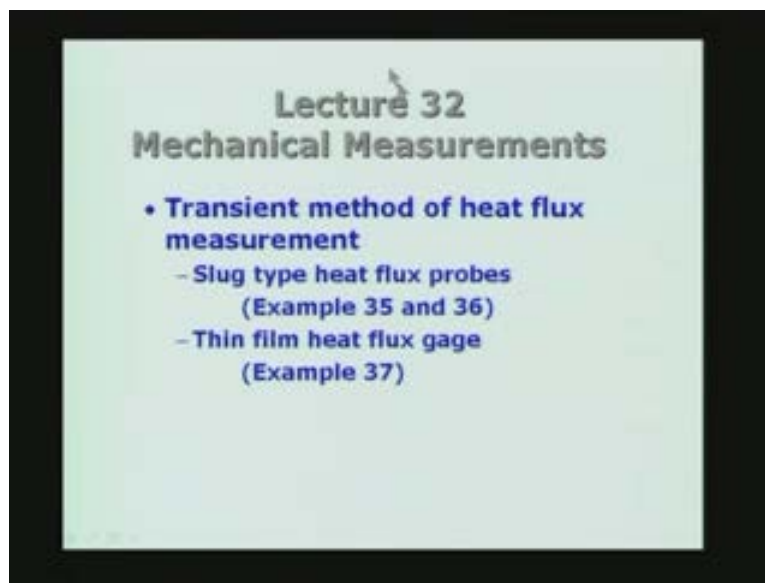


Mechanical Measurements and Metrology
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Module - 3
Lecture - 32
Transient Method of Heat Flux Measurement

So this will be lecture number 32 on our on going series of mechanical measurements. We have been discussing about how to measure the heat flux. And in fact looked at a few of them, and the transient method of heat flux measurement happens to be one of the important techniques that is useful in practice.

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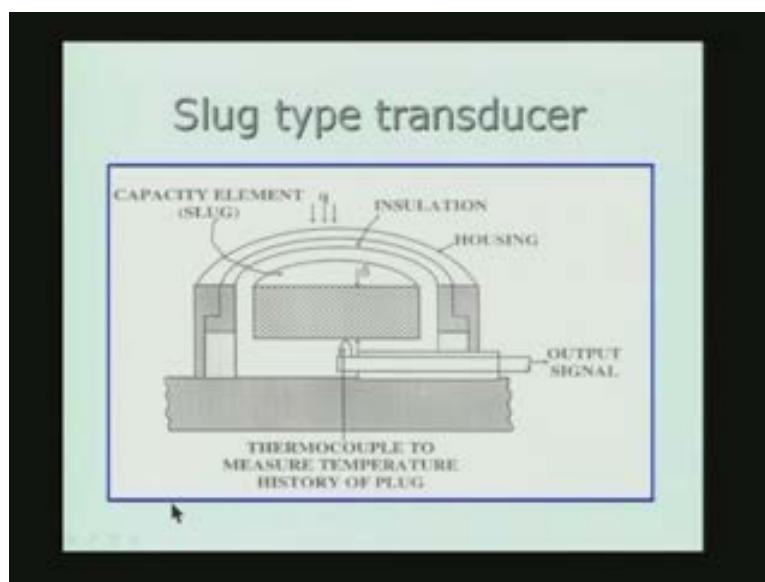


The reason why transient methods are useful is that, many experiments do not last for too long. Especially those which are useful for very high heat flux cannot be a continuous process. For example, if you have a flow inside a nozzle, in the case of space applications you have nozzles with very high temperature, gases moving through the nozzle passages and the entire experiment may last only for a few seconds. Sometimes in high speed internal flow itself may be only for a few seconds, and it some times it may be even for a few milliseconds or microseconds. So the idea is to measure the heat flux during the very short period for which the

experiment itself takes place. Therefore the transient method of heat flux measurement is directed towards measurement of heat flux in those applications where the process itself is going to last for a very short period of time especially high heat flux environment because they cannot be sustained for over a long period of time.

Let us look at two types of transient methods. One it is the slug type heat flux sensor and the second one is thin film heat flux gage. This thin film comes up again and again. And of course the way we are going to use it here is to use it in the transient mode of operation. Let us take one or two examples which will be giving some idea about the numbers which characterize this heat flux sensors. So let us look at the slug type transducer.

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Basically the slug type transducer consists of a slug of material. This is the material which we call as a slug or the heat capacity element. So the mass of the slug multiplied by specific heat of the material is actually the capacity of this element. If the slug temperature does not go through a very large variation during the operational slug type transducer the specific heat may be more or less assumed to be constant.

Of course if the temperature variations are very larger than to you have account for this specific heat variation with respect to temperature. Basically what we are going to do is to look at, for example if the slug has a top surface which is exposed to heat flux which has to be measured and we have fixed the thermocouple, now in fact the thermocouple in principle could be placed anywhere in side the slug but in this particular case it is placed at the back surface. And the temperature response of the back surface of the slug is actually going to be measured as a function of time and indirectly we are going to find out what is the q which gave rise to the temperature increase.

We will look at two cases; one where the slug is so thin in the direction parallel to the applied heat flux. If this thickness is very small and if the thermal conductivity of the material is large enough, then we can assume that the time taken for the temperature information from the top to the bottom is very small. That means the entire slug at any time t after the start of the experiment can be assumed to be a uniform value of temperature. This is one assumption which can be made in case Δ is small and the thermal conductivity of the material is large enough. That means the entire slug will have a uniform raise in temperature throughout.

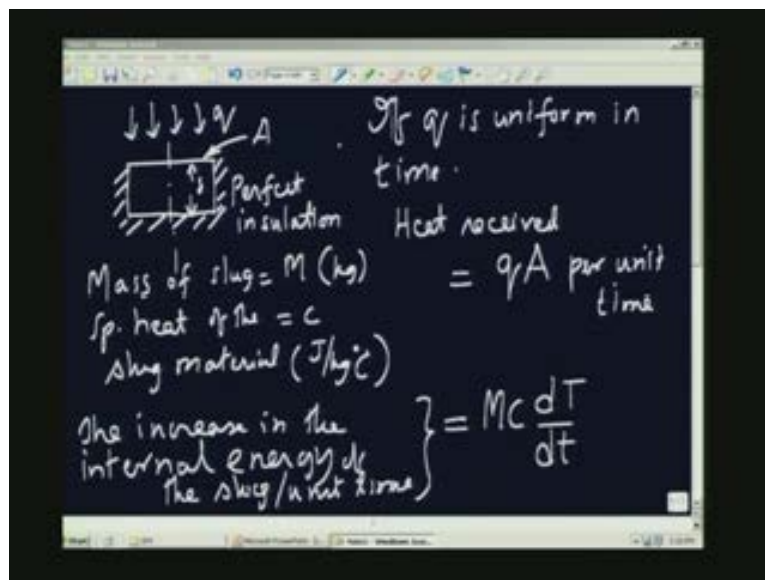
Of course in practice you will have to insulate it or isolate it, isolate the slug from the rest of the universe or rest of the space and so what you have is a casing or the housing and the entire slug transducer is mounted on a base and the temperature of the casing of this housing is assumed to be held at a value which is probably equal to the value of the temperature of the slug before the start of the experiment. So, initial temperature and the final temperature for the housing is supposed to be the same. In fact the housing at the slug is not in contact and it requires some way of isolation. And the housing itself is a good instillation so thermally it insulates the slug.

Of course, there may be a small residual amount of heat transfer or heat loss because no perfect insulation is found in a practice. Therefore we can allow for the small loss by changing the analysis slightly by including a certain small loss coefficient. Here we have q , let us assume the heat flux to be uniform within this diameter of the slug and it is illuminating the top or it is applied on the top surface which is a circle of some diameter and the q multiplied by the area of the slug which is exposed to the heat flux is the amount of heat entering this slug this must be equal to the rate of change of the internal energy of the slug which can be determined by the mass specific heat and the temperature raise product.

Let us do a small analysis and find out how we are going to account for the heat loss. We have the slug shown here as a rectangle but actually it is a cylindrical specimen. We can assume that it is more or less well insulated on the sides and we have a certain thickness δ and then we have the q which is coming in at the top surface and the area of the surface is A . So the slug has got a mass of slug is equal to M and the specific heat of the slug material is c . This will be in so many kilograms and this will be in Joules by Kg degree Celsius.

If q is uniform in time, it uniformly impinges on the front surface of the slug, the heat received is nothing but qA per unit time per sec. So the increase in the enthalpy or slug or internal energy of the slug or a solid material there is no difference of this slug again per unit time will be nothing but the rate of change of the internal energy, this will be assuming the mass and specific heat to be constant M and c can be taken outside dT by dt . This is under ideal conditions where this is perfect insulation. And also I am ignoring any heat loss from the front surface which is receiving the heat. Therefore there is no heat loss from the front surface. Of course, these two assumptions are not going to be perfectly valid but still we may have to apply a small correction later on.

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So all we have to do is equate the two. So we will see that $Mc \frac{dT}{dt}$ is equal to qA or $\frac{dT}{dt}$ is equal to $\frac{qA}{Mc}$ and initially at t is equal to 0 we are assuming

that T is equal to T_0 which is the temperature of the slug at t is equal to 0. Therefore I can say that if you integrate this one you will get T is equal to qA by Mc (t plus some constant integration) and putting T is equal to t is equal to T_0 I will say that what is going to happen. In other words, the temperature increase T minus T_0 is equal to $(qA \text{ by } Mc)$ into t is a linear variation. Of course this is not going to be strictly correct but it is a good approximation in the case of a thin slug which is going to be used as a heat flux gage.

Now suppose we assume if there is some heat loss how can we account for it? We can account for it by saying that, heat loss is proportional to the temperature difference between the slug and the surrounding to which it going to lose heat. And in this case, I am going to assume that the surrounding is some temperature T_{infinity} which we can adjust later.

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$$\begin{aligned} \int Mc \frac{dT}{dt} &= qA \quad \Rightarrow \quad \frac{dT}{dt} = \frac{qA}{Mc} \\ t=0: T &= T_0 \quad T = \left[\frac{qA}{Mc} t + T_0 \right] \\ (T - T_0) &= \frac{qA}{Mc} t \rightarrow \text{Linear variation} \\ \text{If there is some heat loss.} \\ q_L &\rightarrow \underbrace{K_L}_{\text{W/}^\circ\text{C}} (T - T_\infty) \end{aligned}$$

Actually heat loss may take place all around the region surrounding the slug that is in all directions. But what I will do is, I will equate it to an equivalent heat loss per unit area from the top surface. I will base it on the area of A which is receiving the heat. Therefore I will adjust the coefficient such that the heat loss to the surroundings is given by q_L the heat loss is proportional to some coefficient K_L which we call as lost coefficient (T minus T_{infinity}) where infinity is a suitably chosen temperature. And this K_L is actually in W by degree Celsius. That means it

is like the product of a heat transfer coefficient and the area in this case, I am basing it on the area A . Let us see the effect of this. For that let us rewrite the equation. If you go back here we will see that $Mc \frac{dT}{dt}$ is equal to qA but because there is a loss I have to subtract the last part from qA . So, if we do that the equation will change as shown in the next slide.

So we have (Refer Slide Time: 16:16) $Mc \frac{dT}{dt}$ is the rate of change of the internal energy of the sensor is equal to qA which is the heat flux impinging on the sensor minus the last part K_L into $(T - T_{\infty})$. This will be the equation. So we will say this is the gain and this is the loss and of course this is the net effect. The assumption I am going to make is if we divide throughout by Mc you will see that we are going to get the following equation is equal to qA by Mc minus the last coefficient K_L into $(T - T_{\infty})$ by Mc . This is a small quantity. What we are trying to argue is that if this is small amount of loss is present this term is going to be much smaller than the first term. So I will say that is small and I will indicate its magnitude by ϵ . The idea is that the $(T - T_{\infty}) \epsilon$ is small compared to the first term. Of course then you will see that the dominant effect will be the effect of the input heat flux and this will be a small change in the response.

So I can do the following. I will assume that T is given by some dominant term $T^{(0)}$ plus ϵ into $T^{(1)}$ and of course higher order terms will be present with ϵ square and so on. I will say plus order ϵ square neglected, neglect this term. So all have to do is to substitute this into the equation here, and then you will see that, $\frac{dT}{dt}$ will become $\frac{dT^{(0)}}{dt}$ plus ϵ into $\frac{dT^{(1)}}{dt}$. The left hand side is going to become this and on the right hand side I have this which is a constant and this is in the order of ϵ . Therefore this term must be equal to this and the second term must be equal to the second term here.

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$$\begin{aligned}
 \underbrace{Mc \frac{dT}{dt}}_{\text{Nutt}} &= \underbrace{qA}_{\text{Gain}} - \underbrace{K_L(T-T_\infty)}_{\text{Loss}} \\
 \frac{dT}{dt} &= \frac{qA}{Mc} - \underbrace{\frac{K_L}{Mc}}_{\text{small } \epsilon} (T - T_\infty) \\
 T &= T^{(0)} + \epsilon T^{(1)} + \underbrace{O(\epsilon^2)}_{\text{neglected}} \quad (\text{small change}) \\
 \frac{dT}{dt} &= \frac{dT^{(0)}}{dt} + \epsilon \frac{dT^{(1)}}{dt}
 \end{aligned}$$

So you can see that $dT^{(0)}$ by dt is equal to qA by Mc and $dT^{(1)}$ by dt which is now multiplied by epsilon, if you go back to the previous slide, you will see that $dT^{(0)}$ by dt plus epsilon into $(dT^{(1)}$ by $dt)$, epsilon is multiplying this, epsilon is also multiplying this therefore you will see that what I will get is T minus T_{infinity} with a negative sign. That negative sign is coming from the fact that there is a lost term. So this can be integrated as we did earlier assuming that T is equal to T_0 at t is equal to 0. Therefore $T^{(1)}$ is equal to 0 at t equal to 0, because t is equal to T_0 . Therefore we require T_1 is equal to 0.

And if we integrate $T^{(0)}$ will come out to be $(qA$ by $Mc)$ into t plus T_0 and if I integrate this $dT^{(1)}$ by dt is equal to this one so what I will do is I will substitute it here. So $dT^{(1)}$ by dt is equal to minus T_0 , is qA by Mc (t) plus T_0 now minus T_{infinity} . So if you assume that the T_0 and T_{infinity} are the same is equal to 0 if T_{infinity} is equal to T_0 otherwise that will also float around. So now I can integrate the second equation here. This will give you $T^{(1)}$ is equal to minus qA t square by $2Mc$ plus a constant of integration and because $T^{(1)}$ is equal to 0 at t equal to 0 plus C_1 is equal to 0 by the initial condition. Therefore I can write the solution to this problem by adding these two terms.

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Handwritten mathematical derivations on a blackboard background:

$$\frac{dT^{(0)}}{dt} = \frac{qA}{Mc}, \quad \frac{dT^{(1)}}{dt} = -(T^{(0)} - T_{\infty})$$

$$T = T_0 \text{ at } t = 0 \quad \therefore T^{(1)} = 0 \text{ at } t = 0$$

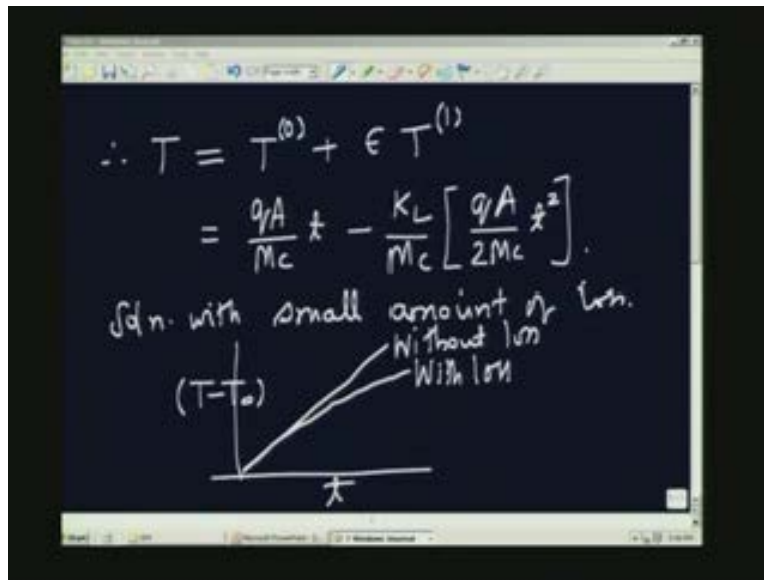
$$T^{(0)} = \frac{qA}{Mc}t + T_0$$

$$\frac{dT^{(1)}}{dt} = - \left[\frac{qA}{Mc}t + T_0 - T_{\infty} \right]$$

$$T^{(1)} = - \frac{qA}{2Mc}t^2 + \cancel{qA}T_0 \text{ by the initial condition.}$$

Therefore T is equal to (Refer Slide Time: 20:26) $T^{(0)}$ plus epsilon $T^{(1)}$ and $T^{(0)}$ is $(qA \text{ by } Mc) \text{ into } t \text{ minus epsilon is } K_L \text{ by } Mc \text{ into } (qA \text{ by } 2Mc (t \text{ square}))$. So this becomes the solution with a small amount of loss. So if I have to make a lot of response, I will get the following, without loss I will get a straight line this is without loss, what I am plotting is the temperature T minus T_0 on the y axis and time t on the x axis and with loss it will become slightly non linear it will go like this. Of course the difference will be very small and as the time increases you can see that this is a quadratic term going to become important when t is large Here is example number 35. I will work out the response. We will first look at the formulation.

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Handwritten equations and a graph on a chalkboard:

$$\therefore T = T^{(0)} + \epsilon T^{(1)}$$

$$= \frac{qA}{M_c} t - \frac{K_L}{M_c} \left[\frac{qA}{2M_c} t^2 \right]$$

sdn. with small amount of loss.

Graph showing $(T - T_\infty)$ vs t . Two curves are plotted: "Without loss" (a straight line) and "With loss" (a curve that starts at the origin and follows the straight line initially but then curves downwards, staying below the "Without loss" line).

We have a slug type heat flux sensor which is made of copper and it is 3 mm thick. It receives the heat flux of 10000W by m square on the exposed face. Initially the slug and the casing are at the same temperature equal to 30 degree Celsius. This is nothing but T_0 and T_{infinity} assuming to be 30 degree and the maximum temperature increase allowed for the slug is given as 50 degree Celsius. We do not want to go beyond 50 degree for the slug temperature. And what is the duration for which the sensor can be used is to be found out and we want to assume the loss coefficient area product that is K_L of 0.5W by degree Celsius. Over the small change in the temperature we can assume the loss coefficient to which one by this. Example 35:

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Example 35: (Solution on the board)

- A slug type heat flux sensor is made of copper and is 3 mm thick. It receives a heat flux of 10000 W/m^2 on the exposed face. Initially the slug and the casing are at the same temperature equal to 30°C . The maximum temperature increase allowed for the slug is 50°C . What is the duration for which the sensor can function? Assume a loss coefficient area product of $0.5 \text{ W/}^\circ\text{C}$.

It is just the use of the formula we derived previously. And all I will do is, I will just label all the things given. The slug is copper, the density is $8890 \text{ kg by m cube}$ and the specific heat is $398 \text{ joules by kg Kelvin}$ and the heat flux given in the problem, q is equal to $10000 \text{ W by m square}$ this is specified in the problem and the initial temperature, T_0 is same as the casing temperature which we will take as 30 degree Celsius and the T_{max} minus T_0 is the raise in temperature is given as 50 degree Celsius difference in temperature between these slugs at the maximum value of the temperature minus the initial value given as 50 degree Celsius. So let us calculate the various quantities which appear there qA by Mc .

Of course A is not mentioned in the problem. So we can calculate based on A is equal to 1 m square . In actual practice A will be a very small quantity and anyway it does not come into the picture at all. So what we require is qA by Mc . If I put A is equal to 1 m square everywhere it will be A is equal to 1 . So q is 10 to the power 4 or 10000 by mass of the slug thickness given as 3 mm and you see that the mass will be nothing but ρ into volume and volume is nothing but A into Δ so this becomes simply ρ into Δ because A will cancel of with that A . That is why we do not require the A in this problem. So this is what happens, 10000 by $(8890$ into 0.003 into $398)$ or the value of C comes out to be 0.942 and the units is degree Celsius per second, the degree Celsius per second is the unit of this quantity. And K_L by Mc is given by 0.5 by, and K_L has actually absorbed that A . Therefore A

will cancel off with that one so K_L can be simply taken as the 0.5 by M will be the product of density and thickness ρ into δ into C the same denominator so 8890 into 0.003 into 398 and this comes to 0.000047 and unit is per second. So the two characteristic numbers describing the problem are obtained. So all I have to do is now substitute into the solution.

(Refer Slide Time: 26:36)

Example 35 Slug is of copper. $A=1\text{m}^2$
 $\rho = 8890 \text{ kg/m}^3$ $C = 398 \text{ J/kgK}$
 Heat flux is $q = 10000 \frac{\text{W}}{\text{m}^2}$ $\delta = 3\text{mm}$
 $T_0 = T_\infty = 30^\circ\text{C}$ $(T_{\text{max}} - T_0) = 50^\circ\text{C}$
 $\frac{qA}{\rho \delta C} = \frac{10000}{(8890)(0.003)(398)} = 0.942 \frac{^\circ\text{C}}{\text{s}}$
 $\frac{K_L}{MC} = \frac{0.5}{(8890)(0.003)(398)} = 0.000047/\text{s}$

The solution is given as T is equal to qA by Mc into t minus K_L by Mc into qA by $2Mc$ into t square so what I have to do is I have put T equal to T_{max} minus T_0 is equal to this and that means t is equal to t_{max} and this will also be t_{max}^2 . So all I have to do is substitute the values T_{max} minus T_0 which is given, qA by Mc which is calculated, K_L by Mc which is calculated and then we will have to work it out. So the equation is T_{max} minus T_0 is 50 and that must be is equal to 0.942 into t minus 0.00047 into 0.942 into t_{max} square by 2. That is the period for which the heat flux sensor can be used without exceeding the temperature which is specified.

And you see that this equation is peculiar, this is the dominant term, and this is the small term. So what I can do is I can rewrite in the form, t_{max} is equal to 50 by 0.942 so I am just retaining it here. I am dividing this and I am sending it to the other side that is, plus 0.00047 by 2 into t_{max} square. So, if we ignore this we will get the value of first approximation of t and then you put it here you will get a better approximation. If you say t is equal to $t^{(0)}$ is roughly is equal to 50 by 0.942

so it is that many seconds, it comes to 53.08 seconds. And now what I do is I substitute it here and improve it so t_{\max} improved will be is equal to 53.08 plus 0.00047 by 2 into 53.08 square and this turns out to be 53.15 seconds because the loss coefficient is very small. The time over which it can be used without exceeding the temperature specified for this slug is about 53 seconds.

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The image shows a digital blackboard with handwritten mathematical equations. The first equation is $50 = 0.942 t_{\max} - \frac{(0.000047)(0.942)^2 t_{\max}^2}{2}$. The first term is labeled 'Dominant' and the second term is labeled 'small'. Below this, the equation is rearranged to $t_{\max} = \frac{50}{0.942} + \frac{0.000047}{2} t_{\max}^2$. Then, an initial approximation is shown: $t_{\max}^{(0)} \approx \frac{50}{0.942} \text{ s} = 53.08 \text{ s}$. Finally, the improved value is calculated: $t_{\max} = 53.08 + \frac{0.000047}{2} \cdot 53.08^2$, which results in 53.15 s .

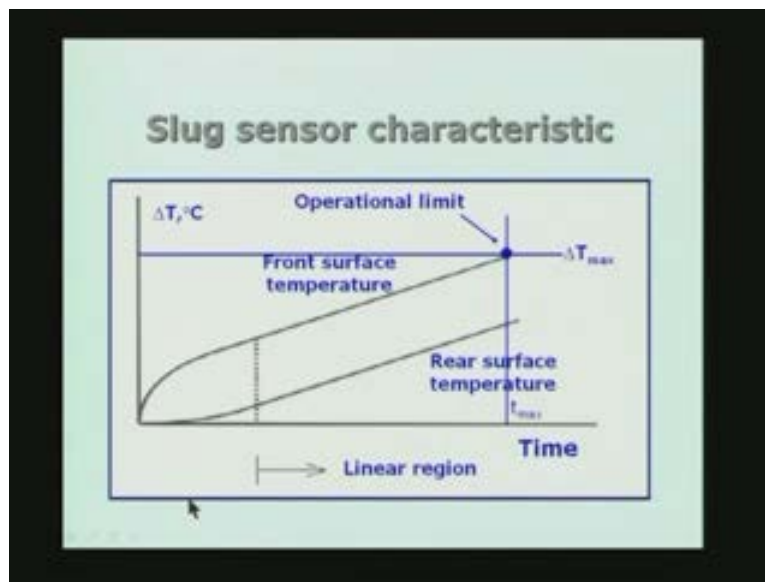
So roughly, we can say about fifty seconds we can use this slug without exceeding the temperature which is specified. Actually in any experiment involving the measurement of heat flux using transient techniques, we are going to conduct the experiment only to a short period. In this particular case we will have to stop the experiment at the end of fifty seconds. So with this background let us look at the second part of the problem. In the previous case, if you remember this slug was assumed to be thin so that the temperature throughout this slug was same at any given time t . there was no variation of the temperature with the slug. Within the slug itself there was no temperature variation at any give time t . That means basically we have assumed that the slug is like a lump system.

But in practice, what will happen is, if the slug is not going to satisfy this requirement then there will be certain lag in the time between the front surface temperature and the back surface temperature. So I have just indicated what is going to be expected in actual practice. We have a slug which is not thin in the

sense we used earlier. The front surface and back surface temperatures are going to follow slightly different ways.

Initially the front surface temperature will go faster, and then, it will go more or less at a linear rate and the back surface temperature will lag behind and then again it will catch up and it will go in a straight line variation with respect to time or linear variation with respect to time. And if I again have certain operation limit for the transducer, so I can say that up to some t is equal to t_{\max} , I can use this slug sensor so that the temperature limit specified is not exceeded. Secondly, I also have to find out what is the time over which the back surface temperature is going to be linear. So initially, there is nonlinearity and then it becomes linear. The analysis is somewhat complex but it can be done. And we can show that the following formulae are going to be useful in that one.

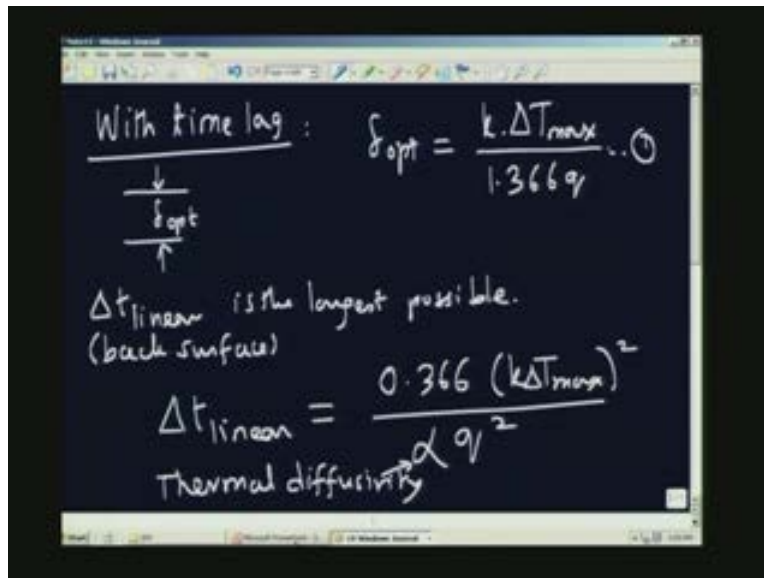
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So with time lag, it can be shown that, there is a certain thickness which you call as the optimum thickness of the slug such that the portion wherein the temperature is more or less going linearly, we will call it as Δt linear for the back surface is the largest possible, given the highest temperature the slug can be exposed. So what is this Δt optimum? It is given by the following formula: If thermal conductivity of the material of slug is k and the temperature maximum it can withstand is ΔT_{\max} that is maximum temperature minus initial temperature

divided by 1.366 into q where q is the input heat flux. This is one formula. The second relationship is that the Δt linear is then given by $0.366 k T_{\max}$ or $(k \Delta T_{\max})$ square by q square times α where α is thermal diffusivity of the slug material. Again a very simple problem can tell us what is going on. Here is example 36:

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With time lag : $\delta_{opt} = \frac{k \cdot \Delta T_{\max}}{1.366 q} \dots \textcircled{1}$

δ_{opt}

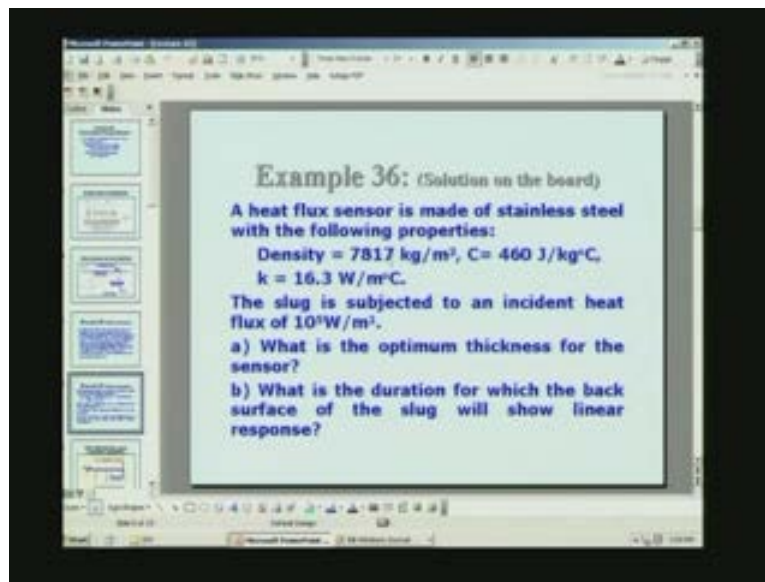
Δt_{linear} is the longest possible.
(back surface)

$\Delta t_{\text{linear}} = \frac{0.366 (k \Delta T_{\max})^2}{\alpha q^2}$

Thermal diffusivity α

So we have a heat flux sensor which is made of stainless steel in this case. And the properties of stainless steel are also specified here. Density is 7817 kg by m cube Specific heat is 460 Joules by kg degree Celsius and the thermal conductivity is 16.3W by m degree Celsius.

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Specific heat is 460 Joules by kg degree Celsius and the thermal conductivity is 16.3W by m degree Celsius. In example 35 where a copper slug and copper has a very high thermal conductivity of about 400W by m degree Celsius. And we are replacing it with the stainless steel because stainless steel has a much lower thermal conductivity, and also lower thermal diffusivity therefore when the copper sensor is useful for satisfying the requirement that is thin as a sensor with more or less uniform temperature throughout the stainless steel sensor will not satisfy the requirements because of the small value of the thermal conductivity and correspondingly the small value of thermal diffusivity. So we would like to find out the optimum thickness in this case when the slug is subjected to an incident heat flux 10 power 5 W by m square. I am just increasing the heat flux, I have increased the heat flux to 10 power 5 because the sensor is going to be useful for measuring very high heat fluxes.

Therefore, I have taken 10 power 5 W by m square and I want to find out what is the optimum thickness for the sensor and also what is the duration for which the back surface of the slug will show linear response. We are just plugging in the values into the formulae to find out what is happening. Here is example number 36: So, $\Delta T_{\text{optimum}}$ is equal to k into ΔT_{max} by 1.366 into q and k is 16.3. I am using everything in SI units therefore I do not have to worry about them. ΔT is given as 100 degree by 1.366 into q which is 10 power 5. So, in the problem

we can simply change delta T available to 100 instead of 50 and this gives you 0.012m or 1.2 cm for the thickness of the surface. It is a fairly thick slug. In the case of copper slug it is only 3 mm, it has got higher thermal conductivity also. Now I have made it thicker and also this thermal conductivity is lower. Therefore it becomes a candidate for the second type of application.

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Example 36

$$\delta_{opt} = \frac{k \Delta T_{max}}{1.366 q} = \frac{(16.3)(100)}{(1.366)(10^5)} = 0.012m$$

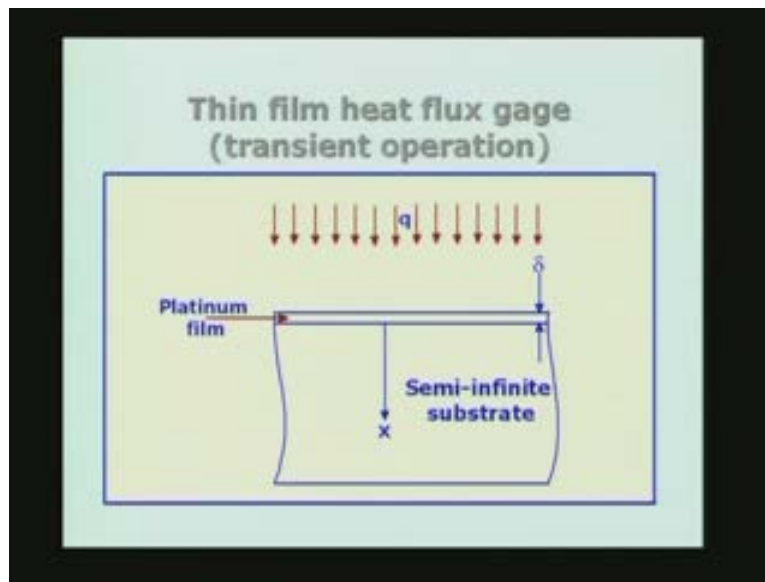
or 1.2 cm

$$\Delta t_{linear} = \frac{(0.366)(16.3 \times 100)^2}{(10^5)^2 \left(\frac{16.3}{7817 \times 460} \right)} \text{ s}$$

$$= \underline{21.5 \text{ s}}$$

So we can also calculate the delta T linear. So (.366) into (16.3 into 100) whole square by alpha q square, q is (10 power 5) whole square alpha itself is given by k by rho into C instead of, 16.3 by rho is 7817 multiplied by C which is 460 which were given earlier and this will be in seconds and this comes to about 21.5 seconds. So the slug can operate for about little more than 20 seconds without exceeding the 100 degree temperature rise which is specified in the problem. So the sensor can be used to measure the heat flux of 10 power 5W by m square for a brief period of about 20 seconds in this particular case. So let us look at the second type of application which we call as a thin film heat flux gage and these are used for measurement of transients over very small periods of time.

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You had a copper of 3 mm thickness, a stainless steel of about 12 mm thickness but now we are not talking about millimeters of thickness but we are talking about very thin gage. So what is the difference between the present thin film heat flux gage and the previous slug here? For example, I can have the platinum film of very small thickness δ which is in the micron region may be a few microns may be ten microns. And we have a substrate which is our material which is chosen suitably to have the desired thermal property and I am going to subject the front surface of the platinum film to the heat flux which is to be measured. Here thickness means the thermal thickness of substrate, so if the thickness of the substrate is large enough then the platinum film will be like it is non existent as far as the heat transfer is concerned, it does not take part in the heat transfer it simply responds to the temperature of the surface and therefore the platinum film is used because it's resistance will change with temperature and if I measure the resistance of the platinum film as a function of time. I am actually measuring the surface temperature as a function of time. So, if the heat flux is constant the surface temperature will vary in a particular fashion.

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Response for constant q

Response within the substrate:

$$\frac{T(x,t) - T_s}{q_s} = \frac{k}{\sqrt{\pi \alpha t}} e^{-\eta^2} - \text{erfc}(\eta)$$

where $\eta = \frac{x}{2\sqrt{\alpha t}}$

Response at the surface (of the platinum film):

$$\frac{k T_s}{q_s} = \sqrt{\frac{4\alpha t}{\pi}} = 1.128\sqrt{\alpha t}$$

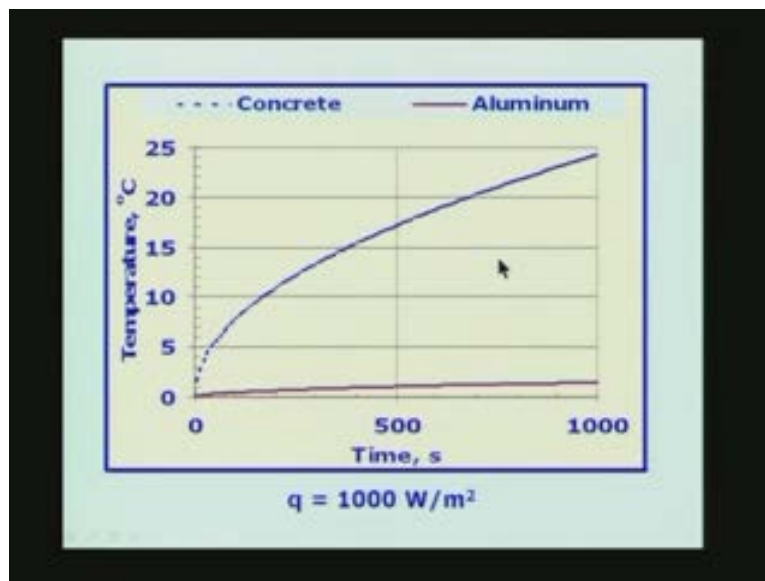
The response of the material, we are not talking about response of the platinum now platinum is only a passive thing which is going to just follow the temperature of the surface. The temperature of the surface is determined by the rate at which heat is coming in, and the rate at which is heat is removed from the surface by the subscript material. And we can in fact show by very simple solution to the one dimensional heat equation that the temperature of the substrate, so, if you go back to the previous slide I have measured x in the direction perpendicular to the surface and I am now talking about the temperature within the substrate at any location x at a given time t.

So $T(x,t)$ into k by q_s where k is thermal conductivity of the substrate material, q_s is the heat flux input at the surface is equal to square root of $4\alpha t$ by π where α is the thermal diffusivity of the substrate material $e^{-\eta^2}$ where η is x by $2\sqrt{\alpha t}$ is a non dimensional variable, x is the distance, t is the time, α is thermal diffusivity minus x times the complementary error function of the η .

In fact, if we put x is equal to 0 in this equation you will get the surface temperature and the surface temperature of the substrate is exactly equal to the temperature of the film because it is very thin film which is formed on the surface. It has got very little inertia and it will follow the temperature of the surface

faithfully, and in fact, temperature of the surface follows a square root of time relationship. So kT_s by q_s if I put x is equal to 0 in this equation it will give you square root of $4\alpha t$ by π and this can be written as 1.128 square root of αt α is thermal diffusivity material of the substrate. Let us look at the typical variation of the temperature of the surface. I have taken two materials; one is concrete and the other one is aluminum just to indicate what we are going to expect. In the case of concrete very little heat is transferred to the material inside. Therefore the surface temperature increases rapidly and you can see that it is parabolic and it is a square root of time dependence.

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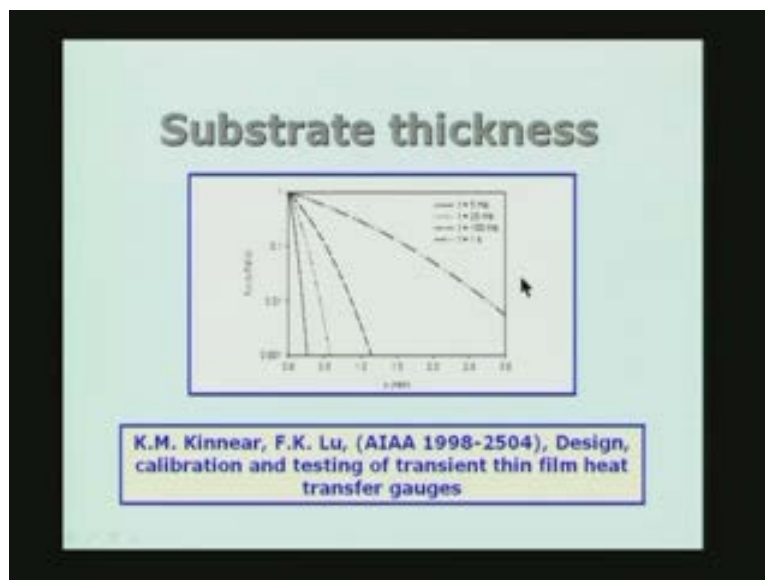
However, if we use aluminum for example, the surface temperature increase is very marginally because most of the heat is conducted into the material and therefore for the measurement of heat flux if you want a sizable temperature change I must use a material with small thermal diffusivity, small conductivity and so on. And now if I compare the response at a depth compared to the response of the surface I just have to take the ratio of the two quantities.

(Refer Slide Time: 44:26)

Response at a depth x compared to response at surface

$$\frac{T(x,t)}{T(0,t)} = e^{-\frac{x^2}{4\alpha t}} - \sqrt{\pi} \frac{x}{2\sqrt{\alpha t}} \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

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It is useful in determining the substrate thickness. What is the thickness of the substrate required given the property of the substrate such that we can assume the substrate to be semi infinite solid. That means the surface temperature will show the square root of time dependence, only if substrate is thick enough, to justify this thing. So I have taken this figure from the paper by Kinnear and Lu which

appeared in AIAA 1998 - 2504 design and calibration testing of transient thin film heat transfer gauges. So I have four different curves here drawn at different times.

First one is drawn at t is equal to 5 milliseconds the ratio of the two and within 5 milliseconds the substrate which is even more than about 0.25 mm is thick enough, whereas, if you are going to have t is equal to 1 second even 3 mm is not thick enough. So the thickness of the substrate has to be so chosen that within the duration of the experiment during which we are going to apply the heat flux at the surface and make the measurement, the substrate appears to be a semi infinite medium. What is the indication of that? The temperature at the maximum depth available will be the thickness of the material and at that thickness value this should be as small as possible compared to this surface.

In this case, if you say that about 0.1 or 0.01% or less then you can see that 3 mm may be just enough in this case up to t is equal to 1 second. Suppose I talk about very rapid experiments which are conducted in a very short time I may even go for 5 milliseconds or even less so you see that you need not have a very thick substrate, the thickness of the substrate will be very small and still it will appear as a semi infinite medium as far as the response of the temperature of the surface is concerned.

So let us look at some of the materials which are used. For the electrically insulating substrates, the platinum film which is basically applied on the surface of the substrate is going to be a conducting material, whose resistance is what I am going to measure, and therefore the substrate should be electrically insulating. And it so happens that some of the materials which have been used are given here. MACOR is a some kind of plastic material, PYREX is a glass material, Quartz is again a plastic material.

(Refer Slide Time: 47:00)

Electrically insulating substrates				
Insulator	ρ (g/cm ³)	c (J/g K)	k (J/cm s K)	$(\rho ck)^{1/2}$ (J/cm ² K/s ^{1/2})
MACOR	2.52	0.790	0.0146	0.171
PYREX	2.22	0.775	0.0136	0.153
Quartz	2.21	0.755	0.0140	0.153

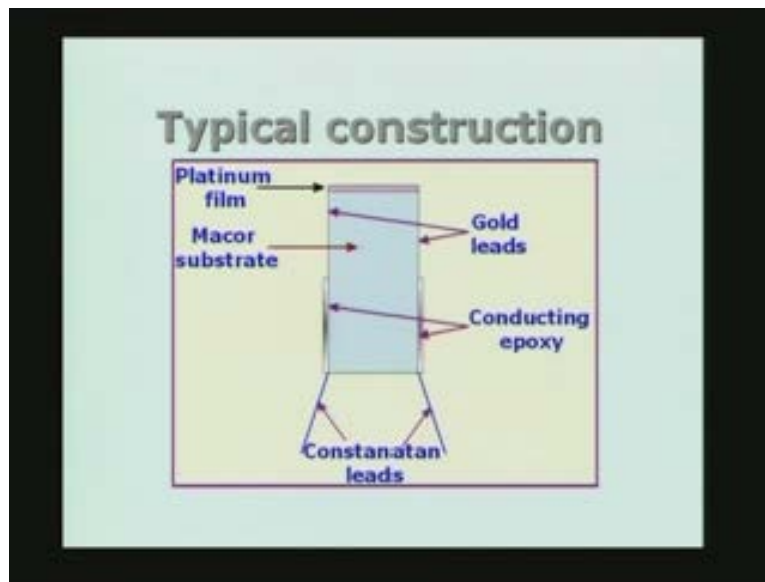
These materials are used basically in making such a heat flux sensor and the density, specific heat, thermal conductivity and the product of rho, c, k to the power 1 by 2 the square root of the product of rho, c and k is also given here and the units are grams by centimeter cube joules per gram Kelvin here, joules per centimeter second Kelvin and if you require you can convert this two SI units, joules by cm square by Kelvin by s to the power 1 by 2. In this last column 0.171, 0.153, 0.153 are all the range of values which are required for the substrate. And if I look at the film material like platinum the value is 1.4.

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Film materials				
Insulator	ρ (g/cm ³)	c (J/g·K)	k (J/cm·s·K)	$(\rho ck)^{-1}$ (J/cm ³ ·K/s ⁻¹)
Platinum	21.5	0.13	0.70	1.40
Nickel	8.90	0.45	0.84	1.83
Copper	8.90	0.38	3.97	3.66

Therefore there is ten times difference in the values of ($\rho c k$) product. I have given platinum, nickel and copper. Platinum is a very useful material because you have already seen that platinum is used as a resistance, RTD material is also platinum and its properties are very well known and so on. So, platinum film sensors are used in practice. The typical construction (Refer Slide Time: 49:13) for the sensor will be a thin film of platinum which is usually obtained by either painting or by paper vacuum deposition of the material as a very thin platinum film on the substrate in this case MACOR substrate.

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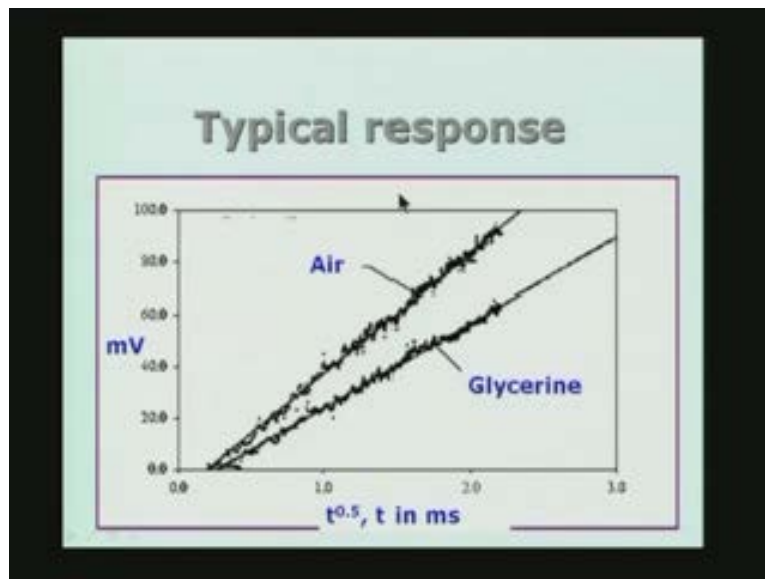
We take two gold leads from the two ends of the platinum film and then we have conducting epoxy and constantan leads are attached to that. Therefore essentially, this is the resistance thermometer, the film itself behaving or acting as a resistance thermometer, and when the temperature of the surface is exposed to the heat flux it is measured by following the resistance. The resistance increases the temperature more or less linearly and if you do not allow the temperature to swing by very large value the linear assumption may be justified.

So if we assume that initially the temperature is room temperature and after the heat flux is imposed on the surface the temperature will go up all I have to do is measure the resistance as function of time and that will be like measuring the temperature as a function of time and it can be calibrated suitably. So a typical response of a sensor of this type I have given in the form of electrical output millivolt and on the x axis, I have got square root of time and time is in milliseconds and here I have taken the square root of time.

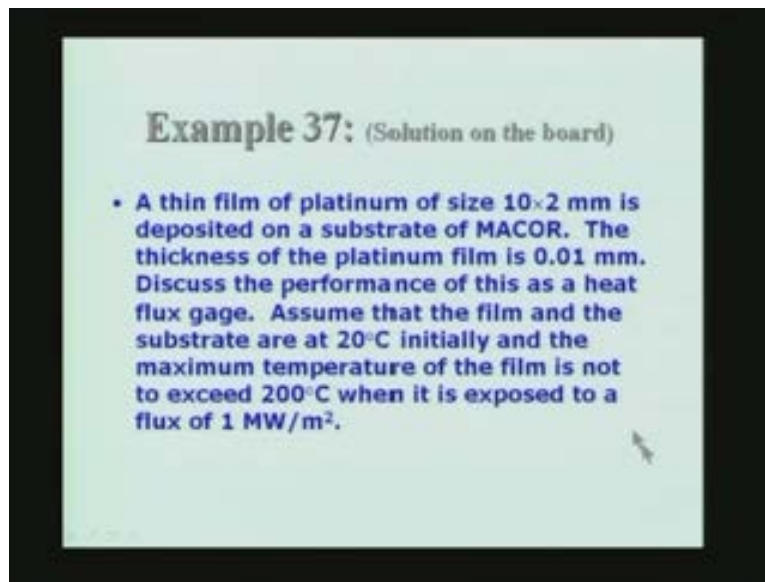
Therefore if you plot the temperature of the surface as a function of square root of time you are going to get a linear relationship and you can see that the linear relationship is more or less obtained except probably at a very small time there may be some problem there. But you can see that there is also some noise in this particular measurement, but the response is very closely the square root of time

response. The experiment has been conducted in two different media, air and glycerine and you can see in case of Glycerin heat transfer is much more rapid and therefore you have a smaller slope for the line whereas in air it is a larger slope. With this basically we will look at one simple solution so that the numbers become clear to us.

(Refer Slide Time: 50:42)



(Refer Slide Time: 51:45)



So, if we have thin film of platinum of size 10 by 2 mm that is 10 mm long and 2 mm in height it is deposited as a substrate of MACOR. The thickness of the platinum film is 0.01 mm 10 microns, discuss the performance of this as a heat flux gage. So, assume that the film and the substrate are at 20 degree Celsius initially and the maximum temperature of the film is not to exceed 200 degree Celsius when it is exposed to a flux of 1 MW by m square .So I am now slowly increasing the heat flux, I started at 10000 into 10 power 5 in the previous example and in this last example we are having the MW by m square. This is example 37, thin film sensor.

The platinum has have got a length of 10 mm by 2 mm the length and this is the width and multiplied by 0.01 mm and this is your delta. So platinum has a resistivity, it is also given the symbol rho this electrical resistivity is 10.5 into 10 to the power minus 8 ohmmeter at 20 degree Celsius and this alpha is the coefficient of resistance with respect to temperature, the temperature coefficient resistance is 0.00392 by degree Celsius. So I can actually calculate the resistance of the element. Initially the temperature is,20 degree Celsius and already I am given rho at 20 degree so I can calculate the resistance at 20 degree R_{20} is rho L by A, rho is your 10.5 into 10 to the power minus 8 and L is 10 mm. Therefore that will be 0.01 meter divided by area of cross section will be 2 mm that is 0.002 into 0.01 mm into 10 to the power minus 5, this turns out to be 0.53 ohms, the

resistance is 0.53. And in fact I can calculate the resistance at 200 degree when the thin film sensor is allowed to go up to that temperature the value will be just R_{20} into $1 + \alpha(200 - 20)$. So all I have to do is plug in all the values and this comes to about 0.904 ohms. If the output of the sensor is proportional to the resistance you will see that it will go up almost by a factor of 2 during the operation of the sensor in this particular example.

(Refer Slide Time: 53:45)

Example 37 Thin film sensor
 $10 \text{ mm} \times 2 \text{ mm} \times 0.01 \text{ mm}$ $T_0 = 20^\circ\text{C}$
 $L \cdot W \cdot \delta$
 $\rho_t: \rho = 10.5 \times 10^{-8} \Omega\text{-m at } 20^\circ\text{C}$
 $\alpha = 0.00392 / ^\circ\text{C}$
 $\text{Resistance: } R_{20} = \rho \frac{L}{A} = \frac{10.5 \times 10^{-8}}{0.002 \times 10^{-5}} = 0.53 \Omega$
 $R_{200} = R_{20}(1 + \alpha(200 - 20)) = 0.904 \Omega$

Now I can calculate for the heat flux which is incident 10^6 W by m^2 that is 1 MW by m^2 that is given and $k \Delta T_s$ by q is equal to 1.128 square root of αt where α and k these are the properties of the MACOR which was given in the table form and if the ΔT_s is maximum value, then I have to find out what is t_{\max} . I will just say ΔT_s is the maximum value. So I can write t_{\max} , this square root of α by k is nothing but square root of k by ρ into C where ρ is the density here divided by k is nothing 1 by square root of $k \rho$ into C . That is the reason we gave the product of these three quantities in the table. So all I will do is I will just say that this is equal to $(k \text{ into } \rho \text{ into } C) \text{ into } (\Delta T_s (\max) \text{ by } 1.128 q) \text{ whole square}$ and I will just substitute k as 1.46 for the MACOR, ρ is 2520 and C is $(790 \text{ into } 200 \text{ minus } 20 \text{ by } 1.128 \text{ into } 10 \text{ to the power } 6) \text{ whole square}$ and this here was ΔT_{\max} and T_{\max} comes to about 0.074 s or about 7 ms . So the sensor will be able to withstand this particular heat flux for about 7 ms . Now let us go back and see from the curve given here. We can look at the t is equal to

7ms is between these two the 5ms and 25ms, the 5ms is this and this is 25ms. So you see that if I make the substrate about .5 mm it is going to satisfy the requirement so a very thin layer of MACOR and surface of which I am going to have this film of 10 microns deposited will be a good sensor for this particular application. Thank you.