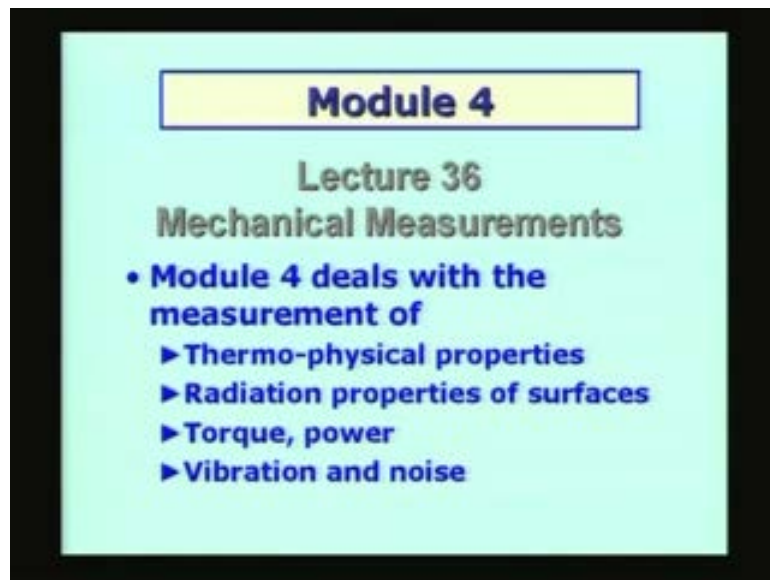


**Mechanical Measurements and Metrology**  
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**Department of Mechanical Engineering**  
**Indian Institute of Technology, Madras**  
**Module - 4**  
**Lecture - 36**  
**Measurement of Thermo-Physical Properties**

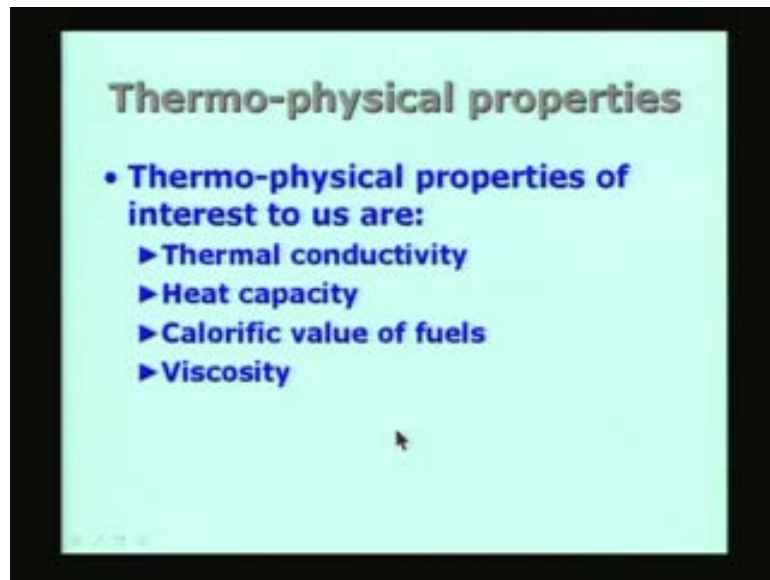
So this will be lecture 36 on the ongoing series on mechanical measurements and also the beginning of the module 4 which basically deals with some specific topics which are of interest in measurement science. The things we will be interested in doing will be the thermo-physical properties of materials.

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Then we will look at the radiation properties of surfaces and also look at the measurements of torque, power and such quantities which form a part of power sources. Then we will also look at measurements of vibration and noise. The first topic I am going to look at is the measurement of thermo-physical properties. In my opinion these are the once which require a little bit of understanding and the measurements are somewhat involved.

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The quantity which will be of interest in thermal science basically are the properties of the materials like thermal conductivity which of course could be for any of the states of matter, solid, liquid or gas and each one of them has a specific problem attached with them. Therefore we suddenly look at the measurement of thermal quantity of various materials in the three states of matter. We will be interested in thermal capacity or heat capacity of the materials and that will be dealt with subsequently.

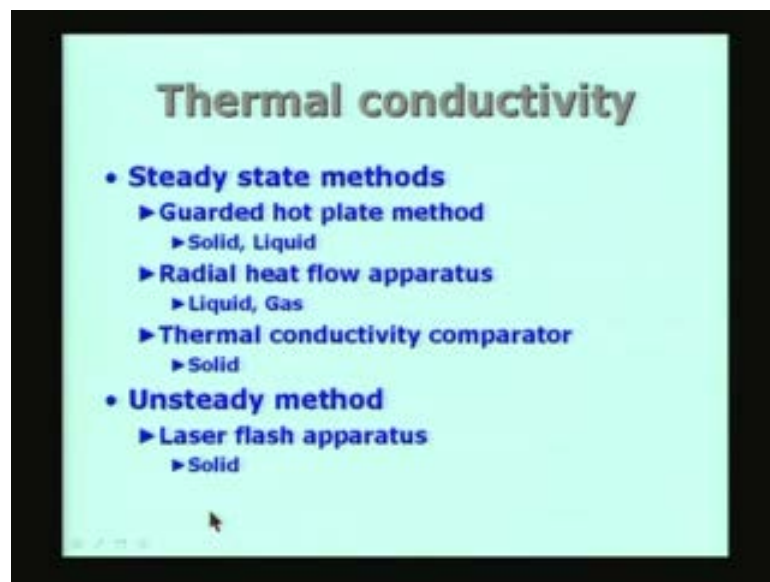
And one of the important things we require when we talk about power plants either based on fossil fuels or either gases or liquid fluids and so on is the measurement of calorific value of fuels. So we deal with one or two examples of how to measure the calorific value. Basically heat capacity and calorific value of fuels are basically of the same type of measurements. Then of course we look at viscosity of fluids. Again they can be of different states of matter like solid, liquid and gases.

Of course if one would like to completely look at all the thermo-physical properties there are many other interesting properties like specific gravity, density, thermal diffusivity, then molecular diffusivity and so on. We will look at selected topics which will bring out the features which are common to all these measurements. Therefore the properties I am going to consider are only representative but they cover more or less all the nuances which you will find when you want to measure any of the thermal physical properties

other than the ones which I have listed here. The first thing I am going to look at is the measurement of thermal conductivity. Thermal conductivity as you very well know is a fundamental quantity of interest.

Whenever we talk about heat transfer in any material whether it is liquid, gas or solid and of course when you talk about conduction and, only conduction heat transfer taking place it is only in the case of a solid, assuming that of course, the solid is stationary and there is no possibility of much movement in the macroscopic sense. Solid is the only case where thermal conduction can be actually studied in great detail. In the case of both liquids and gases immediately when you have a temperature gradient in the system there can be some amount of movement because of density changes or differences. Therefore we have bulk motion.

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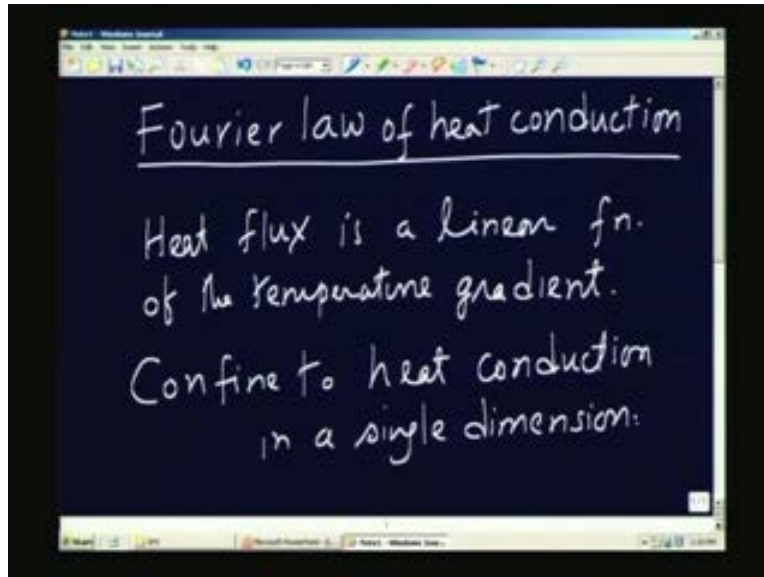
The main problem which we have to address when we talk about the measurements of thermal conductivity of liquid and gases is the mechanism or how we are going to suppress the fluid motions, how we are going to minimize the effect of convection etc so that we can study the conductivity or the conduction phenomena in isolation. This will be the major difficulty. So the various techniques available can be classified broadly into two categories. We call them the steady state methods. And of course we have the unsteady or transient techniques.

Steady state methods generally take a lot of time in terms of the time that is required to bring the process from the beginning to the steady state and maintain the system in the steady state. The steady state methods are usually time consuming, and some of these methods will require days of time not even hours but several days will require for the technique to come to the steady state condition. Then in the unsteady method, the advantage is that the entire experiment is conducted very fast and a very few may be in a few milliseconds or few seconds or within a few minutes. Therefore there are some important advantages with unsteady methods.

But unsteady methods are more expensive in terms of the equipment required generally. Among the steady state methods for measurement of thermal conductivity the most important one we have is the guarded hot plate method, which is considered the primary standard for the measurement of thermal conductivity. We can use it for both for the cases solid and liquid. We have what is called the radial heat flow operators which use a very small gap between two cylinders and the specimen either in the form of liquid and gas is taken in the gap, and we allow heat transfer to take place across the gap under a temperature difference.

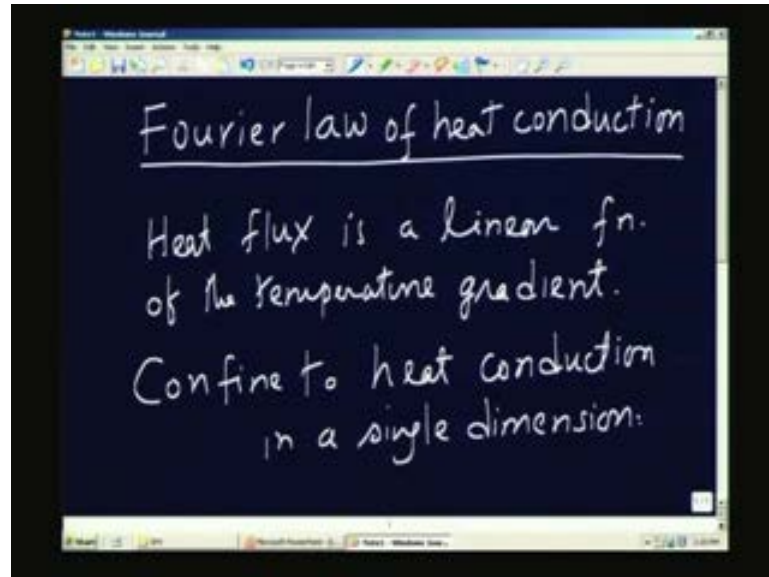
And in fact that is one way of suppressing any convection phenomena taking place or convection taking place within the sample. The third method I am going to discuss is the thermal conductivity comparator which will be with respect to a measurement of thermal conductivity of solids. So the first topic I am going to look at is the guarded hot plate method and then I will also tell how it is going to be useful for both solids and liquids. Then we will look at radial heat flow apparatus which will be useful for both liquids and gases and then we will go on to thermal conductivity comparator. Let us look at the background on the thermal conductivity. We will also look at the **Fourier** law of heat conduction which is the basic law which is going to be used in conduction heat transfer.

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Fourier law of heat conduction states that, the heat flux is a linear function of the temperature gradient. Of course, temperature is the scalar quantity and the temperature gradient of course, will be a vector quantity and heat flux is also a vector quantity because it has got both magnitude and direction. And what you done in practice is not to consider heat transfer in an arbitrary temperature field but in one dimensional temperature field. So what we will do is we will consign to heat conduction in one direction or in one dimension in one dimension or a single dimension. What does it mean? It means that temperature is a function of only one space coordinate and we can call it as  $x$  or  $y$  or  $z$  and the temperature field varies only with respect to one coordinate which is  $x$  for example. So, if I look at the Fourier law of heat conduction which is now written for a single direction, here it is shown by having a material medium in the form of a slab. So we will say this is a slab and we have the direction of heat transfer indicated by this.

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So we will say heat flux is in the  $x$  direction so the temperature will vary from the left to right. So, if temperature is something here it will vary like that so this is  $T_0$  and we will say this is  $T_L$  and the thickness of the slab is  $L$ , We will write the equation as,  $q$  in the  $x$  direction is given by  $k$  the thermal conductivity of the solid or the medium multiplied by  $T_0$  minus  $T_L$  by  $L$  and you can immediately see that the temperature gradient has become  $T_0$  minus  $T_L$  by  $L$  is the temperature gradient and  $k$  is the factor thermal conductivity which is to be determined  $q_x$  is the heat flux by conduction.

We will see that the units are going to be  $W$  by  $m$  square here rate at which heat is transfer per unit area per unit time the rate is transfer per unit area, that means the amount of energy crossing per unit time an unit area of the slab  $T_0$  and  $T_L$  are the temperatures of the left and the right end of the slab and  $L$  is the thickness of the slab. Of course, I am assuming that this is a slowly varying function of  $T$  or slowly varying with  $T$  or may be not at all varying with  $T$ . If I make sure the temperature difference  $T_0$  minus  $T_L$  the temperature difference across the slab is very small.

I can say that thermal conductivity I am going to determine is, actually at the mean temperature of the solid or the medium which is  $T_0$  plus  $T_L$  by 2 and it is very close to the difference  $T_0$  minus  $T_L$  is very small compared to the mean value. So I can say that, if  $T_0$  minus  $T_L$  is very small compared to  $T_0$  plus  $T_L$ , then I can assume that thermal conductivity is the thermal

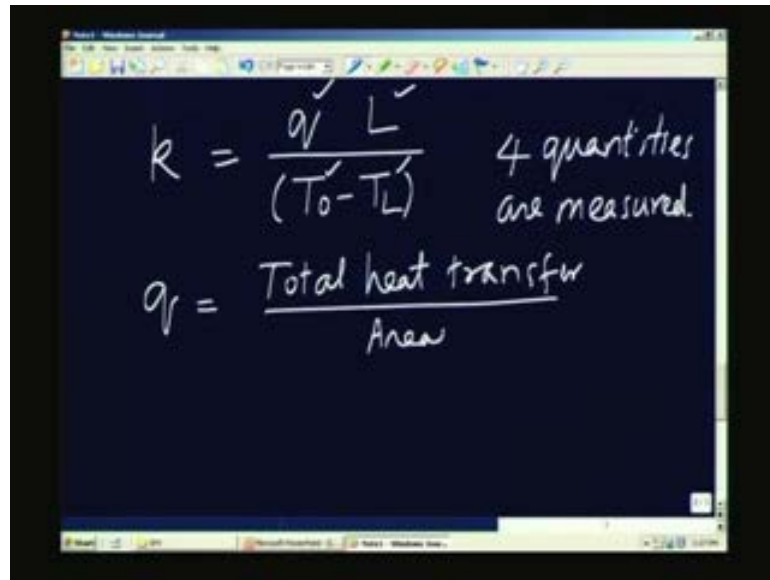
conductivity at the average value. So  $k$  is nothing but the thermal conductivity mean value of  $k$  of the thermal conductivity. If I maintain a temperature difference of the order of 5 degree for example between the two ends of this particular slab material then it is close enough to being at the mean value with the perturbation or the change with respect to mean value is only plus or minus 2 degree or 3 degree.

So it will satisfy the requirements that, the thermal conductivity I am going to determine is at the mean value. Why I am talking about this is, if we have a medium whose thermal conductivity varies significantly with temperature, I have to conduct the experiment such that at any heating level or any mean value of the temperature. The perturbation of the variation of the temperature across the slab should be small compared to the mean temperature of the solid. That is how you have to actually maintain the condition. So the thermal conductivity can be obtained as a point function or a function of temperature where the temperature is identified with the mean value. So, with this background let us look at what one has to do in any experiment where I want to measure the thermal conductivity.

One I have to measure  $q_x$  or the heat flux. I have to measure the temperatures  $T_0, T_L$ . These two temperatures have to be maintained, measured and I have to measure the length or the thickness of the solid or the medium in this particular case. So I have to do four measurements  $q, T_0, T_L$ , and  $L$ . 4 measurements are required before I can obtain the value of  $k$ . Actually what I am going to do is, I am going to write it in the following way.

I will say that  $k$  is equal to  $qL$  by  $(T_0 \text{ minus } T_L)$ , that is the expression for thermal conductivity. It will require the measurements of  $q, L, T_0$ , and  $T_L$ . Four quantities are required, four quantities are measured. Of course, in practice, the temperature is measured using for example thermocouples can be used. The length has been measured using an accurate length measuring instruments. The  $q$  sometimes is the most difficult to measure because  $q$  itself is the total heat transfer divided by the area through which the heat is transferred to the major thing. One has to remember that we have to maintain one dimensional temperature field. That means we have to see that, the temperature varies only with respect to  $x$ , and there is no significant temperature variation in this direction.

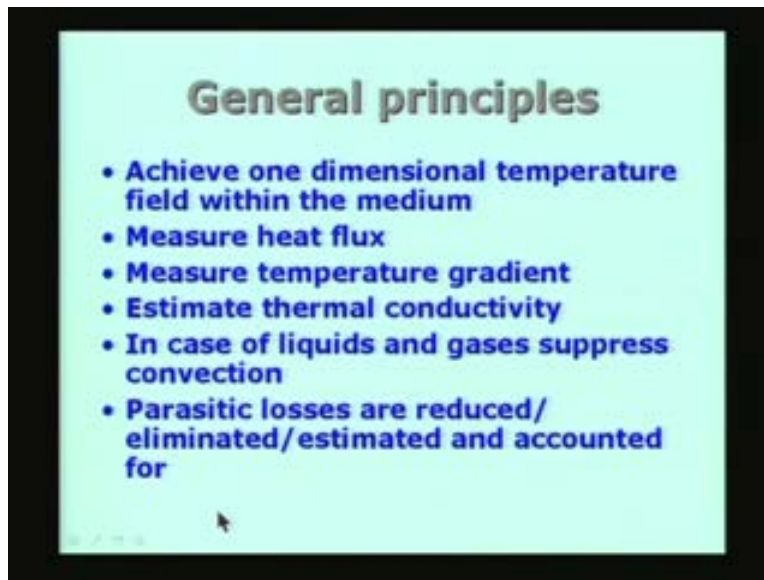
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A digital blackboard with a white border and a toolbar at the top. It contains two handwritten equations in white ink. The first equation is  $k = \frac{q_l L}{(T_o - T_L)}$  with a checkmark above the  $q_l$  and  $L$ . To its right, the text "4 quantities are measured." is written. The second equation is  $q_l = \frac{\text{Total heat transfer}}{\text{Area}}$ .
$$k = \frac{q_l L}{(T_o - T_L)} \quad \text{4 quantities are measured.}$$
$$q_l = \frac{\text{Total heat transfer}}{\text{Area}}$$

So the transverse direction there is no significant temperature variation. Only variation should be along this direction. That will be guaranteed only if  $q$  is truly in the direction which is shown. There cannot be any heat loss in a transverse direction. Of course, if we have an infinitely large slab it is easy to maintain that in practice, the infinitely loss slab is not possible. So we always have a finite size slab. When the sample size is finite it is difficult to maintain the one dimensional heat transfer or a heat transfer in a direction as it is shown here. That is the major thing which we have to look at. So, how to obtain the heat transfer in a particular direction in one dimensional without any losses in the transverse direction is the major thing one has to look at. Now let us look at the principles involved in doing that. Let us try to do that with the guarded hot plate operator.



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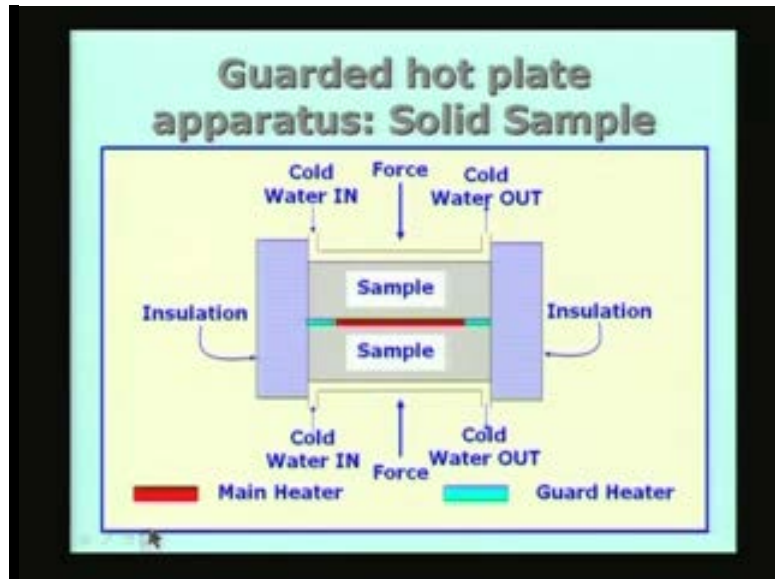
The first goal is to achieve one dimensional temperature field within the medium. I have to measure the heat flux that means the amount of heat transfer per unit area in unit time, I have to measure the temperature gradient either by measuring the temperatures individually and the length separately and then taking the ratio I have to use these four quantities  $q$ ,  $L$ ,  $T_0$ , and  $T_L$ , I will be able to estimate the thermal conductivity. The additional problems when we come to liquids and gases are how are we going to suppress convection.

In the case of solids the advantage is that, there are no possibilities of any movement of the solids in the macroscopic sense of the term. Therefore I do not have to worry about the suppression of the convection. In the case of the suppress of the convection either I can use a very short time for the experiment as in the unsteady or transient methods or I have to take special precautions so that these movements are not going to be present. Finally we have to look at the parasitic losses, even though I am saying that we will achieve the one dimensional temperature field and so on.

How closely are we able to do that? There will be always some parasitic losses, these have to be reduced or eliminated or in the worst case we have to estimate what these losses are, and account for these losses so that the estimated value of thermal conductivity is of acceptable level of accuracy. (Refer Slide Time: 20:21) Let us look at one simple arrangement. In fact,

earlier when we talked about the measurement of heat flux we already looked at the concept of the guard. And if you recall from our discussion a guard heater is going to help in reducing the losses in a transverse direction and therefore make the temperature field of the heat flux one directional or one dimensional.

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So what we have done in the case of guarded hot plate apparatus is, now for a solid sample, I am taking the example of solid sample. I take two samples of identical size and thickness. This is a sample, this is also a sample, it is of the same material, same size, physical dimensions are the same and now I am going to use this in the symmetric arrangement. So I have a heater in between these two samples, it is in a sandwich kind of a construction like this and I have got what is called the main heater. The red symbol is called the main heater and the blue color thing is called the guard heater.

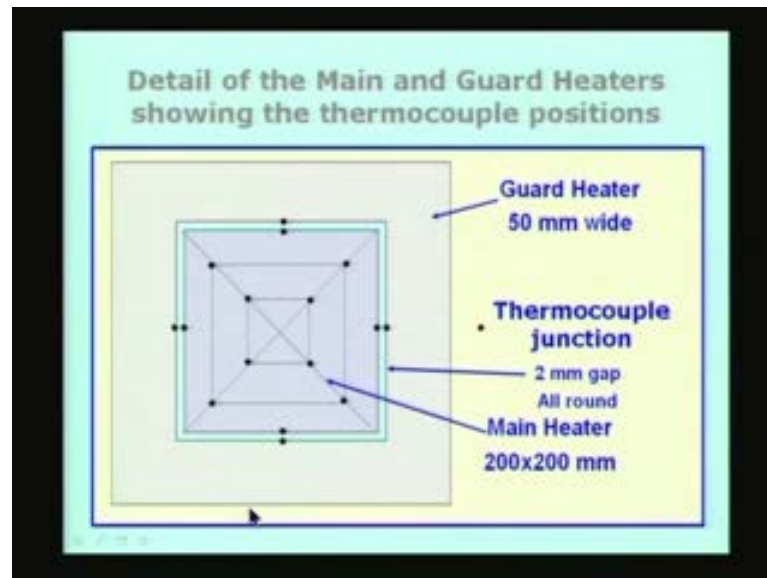
So we have the sample on the two sides. The sample is going to occupy the entire width which is the size of the main heater plus the guard heater. And the sample is placed symmetrically on the top and the bottom and what I am doing is the other's surface of the sample at the top as well as the bottom are cooled by cold water which is coming from the cooler. Where the water comes in the cold water I will go have after taking out the heat which is transferred from the heater across the sample by the conduction it will remove the heat and go out. Similarly, symmetrically on the bottom.

And in order to further reduce any heat loss or heat transfer in a transfer direction I am going to provide a large thickness of insulation of very low thermal conductivity. In fact this figure shows another quantity of the force I am applying, the force in this direction shown like this so that the contact between the sample and the hot plate as well as the cold plate here is guaranteed so that the contact resistance will be very small. The contact resistance of the temperature we measure will also contain extra resistance due to the contact resistance and therefore the method will not give you accurate value of the thermal conductivity. Therefore in order to improve the material method, we can even apply a force which is newly done by putting a screw and then tightening that so that you get sufficient pressure so that the sample is intimate contact is surface of the heater as well as the cold plate. Now we can easily see what we are trying to do. We are trying to introduce the heat at the center in the symmetric fashion.

We can assume that if we put some  $q$  watts into the heater,  $q$  by 2 will go through the top sample,  $q$  by 2 will go through the bottom sample assuming that both of them are identical in shape and are made of the same material. That means the same thermal conductivity of the heat from the sample will go from the upstairs and half will go from near and of course the amount of heat put in I will measure using the amount of electrical heat supplied to the heaters.

I also have the guard heater which is surrounding the main heater and what I will be doing is, I will supply enough heat to the guard heater so that, there is a physical gap between the main heater and the guard heater which is  $\Delta T$ . The temperature difference is very small or equal to 0. In the case of ideal situation there should be no temperature difference across the gap. The temperature difference across the gap is 0. That means there is no heat loss from the main heater in the direction perpendicular to this axis. Let us look at the typical construction of the guard and main heater. This sketch indicates what is happening.

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So we have the main heater which is something like 200 mm by 200 mm, this is the smallest size. We can in fact have much larger size. If one wants to, in fact it can be even 1 m by 1 m in some situation. This is necessary to use very large sizes. Of course, if we use large size the large amount of heat must be supplied and also we require very large sample and so on. So, the minimum is something like 200 mm by 200 mm that is the main heater and then you have a 50 mm wide guard heater all around the main heater with a small narrow gap. Of course the gap I have shown here is too large. The gap itself can be something like couple of millimeters 2 mm in this case. So I have a 50 mm wide guard heater then a 200 mm by 200 mm main heater and we fix the thermocouples as shown here. Across the gap, I have got two thermocouples. These closed circles are thermocouple junctions. I have two junctions here, two junctions here, two junctions here, two junctions here and two junctions here and the  $\Delta T$  the temperature difference is indicated by this.

The two thermocouple junction pairs must be ideally equal to 0. That means in a particular experiment I am going to dial up the amount of heat I am going to give to main heater to a particular value then I will adjust the amount of heat supplied to the guard heater such that the  $\Delta T$  across this gap is going to be 0 so I must have some kind of a controller which will monitor the  $\Delta T$  across this gap and if the  $\Delta T$  goes up, it will reduce if the  $\Delta T$  goes down that is with respect to 0, if it is one direction it will

depend on the direction of the  $\Delta T$ . It will either increase or decrease the heat supplied to the guard heater so that it comes back to  $\Delta T$  is equal to 0.

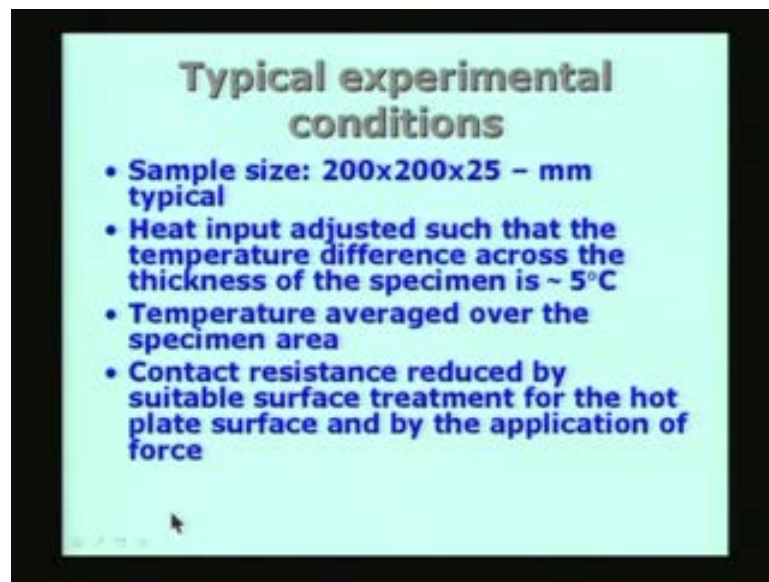
So this is something which has to be built into the system, and the other point is that we are going to measure the temperature not at a single point but we are going to measure the temperature as I have shown here. For example, eight temperatures are measured on one of the surfaces and of course, there is another surface on the other side and also another surface on the cool side. So, on each side I am going to have several thermocouple junctions like this, and then I am going to average them.

In fact I can do the average by simply connecting all of them. If you connect them all in parallel we will get the average value of the temperature because we are expecting here of the temperature more or less of the same uniform flow but there may be a small perturbation small change so we would like to find out what is the mean temperature on this side because for heat transfer calculations, I need the mean temperature on the surface. Similarly, on the cold surface, the thermocouples usually are built in on to the heater and they are not going to be attached to the sample and the sample is simply kept there, and the sample between the sample and the heater you may give some kind of surface treatment, you make it very smooth and use some kind of a film of material of high thermal conductivity so that the contact resistance is broad way.

Therefore the assumption we can make is that the temperature of the surface of the heater and the surface of the sample which is in contact with it are exactly the same. When I say,  $T_0$  in the expression for thermal conductivity we have  $T_0$  where  $T_0$  corresponds to the temperature of the heater and  $T_L$  will correspond to temperature of the cold surface which will be the same as temperature of the surface of the sample which is in contact with the cold surface. That is the reason why we have so many thermocouples and we measure as I indicated,  $\Delta T$  across the gap must be equal to 0. So the heat input to the guard is adjusted such that  $\Delta T$  is 0, then you allow the entire thing come to steady state, and generally it takes about two to three days for the steady state operation steady state to be reached for a guarded heat plate apparatus. That means it is a very time consuming proposition. So, one single thermal conductivity measurement will take several days may be a week before we can get the numbers.

So let us look at the typical experimental conditions where we are going to have the sample size 200 into 200 into 25 this is for the guard region the main heater region plus 50 mm you add on all these sides. So actually the sample size will be 300 by 300 by 200 another 50 on either side. So 300 into 300 into 25 mm is the main heater size. Only I have given this. Typically this heat input is adjusted such that, the temperature difference across the thickness of the specimen is roughly equal to 5 degree. We do not want to have very large temperature difference across the thickness of the sample.

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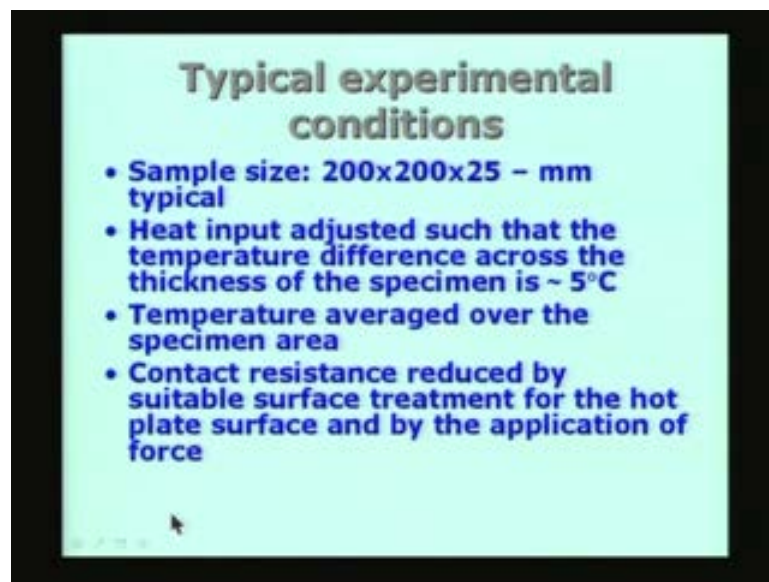


The question now arises as to what we do if we have high thermal conductivity material. What we do when thermal conductivity at the material is very low. You can immediately say that the thermal conductivity is small for a given heat transfer and we require larger temperature difference. This is stated by using the Fourier law. You require large temperature difference or putting it in another way if the thermal conductivity is small for a given temperature difference the amount of heat input is small. So if you want to have a 5 degree Kelvin 5 degree Celsius difference across the thickness of the material you have to adjust for knowing roughly the thermal conductivity value material. We can find out what is the amount of heat we have to supply. That will vary depending on the material. The other condition, or the other control, I can have is thickness of the specimen. If the thermal conductivity is very small, I can take a smaller thickness of the sample, and if the thermal conductivity is large, I can take a slightly thicker

material.

So the sample thickness can be adjusted. Even though I have said 25 mm typical value it could vary depending on the particular material whose thermal conductivity I am going to measure. As indicated earlier, the temperature is averaged over the specimen area by having several thermocouples, all parallel, so that you get the average value of the temperature contact resistance reduced by suitable surface treatment for the hot plate as well as for the other surface by the application of force. The force is going to help in actually bringing out a good physical contact. A good physical contact coupled with this surface treatment will also mean that it is a good thermal contact. Therefore the contact resistance can be reduced by this particular method.

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Here is example 41. So we have a guarded hot plate apparatus which is used to measure the thermal conductivity of an insulating material, the specimen thickness is 25 plus or minus 0.5 mm. So now I am now going to look at the typical case, where the thickness is measured with plus or minus 0.5 mm precision, 25 plus or minus 0.5 mm, the heat flux is measured within 1% and the nominal value has been obtained as 80W by m square this is the value, the temperature drop across the specimen under the steady state that has been measured to be 5 plus or minus 0.2 degree Celsius.

Therefore, what I have done is I have got the nominal value plus or minus the expected error either 0.5 mm in the case of thickness, 1% in the case of heat flux and 0.2 plus or minus 0.2 degree Celsius in the case of the temperature difference. Determine the thermal conductivity of the sample along with its uncertainty. This is example 41. The heat flux is directly given. In practice, one would have to measure the total heat supplied to the main heater divided by the area of the main heater, to get the heat flux. In this case it is directly given. Therefore, I will write down  $q$  as given 80W by m square this is the nominal value and the error is plus or minus 1% in brackets. Then I am given the thickness of the sample  $L$  which is 0.025, 25 mm is 0.025 m plus or minus 0.0005 m, 0.5 mm plus or minus 0.5 mm comes out to be plus or minus 0.0005 m and I am writing everything in terms of SI units and the  $\Delta T$  has been measured as 5 degree centigrade plus or minus 0.2 degree Celsius.

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Example 41

$$q = 80 \frac{\text{W}}{\text{m}^2} (\pm 1\%) \quad L = 0.025 \text{ m} (\pm 0.0005)$$

$$\Delta T = 5^\circ\text{C} (\pm 0.2^\circ\text{C})$$

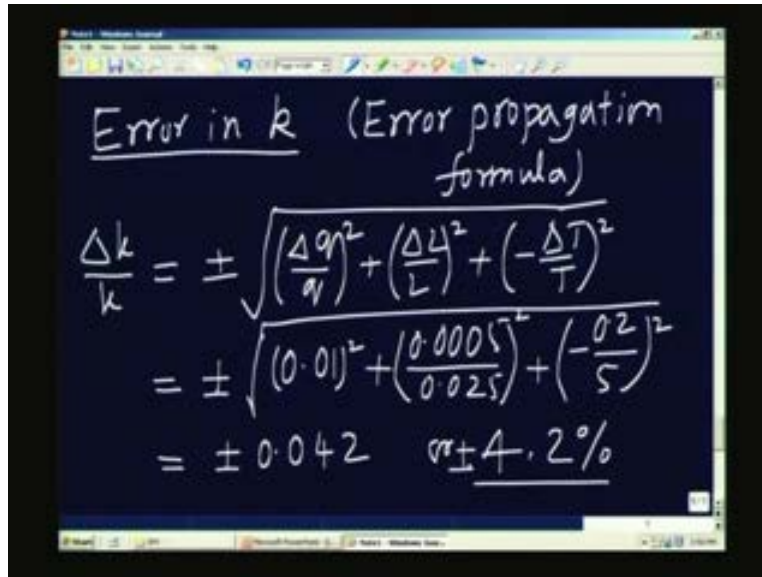
Nominal  $k$   $k = \frac{qL}{(\Delta T)} = \frac{80 \times 0.025}{5}$

$$= 0.4 \text{ W/m}^\circ\text{C}$$

So the nominal  $k$ , I can calculate using the Fourier law of heat conduction which is  $qL$  by  $\Delta T$  and,  $T_0$  minus  $T_L$  is nothing but your  $\Delta T$  and this will be 80 into 0.025 by 5 this comes to, 0.4W by m degree Celsius this is the nominal value. It is a very simple straight forward, plug into the expression and gets the value of the thermal conductivity. The next part is to look at the error in  $k$ .



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The image shows a handwritten derivation of the error propagation formula for thermal conductivity  $k$ . The title is "Error in k (Error propagation formula)". The formula is written as follows:

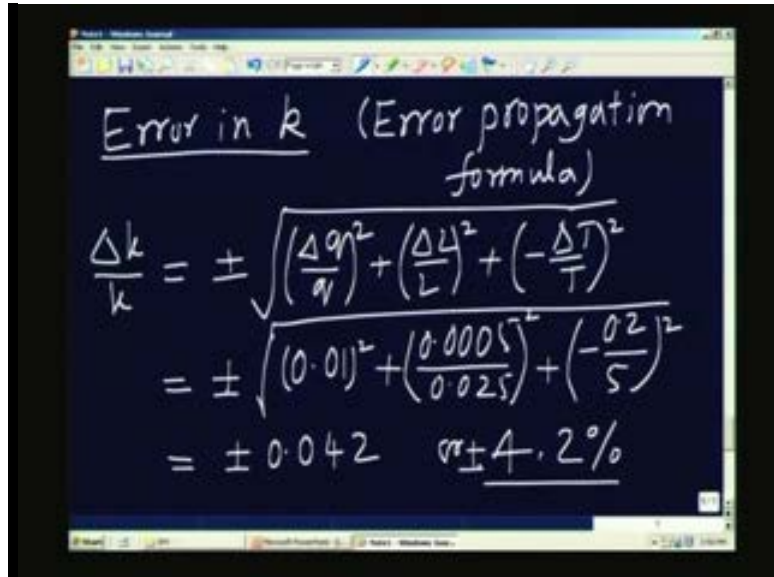
$$\frac{\Delta k}{k} = \pm \sqrt{\left(\frac{\Delta q}{q}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(-\frac{\Delta T}{T}\right)^2}$$
$$= \pm \sqrt{(0.01)^2 + \left(\frac{0.0005}{0.025}\right)^2 + \left(\frac{-0.2}{5}\right)^2}$$
$$= \pm 0.042 \quad \text{or } \pm 4.2\%$$

I have to use the error propagation formula. We came across the error propagation formula, when we discussed the estimation errors in measured quantities. In this case, we are measuring  $q$ , the heat transfer per unit time when measuring this thickness and we are also measuring  $\Delta T$ . All of them are prone to errors, and you can see that in this case, because  $k$  is related to the other quantities in terms of product of  $q$  and  $L$  by  $\Delta T$ , I can use logarithmic differentiation and obtain the value of  $\Delta k$  by  $k$  will be nothing but plus or minus (square root of  $\Delta q$  by  $q$ ) whole square plus ( $\Delta L$  by  $L$ ) whole square plus (minus  $\Delta T$  by  $T$ ) whole square.

Of course, it is because square of this minus  $\Delta T$  by  $T$  is not going to make any difference. So for this, I can write the quantity  $\Delta q$  is 1% so  $\Delta q$  by  $q$  is nothing but (0.01) whole square, 1% is 0.01  $\Delta L$  by  $L$ , I can substitute the values as (0.0005 by 0.025) whole square, plus (minus 0.2 by 5) whole square, all are ratios. So when you do that the entire thing comes to plus or minus 0.042, so  $\Delta k$  by  $k$  is equal to 0.042 or some 4.2% this is a very typical value of error with when use a guarded hot plate apparatus, according to normal standards which are specified for the guarded hot plate apparatus, about 5% plus or minus 5% is the expected uncertainty in the value of measured thermal conductivity and in this particular example, it is within that particular value within the plus or minus 5%. So I can say, this is plus or minus 4.2%. In fact I can now specify the value of the thermal

conductivity of the solid which will be 0.4 plus or minus 4.2% in W by m degree Celsius.

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The image shows a handwritten derivation of the error propagation formula for thermal conductivity  $k$ . The title is "Error in k (Error propagation formula)". The formula is written as follows:

$$\frac{\Delta k}{k} = \pm \sqrt{\left(\frac{\Delta q}{q}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(-\frac{\Delta T}{T}\right)^2}$$
$$= \pm \sqrt{(0.01)^2 + \left(\frac{0.0005}{0.025}\right)^2 + \left(-\frac{0.2}{5}\right)^2}$$
$$= \pm 0.042 \quad \text{or } \pm 4.2\%$$

So I can specify it as 4.2% plus or minus 0.4 plus or minus 4.2% that is the value which I will get using this particular method. So what we have done is to look at measurement of thermal conductivity of a solid using the guarded hot plate apparatus, and we have taken an example, and seen how to work it out and also seen the type of expected error in these measurements.

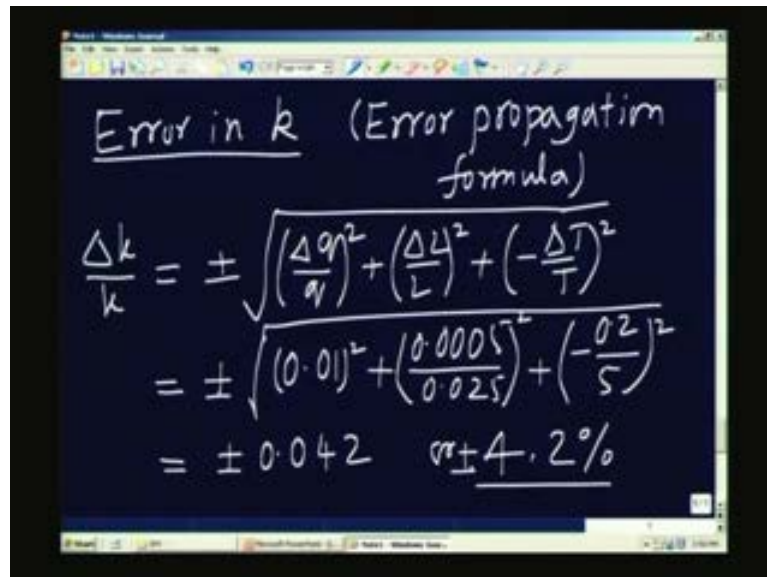
And we recapitulate what we have already talked about when we are talking about measurement of temperature, measurement of heat flux and so on. The kind of numbers I have used in this particular example, are typically the numbers which we discussed earlier. Then we know that in the case of measurements of temperature difference plus or minus 0.2 degree is an achievable value. Similarly, with respect to heat flux and with respect to a specimen thickness also the plus or minus 0.5 mm is a very nice value.

Actually the reason why it is coming out plus or minus 0.5 is not because we are unable to measure the thickness but because many times the guarded hot plate apparatus are used for the measurement of the thermal conductivity of sample as received from the manufacturer. You just cut a small portion of a slab usually in the form of slab or plate and you do not do much of a surface

treatment or no machining is done. Usually, it is just taken as a slab as is obtained from the manufacturer and many times we use the method.

In this case, for example, the material has got a thermal conductivity of 0.4 which is characteristic of an insulating material and usually these thicknesses are not very closely specified for such a thing. Therefore plus or minus 0.5 mm seems to be a reasonable number for this. This is not measurement related; it may be actually the material itself the thickness does vary with this kind of a difference from position to position. Let us look at what we have to do, if you want to use the guarded hot plate apparatus for a liquid sample.

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The image shows a handwritten calculation on a dark background, likely a chalkboard or a digital whiteboard. The title is "Error in k (Error propagation formula)". The formula for the relative error in k is given as:

$$\frac{\Delta k}{k} = \pm \sqrt{\left(\frac{\Delta \rho}{\rho}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(-\frac{\Delta T}{T}\right)^2}$$

The next line shows the substitution of values:

$$= \pm \sqrt{(0.01)^2 + \left(\frac{0.0005}{0.025}\right)^2 + \left(-\frac{0.2}{5}\right)^2}$$

The final result is:

$$= \pm 0.042 \quad \text{or } \pm 4.2\%$$

When we use a liquid sample there is the possibility of macroscopic movement of the liquid. Suppose, I have a situation like this, where I have a high temperature as indicated here, the low temperature here. Suppose I have a liquid here, then normally the temperature at a high temperature in the density is smaller and low temperature in the density is higher and therefore the liquid will be lighter near the hot wall and it will tend to move away. Then of course cooler liquid will come downwards. Therefore you expect some kind of a motion like this, an up and down motion. So this is referred to as natural convection.

Now, what is the consequence of the natural convection?

The consequence is that the heat transfer is not simply by conduction but it is also because of the bulk motion. Therefore the heat transfer is actually much more than what would be if the heat transfer is by pure conduction alone. So, if I make an experiment as shown here with high temperature at the bottom and low temperature at the top I am certainly not going to measure the thermal conductivity I am measuring something which is not simply the thermal conductivity it includes also this effect of the thermal motion. So you can immediately see that if I were to put the high temperature at the top and low temperature at the bottom much of this movement can be completely restricted. That means the natural convection which will occur because of the density changes or density gradients can be reduced or eliminated. This is in fact the first thing which we have to do.

The second thing is, if the thickness of the liquid layer is very small it also inhibits this natural convection. Therefore, I can do two things, I will take the liquid in the form of a very thin layer may be a few millimeters or even less than a millimeter so that, there is not enough chance for this natural convection or the convection motion to set in. And secondly, I am going to put the high temperature at the top and low temperature at the bottom as shown in the next slide, where I have the main heater here, and because I want to put the high temperature at the top and the low temperature at the bottom, I cannot of course use the symmetric arrangement which we had earlier. If you remember in the case of solid we had two samples, one at the top and one at the bottom and half the heat went from the top side and half from the bottom side.

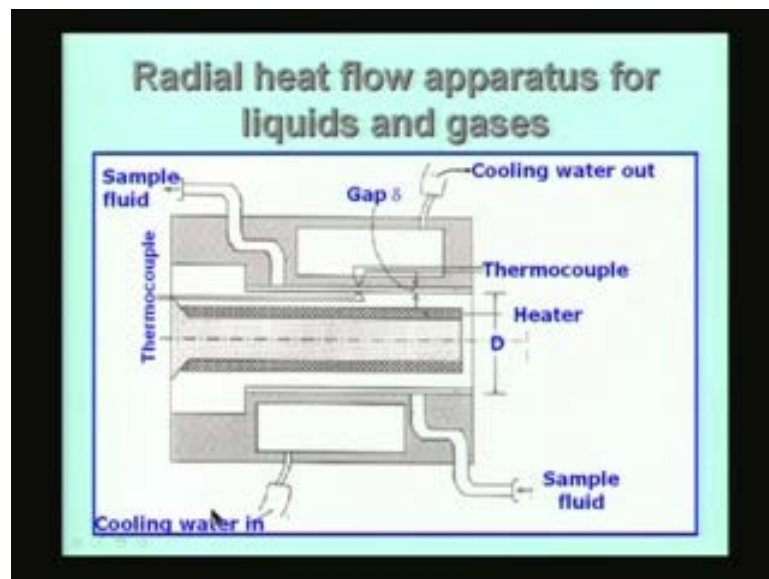
Here I cannot have that facility because I want to have the high temperature at the top and the low temperature at the bottom. The only way I can do is to have a single layer of the sample. So I have a single sample layer, and therefore the main heater in this case will have to be backed up by a guard heater. There is a guard heater here and the main heater here, now what I will do is, I will monitor the temperature difference across these two, and if it is made equal to 0 by suitably controlling the amount of heat I am going to give it to the guard heater, then I can assume that all the heat that is generated in the main heater has to go only from the bottom through the liquid layer.

In this case, the guard is doing to something different from what was done in the previous case. In the previous case, it was trying to make it one dimensional by preventing the heat loss from the lateral side, but here it is

actually preventing heat loss from one side all together so that there is a purely conductive heat transfer across the layer of the liquid here, and you also know that when liquid gets heated, it tends to expand and the volume changes, and therefore I must allow for the volume changes by having small gallery here, where the excess liquid after expansion will be collected in this.

And of course I am going to prevent heat transfer in the lateral direction by having insulation as shown here. And the bottom is cooled by cold water coming in at a low temperature and going out after receiving the heat from the main heater which passes through the liquid sample and it goes out. So the difference between the guarded hot plate apparatus for liquids and solids is very clear. In the case of liquids, I must have heat transfer from one side that is in the up and down direction unlike in the other case, where we had both up and down and down and up both directions but here we have only one direction, and the main heater is guarded by the guard heater which prevents heat transfer from the top all together. So the heat transfer is only from one direction. All the heat I am going to give to the main heater it goes through the liquid sample and if I take a very thin layer of the liquid as shown here, then I can assume one dimensional heat transfer to the liquid and the same expression which we used earlier is going to be valid for the particular case also. There is another way of doing the whole thing which is the radial heat flow apparatus, and it is used for both liquids and gases.

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The other two cases were in the form of slab. It was either a solid material in the form of a slab or the sample of liquid in the form of a layer which was a planar layer. In the case of radial heat flow what I have is a cartridge heater in the form of a cylinder, this is the cartridge heater which is in the form of a cylinder and there is an annular region, this thin region here of gap equal to  $\delta$ , this is a cylinder and the diameter of the cylinder is  $D$ , and there is a narrow gap equal to  $\delta$  through which heat transfer takes place from the hot plug to the outside, the outside of cylinder is actually cooled by cold water coming in as indicated here and it is leaving out.

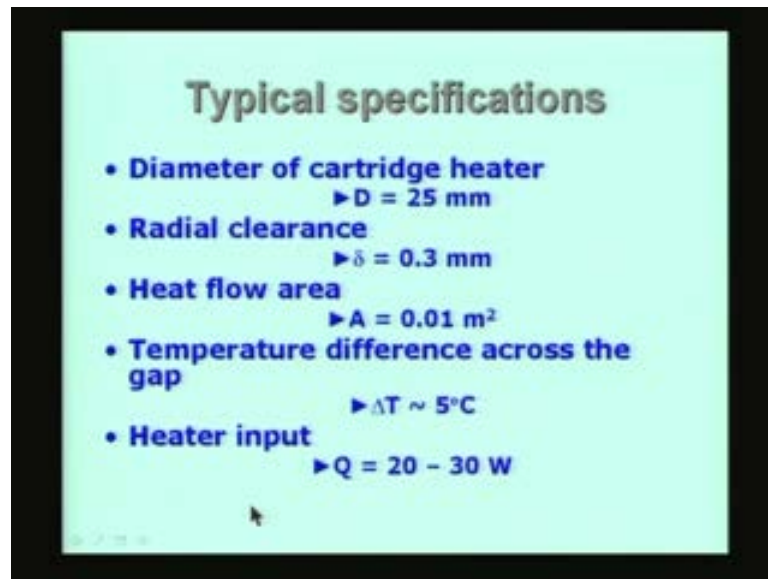
The sample itself is taken in this narrow gap between the inner cylinder and the outer cylinder in the form of an annulus. The sample fluid either liquid or gas has to be filling this narrow gap and usually the gap is very small, the gap thickness could be some millimeter in diameter may be 0.5 mm or 0.3 mm, therefore it is very difficult to fill that gap. Therefore, usually the sample fluid is forced through this entrance here, by using a syringe or some such method, and then it is made to fill the entire gap. And when either the liquid or gas comes out from here, we know that the entire thing is filled with the sample liquid or sample gas.

Of course, in the case of gas what we do is, we allow the gas whose thermal conductivity we want to measure to freely flow through the system for a certain length of time so that we know that it has displaced all the other material which was inside and finally after some time we can expect the gap to be filled with the gas whose thermal conductivity we want to measure. Of course, then you close the entrance and the exit with two valves which may be there, and then the gap is now full of the gas whose thermal conductivity I want to measure.

The theory of this is very simple. I have a diameter  $D$  which is typically of 25 mm, or may be a little more may be 40 mm and the thickness of the gap or the gas the gap thickness is very small sub millimeters therefore we can assume that the layer here is almost like a planar layer, because the area change from inside to the outside is so insignificant, if the diameter is very large compared to the gap, if  $\delta$  is very small compared to  $D$ , we can assume that the gap is just like a planar layer as far as the heat transfer the conduction is concerned. So I put a thermocouple inside, and I put a thermocouple outside, this is the thermocouple pair which is going to measure the temperature difference across the gap between the inner cylinder and outer cylinder.

And, we also know the area of heat transfer which will be the length of this gap, this annulus length into  $\pi d$ , so that  $\pi d$  is the circumference by length multiplied by this gap thickness and that will actually give you the volume of the sample which is taken here. So the area will be actually ( $\pi d$  into  $l$ ) that is the heat transfer area, the gap is delta so we can immediately apply the Fourier law of heat conduction and then measure the thermal conductivity. But before we do that let us just look at the typical numbers.

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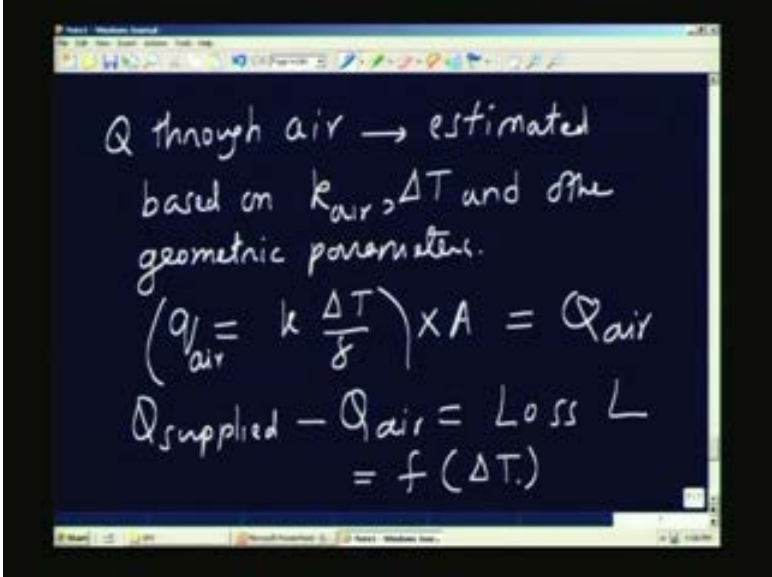
The diameter of the cartridge is roughly 25 mm, radial clearance is 0.3 mm, the heat flow area is that ( $\pi d$  into  $l$ ) which is about 0.01m square, the temperature difference across the gap is roughly of the order of 5 degree, the heater input can be between 20 and 30W depending on the liquid or gases which we are going to take inside. If it is gas, the heat transfer will be much lower and if it is a liquid, it could be even 20 to 30W of heat which is put in. And the heat is put in the form of electrical heat the cartridge heat consigns an electrical resistance inside and you measure the heat supplied by the amount of electrical power we are going to supply to the heater. So, if we know the heater resistance and the voltage across the leads of the heater, I can find out the amount of heat input, and that will be exactly equal to the amount heat which is going through the narrow gap through the sample by conduction plus any losses.

The difference between the previous method where I had a guard to reduce the heat transfer or heat losses and here in this particular case the losses are not 0, the losses have to be estimated. If you remember I said the losses can be reduced or eliminated or estimated and accounted for. So in the present case, what I have to do is to find out what is the loss. Suppose I take either liquid or a gas of known thermal conductivity in this gap, the easiest thing is to take air, we do not have to do anything as air is everywhere so you just allow the entire gap to be filled with air so the gap here is full of air and the thermal conductivity of air has been measured by various people, and very accurate values of thermal conductivity are available from the tables given in many books on heat transfer so you can find out what is thermal conductivity of air very accurately.

So all I have to do is give different amounts of heat into the heater, find out by finding out what is the temperature difference across the gap. So we know the temperature difference across the gap, we also know the thermal conductivity of air at that particular mean temperature. So you measure the mean temperature between the two, and we know what the thermal conductivity of the air is at that temperature. Therefore, I can find out how much heat is transfer from the inside to the outside by conduction through the air. So, now I will find out this amount, and then I will also measure the amount of heat input into the cartridge heater. So if I take the difference between these two that will give the loss. So,  $Q$  through air is estimated based on  $k$  of air  $\Delta T$  and other geometric parameters.



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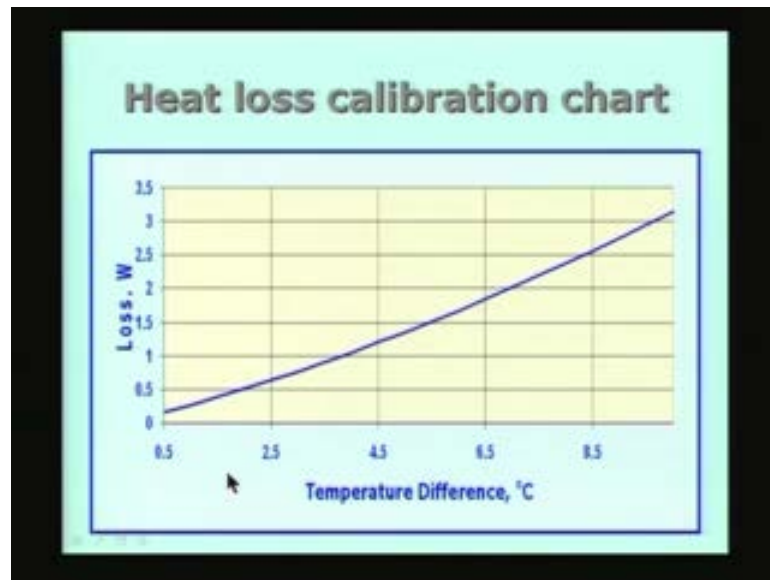
Q through air  $\rightarrow$  estimated  
based on  $k_{air} \Delta T$  and other  
geometric parameters.

$$\left( q_{air} = k \frac{\Delta T}{\delta} \right) \times A = Q_{air}$$
$$Q_{supplied} - Q_{air} = Loss \quad L$$
$$= f(\Delta T)$$

So we know that Q through air will be  $(k \text{ into } \Delta T) \text{ by } \delta$  because, we are assuming that the radial clearance is very small. So it is almost like a slab heat transfer through a plain parallel layer so  $(k \text{ into } \Delta T) \text{ by } \delta$ . And if I multiply this by A, I will get Q through air,  $(Q_{air} \text{ into } A)$  the heat transfer area will give you  $Q_{air}$  and  $Q_{supplied} - Q_{air}$  is equal to the loss, L and this will be a function of  $\Delta T$ , the temperature difference across the gap. So the theoretical basis is the following.

When I have a different sample not air in that gap I am assuming that if the temperature difference across the gap is the same as in the case of air experiment the amount of heat loss is not going to change much. So the experiment with air whose thermal conductivity is known is going to give me a handle on the problem of heat loss from the system which will be a function of a  $\Delta T$ . So if call  $\Delta T$  is  $\theta$ , this becomes a function of  $\theta$ . And in fact, I have data for a particular set up in the form of heat loss calibration chart. You can see that as the temperature difference changes from 0.5 to about 10 degree, here you can see that the loss of heat due to the heat loss from the system goes from here about 3 to 4W.

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So, suppose I am performing an experiment with a liquid or liquid of unknown thermal conductivity, I find out the temperature difference and then corresponding value of loss I get from here, and I subtract this from the total amount of heat I am going to supply through the plug and that will be the actual amount of heat transfer through the sample liquid I have taken in the gap between inner and the outer cylinder. So I can now find out what is the amount of heat transfer by conduction alone, and then obtain the thermal conductivity.

The second point is, because of the gap between the two is very small and the  $\Delta T$  is also very small generally of this order there is no appreciable movement of the liquid. Therefore I can assume that is purely conductive heat transfer. Thank you.