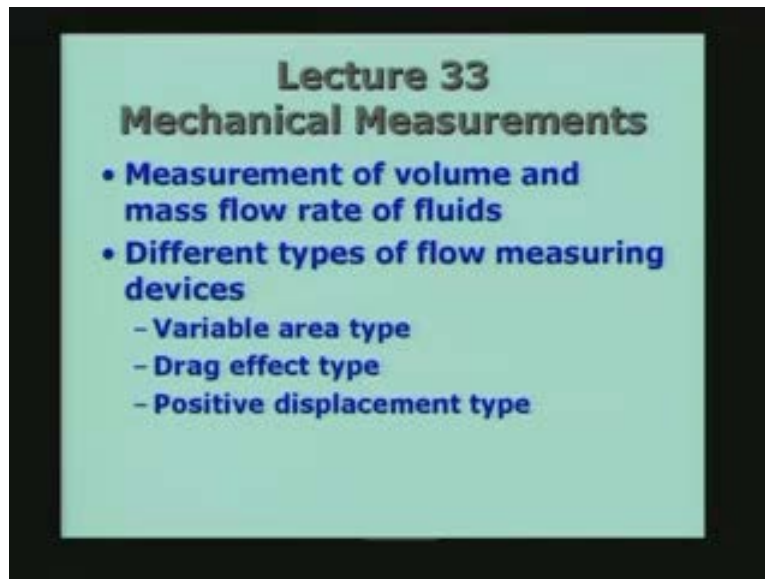


Mechanical Measurements and Metrology
Prof. S. P. Venkateshan
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Module - 3
Lecture - 33
Measurement of Volume and Mass Flow Rate of Fluids

Welcome to lecture number 33 on the ongoing series on mechanical measurements. In this lecture I am going to start looking at measurement of volume and mass flow rate of fluids namely both liquids and gases. We will look at different types of flow measuring devices. Broadly they can be classified as variable area type, the drag effect type and the positive displacement type.

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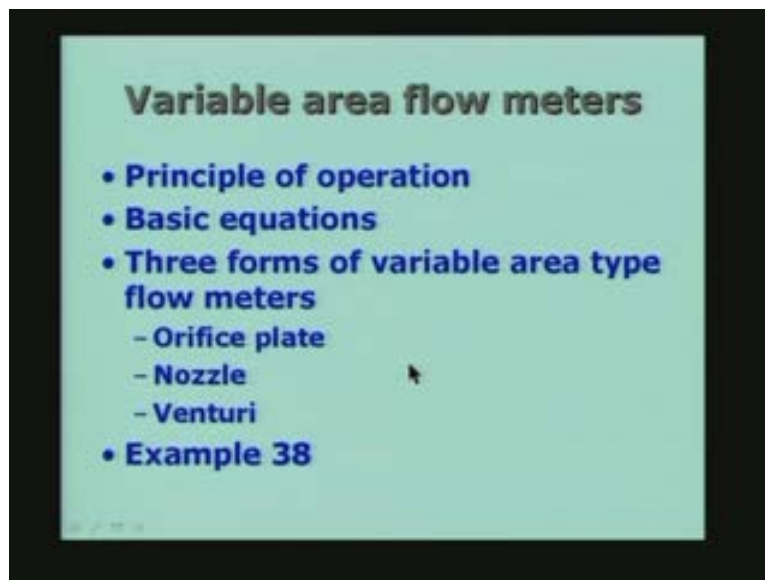


So the present lecture, will more or less confine itself to the discussion of variable area type flow measuring devices. And in the next slide I have shown what we are going to do in the present

lecture. So we look at the principle of operation variable area flow meters. Basically, what we do in the variable area flow meter is to deliberately create a change in the area of the duct through which the fluid is flowing mostly circular pipes. We deliberately introduce a change in the area and because of the change in the area the velocity of fluid is going to undergo a change and because of the change in the velocity there will be a change in the static pressure and we measure the static pressure and relate it to the flow which is being measured or the flow rate which is being measured.

Of course, we will have two different cases. One is compressible corresponding to either liquids or gases flowing at very low velocities and if the velocities are not very small or moderately high then some compressibility effect may be important in the case of gas flows.

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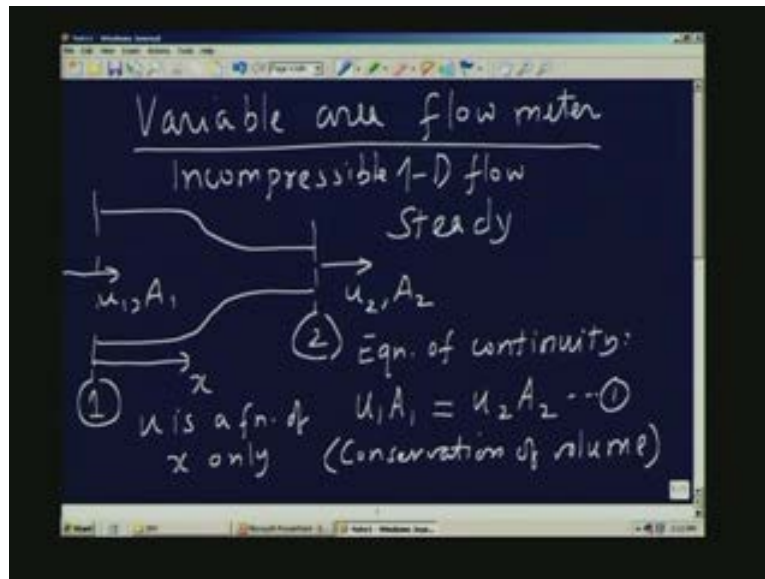
So we will look at the basic equations which describe the variable area flow meters and then we will discuss three types of variable area flow meters which are normally employed such as the orifice plate, the nozzle, and, the venturi. Example number thirty eight will discuss a problem

involving these three devices. This is the principle of operation of a variable area flow meter device. Basically what we do is, we assume that the flow is a set of ideal fluid it that means there is no viscosity and then we look at the flow through a tube of variable cross section or variable area for the flow and assume that the flow is one dimensional and steady. We will deliberately introduce some change in area so let us assume that this is the region in which we are interested.

We will call this is as station 1, we will call this is as station 2 and we shall assume at present that the flow is entering with a velocity u_1 and the area of cross section is A_1 . And at section 2 the values are going to be u_2 and the area of cross section A_2 . We are assuming one dimensional flow which means that the velocity is a function only of coordinate x so u is a function of x only. The velocity is supposed to be uniform across the cross section that is this cross section as well as this cross section and we will assume that viscous viscous effects are very negligible. Of course we will introduce correction account for viscous effect later on and we will assume that the flow is steady.

So the equations which govern the flow of fluid the variable area duct or variable area cross section device like this or the equation of continuity and to start with let us assume that the flow is incompressible that means the density is constant. Therefore equation of continuity or mass conservation equation requires that the volume also be conserved. So there is no change in the volume, and therefore, we can say that the volume flow rate which is constant across the two section is given by $u_1 A_1$ that volume flow rate across the section 1 equal to $u_2 A_2$ across the section 2. So we can call this as equation 1 which is actually conservation of volume because density is constant, conservation of mass is satisfied once the conservation of volume is satisfied. The second equation I am going to use is the momentum equation which can be actually integrated assuming that the velocity is constant across the cross section. We can integrate the momentum equation which is nothing but a statement of conservation of momentum in the following form.

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So p_1 by ρ plus u_1 square by 2 equal to p_2 by ρ plus u_2 square by 2, this is equation 2. So we have two equations, the equation of conservation of mass which became equation of conservation of volume flow rate and the second is equation of momentum conservation, which on integration actually gives you this equation assuming that the velocity is uniformly across the cross section and it is function only of the location along the variable cross sectional area duct. From equation 1, I can do the following. You remember, $u_2 A_2$ equal to $u_1 A_1$ and therefore, I can say u_2 equal to u_1 into A_1 by A_2 . So I can substitute this into equation 2 so this I am going to put in here and what I will get is p_1 minus p_2 by ρ . I am just taking p_2 by ρ to the other side equal to u_2 square minus u_1 square by 2 and u_2 I am going to substitute this and this becomes u_1 square into $(A_1$ by $A_2)$ square minus u_1 square by 2 which will become as u_1 square by 2 is common into $(A_1$ by $A_2)$ square minus 1. This is the equation 3 obtained by combining equation 1 and 2.

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Handwritten derivation on a blue background:

Mom. eqn:

$$\frac{p_1}{\rho} + \frac{u_1^2}{2} = \frac{p_2}{\rho} + \frac{u_2^2}{2} \dots (2)$$

Continuity equation: $u_2 A_2 = u_1 A_1 \therefore u_2 = u_1 \frac{A_1}{A_2}$

$$\frac{p_1 - p_2}{\rho} = \frac{u_2^2 - u_1^2}{2} = \frac{u_1^2 \left(\frac{A_1}{A_2} \right)^2 - u_1^2}{2}$$

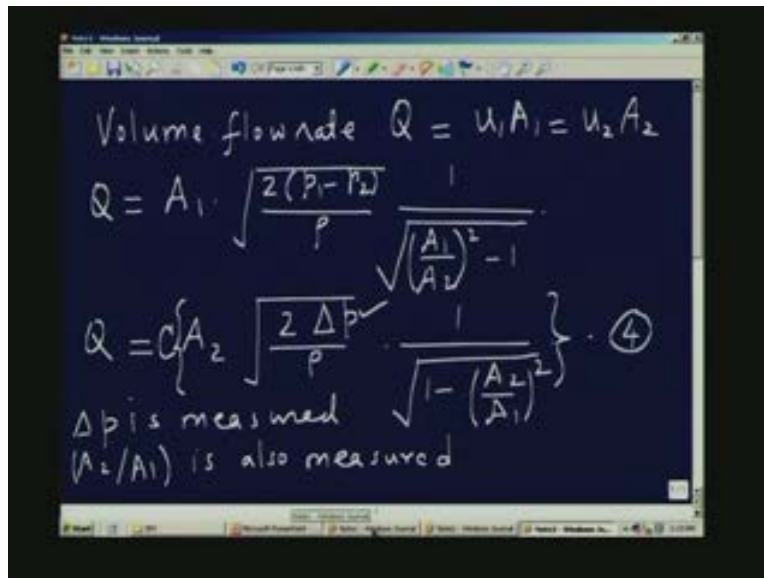
$$= \frac{u_1^2}{2} \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right] \dots (3)$$

Let us now look at the volume flow rate (Refer Slide Time: 13:58) which is nothing but Q equal to either $u_1 A_1$ which is also equal to $u_2 A_2$. I will use the expression for u_1 from equation 3. From equation 3, I can solve for u_1 in terms of the other quantities and substitute to get $A_1(u_1)$ which is nothing but square root of $2(p_1 - p_2) / \rho \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right]$. This can also be written down as A_1 instead of A_1 I will say A_2 (A_1 by A_2) and then I will adjust it with the denominator here. Therefore this also can be written as A_2 square root of $2 \Delta p / \rho$ and $p_1 - p_2$, I am going to call Δp and instead of A_1 I am putting A_2 here so I will adjust it by saying that this equal to $(1 - A_2 / A_1)$ whole square. So Q equal to A_2 square root of $2 \Delta p / \rho$ ($1 - A_2 / A_1$) whole square is the formula I am going to get.

So, in a flow measuring device based on variable, area, principle I am going to measure this, Δp is measured, ratio A_2 / A_1 is also measured because I know the change in the area so I know what the area is at A_1 at section 1 and I know what the area is at section 2. The assumption

is A_2 is less than A_1 , the section 1 has a larger area and A_2 is the smaller area. So I am going to basically use the measured pressure difference Δp and the value of A_2 by A_1 which characterizes the variable area device and use this equation which I will call as equation 4 as the basis for measurement of the volumetric flow rate assuming of course, that the fluid is incompressible or if the fluid is gas for example the velocity is small compared to the speed of sound such that the incompressibility assumption is valid. Earlier in the case of pitot tube also we have seen that. In order to make this equation 4 valid for real fluid flowing through the periphery of the device we can introduce a coefficient of discharge. So what I will do is, I will simply multiply this by C the coefficient which is chosen such that the actual volumetric flow rate is actually given not by the ideal expression, this we will call an ideal expression multiplied by some coefficient which is to adjust for the non-idealness of the fluid flow.

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Volume flow rate $Q = u_1 A_1 = u_2 A_2$

$$Q = A_1 \cdot \sqrt{\frac{2(p_1 - p_2)}{\rho}} \cdot \frac{1}{\sqrt{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

$$Q = C \left\{ A_2 \sqrt{\frac{2 \Delta p}{\rho}} \cdot \frac{1}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \right\} \quad (4)$$

Δp is measured
 (A_2/A_1) is also measured

When writing the equation I wrote down the pressure as p_1 here, and I also wrote the pressure as p_2 here. The question is, what we are going to do in practice is to measure the pressure at the wall and here, we are going to measure p_2 at the wall. If there is some viscous effect, the viscous

effect is going to be confined to region close to the boundary. It can be shown from fluid mechanics principle that, the pressure measured at the wall is equal to the pressure just outside the boundary layer which is formed next to the boundary. Therefore the pressure you measure here is exactly same as the pressure in bulk of the fluid. That is basic assumption made in use of the variable area device for measurement of volumetric flow rate. So I will say C is the coefficient of discharge, A_2 is the area of cross section of the smaller cross section, Δp is pressure difference between section 1 and section 2 or station 1 and station 2, ρ is the density of the fluid which is flowing through the duct, $1/\sqrt{1 - A_2/A_1}$ whole square. Parameter is usually referred to as the parameter M or it is also called the velocity of approach factor.

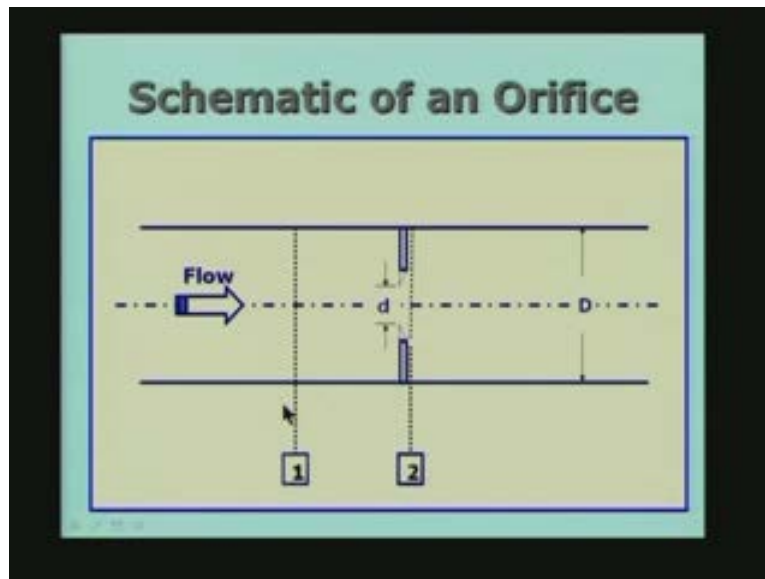
Why is it called velocity of approach factor?

If A_2 is very small compared to A_1 we can actually neglect that term. That means the velocity at section two is much larger than the velocity at section one. Therefore if we are not neglecting that term that means that A_2 is not much smaller than A_1 then we are taking into account that there is a small velocity in the larger area section and we are accounting for that. Therefore it is called the velocity of approach factor. So, if you call it as M and this is C so I will rewrite this equation in a slightly different form. So I will say Q equal to (Refer Slide Time: 18:51) C into M into A_2 into square root of $2\Delta p$ by ρ . And this is the coefficient of discharge and this quantity C into M the product of coefficient of discharge and the velocity of approach factor, is referred to as the flow coefficient which is usually given the symbol K . So we can also write this is as KA_2 into square root of $2\Delta p$ by ρ . We can call this equation 6.

We can also do the following. For the case of compressible flow, actually what we are talking about is flow velocity is not too small compared to the velocity of sound so that some amount of compressibility effects need to be taken into account. In that case, we will introduce another parameter called Y which is called the expansion factor. Therefore equation 6 can be rewritten

with a factor Y here. I am just going to put the value of y there and I will also put the Y here. This Y is going to account for small amount of compressible effect which may be present because the velocity is not very small as compared to the speed of sound in which case of course you would use incompressible flow assumption but small amount compressibility is there that can be accounted by the factor Y . Basically the variable area flow meter operation depends on equation number 6. Now let us look at the different types of variable area type meters which are used and how they look. I have labeled them here such as the orifice plate, nozzle and the venture.

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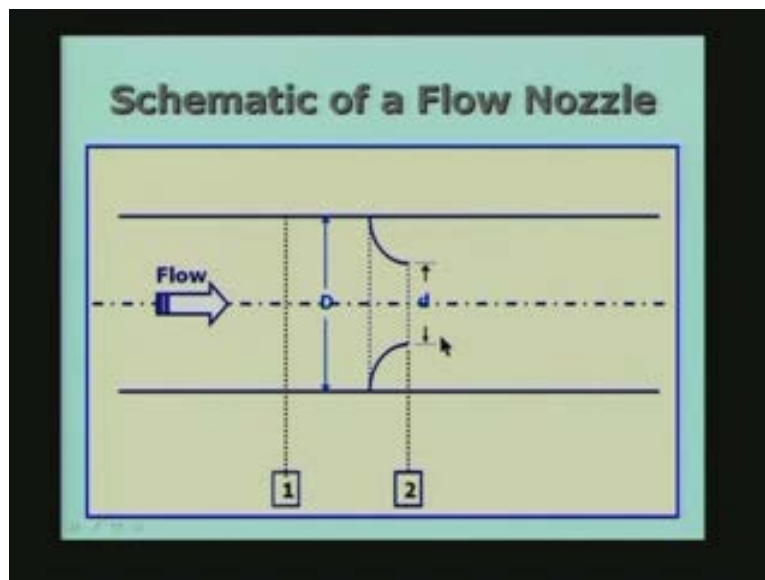
How do we create the variable area here?

I am actually having a tube of diameter D , and flow is taking place from this side. I have identified an obstruction called the variable area device has an obstruction in the form of an orifice with smaller diameter d , this d is less than D . The physical area in this case D represents the area in the pipe and corresponding to d the corresponding area will be the orifice area. And

what we have done is deliberately we have introduced the area change here by introducing a small plate with a hole in it. Therefore, because the flow has to pass through this small hole it is going to accelerate when it comes towards the hole. How does it accelerate?

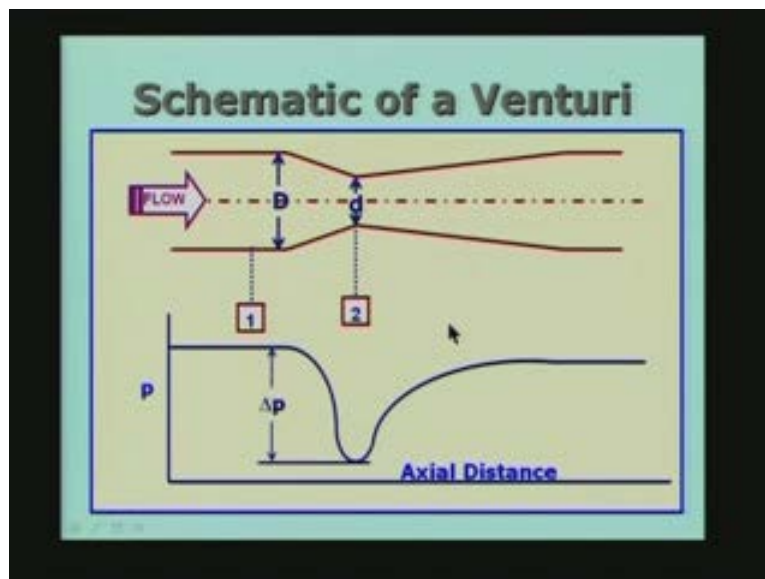
The pressure energy is converted to the kinetic energy because the increase in kinetic energy comes from the pressure. The potential energy is converted to kinetic energy. Therefore the velocity is larger here as compared to here and that is what happens. Now we can choose two locations one and two. In this case, I have taken one upstream of the orifice; usually it is one diameter ahead of the orifice and the second one I can choose very close to the orifice like what I have done. So, what I will do is, I will measure the pressure difference between p_1 and p_2 and use that equation 6 we had which gives the volumetric flow rate in terms of the measured pressure and the known diameter or the corresponding areas we can measure the volumetric flow rate. So the schematic of an orifice is simply showing that there is a small area due to an orifice which is introduced in the pipeline so that the flow accelerates, the pressure changes and we measure the pressure difference and then we relate it to the volumetric flow rate.

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The second arrangement is much better which is to have a nozzle which is specifically shaped so that the streamlines are going to follow a certain contour and in this case, we are going to have much better coefficient of discharge and the pressure loss due to this flow nozzle is much smaller than due to the orifice plate. So here again what you see is that I am measuring the pressure at section 1 and the pressure at section two which may be at the exit of the nozzle or some other location which is specified and we measure the pressure difference again related to the volumetric flow rate. Again, I have used the same symbol D for the pipe diameter and d for the smallest section of the nozzle. This is the exit section and this is the smallest section. The third arrangement which is one of the best ways of doing is to have a venturi which is a converging diverging nozzle.

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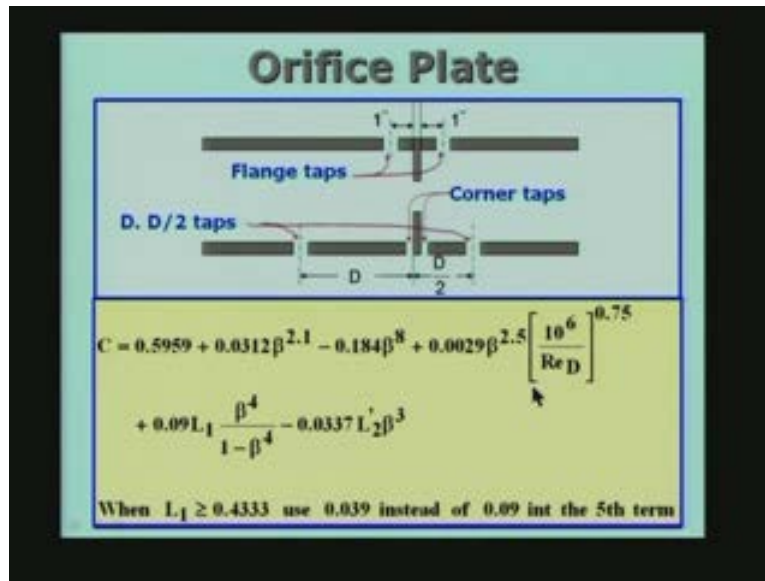
So it starts converging from D to a small diameter d and again it starts diverging and it goes back to the original diameter of the thing. Of course the disadvantage of venturi is that it requires a long length for the measuring device, the venturi itself is very long, from here to here is a fairly long thin, way compared to either the orifice plate which is just a small plate with a hole in it, or

you take the nozzle which is a short thing whereas here it is a very long thing. So, the disadvantage of a venturi is it requires a very long length for installation.

Let us look at what is happening. I have just schematically shown, the change of the pressure or the variation of the pressure along the direction of flow. The direction of flow is from left to right as it is shown here and the pressure is uniform till it comes to the variable area section and it starts reducing and at section two it has got the minimum value because the velocity is highest here. And at the diverging section the velocity again starts decreasing and the pressure will start increasing.

But you will see that the pressure here is slightly different from pressure here because the pressure does not come back to the original value but there is a slight difference between p_1 and P at the exit. This difference between this pressure and this pressure is called the irrecoverable pressure value. And later we will see how the venturi nozzle and the orifice plate varies with different flow metering device. But essentially any variable device (slide time:25:25) will have pressure variation like this and to some extent pressure recovery will take place but the pressure will always be lower than the pressure upstream of the flow measuring device because there is the so called irrecoverable pressure loss. So let us look at the three devices in some more detail, the orifice plate. The orifice plate is essentially a tube with a small obstruction like this. This is the schematic, and I have indicated three different ways of measuring the Δp locating the pressure tap for p_1 and p_2 .

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If I look at these, these are called the flange taps. Why is it called flange taps is because the orifice plate is going to be assembled as a thin sheet of material with a small hole in the center which is going to be held between two flanges and the two pressures taps are going to be very close to the flanges. So essentially they are called flange taps. This is 1 inch and 1 inch, if you want you can take it as 2.5 cm and 2.5 cm in the case of the SI system. This is one arrangement; you measure the delta p between two flange taps as usual. The second one is referred to as **D,D/2** taps. The upstream tap is at a distance of D from the orifice and the second tap is D by 2 from the orifice as **is shown**, D and D by 2 taps so this is one and this is two here.

The third arrangement which can be done is called the corner taps. And in this case, the two taps are right adjacent to the orifice plate and the two sides of the plate. There are three different ways of using the taps for the pressure management. So what is the tap I am talking about? It is a small diameter hole normally 1, 2, 3 mm in diameter to which one of the manometer legs is connected and the other manometer leg is connected to this, this is the essential orifice. If you do not want to use manometer because it is not convenient or it is not possible to use because the pressure

difference is too small then you can use two pressure transducers or one differential pressure transducer and this tap will be communicated into one side of the differential pressure transducer and this tap will correspondingly go to the other side of the differential pressure transducer. So the choice of whether you are going to use a manometer or differential pressure transducer depends on the maximum pressure difference you are going to encounter in a particular application.

Of course, it depends on the velocity, the fluid and so on. So, depending on the range of the pressure difference, we are going to get, you can choose the proper pressure measuring device, either a manometer or an inclined tube manometer if the pressure tap is very small or we can choose the manometer liquid suitably so that it gives you a large reading and so on and so forth. Depending on the location of the taps, I will refer to this length as L_1 and this length as L_2 , L_1 is the distance from the orifice plate to the upstream tap, and L_2 prime is the distance from this space to the other tap the second tap.

The coefficient of discharge has been measured for standard orifices and it depends on two factors. One, it depends on what is called beta, beta is the ratio of the diameter of the orifice to the diameter of the pipe d by D that is your beta. In fact, you can see here, d and D the ratio is, the d by D is called the beta, beta is the symbol used for all the three different types of flow meters namely, the orifice plate, the nozzle and as well as the venturi, so it depends on beta. Secondly, it depends on the Reynolds number of the fluid flow within the pipe in this portion.

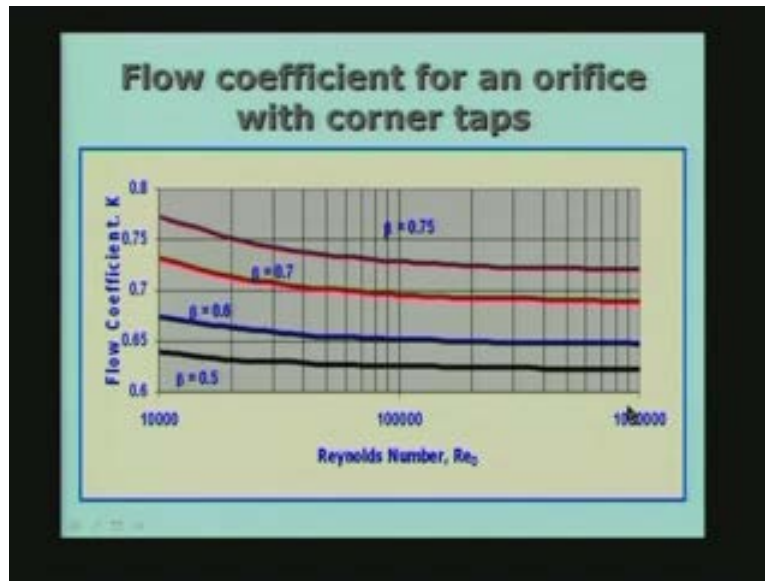
Hence what is the problem with this equation?

This is incidentally called the Stolz equation. Stolz is the name of the person. The equation says that, the coefficient of discharge 0.59959 the dominant term plus some contributions due to the value of beta then the L_1 , L_2 etc and then we have a term which depends on the Reynolds number because Reynolds number is not known. When you want to measure the flow rate we

don't know the velocity therefore we do not know the Reynolds number so this may require a little bit of iterative procedure because Reynolds number has to be guessed or taken for some value and then find out the velocity and then find out the Reynolds number again and so on and that will require some little bit of iterative solution.

For example: L_1 is equal to 0.4333, L_1 is nothing but the ratio of this distance to the diameter of the pipe. It is a non-dimensional parameter, L_1 for example if it is greater than 0.4333 you have to use in this term instead of 0.09 you use 0.039 in this term and beta is d by D and this will give you a value of C . Normally what we do in the case of an orifice plate is we can start with a value of C equal to 0.6 as the starting value because the value of C is close to that value and then after evaluating the velocity from the pressure drop data you come back here calculating Reynolds number based on the evaluated velocity, and then correct for the C value by using this Stolz and may be one or two iterations will be sufficient for this purpose. In fact what I have done is I have plotted the flow coefficient for an orifice with corner taps. Corner taps means L_1 equal to L_2 equal to 0. So what I have is the flow coefficient. If you remember, the flow coefficient is nothing but the product of the coefficient to discharge and the velocity of approach factor.

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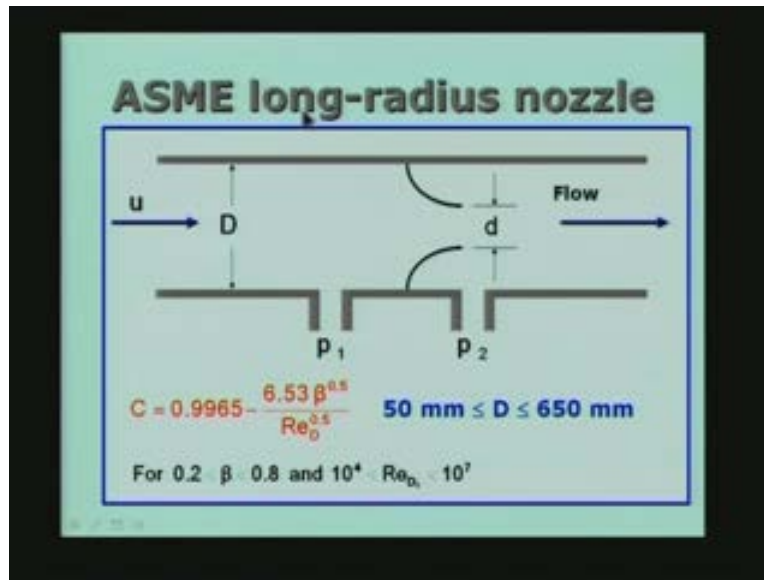


All I have done is I have taken the Stolz equation evaluated the **CD multiplied by K** value and plotted it. For beta it is different values so beta equal to 0.5, 0.6, 0.7 and 0.75 and with different values for the Reynolds number. So what we see here is very interesting. The flow coefficient is almost constant for the values of Reynolds number greater than about 10 to the power 5. So in the particular installation, the Reynolds number in the pipe is more than 10 to the power 5 you can use a constant value for CD and that is the advantage of that. Of course the actual value for CD depends on beta but for different values of beta I have different values of coefficient but they also show some kind of asymptotic behavior which means that for Reynolds number greater than about ten to the power of five so hundred thousand is 10 to the power 5 is more or less independent of the velocity or independent of the Reynolds number.

Of course Reynolds number is calculated based on the velocity, the diameter of the pipe and also the particular fluid which is flowing. So the fluid property also is coming into this by itself. The second type of device we can think of is called the ASME long radius nozzle. ASME is American Society for Mechanical Engineering who has standardized the design. Essentially what

we have is p_1 upstream of the nozzle, this is the nozzle, p_2 is the down stream of the nozzle and for the ASME long radius for the nozzle the coefficient of discharge is given by 0.9965 minus a small quantity 6.53 the beta to the power of 0.5 by the Reynolds number to the power of 0.5.

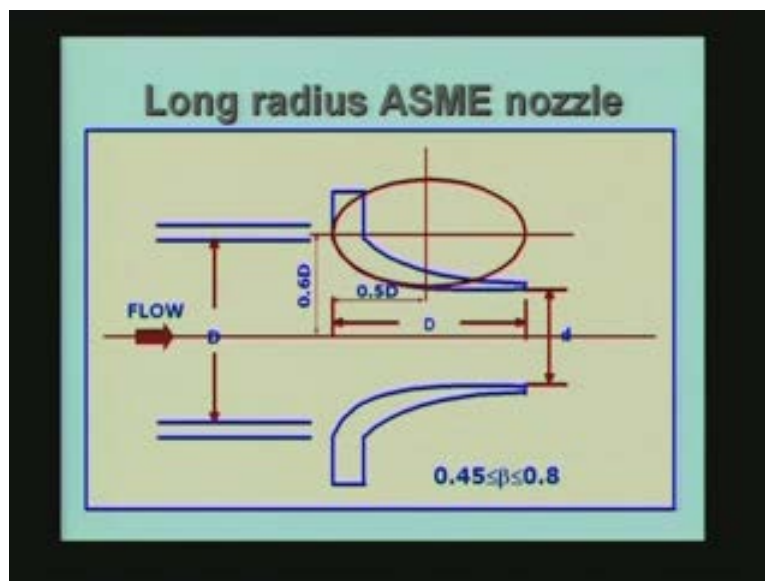
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Again you see that C depends on the Reynolds number and the value of beta and the diameter of this could be any where between 50 mm and 650 mm. We are talking about fairly large range of values for the diameters. And 50 mm is the typical size at with many engineering applications, 650 mm for example can be met within large scale installations like water supply or natural gas flowing through the pipeline and so on you can have very large diameter pipe and water supply is one example. So, if you go back to the sketch you will see that the long radius nozzle can have values of beta between 0.2 and 0.8 and Reynolds number between 10 power 4 and 10 power 7. Normally less than 10 power 4 we do not use any of these devices. In most applications the Reynolds number is certainly more than 10 power 4. Let me just digressively look at this 0.2 to 0.8. Why do we have such a large range of values for beta? If you go back to the equation which determines the flow rate from the delta p, delta p is actually created by the change in the area and

if the beta is small the change in area is larger and the delta p is larger. Therefore in most applications where we want to use a variable area metering device for measuring velocity or flow rate we must make sure that the delta p which we are going to obtain is large enough to be measured easily by using an available pressure measuring device. Therefore depending on the velocity achieved in that particular thing you may choose different values of beta or you choose appropriate value of beta such that the pressure difference is measurable and easily measurable at that. That is the reason why we have different values of beta. And you can choose anywhere between 0.2 and 0.8 for this particular case as I have indicated which is also true for other devices. Again I have shown the proportion of a long radius ASME nozzle.

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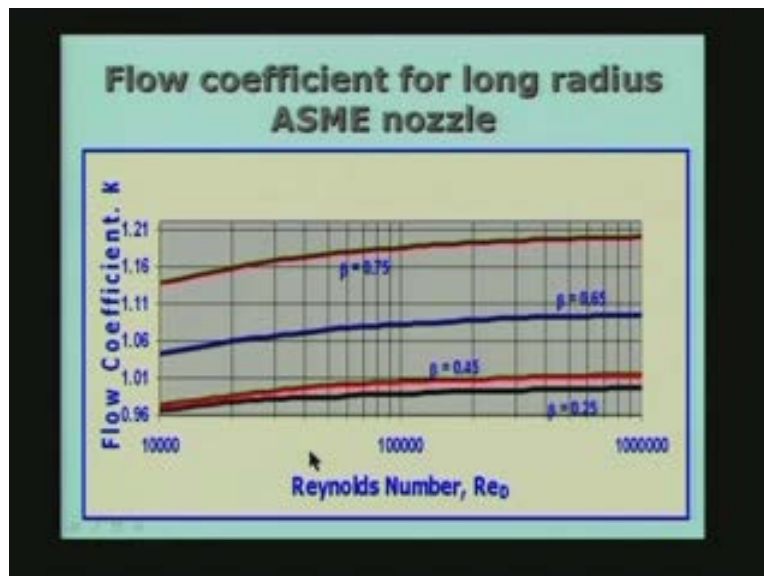


This is the nozzle as shown here, exit diameter is d , inlet diameter is D here and the nozzle itself is controlled as the part of the ellipse, from here to here is part of ellipse and the major axis equal to D is the diameter D displaying this D and the semi major axis will be $0.6D$ minus $0.5D$ into 2 because that will give this. Therefore this is the standard procedure for obtaining the long radius

ASME nozzle. And the long radius ASME nozzle is recommended for beta between 0.45 and 0.8. These are the values between which this can be used.

And what is important for us is only the inner contour; the outer contour is of no significance. And the pressure is measured upstream of this space and close to this or some other space depending on the particular design. So the flow coefficient for long radius ASME nozzle is again shown here. If you remember, the flow coefficient is nothing but the form of the product of coefficient of discharge, and the velocity of approach factor. And the velocity of approach factor is given by $(1 - \beta^4)^{-0.5}$ whole square A_2 by A_1 is now $(d/D)^4$ whole square because area is proportional to diameter square. Therefore actually the velocity of approach parameter is nothing but $(1 - \beta^4)^{-0.5}$ as we can easily see that. So, if we choose a particular value of beta the velocity of approach factor gets fixed then CD is going to determine the product of CD the velocity of approach factor.

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Therefore flow coefficient is nothing but the product of these two and if you remember the value of C_D for the nozzle is very close to the unity. That means the value of the flow coefficient is almost close to the value of the velocity of approach factor itself. Of course, there is a small Reynolds number dependence and of course asymptotically it will reach the value equal to M or very close to M . So this is for the ASME nozzle. So, if you compare the orifice plate and the nozzle the orifice plate has a low value of the flow coefficient and if you go back to equation 6 as showed the dependence of Q versus the rest of the thing, if the coefficient which is smaller or if the flow coefficient is smaller it means that the Δp generated will be higher.

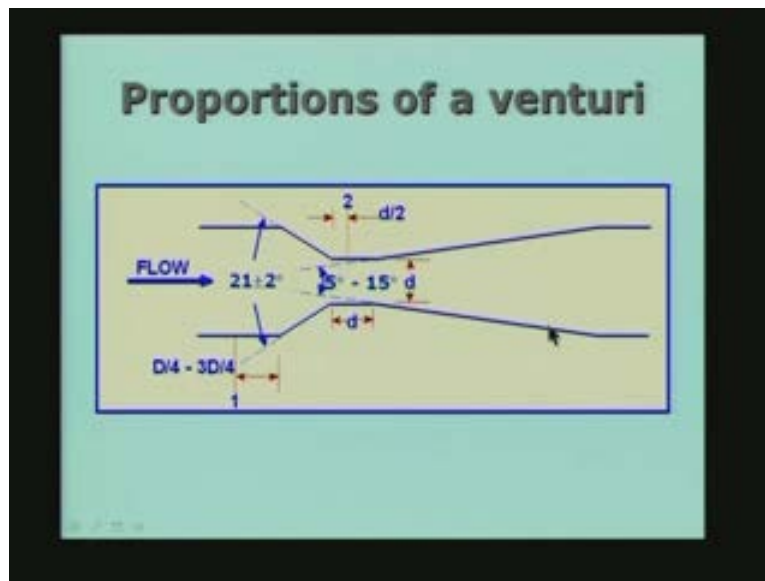
So the orifice plate is going to give you a larger pressure drop or pressure difference or measurable pressure difference as compared to the nozzle in this case. In the case of venturi also the same thing is possible or the same thing holds. The proportions for venturi are given here. It consists of two sections; the converging section then a landing or a flat region where the diameter is uniform equal to d and then it starts diverging again. The angle of convergence can be 21 ± 2 degree which is the allowed value. The diverging section can have an angle of 5 degree to 15 degrees and any angle can be chosen.

So, if you choose the angle of 5 degree the length will be very large. If you take a 15 degree angle then this will be shorter. So, if you want a compact design for the venturi you go for a larger angle at the diverging section. But if you do not mind having a very long venturi you can go for 5 degree nozzle or any value between 5 degree and 15 degree. There is a landing which is equal to diameter of the section here, and the second tap is positioned at $d/2$. Here $d/2$ is at the center of this flat portion. And the upstream tap is anywhere between $D/4$ and $3D/4$ where D is a pipe diameter from the start of the converging section.

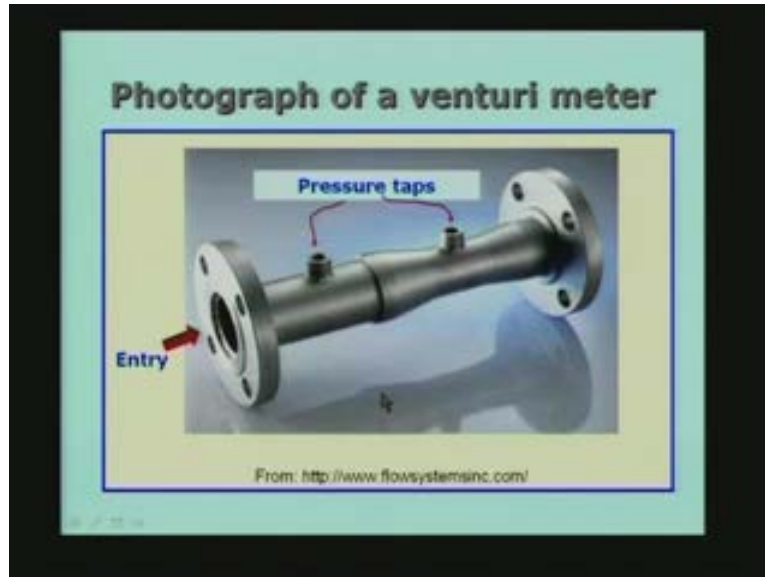
Just before the converging section starts you can have it anywhere between $D/4$ and $3D/4$ as indicated. This is the proportion for a standard venturi as per ASME specification. The flow is taking place this side. Here is a small photograph of the venturi meter which is commercially

made. It is taken from the website of flow system incorporated. They are the manufactures of the venturis. Here you can see the venturi meter which is made extremely well. You have the entry on the left side, the pressure tap is shown here, this is the pressure tap 1, this is the pressure tap 2 and you can see here, there is a portion which is striped without change in the area and in the middle of that and there are two flanges one at the beginning, and one at the other end and this entire thing is placed in between, you make a small gap in the pipeline and introduce this in the pipeline for the flow measurement. So these are the two pressure taps which are used for measuring the delta p related to the chew of the venturi meter or the velocity flow rate.

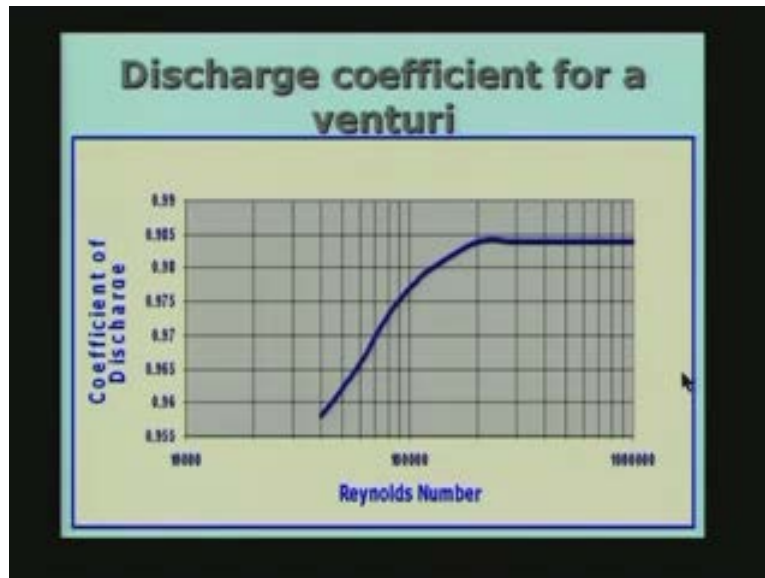
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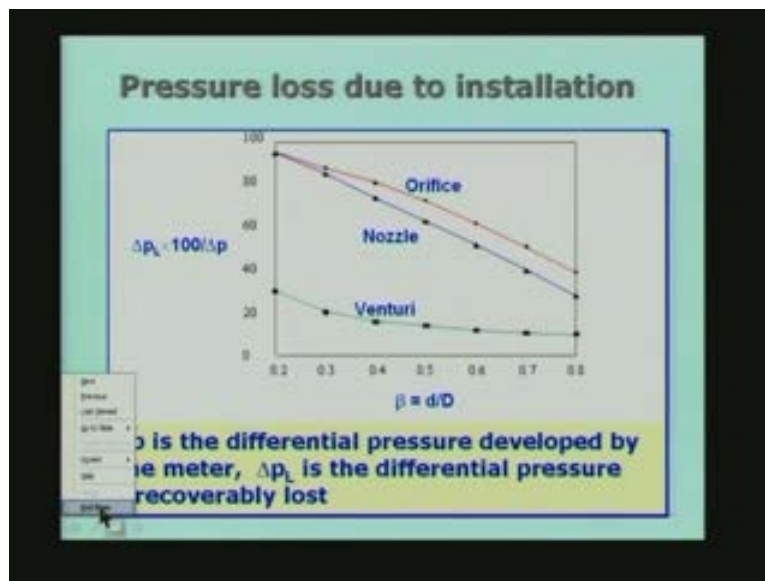
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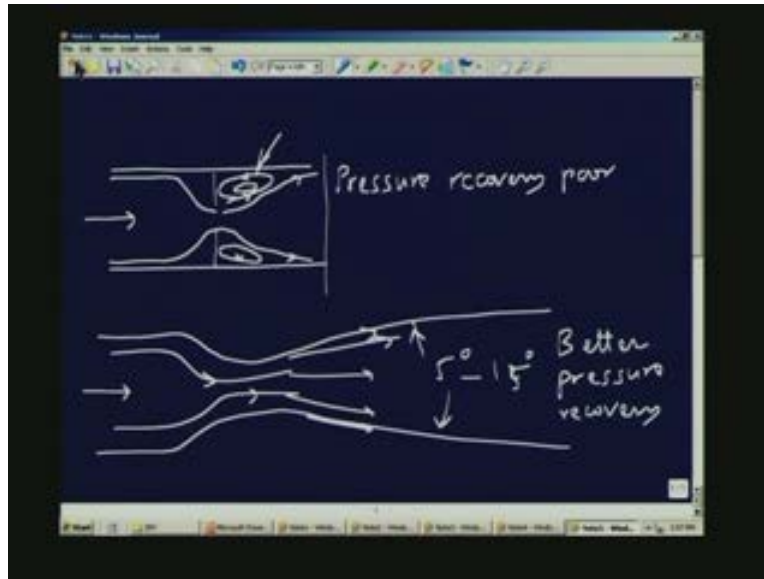
And in fact the shell coefficient of the venturi shows some kind of a variation like this. This is from the measured data. The coefficient to discharge is a function of Reynolds number again and

you will see that it is starting with a fairly large value of 0.96 and if the Reynolds number is more than about 2×10^5 it levels off at about 0.984 or so. Now we will look at why we have two or three different types of meters when one of them can possibly adequate. Actually the pressure changes from the entry to the second section the pressure goes down, because of the change in the velocity and then afterwards, the flow is again going to expand to fill the pipe so the pressure has to recover.

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The pressure recovery process is not the same in all the three cases. Let us look at the pressure recovery. If I take a flow in an orifice plate, I will just indicate the orifice by a small hole here, the flow is taking place from this side, the flow is going to come like this and it is going to expand like this and it forms some kind of a circulation pattern here. What happens in the circulation pattern is, the kinetic energy is converted into the internal energy. Therefore some amount of pressure ahead is swallowed up by this activity. Therefore pressure recovery here is not good the pressure recovery is poor, because of this flow which is generated, the flow separation. If you take a look at the **venturi**, this is a venturi there is no flow separation. It goes nicely like this, follows the **contour**, the flow is not going to have any of these things.

The flow separation and the formation of a vortex etc., is not there. Therefore there is no pressure and the pressure recovery is much better. That means, if you choose the exit angle that 5 degree to 12 degree or 15 degree whatever we mentioned if you choose within these limits, the pressure recovery is much better than in this case because in this case there is a scope for development of this separated flow and the circulating flow which is going to dissipate some of the kinetic

energy. In fact the nozzle also is a good device because it is going to behave much better. In fact you can see that the orifice, nozzle and the venturi. if you look at the pressure loss is due to the installation because total recovery is not possible. In the case orifice you see that of course it is some kind of beta and smaller the beta the worse is the pressure loss. As the beta becomes larger and larger the pressure drop which is created is also small and the amount of pressure drop which is not recovered is also small.

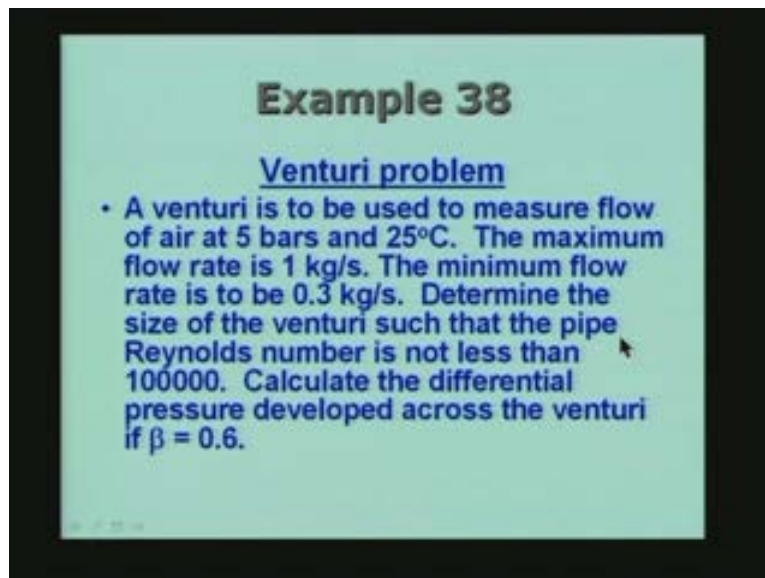
Let us look at how we have plotted here. Δp is the differential pressure developed by the meter. This is p_1 minus p_2 and Δp_L is the differential pressure irrecoverably lost. That means this is the pressure at the exit of the device minus the upstream pressure minus the exit pressure. So this is nothing but, the pressure difference created by the device because of the acceleration of the flow, the converging section divided by the pressure drop across the device from inlet to the exit divided by the pressure difference created by the converging section.

For the orifice, almost even for beta equal to 0.8 almost 40% of the pressure drop is irrecoverable. That means 40% of the pressure drop which occurred in the converging section is going to be irrecoverably lost. The nozzle is slightly better but you see that venturi is the best because even in the worst case of beta equal to 0.2 only about 30% of the value is going to be lost and as you go to higher values of beta asymptotically it is going to some 5 to 10%. Therefore the reason why we have three different types of variable area type devices is because orifice is not very good from the point view of the pressure loss due to the installation, nozzle is slightly superior but venturi is of course the best. But orifice occupies the smallest length, nozzle is next to that and of course venturi requires a large length for the installation. So, if you are particular about the length of the installation then you can go for orifice but then you have to sacrifice the pressure loss.

Another point is, the nozzle and the venturi both have a very high coefficient to discharge compared to the orifice which is a major plus point for the nozzle and the venturi. The

coefficient discharge is higher and also the variation is small, whereas the orifice has very large variations, starts with about 0.6. The third point is that, the orifice is very inexpensive, it requires simply a small sheet of material in which you drill a hole and the nozzle requires the contouring it is very expensive, and the venturi is very well made and it requires a lot of machining and so on, it is most expensive. So the expense goes up in this direction and as the expense goes up the pressure loss also goes down. Therefore based on the requirement you have to decide whether to use the orifice, nozzle or venturi. Cost-wise also the orifice is the cheapest, the nozzle is more expensive and venturi is the most expensive amongst the three different types of measuring devices. Here is a simple problem. You want to use a venturi in the measurement of flow of air at five bars and twenty five degree Celsius and the maximum flow rate is given as 1 kg by s and the minimum flow rate is also mentioned as 0.3 kg by s.

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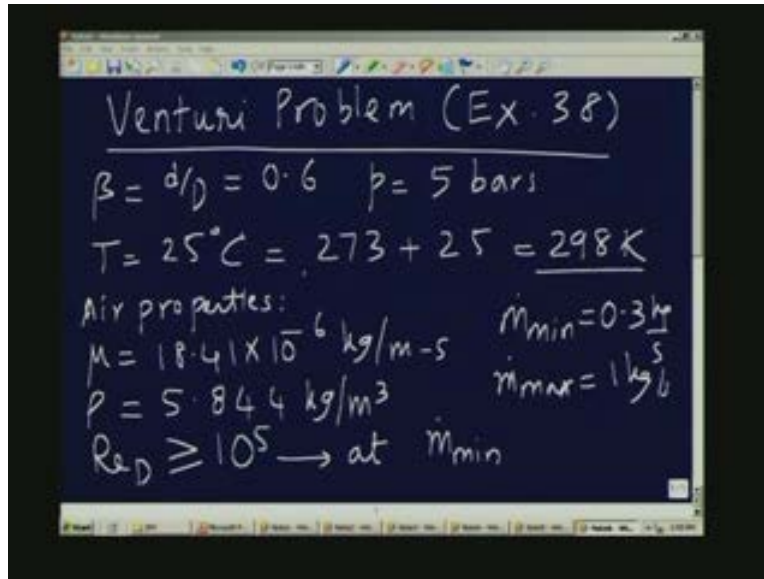
Example 38

Venturi problem

- A venturi is to be used to measure flow of air at 5 bars and 25°C. The maximum flow rate is 1 kg/s. The minimum flow rate is to be 0.3 kg/s. Determine the size of the venturi such that the pipe Reynolds number is not less than 100000. Calculate the differential pressure developed across the venturi if $\beta = 0.6$.

And we want to find out what is the size of the venturi such that the pipe Reynolds number is not less than 100,000. In that case you want to calculate the differential pressure developed across the venturi if the beta is point six. This is example 38.

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Venturi Problem (Ex. 38)

$$\beta = d/D = 0.6 \quad p = 5 \text{ bars}$$
$$T = 25^\circ\text{C} = 273 + 25 = 298 \text{ K}$$

Air properties:

$$\mu = 18.41 \times 10^{-6} \text{ kg/m-s} \quad \dot{m}_{\min} = 0.3 \frac{\text{kg}}{\text{s}}$$
$$\rho = 5.844 \text{ kg/m}^3 \quad \dot{m}_{\max} = 1 \frac{\text{kg}}{\text{s}}$$
$$Re_D \geq 10^5 \rightarrow \text{at } \dot{m}_{\min}$$

We are given the beta as 0.6 is nothing but d by D equal to 0.6, the pressure is 5 bars and temperature is given as 25 degree and I will convert it to Kelvin that is 298 Kelvin. So the air properties are evaluated at this pressure equal to 5 bars and 298K. In fact I have taken the properties from the table of properties and directly I will give the values. The value of viscosity is 18.41 into 10 power minus 6 kg by ms and the density at 5 bars is given by 5.844 kg by m cube. So the problem specifies that the pipe Reynolds number should be greater than or equal to 10 power 5. And we will say that the minimum mass flow rate is 0.3 kg by s and the maximum possible is 1 kg by s .

So, if the Reynolds number has to be greater than or equal to 10 power 5 this will be at \dot{m}_{\min} . For the minimum mass flow rate, I must choose the Reynolds number such that it is equal to or greater than 10 power 5 so that will be the limiting value. So, if you remember (Refer Slide Time: 57:00) Reynolds number based on the diameter is given by $4 \dot{m}_{\min}$ by $(\pi D \mu)$. So this is equal to 10 to the power 5 equal to $4 \text{ into } 0.3 \text{ kg}$ by s by $\pi \text{ into } D$, and that we do not know and that is what we want to determine, μ is given by 18.41 into 10 to the power

minus 6 and solve for D it comes to 0.207m, some 20.7 cm. And we are also given beta equal to d by D equal to 0.6 and 0.6 equal to d by D. Therefore d equal to 0.6 into 0.207m and this value is 0.124m. So the venturi meter has got an inner diameter at entry of 0.207m, the smallest cross section has a value of 0.124m. In fact, now I can calculate the delta p which is going to develop. For that we know that, the Reynolds number equal to 10 power 5 for the minimum value the corresponding C value I can take from the graph which I showed earlier. So, if you see 100000 this is the value and the value it is corresponding to is something like 0.756 or so 975 977. So we can take that value from this graph. And because Reynolds number is already given, I can simply take the value from that, and therefore this value of C equal to 0.977 as I just now indicated. So if the value of C is given, I can calculate now M, the velocity of approach factor is 1 by square root of 1 minus beta power 4 1 by square root of 1 minus 0.6 to the power 4 and the value of M happens to be 1.072.

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Handwritten calculations on a digital screen:

$$Re_D = \frac{4 \dot{m}_{min}}{\pi D \mu} \cdot 10^5 = \frac{4 \times 0.3}{\pi \times D \times 18.4 \times 10^{-2}}$$

Solve for $D = 0.207 \text{ m} \checkmark$

$$0.6 = d/D \therefore d = 0.6 \times 0.207 \text{ m} = 0.124 \text{ m} \checkmark$$

$$Re_D = 10^5 \rightarrow C = 0.977$$

$$M = \frac{1}{\sqrt{1 - \beta^4}} = \frac{1}{\sqrt{1 - 0.6^4}} = 1.072$$

So, I use equation 6 of course, with the assumption that γ equal to 1. There is no expansion factor, the compressibility effect is not there. Therefore I can say delta p minimum by using that

equation is \dot{m}_{minimum} by CMA at the throat because $(s)^2$ square multiplied by $\frac{1}{2}$ times ρ . So I will just substitute the values; A_t is throat area which is given by $\frac{\pi D^2}{4}$ and that happens to be 0.012m square, D is 5 into d square by 4 so this will be 0.3 by C is 0.977 (1.072 into 0.012) whole square $\frac{1}{2}$ into 5.844 that is the density and this will be in so many Pascals and the value comes out to be 47.4 Pascals. So the pressure drop which is measured comes to 47.4 Pascals at the minimum of the flow rate. Thank you.