that is, the data can be saved into a computer and the analysis can be done by using a program which you can develop or available programs and then you can analyze the data at leisure after conducting the experiment. Because the experiment itself is conducted in very short period, and the processing of data takes place later on. Previously we have been talking only about the measurement of pressure, measurement of temperature, measurement of flow rates and we have talked about radiation quantities, we have talked about thermo chemical properties but now we have really tried to bring everything together in one setup, where all the elements we discussed in isolation are coming together in a group.

So this is a system level experiment where lots of different instruments are used, and each instrument is going to give you a particular reading and how to integrate this. So the idea is to just to indicate what kind of a situation will prevail in real life. So with this background, you can see also that, we will be measuring the power output of the engine. For example, we may operate the engine at various speeds.

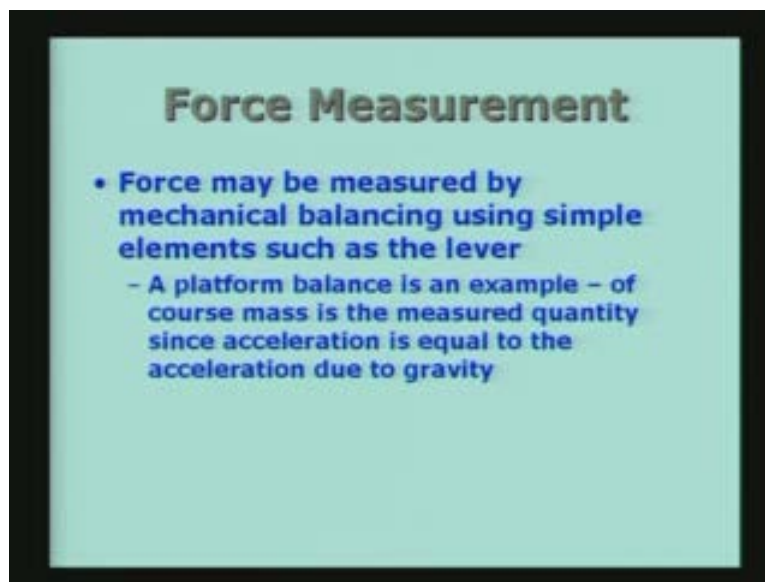
How do you do that?

You have to run this engine at various speeds, and also allow some output to vary by loading the engine. That means that we are going to subject the



engine to a load. A load is usually applied through what is called a dynamometer, and then with this as the quantity is of interest to us let us also know how these are going to affect the various parameters which we are measuring here. The various quantities we measure using these instruments here are linked to the output in terms of the speed and the load it carries or the power the engine produces. A typical layout for engine studies is what is shown here. In fact we can do it with any other system. This is taken from an engine side; you could also do it for a gas turbine laboratory or a steam turbine laboratory. Hence everywhere you can have a similar setup.

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Let us now look at force measurement. So the force may be measured basically, by mechanical balancing using simple elements such as the lever. Force measurement can be done when it is not a dynamic situation. When the situation involves force measurement it could be through equilibration or when situation is one of equilibrium so you balance the force for example, the measurement of weight of a body or the mass of the body.

Actually mass is going to be measured using a balance. So force may be measured by mechanical balancing using simple elements such as the lever and typical example is a platform balance, where what we are interested in measuring is actually the mass of the object or mass of the body but we use the forces generated because in the gravitational field any mass will experience a force proportional to the mass so it is the mass times the

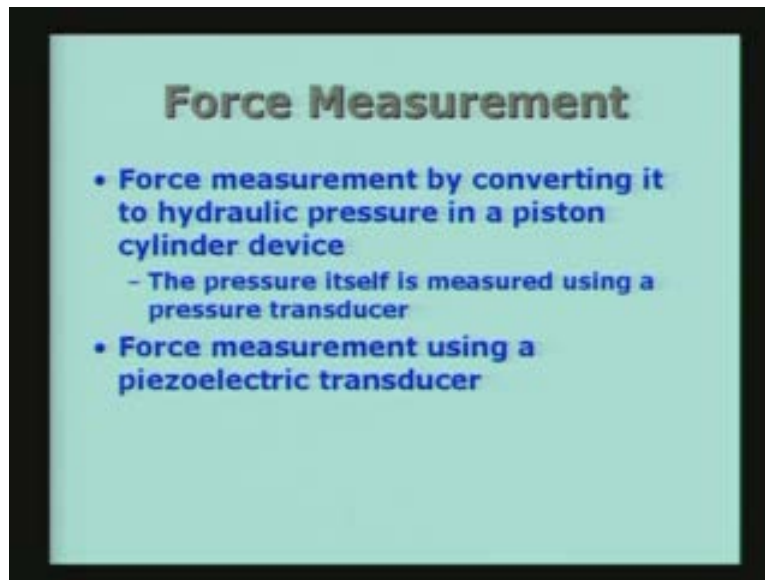
gravitational constant. So, if the apparatus is very small in size, the gravitational acceleration is more or less constant, and therefore that does not come into the picture so the mass can be directly measured using that.

The second thing is to use a transducer that transforms force to a displacement. This is a very common example, and is one of the simplest methods to be used. If you have a force, and if it is applied on a spring element and the spring element will undergo a change in length, it will undergo a displacement. For example, the spring element may be an actual spring as in a spring balance, or it could be an elastic member that undergoes a strain because of the force applied on it. The strain is nothing but the displacement, and the spring element is an elastic member, it may be in the form of a rod, or it may be in the form of a beam, or it may be in the form of a ring, so any of these elements are possible.

Therefore, the displacement is because the material behaves as an elastic material and when the strain is very small the material behaves as an elastic material, and the force or the stress is proportional to strength. This is called Hook's law. There is a linear relationship, between the applied stress and the strain which is generated because of the springiness of the element. It may be either a regular spring as in the spring balance or it could be a spring element in the form of a structural member.

Another technique which can be used to measure force is to convert it to hydraulic pressure. Convert it to hydraulic pressure. So, if I have a piston cylinder arrangement if I apply the pressure on the piston it will increase the pressure of the hydraulic fluid which is filling the cylinder.

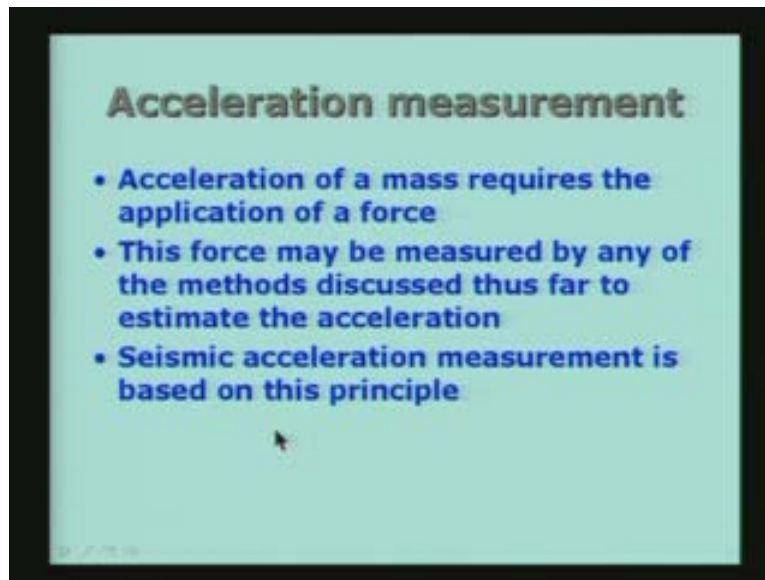
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So, if I have a piston cylinder arrangement, if I apply the pressure on the piston it will increase the pressure of the hydraulic fluid which is filling the cylinder. You apply the pressure or you apply the force on the piston and the force divided by the piston area is going to be the corresponding pressure and this pressure is going to be the pressure of the hydraulic fluid and these can be measured by using a pressure transducer.

In fact we have already seen different types of pressure transducers and we know how to use them. A pressure transducer can be used for the measurement of force by converting a force. Instead of converting it to a displacement as we did in the earlier case, here we are going to convert it to a pressure. It can also be measured by using what is called a piezoelectric transducer directly. When we are talking about force the obvious thing to talk about is also the acceleration. The measurement of acceleration is very important in many applications.

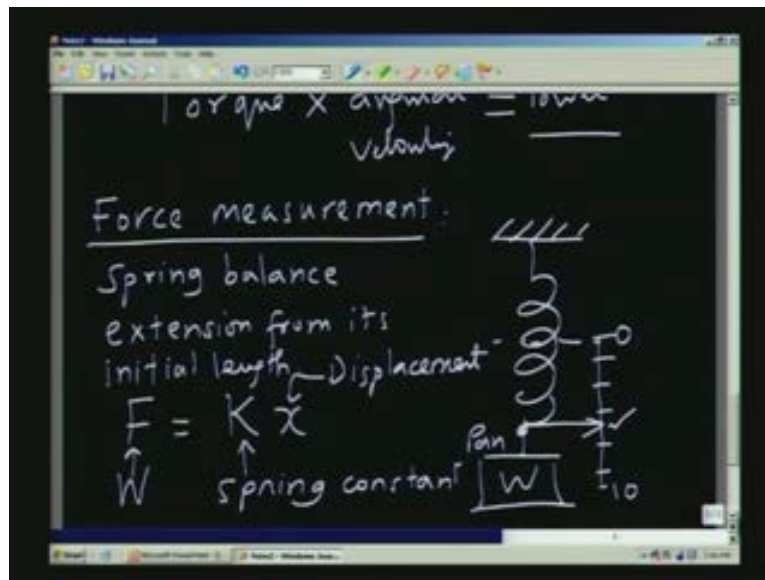
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The Newton's law of motion says that, force is equal to mass times the acceleration so it is based on this. Suppose, I have a mass and it is subjected to an acceleration it requires a force to do that. So, by measuring the force, and knowing the mass of the system the acceleration can be calculated. So that is one way of doing it. The force may be measured by any of the methods. Therefore the force measurement and the acceleration measurement is more or less inter related. For example, seismic acceleration measurement is based on this principle.

So we subject what is called a seismic mass to acceleration and this generates a force, this force is measured by a spring which is going to be displaced, because of this force acting on the seismic mass. So the measurement is actually the displacement of a spring, and that is equivalent to a force, and this force is equivalent to an acceleration multiplied by the mass, and knowing the mass I can find out what the acceleration is. This is a very simple way of doing it. Now let us look at typical cases, so let us discuss about force measurement using simple methods and then we will go on to force transducers and so on. We will discuss some of the issues.

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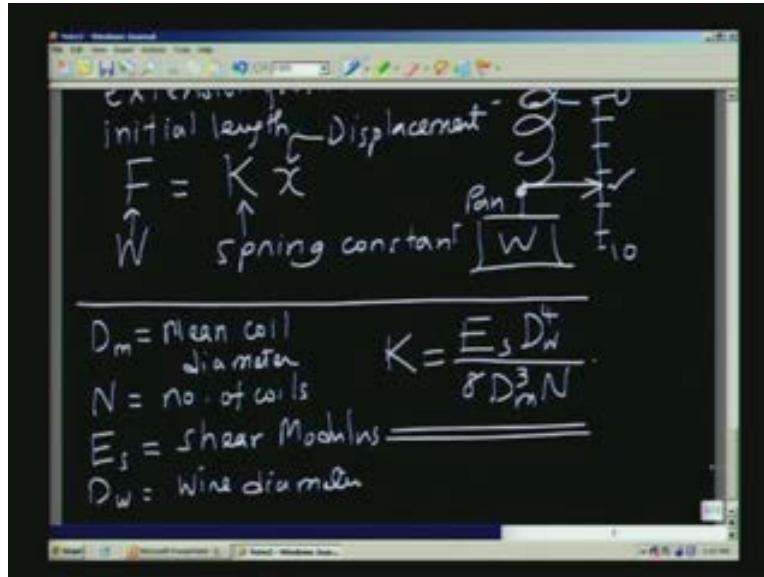
For example, I talked about spring balance. This is very commonly used. we have a support which is fixed and we have spring and then we have the W, the weight which is hanging from it and you can put some kind of a pointer here attached to the base of the spring and you have a scale here. This is 0 and may be this corresponds to 10 so this is the weight.

Therefore in spring balance what happens is, when there is no weight here it will be in this position an unextended position, it is extending quite a bit here so the spring extends from its initial length, the initial length means unloaded length, when there is no load on it, there is a certain length  $l$  and it extends by a certain amount, and we know that the spring has a linear relationship. Therefore  $F$  is equal to  $K$  into  $x$ , where,  $K$  is called the spring constant and  $x$  is the displacement. In this particular case, it is nothing but the weight of the object which is put in the pan, this is the pan.

When I said extension from its initial length, the initial length will mean when the pan is also present. So, the pan has its own weight and you adjust the zero such that with just the pan the position is zero, and from that onwards, whatever takes place is due to the weight of the object which is hanging from the support point from the string. Let us look at the spring and how it is made. The spring has got a certain material it has got a mean coil of diameter  $D_m$ , number of coils is  $N$ , and then we have the shear modulus sometimes the symbol  $G$  is used for that. Here I am just using shear modulus

$E_s$ , and the wire diameter is  $D_w$  so the spring constant  $K$  is given by the shear modulus ( $E_s$  into  $D_w$  power 4) by  $(8 D_m \text{ cube } N)$ . This gives you the spring constant.

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From this equation  $F$  is equal to  $K$  into  $x$ , we can see that this  $x$  is nothing but,  $F$  by  $K$  and the magnitude of the displacement is, inversely proportional to the spring constant. So if you want to have a reasonably a small value for the displacement, one would like to go for a large spring constant or another way if we can put it is the following. If you want to measure large forces, we will require large spring constant. That means the same spring balance I cannot use with infinite position, I cannot use it for measuring a very small force or a very small weight. I cannot also use it for measuring a very large weight. So the behavior of the spring depends on the spring constant, and the spring constant is to be chosen to cater to a particular range of the measured value of the force. If you want very large forces to be measured with a reasonable magnitude for the displacement, I will have to choose the  $K$  properly to suit that particular application.



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$F = K \propto \underbrace{x}_{\text{Magnitude}} = F/K$ . Coiled Spring

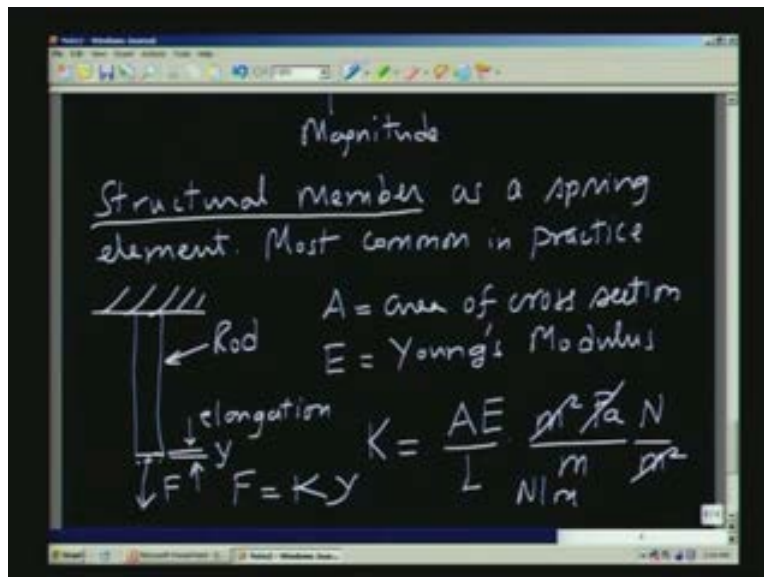
Structural member as a spring element. Most common in practice

And also you will see that, if you see this  $F$  is the force and  $K$  is this so the displacement is proportional to  $F$  by  $K$  and larger the spring constant the smaller the magnitude of  $x$ . Therefore, the sensitivity is also determined by this  $F$  by  $K$ . The spring constant is going to determine the sensitivity of the particular spring material. This was a coiled spring, what we showed earlier was a coiled spring made of a suitable material, and usually you should not allow the spring to extend too much, because it may undergo permanent deformation. So, if you subject it to a load which is too large, it may actually undergo a permanent set and therefore it becomes a useless instrument.

Therefore one has to remember that there is a certain maximum value of force or weight which you can measure with a particular spring. The second way of doing the same thing is to view the structural member as a spring element. This is by far the most common in practice, so what do we mean by a structural member? It could be of different types. For example, if I have an elastic element which is in the form of a small length of a rod, I have one end fixed rigidly, and I have a rod, maybe it is a circular cross section for example so I applied a force at the free end the force is applied to the end which is free. Now what will happen is that because of the force it will undergo a displacement so this will move slightly downwards just like in the case of a spring.

So this displacement the elongation if you want to call it the elongation is a measure of the force. So if I have the area of cross section is  $A$  and the Young's modulus is  $E$ , for the material of the rod, and if the deflection or elongation is  $Y$  the spring constant is nothing but area of cross section times the Young's modulus divided by the length  $AE$  by  $L$ . You can in fact verify this  $A$  is in  $m^2$ , Young's modulus is in Pascal by  $m$ , I can replace Pascal by  $N$  by  $m^2$ , this  $m^2$  is cancelled so  $N$  by  $m$  so  $F$  is equal to  $K$  into  $Y$ ,  $K$  is in  $N$  by  $m$  so it is a spring constant. So, for a structural member in the form of a rod of certain length  $L$  area of cross section  $A$  and the Young's modulus  $E$  the spring constant can be easily worked out as  $AE$  by  $L$ .

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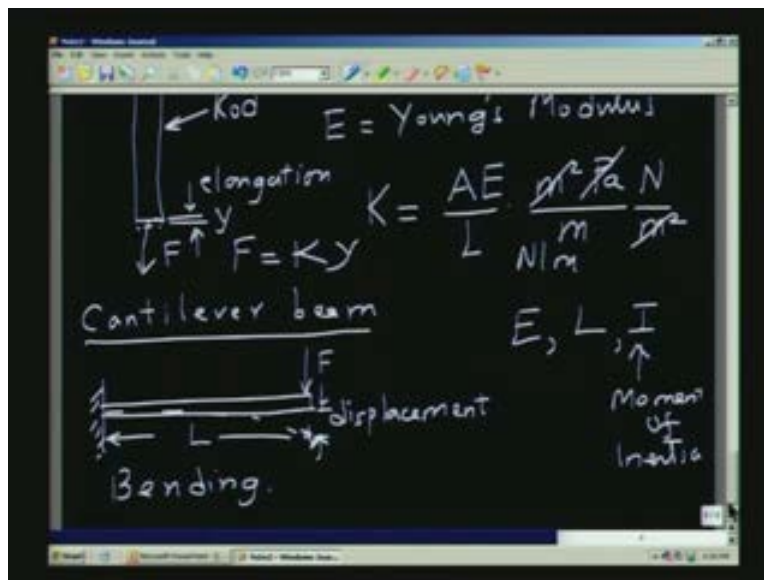


Now you can immediately see how we are going to design a simple element like this structural member which is in the form of a rod. You can choose the material because the particular material will determine the Young's modulus. So you can have different values of  $E$  there, and for getting different values for  $K$ , if I make it a larger cross sectional  $A$ , it becomes stiffer, because  $A$  is the stiffening mechanism. And if you make it longer you can that it is going to become the less stiff. So the  $A$  by  $L$  the area of cross section to length determines the value of  $K$ . Once you have determined the Young's modulus or chosen the particular material the area of cross section to the length that ratio is going to play an important role in

determining the value of K. Of course, in practice, we are talking about very small elongation we are talking about micro strains.

In fact when we talked about measurement of pressure by using a tube the pressure is applied inside the tube and you have a transducer a strain measuring element fixed on to the wall of the tube we were talking about very small strains, we were talking about micro strains. But here we are actually talking about micro strains. We are talking about very large forces being measured by using a very stiff member. Therefore you are going to have very small changes in the elongation. So, to measure a very small elongation we require a very sensitive measurement device. Let us look at another element which can be used for this purpose. The case is that of a cantilever beam which is very often used for measurement of forces.

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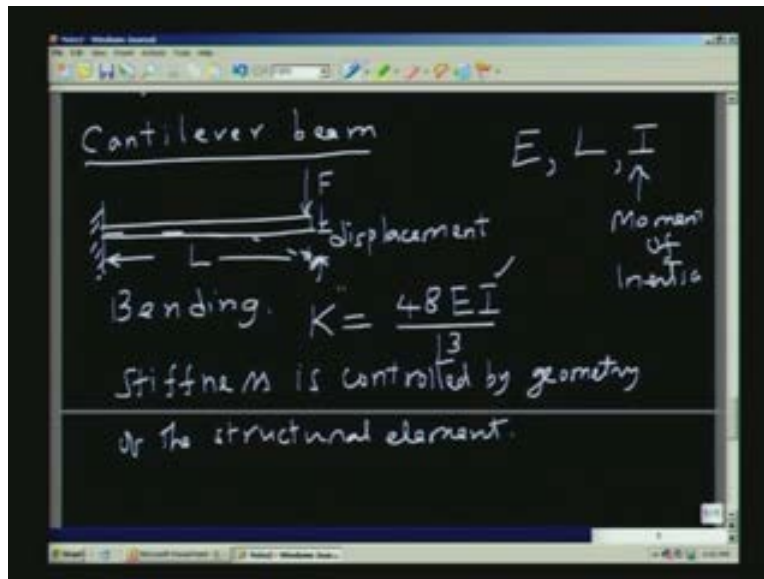


So what we have is a rigid support, a long rod of some suitable cross section you apply the force at the free end, the force is applied at the free end, and of course, it will bend like this, because there will be some displacement as it is shown here. Again in this case if the Young's modulus is  $E$ , and,  $L$  is the length of this cantilever beam, and if  $I$  is the moment of inertia of the cross section, so this is actually a case of bending and we know from our study that the bending of cantilever beam or bending of any element like this is that the moment of inertia of the cross section is going to play an important

role in determining the stiffness of the system. So  $I$  is the moment of inertia, we will call it the moment of inertia.

What I am showing is the case of cantilever beam, the Young's modulus  $E$ , the length of the beam, and the moment of inertia are going to determine the spring constant and the formula is  $K$  is equal to  $(48 \text{ into } E \text{ into } I) \text{ by } L \text{ cube}$ , the spring constant.

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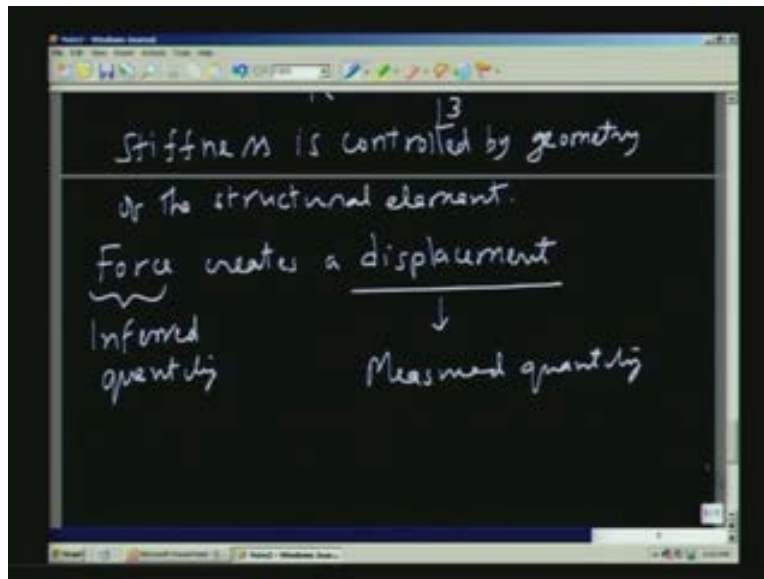


Again you see that the moment of inertia is going to determine, very clearly it is in the numerator, the longer the length the less stiff it is as you can see the length is in the denominator, and if we make it longer it becomes less stiff whereas if you distribute the material of the cantilever beam such that the moment of inertia is larger it is going to give you a larger spring constant.

Of course, once we have chosen the material we do not have any control over the  $E$ , and therefore it is only the geometric control, when I say stiffness, the stiffness means higher spring constant, so very stiff means spring constant is large. Stiffness is controlled by the geometric parameter. In fact it is true in the previous case also. Stiffness is controlled by geometry of the structural element. So, a cantilever beam can be used. In the cantilever beam case the force is determined by  $F$  is equal to  $KY$ , where  $Y$  is the displacement. So, displacement is going to be determined again by the force.

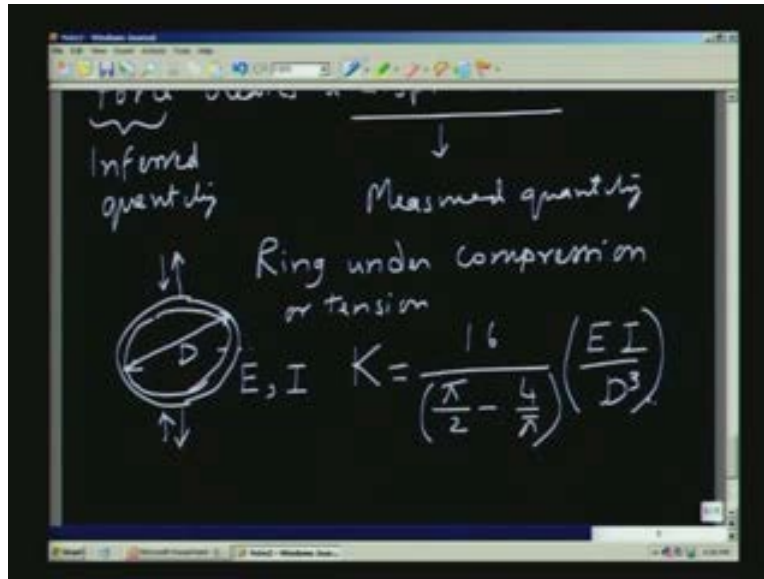
In all the examples you have seen so far the effect of the force is to bring about the displacement. So you can generally say that, force creates a displacement. And in our terminology of measurement, the displacement is the measured quantity. This is just to highlight that, this is the measured quantity, and this is the inferred quantity. So we are measuring force or inferring force, or estimating the force by measuring it's effect in terms of a displacement.

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This actually is a transducer as you can see. You are converting a force into a displacement, and therefore it is a transducer. Let us look at one last element. It is in the form of a ring of material and you can apply the force either in this fashion or in this fashion. You can either compress or extend it. So it is in the form of a ring under compression or tension.

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I am assuming that the ring is a small cross section compared to the diameter it is very thin therefore I will take this as the diameter and if you take the diameter of the ring and E the Young's modulus and I is the moment of inertia of the section about its central axis you can see what is going to happen.

If you apply a force like this that is if you take a section here it is undergoing bending. Therefore the formula which we are going to have is similar to what we had earlier the K value for this particular case works out to 16 by pi by 2 minus 4 by pi into EI by D cube. So what is common in all the three or four cases we have considered now is that force is now converted to a displacement and displacement is measured and from this you work backwards knowing the spring constant of the particular system either a regular spring or one of the structural members we can find out what the force is. Thank you.