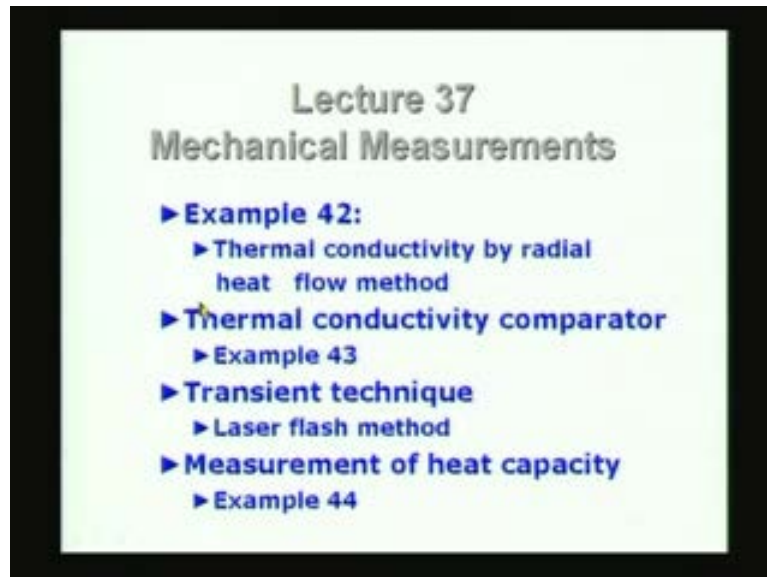


Mechanical Measurements and Metrology
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Module - 4
Lecture - 37
Measurement of Thermal Conductivity

This will be lecture number 37 on Mechanical Measurements. Towards the end of the last lecture we were looking at the measurement of thermal conductivity by the use of radial heat flow apparatus. What we will do in this lecture is start with an example based on data taken from a thermal conductivity apparatus, which is in the radial heat flow method. This will be followed by the discussion of thermal conductivity comparator and example 43. Then let us look at transient technique, the laser flash method which actually measures directly not the thermal conductivity, but the thermal diffusivity.

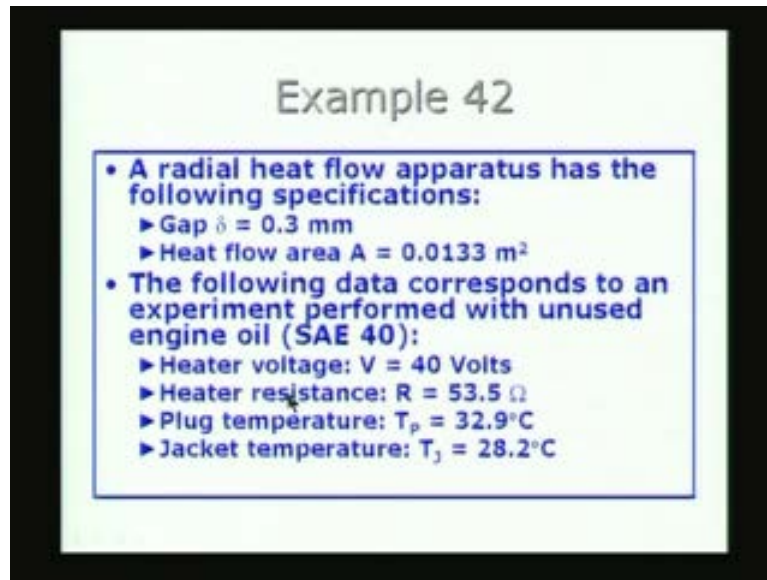
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Then we will say how thermal conductivity can be estimated from these measurements. The example number 42 is considering the data taken from a radial flow apparatus and the delta, the gap between the inner cylindrical plug which is heated, and the outer annular cylinder across which the heat is going to be transferred through a sample which is kept in the gap has a thickness of 0.3 mm which is a very narrow gap between the inner and outer cylinder. The heat flow

area has been measured by measuring the diameter of the plug and the length accurately and it is given as 0.0133m square and the data actually corresponds to an experiment that was performed with unused engine oil. The engine oil itself is characterized by SAE 40

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Example 42

- A radial heat flow apparatus has the following specifications:
 - ▶ Gap $\delta = 0.3 \text{ mm}$
 - ▶ Heat flow area $A = 0.0133 \text{ m}^2$
- The following data corresponds to an experiment performed with unused engine oil (SAE 40):
 - ▶ Heater voltage: $V = 40 \text{ Volts}$
 - ▶ Heater resistance: $R = 53.5 \Omega$
 - ▶ Plug temperature: $T_p = 32.9^\circ\text{C}$
 - ▶ Jacket temperature: $T_j = 28.2^\circ\text{C}$

And it is taken from the new can directly and not used earlier. The experiment was conducted by supplying a heater voltage of 40V and heater has the resistance of 53.5 ohm measured accurately. Actually, in the particular case, the manufacturer of the equipment specifies the resistance of the cartridge heater. The plug temperature is measured and the jacket temperature is also measured and the two measurements are given as 32.9 for the plug temperature, and the jacket temperature is 28.2. Therefore you see that the difference in temperature between these two are the driving potential for the heat transfer.

So we are asked to find out (Refer Slide Time: 03:57) the thermal conductivity of the oil sample. And also, we would like to find out the errors in the measurement, we are interested in finding out the error bar on the thermal conductivity given that the following uncertainties in the values which are measured are given. The delta V, the voltage is measured with plus or minus 0.5V which may be improved with better apparatus. But in this case plus minus 0.5V and delta T is known to be within plus minus 0.2 degree Celsius.

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Example 42
(Continued)

- What is the thermal conductivity of the oil sample?
- If the measured parameters have the following uncertainties what will be the uncertainty in the estimated value of the thermal conductivity?
 - ▶ $\Delta V = \pm 0.5 \text{ V}$
 - ▶ $\Delta T = \pm 0.2^\circ\text{C}$

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Example 42
(Continued)

- Assume that the heat loss is a function of $\theta = T_p - T_j$ given by (with an uncertainty of $\pm 5\%$)

$$L = 0.0511 + 0.206 \theta + 0.0118 \theta^2 - 0.000153 \theta^3$$

Solution is worked out on the board

We also are given that a separate experiment has been performed, to determine the heat loss as a function of the temperature difference between the plug and the jacket. We can determine the heat loss as a function of the temperature difference by performing experiment with a medium whose thermal conductivity has been or is known very accurately. So, one easy way of doing that is to perform the experiment with air within the narrow gap between the inner and the outer cylinder and we know the thermal conductivity of air. Therefore for different values of the

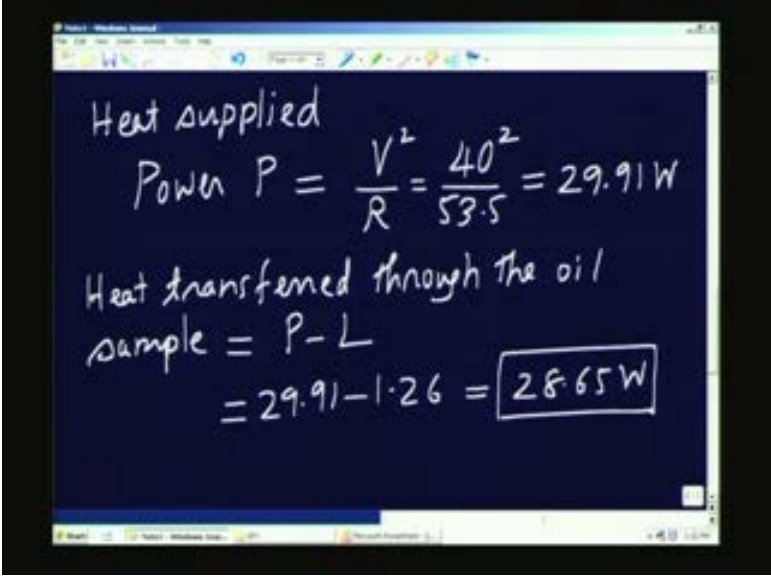
temperature difference between the plug and the jacket we can actually estimate the heat loss by assuming that the thermal conductivity of the medium is known accurately.

So, an experiment of that type has been performed and the loss has been indicated as a function of theta. This expression is given by a cubic in this particular case and the theta can be anywhere between half a degree and 10 degree that is the validity of this expression. Therefore with this background, let us work out the solution to the problem of the determination of the thermal conductivity as well as the estimation of the error in the measured value of the thermal conductivity. We are given the voltage supplied to the heater as 40V plus or minus 0.5 this will be the error bar, I am combining it here. We are also given that ΔT is nothing but the plug temperature minus the jacket temperatures and these two are given as 32.9 minus 28.2 these are the two measured values. This will give you 4.7 degree Celsius and each of the temperatures has got a plus or minus 0.2 degree Celsius uncertainty.

This is also specified in this problem, and now, I can write down the expression for the loss which is a function of theta which is nothing but theta in that expression, all I have to do is to substitute this into that expression and I will get 0.0511 plus 0.206 into 4.7 plus 0.0118 into 4.7 square minus 0.000153 into (4.7) cube and this comes to 1.26W heat loss because of the temperature difference is equal to 4.7 degree between the plug and the jacket is 1.26W.

Now we will calculate the heat supplied to the heater, the heat supplied under the steady state. Of course, this amount of heat supplied has to flow through the specimen oil sample or some of it lost from parasitic losses through other channels. Therefore the heat transfer through the sample, is the difference between the heat supplied and the loss. So heat supplied can be calculated with the electric power given by V^2 divided by R . This will be 40 square divided by the resistance of the 53.5 ohms and this gives you 29.91W.

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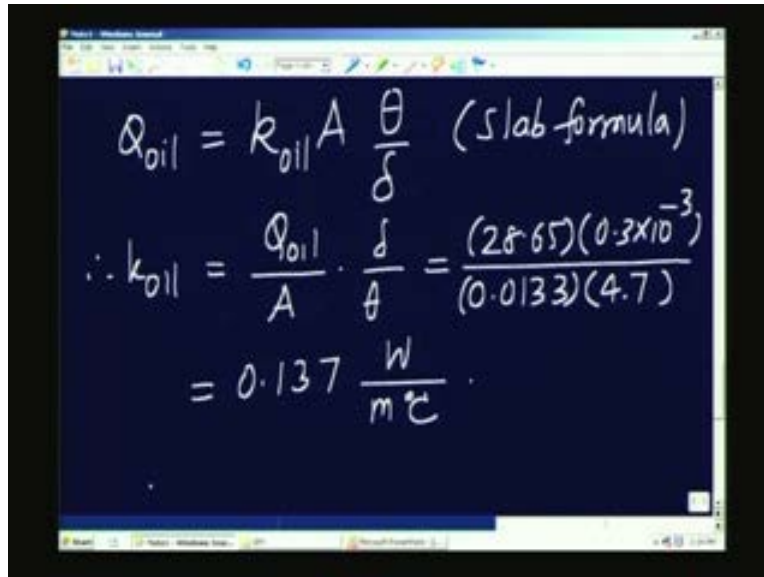


Heat supplied
Power $P = \frac{V^2}{R} = \frac{40^2}{53.5} = 29.91 \text{ W}$
Heat transferred through the oil sample = $P - L$
 $= 29.91 - 1.26 = 28.65 \text{ W}$

So, we can say that, heat transferred through the oil sample which will be difference between P and L will be, 29.91 minus 1.26 and that gives you 28.65W. So we have the heat transfer through the oil sample, and we can now calculate the thermal conductivity of the oil. So, if you look at the thermal conductivity of the oil, the Q passing through the oil Q transfer through the oil must be equal to k of the oil into the area or heat flow into theta and the temperature difference divided by delta, because the delta is very small compared to the diameter of the plug.

We are using the slab formula because, the thickness of the oil film is very small compared to the diameter of the plug and therefore it is almost like a planar slab. Therefore, I can say that, k of oil is equal to Q by A into delta by theta and all I have to do is substitute all the values. This will be 28.65 into delta is 0.3 into 10 to the power minus 3 mm. So I have to convert it to meters. So it will be 10 to the power minus 3 by 0.0133m square that is the area available for the heat flow into 4.7 degree Celsius. So this will work out to be 0.137W by m degree Celsius. This is the first answer we require, the thermal conductivity of the engine oil is 0.137W by m degree Celsius.

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$$Q_{oil} = k_{oil} A \frac{\theta}{\delta} \quad (\text{slab formula})$$
$$\therefore k_{oil} = \frac{Q_{oil}}{A} \cdot \frac{\delta}{\theta} = \frac{(28.65)(0.3 \times 10^{-3})}{(0.0133)(4.7)}$$
$$= 0.137 \frac{W}{m^{\circ}C}$$

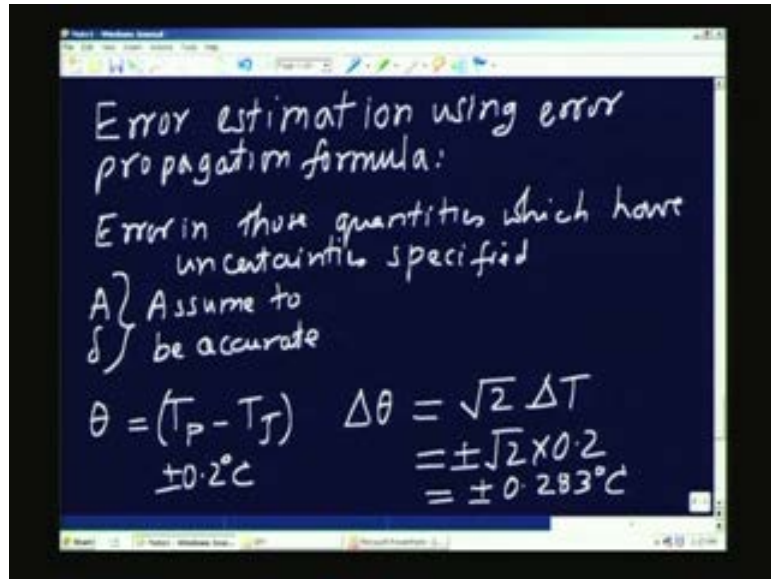
The second part of the problem, requires a calculation of the various uncertainties and the effect of these uncertainties on the value of k which is determined here. So, for that what we have to do is, we have to do an error estimate using error propagation theory. And you also see that, whatever quantities are specified, an uncertainty only will be taken into account. For example, error in area is not taken into account, δ we are not going to take into account these we are going to assume to be highly accurate, so that the error is not going to be due to any of these. Error in those quantities which have uncertainty is specified in the problem.

For example, if we take θ , if we difference between T_p and T_j each one is individually having plus or minus 0.2 degree Celsius and you will immediately see that $\delta \theta$ or the error in θ is nothing but square root of 2 times error in any one of these temperatures. This can be verified easily, and therefore, this will be plus or minus square root of 2 into 0.2 degree Celsius and that comes to 0.283 degree Celsius. This is the error expected in the value of $\delta \theta$ because each of the temperature measure is prone to an error the error in $\delta \theta$ is now equal to square root of 2 times the error in each one of the individual temperature measurements.

We also know that Q is equal to P minus L and this has got a plus or minus 5% specification, P has an error because we have an error in the value of the voltage which is measured. We are again assuming that the resistance has no error specified

on that so we have Q is equal to (V square by R) minus L. Therefore, I can find out the partial derivatives, and, $\frac{dQ}{dL}$ is equal to minus 1.

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Handwritten notes on a digital screen:

Error estimation using error propagation formula:
 Error in those quantities which have uncertainty specified

A } Assume to
 δ } be accurate

$\theta = (T_P - T_J) \quad \Delta\theta = \sqrt{2} \Delta T$
 $\pm 0.2^\circ\text{C} \quad = \pm \sqrt{2} \times 0.2$
 $= \pm 0.283^\circ\text{C}$

Now, I can use the error propagation formula, substitute values of the voltage and the resistance and therefore delta Q is equal to plus or minus square root of....., we are going to use the error propagation formula whole square and we can put it here $\frac{dQ}{dV}$ is equal to $2V$ by R , (2 into 40 by 53.5), delta V is 0.5 whole square plus $\frac{dQ}{dL}$ is minus 1 into delta L , is 5% that means we have to put it as 5 by 100 into 1.26 which is the value we have identified earlier as the loss which gives you plus or minus 0.75W there is an error the uncertainty of the measured value of the heat transfer through the sample of plus or minus 0.75W because of the error in the measured voltage, as well as the error in the estimated value of the heat loss from the system.

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$$Q = \frac{P}{L} \quad Q = \frac{V^2}{R} - L$$

\uparrow \uparrow
 Error $\pm 5\%$
 in V

$$\frac{\partial Q}{\partial V} = \frac{2V}{R} \quad \frac{\partial Q}{\partial L} = -1$$

$$\Delta Q = \pm \sqrt{\left(\frac{\partial Q}{\partial V} \cdot \Delta V\right)^2 + \left(\frac{\partial Q}{\partial L} \cdot \Delta L\right)^2}$$

$$= \pm \sqrt{\left(\frac{2 \times 40}{53.5} \times 0.5\right)^2 + \left(-1 \times \frac{5}{100} \times 1.26\right)^2} = \pm 0.75W$$

So now we have all the things we require to determine the error in k. So we can just go back to the expression and see k of oil is Q passing through oil divided by the area available for heat flow, multiplied by delta divided by theta, A and delta does not have any error in them this is also in the form of product of quantities divided by product of quantities. So I can use the logarithmic differentiations. Therefore delta k I will drop the subscript you know we are talking only about the k of oil delta k by k is equal to plus or minus (square root of delta Q by Q) whole square plus (delta theta by theta) whole square and we have determined all the quantities earlier. So this will be plus or minus square root of delta Q is 0.75 by 28.65 which is the estimated value of Q plus delta theta by theta is (0.283 by 4.7) whole square this will give you if you want you can multiply by 100 you can get the percentage value so this will be 6.6%.

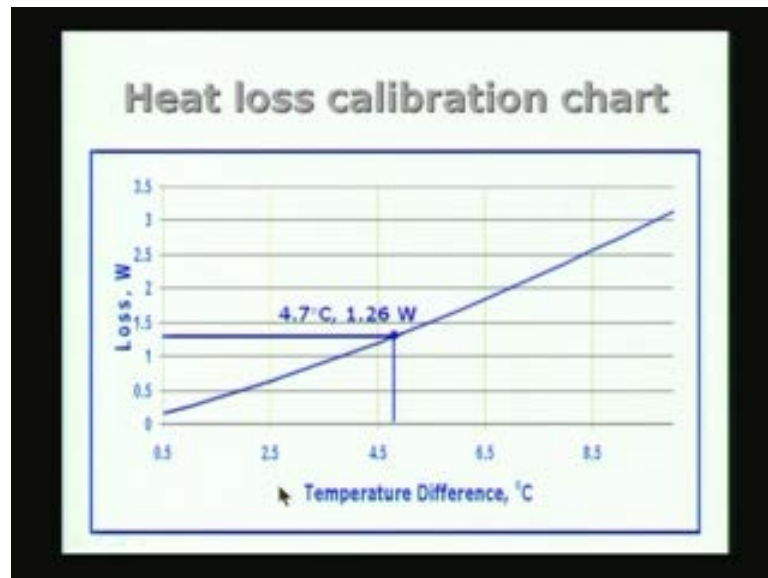
The answer is k_{oil} is equal to 0.137, which was determined earlier, plus or minus 6.6% of this value, which works out to 2.009W by m degree Celsius. In evaluating the error bar or the uncertainty of the thermal conductivity of the oil, we have taken into account the errors or the uncertainties specified in the value of Q and theta, and all other values have been assumed to have no answer of this because they are not specified in this problem.

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The image shows a digital screen with handwritten mathematical derivations. The first equation is $k_{oil} = \frac{Q_{oil}}{A} \frac{\delta}{\theta}$ with the note "(logarithmic differentiation)". The second equation is the relative uncertainty formula: $\frac{\Delta k}{k} = \pm \sqrt{\left(\frac{\Delta Q}{Q}\right)^2 + \left(\frac{\Delta \theta}{\theta}\right)^2}$. The third equation shows the numerical calculation: $= \pm \sqrt{\left(\frac{0.75}{28.65}\right)^2 + \left(\frac{0.283}{4.7}\right)^2} = \boxed{\pm 6.6\%}$. The final equation is the result for k_{oil} : $k_{oil} = [0.137 \pm 0.009] \frac{W}{m^2 \cdot ^\circ C}$.

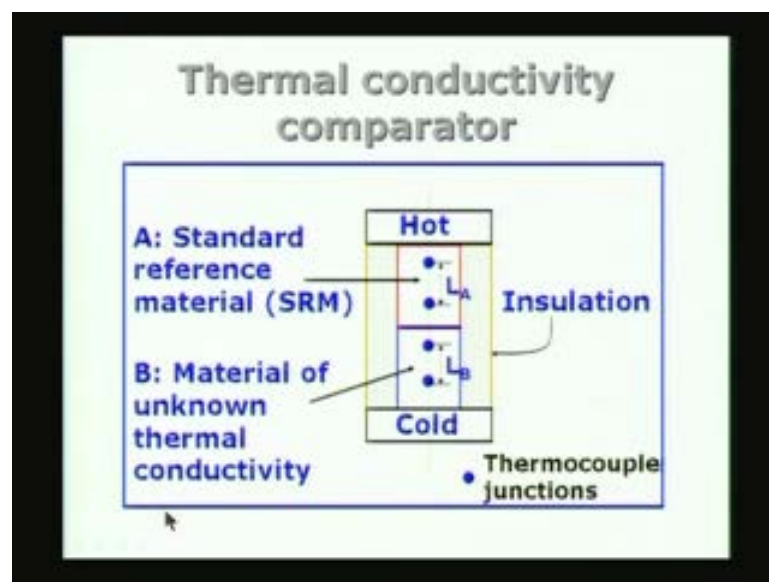
Let us look at the loss term, which is given as a function of theta and we can see that, I have plotted the function loss verses the theta which was the cubic expression which was given and in the specific problem we have considered the temperature difference as 4.7 and the corresponding value of the loss is 1.26. And if you remember, the value of delta Q the uncertainty in the value of measured Q is about plus or minus 0.75W and you can see that the loss here is even larger than the uncertainty which is specified in the problem. Therefore it is essential that the loss which is about 1.26W in this particular case, is taken it account because we do not do that and it will incur very large error in the value of k which is determined even though the k itself has got an error of about 6.6% because of the measured uncertainty in the quantity.

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Of course, that includes also uncertainty in the loss. When we calculate the uncertainty you have taken into account the uncertainty in loss. So with this, we will take a look at another way of measuring thermal conductivity using what is called the thermal conductivity comparator.

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It is very useful when you have the sample available in the form of rods that is cylindrical specimens are available and if you have the Standard Reference Material

SRM, whose thermal conductivity has been documented extensively and available in the literature, then we take that Standard Reference Material as one of the materials and prepare as shown in this particular figure. Take a specimen B of material whose thermal conductivity is not known, and the same diameter as the standard reference material put them together, stack them like this one above the other, and insulate on the lateral side so that there is no heat loss and you subject it to a temperature difference as high temperature of the top and low temperature of the bottom.

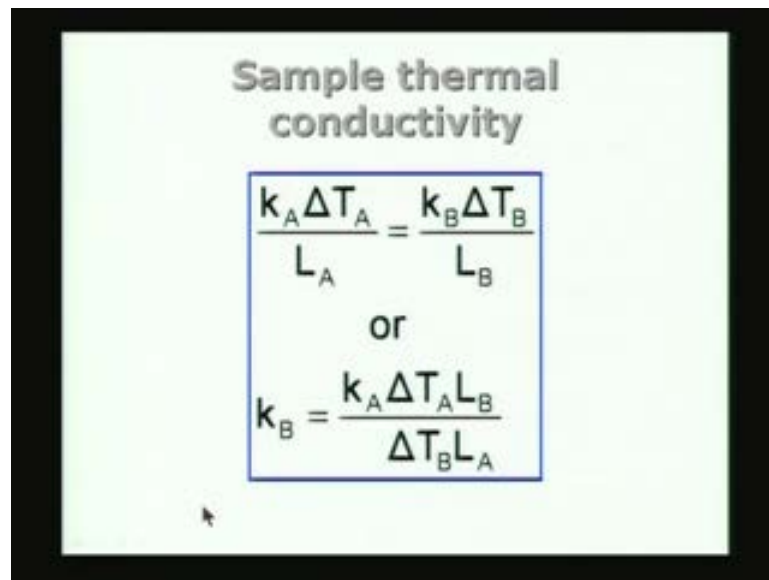
Now, what we are doing by the thermal conductivity comparator is to place the two materials. The Standard Reference Material whose thermal conductivity is known and the material of the thermal conductivity is to determine in series and the heat transfer which occurs through this Standard Reference Material is exactly the same as the heat transfer that take place through the material whose thermal conductivity is to be determined. So the common factor in these two cases the Standard Reference Material and the thermal material b is that the same amount of heat is transferred through the sample as well as the Standard Reference Material assuming that there are no losses in the lateral direction impact instead of insulation.

We can in fact, use vacuum as a possible choice because vacuum will give you a good insulation. And of course the vacuum chamber or the walls of the chamber can be maintained at a temperature which is close to the temperatures of these two media here. These two are the samples of the reference material. If you can maintain a small differential temperature between these two and the walls of the vacuum chamber heat loss of the latter direction can be reduced significantly. Therefore whatever heat transfer takes place through the SRM also takes place through this. So the common factor is the heat transfer which is the same in these two cases and we invoke the Fourier law by measuring the temperature at these four points as indicated and these are the thermocouple junctions.

The distance between the first two thermocouples in the SRM is L_A and the case of the material of unknown thermal conductivity is L_B . L_A and L_B are accurately known and of course, I have shown this thermocouple junction to be very big. In practice, they are going to be very thin and very small junctions. They may not be embedded inside the solid, but it may be at the surface. If necessary it is easier to attach at the surface and measure the temperature. You need not drill a hole, otherwise you will have to do that. And of course drilling the hole will probably change the heat transfer pattern slightly. Actually it is not a very desirable thing so what we will do in this thermal conductivity comparator is measure the four temperatures here, here, here and here.

From these two I calculate the ΔT across the length L_A , I calculate across the length L_B and using the fact that the same amount (Refer Slide Time: 25:20) of heat is transferred across the first SRM as well as the sample I can write $k_A \Delta T_A$ by L_A , ΔT_A is measured by two thermocouples, L_A is known, similarly is equal to $k_B \Delta T_B$ by L_B and we rearrange it such that k_B is equal to k_A into $(\Delta T_A L_B$ by $\Delta T_B L_A)$. So all I have to do is to know the thermal conductivity material which is the Standard Reference Material then I measure the two temperature differences, and the two lengths, and I am able to estimate the thermal conductivity of the material whose thermal conductivity is unknown.

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Sample thermal conductivity

$$\frac{k_A \Delta T_A}{L_A} = \frac{k_B \Delta T_B}{L_B}$$

or

$$k_B = \frac{k_A \Delta T_A L_B}{\Delta T_B L_A}$$

So we are comparing the performance of the two materials by imposing the same amount of heat transfer through each one of them.

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Sample thermal conductivity

$$\frac{k_A \Delta T_A}{L_A} = \frac{k_B \Delta T_B}{L_B}$$

or

$$k_B = \frac{k_A \Delta T_A L_B}{\Delta T_B L_A}$$

Here is an example. The thermal conductivity comparator uses a Standard Reference Material whose thermal conductivity is known to be 45 plus or minus 2%W by mKelvin. Here there are two thermocouples and there is high temperature a high thermal conductivity material we are talking about and therefore we will also have to use a relatively large length for the specimen to get adequate temperature drop. So the two thermocouples are placed at 22 plus or minus 0.25, this plus or minus 0.25, is the uncertainty in the location of the thermocouple which indicates a temperature difference of 2.5 plus or minus 0.2, again this is the uncertainty of the thermal temperature difference which is measured.

The material of unknown thermal conductivity is in series with the SRM as indicated in the comparator and indicates the temperature difference of 7.3 plus or minus 0.2Kelvin across the length of 20 plus or minus 0.25 mm, the 22 and 20 so determine the thermal conductivity of the sample and its uncertainty. Here the same amount of heat transfer takes place through the two of them the SRM as well as the sample. That means there is no heat loss in the lateral direction. In fact, in this case the contact between the two of them need not be very perfect, because we are not measuring the over all heat transfer. The temperature difference is measured, the heat transfer itself we need not measure therefore the heat transfer rate is not measured. We are measuring only the temperature differences and the lengths.

That is why this is a very interesting method, because the heat transfer itself is not measured but only the ratios that are required, ΔT_A , ΔT_B , L_A , and L_B these

are the ratios. In fact any comparator or any property measurement will involve only the ratio that is the basic idea. So this will be example 43. All I have to do is to use this expression given in the slide. We know the quantities L_A is 22 mm. If you want you can keep all of them in millimeters because we are going to only take ratios. If you want you can convert it into meters because only ratios are going to come and only if the value of k of the Standard Reference Material is given in the SI units the k determined here will also come out to be in SI units because all others are ratios only. This will be 22 mm, L_B is 20 mm, then we have k_A which is 45 W by m degree Celsius, ΔT_A is measured as 2.5 degree Celsius or Kelvin it does not matter.

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Example 43

$$L_A = 22 \text{ mm} \quad L_B = 20 \text{ mm}$$

$$k_A = 45 \frac{\text{W}}{\text{m}^\circ\text{C}} \quad \Delta T_A = 2.5^\circ\text{C}$$

$$\Delta T_B = 7.3^\circ\text{C}$$

$$k_A \frac{\Delta T_A}{L_A} = k_B \frac{\Delta T_B}{L_B} \text{ or } k_B = \frac{k_A \Delta T_A L_B}{\Delta T_B L_A}$$

$$k_B = \frac{(45)(2.5)(20)}{(7.3)(22)} = 14.01 \frac{\text{W}}{\text{m}^\circ\text{C}}$$

The temperature difference is the same. When you represent temperature difference it can be either Kelvin or degree Celsius. And we also have measured ΔT_B of the second material, as 7.3 Kelvin or degree Celsius. So all I have to do is to use the simple rule, $k_A \Delta T_A$ by L_A is equal to $k_B \Delta T_B$ by L_B or k_B is equal to $k_A \Delta T_A L_B$ by $\Delta T_B L_A$, all I have to do is to substitute these values.

Therefore k_B is equal to 45 into 2.5 into L_B is 20 by 7.3 into 22, so this will give you a value of 14.01 W by m degree Celsius, it can be easily visualized, the L_A and L_B are very close to each other, 20 mm and 22 mm is more or less equal but slightly different. The ΔT_A is smaller than ΔT_B therefore we expect the temperature thermal conductivity of second specimen to be lower than that of the Standard Reference Material which is very clear now. Now, to calculate the uncertainty, I

have to use this expression. It is k_B is equal to k_A into ΔT_A by L_A into L_B by ΔT_B .

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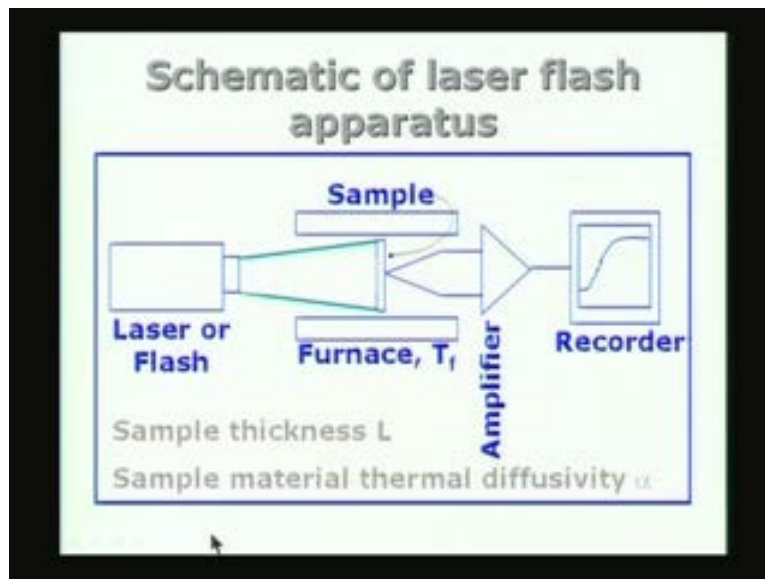
The image shows a handwritten derivation on a digital screen. The first line defines k_B as $k_A \cdot \frac{\Delta T_A}{L_A} \cdot \frac{L_B}{\Delta T_B}$, with a note that k_A has a $\pm 2\%$ error. To the right, it specifies $\Delta T's \rightarrow \pm 0.2^\circ C$ and $L_A, L_B \rightarrow \pm 0.25$. The second line shows the logarithmic differentiation formula for the relative uncertainty in k_B : $\frac{\Delta k_B}{k_B} = \pm \sqrt{\left(\frac{\Delta k_A}{k_A}\right)^2 + \left(\frac{\Delta \Delta T_A}{\Delta T_A}\right)^2 + \left(\frac{\Delta L_B}{L_B}\right)^2 + \left(\frac{\Delta \Delta T_B}{\Delta T_B}\right)^2 + \left(\frac{\Delta L_A}{L_A}\right)^2}$. The third line calculates the final result: $\Delta k_B = \pm \frac{1.24}{\sim 9\%} \frac{W}{m^\circ C} \cdot k_B = (14.01 \pm 1.24) \frac{W}{m^\circ C}$.

So all I have to do is, to determine the error bar on this or uncertainty of k_B . Of course, I assume that k does not have any error, or it is given with the 2% error. So let us use that value so this has got 2% error ΔT_A and ΔT_B are both measured with ΔT that has an error of 0.2 degree plus or minus degree Celsius and L_A, L_B have errors of plus or minus 0.25. I am using the same unit so it does not matter plus or minus 0.25 mm. So all I have to do is to calculate using the logarithmic differentiation method is equal to plus or minus (square root of Δk_A by k_A) whole square plus (ΔT_A by ΔT_A) whole square plus similar terms (ΔL_B by L_B) whole square plus same terms for involving (ΔT_B by T_B) whole square plus (ΔL_A by L_A) whole square due to error in all the quantities.

I will just say that, Δk_B by k_B or Δk_B is equal to plus or minus k_B into this whole thing. If we substitute all the values Δk_B comes to 1.24W by m degree Celsius. Therefore k_B is equal to 14.01 plus or minus 1.24 so many W by m degree Celsius, that is the error bar in this particular problem. The error is quite something like roughly about 9%. We can also discuss this error by looking at how we can reduce the error. For reducing the error what are the sources of the error, sources error of course, the measured values the k_A we have no control, because k_A has got a specified value of 2%. It is the ΔT which needs to be measured more precisely.

If we can measure the temperature difference much better than 0.2 plus or minus 0.2 and also the location of the thermocouple is more precisely known then we can improve the accuracy of the measurements the precision of the measurements of the thermal conductivity of the material B by suitable choice of various suitable improvement we can bring it down to about plus or minus 5% that will be acceptable for most engineering applications. So with this basically we have looked at the measurements of thermal conductivity by steady state methods.

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Now let us look at the measurement using unsteady method. A typical example would be the laser flash apparatus. Let us look at the arrangement of the apparatus. Here is a thin slice of the material whose thermal diffusivity I want to measure, or estimate, and it is in the form of a thin sample. On one side a certain amount of heat is going to be supplied by a laser or a flash which is going to be for a very small length of time may be microseconds or milliseconds, the laser flash will illuminate this side, and the energy observed by the front surface will slowly diffuse through the material. So the energy is supplied to this side of the sample by a flash or a laser which is going to be for a very brief period.

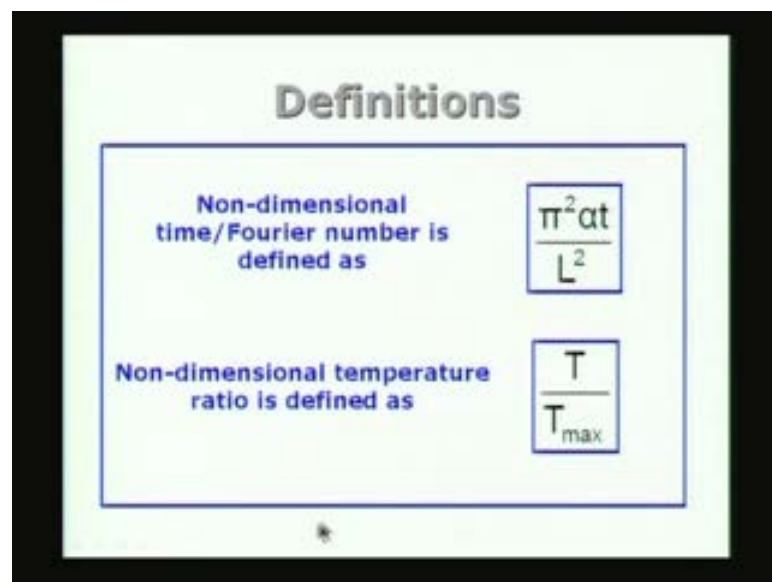
Then at the back of the sample, I have attached a thermal temperature sensor. I am going to study the transient response of the thermo temperature sensor applied at the back, and I will amplify the signal if necessary and record it. There is no need to measure the temperature accurately, but measure the temperature with some effect of the thermocouple response and without worrying about the units I can record.

The sample thickness is the length L , and the sample material thermal diffusivity I am assuming is α which is the ratio of k by ρc so thermal conductivity divided by the density of the specific heat product.

So if I know the specific heat, and the density of the material whose thermal diffusivity I am measuring here, and having measured the thermal diffusivity here I have to multiply by the product of density and specific heat to get the thermal conductivity. So the laser flash apparatus is going to give me only the thermal diffusivity. And if I know the other thermal properties like specific heat and the density I can estimate the thermal conductivity. The method is very simple in terms of its applications.

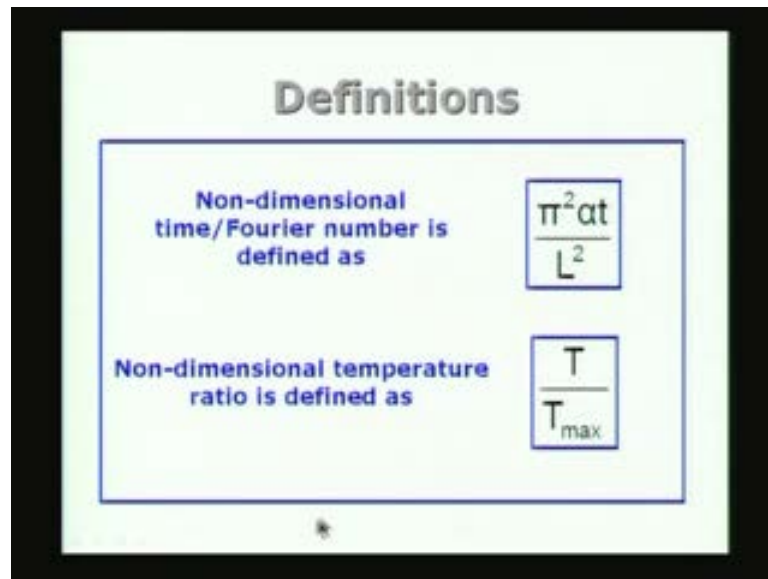
I have laser or flash which goes for a very brief period, dumps certain amount of energy on to one side of the sample; the sample itself is surrounded by a furnace at a constant temperature. For example, if I want to measure the thermal diffusivity material as a function of temperature, all I have to do is to put the sample inside the furnace and bring it to the desired temperature so that the sample as well as the furnace at equilibrium temperature may be of any value we want. And momentarily, I am going to introduce a certain amount of heat on one side which may increase the temperature of that side by only a few degrees 1 or 2 degree above the equilibrium value. So what I will do is, I will just look at the amplified signal from the back of the sample and the pattern of this is going to give me a thermal diffusivity.

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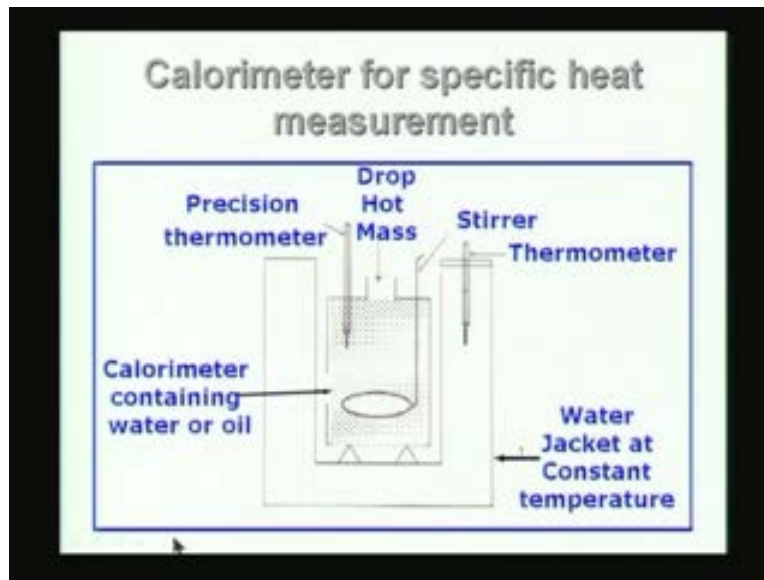
Before we do that, I will introduce two quantities; one is the non dimensional time or it also refers to as the Fourier number which is given by $\pi^2 \alpha t$ divided by square of the length of the thickness of the sample and the non dimensional temperature I am going to define as temperature divided by the maximum temperature recorded from the back surface of the sample. So, if I make a plot of the non dimensional temperature ratio as a function of the non dimensional time or the Fourier number you get a unique curve which has got the property that at a value of non dimensional time is equal to 1.37, the value of the non dimensional temperature ratio is exactly half, so 1.37 is equal to $\pi^2 \alpha t$ by L^2 . So all I have to do is, find out the time at which it happens, you find out when the maximum temperature is going to be equal to half the maximum temperature, half the maximum is here, so from the actual measurement you find out the time at which the temperature ratio is half, and that should correspond to a non dimensional time of 1.37.

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Therefore you take $\pi^2 \alpha t$ which is measured divided by L^2 is equal to 1.37 and from this I will determine the α of the material. The α is equal to 1.37 is equal to $\pi^2 \alpha t$ which is measured divided by L^2 , so I can determine the thermal diffusivity. The method is somewhat expensive, and that is the only major drawback of the method because the laser and the temperature recorder and so on are expensive equipments.

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But it is a very accurate way of determining the thermal diffusivity of the materials. Now let us look at the measurement of the specific heat of a solid. Let us look at this experiment. The idea to discuss the experiment is to indicate how the experiment is designed such that we can take into account the unknown things like heat loss during the experiment. It is estimated by a clever way of measurement. It is an estimation of the heat loss which is central to this particular measurement technique. The calorimeter is an instrument which is used whenever we want to make the measurement of specific heat or latent heat and such other quantities. Any heat capacity measurement requires a calorimeter.

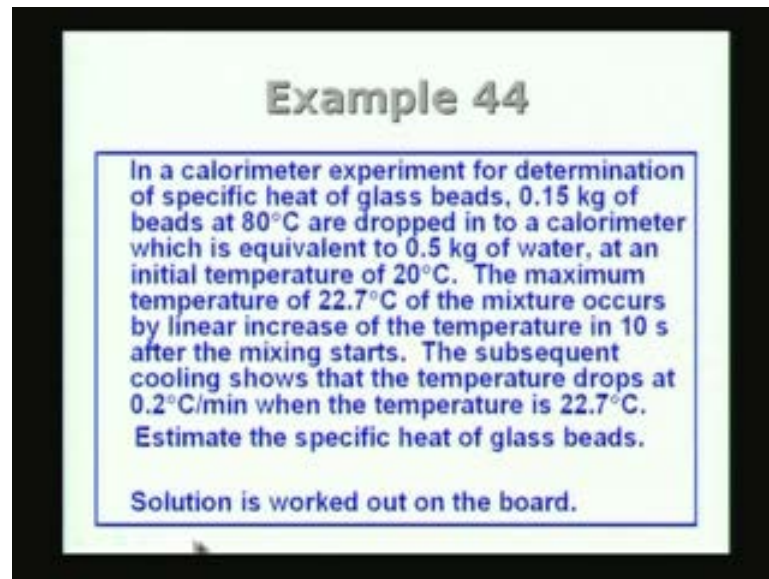
Calorimeter by definition involves the measurement of heat as a quantity which is going to be either going from one system to another system or redistribution of heat and the idea is to measure this accurately and taking into account the losses which may be present in the system. Essentially the calorimeter consists of an inner vessel which we call as the calorimeter and the inner vessel consists of a thin wall vessel with a liquid, in this case it may be water or some oil and the temperature of this oil or water is measured very accurately using a precision thermometer as indicated here.

And there is an arrangement by which we can drop the hot mass whose specific heat you want to measure, the heat can be easily transferred or immediately transferred to the inside by a suitable arrangement for dropping here. And also we have a stirrer which maintains the uniform temperature or that which promotes the

mixing in the calorimeter vessel. Then we surround it, by a second vessel by a water jacket which is maintained at a constant temperature, and that temperature is measured using the thermometer. So the experiment is very simple. You take a certain amount of the material usually in the form of beads or pieces of the material whose specific heat I want to measure; heat it separately in a different environment. For example, you can put the material inside an oven bring it to the desired temperature, and immediately transfer it by dropping it to the calorimeter vessel and immediately after dropping you start noting down the temperature as a function of time.

So, from the time temperature graph, we will be able to estimate the specific heat of the material, which has been dropped after being heated separately and then drop the initial temperature of the calorimeter as well as the water jacket will be the same. We can refer to as T_1 the temperature of the mass which is being dropped can be taken as T_3 , finally the temperature of the calorimeter vessel goes to value is equal to T_2 . So these are the temperatures which we are going to be interested in. So, once this temperature has been measured, I can estimate the specific heat of the material. Let us look at the various features of this method.

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Example 44

In a calorimeter experiment for determination of specific heat of glass beads, 0.15 kg of beads at 80°C are dropped in to a calorimeter which is equivalent to 0.5 kg of water, at an initial temperature of 20°C . The maximum temperature of 22.7°C of the mixture occurs by linear increase of the temperature in 10 s after the mixing starts. The subsequent cooling shows that the temperature drops at $0.2^{\circ}\text{C}/\text{min}$ when the temperature is 22.7°C . Estimate the specific heat of glass beads.

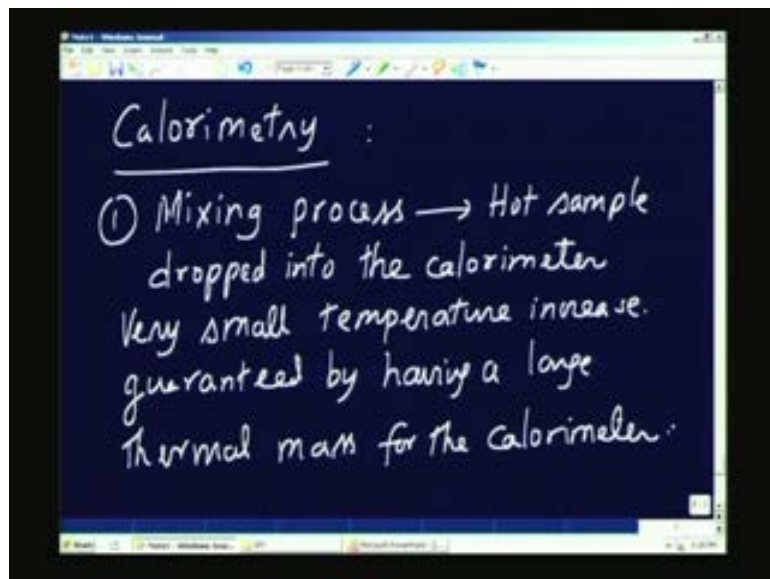
Solution is worked out on the board.

In this experiment, there is the mixing process which means mixing in the thermal sense. Hot sample is dropped into the calorimeter. So the temperature of the mass which is dropped is higher than the temperature of the medium either oil or water taken inside the calorimeter. Therefore the temperature of water will increase. We

are talking about very small temperature increase. How do we guarantee this? We guarantee by taking a small amount of the material whose thermal conductivity is to be measured. We have a large thermal mass for the calorimeter.

So, the increase is guaranteed by having a large thermal mass for the calorimeter. The thermal mass of the calorimeter is given by the mass of the material of the calorimeter that is water plus the walls of the vessel and the specific heat of each one of these. You take the product of mass and specific heat, that is the thermal mass of the system or the calorimeter. So, if we have a large enough value compared to the thermal mass of the drop mass which is going to be dropped into the calorimeter then we will have a small temperature raise. We want the temperature raise to be small, so that losses will be also small. That is the reason why we want to have small temperature raise. In fact from measurement of point of view of temperature a larger temperature change would be desirable but if you want have larger temperature difference the losses also will be larger.

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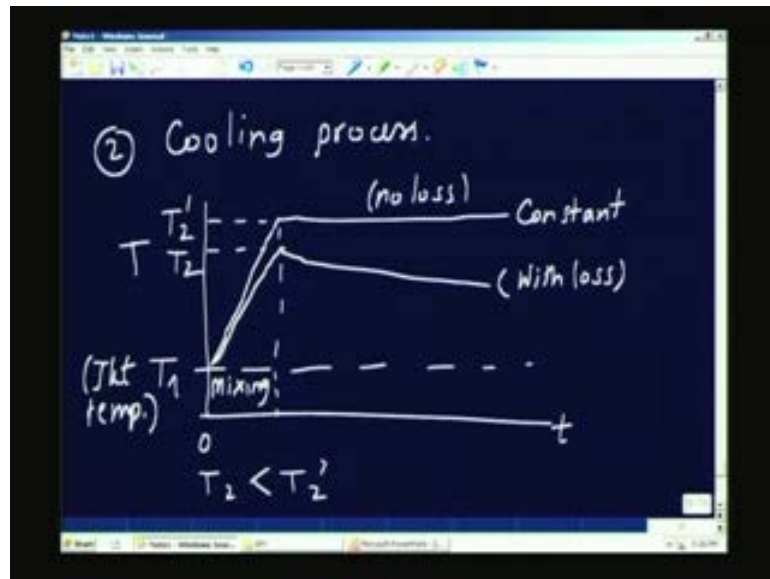


Therefore the estimation of the losses becomes the major problem so you may you have a mixing process which consist of a hot sample which dropped in to the calorimeter, the calorimeter has got the thermal mass which much larger than thermal mass of the material which is being dumped in to it The second part consists of a cooling process. In this sketch here this is the time axis, this is temperature. I have the calorimeter at temperature T_1 which is also the temperature

of jacket, this is also equal to the jacket temperature, it is the same as the jacket temperature.

So let us draw a line like this. Now during the mixing process, I am adding a certain mass of material at a higher temperature into this material into the calorimeter. Therefore the temperature of the calorimeter will show an increase. Let us just assume that this goes up like this.

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So it will go up till it reaches some value as T_2 prime and if there are no losses this temperature will remain fixed at this value. This will be constant if the losses are 0. Hence there is no loss. But what happens in real practice is there will always give some loss. Therefore we expect the temperature to go like this, not to the full value but to some value T_2 which is less than T_2 prime. This is time for which mixing is taking place may be a few seconds, and then temperature will come down like this, this is with loss. So if you have a loss what will happen is that the temperature is going to be lower.

You can say T_2 is less than T_2 prime and with the loss the temperature is going to come down continuously and after sometime if you leave it then it will come back to the room temperature, or the jacket temperature. This temperature will come down, of course it cannot go below that, it will come to the jacket temperature because of the heat loss. That means during the mixing process there is some heat loss. Therefore this temperature did not reach that value. Of course in this figure it

is highly exaggerated, the temperature difference may be very small and then it starts coming down instead of remaining constant. So, if you look at this, this is the error and similarly here this is the error.

So if I want to calculate the specific heat accurately, I must account for this error and make it up. That means I have to add this due to heat loss. So how am I going to account for it? During the mixing process, the amount of heat given up by the hot mass which is dropped into it so, heat given up by hot mass is equal to heat gain by calorimeter plus loss. So I can write, the heat given up by the hot mass, as mass equal to m_g into specific heat C_g product into T_3 minus T_2 and the T_3 minus T_2 has come to the same temperature as that of the calorimeter, at the end of the mixing process the heat gained by the calorimeter is given by the thermal mass of the calorimeter which I will call it as C into $(T_2$ minus $T_1)$ plus I will say loss is L .

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Mixing process:

$$\text{Heat given up by hot mass} = \text{Heat gain by calorimeter} + \text{Loss}$$

$$m_g C_g (T_3 - T_2) = C (T_2 - T_1) + L$$

$$C_g = \frac{C (T_2 - T_1) + L}{m_g (T_3 - T_2)}$$

How do we estimate L ? $L \propto (T - T_1)$
 $L = K (T - T_1)$

This is the equation, I am measuring T_2 , I am measuring T_3 , I know mass of the solid which is transferred, I can actually use this expression to determine C_g which is nothing but C into T_2 minus T_1 plus L by m_g into T_3 minus T_2 . So this is the problem, all these are measured in this, so how do we estimate L ?

We can make a suitable assumption for this, that the loss is, if you remember, in the radial flow heat conduction apparatus we saw that the loss is proportional to ΔT the temperature difference between the inner cylinder and the outer cylinder. Therefore, I can assume here also that the loss must be proportional to the

temperature of the calorimeter minus the jacket temperature. So we know that it has to be determined by the difference in temperature between that. Therefore we can assume that the loss is proportional to the temperature T minus temperature T_1 . Actually I can say that the loss is equal to some K multiplied by T minus T_1 , and if I know what the value of K is..... I will be able to actually determine the error or the loss and then I will be able to determine the specific heat accurately. Thank you.