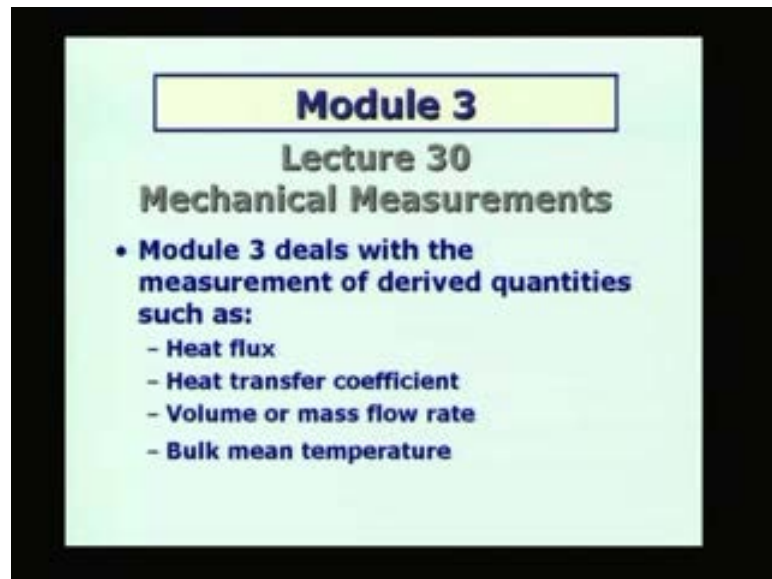


**Mechanical Measurements and Metrology**  
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**Module - 3**  
**Lecture - 30**  
**Measurement of Heat Flux**

This will be lecture number 30 on our ongoing series on Mechanical Measurements. It also happens to be the beginning of module 3 which will deal with measurement of derived quantities like the heat flux, heat transfer coefficient, volume or mass flow rate.

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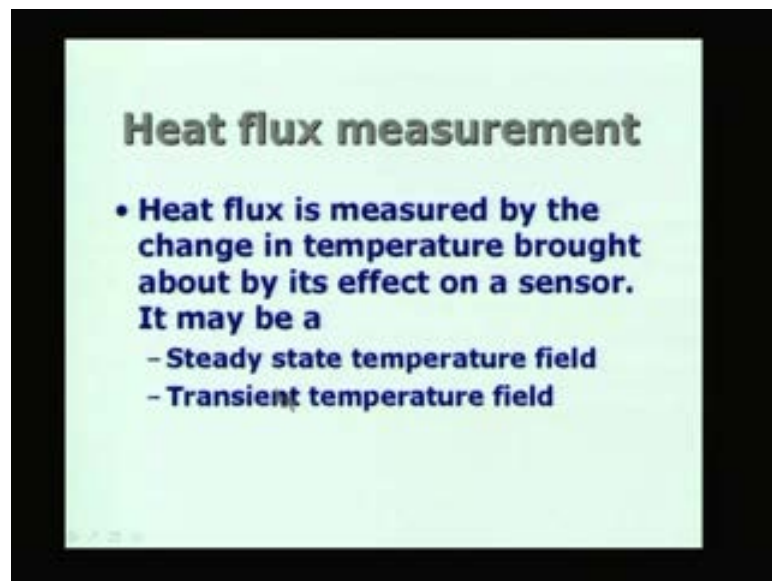


Earlier, we discussed about measurement of velocity but here we are going to talk about volume or mass flow rate and then the so called bulk mean temperature. Some of the measurements we are going to highlight in this module, will be important or will play an important role in thermal engineering. For example, heat flux is a very important quantity which is measured and it may also lead to an indirect fashion to the measurement of heat transfer coefficient. Therefore, these two are some what inter related; the volume or mass flow rate is very important because in most process applications you would like to know the volume flow rate or the mass flow rate.

If there is a pipe or a duct carrying a fluid the velocity will of course vary from point to point within any cross section of the pipe. But we would be interested in finding out the mass flow which crosses any section. Therefore it will be an integrated effect of the velocity through each area element across the cross section. So the volume or mass flow rate either can be directly measured or indirectly measured. But basically it is a very important quantity, in most of the process applications. The bulk mean temperature corresponds to the same case where you have a fluid moving through a duct or a pipe and we would like to find out the mean velocity of the fluid as it is carried along the axis of the tube.

Now let us take a look at the measurement of heat flux. Heat flux measurement can be measured basically by measuring the effect. The heat flux brings about in terms of some kind of a temperature distribution in the sensor. Heat flux is measured by the change in temperature brought about by its effect on a sensor and there can be different types of sensors. For example, we can have a steady state temperature field and therefore we have a sensor which is essentially under steady conditions. You could also measure the heat flux in terms of a transient set up within a medium.

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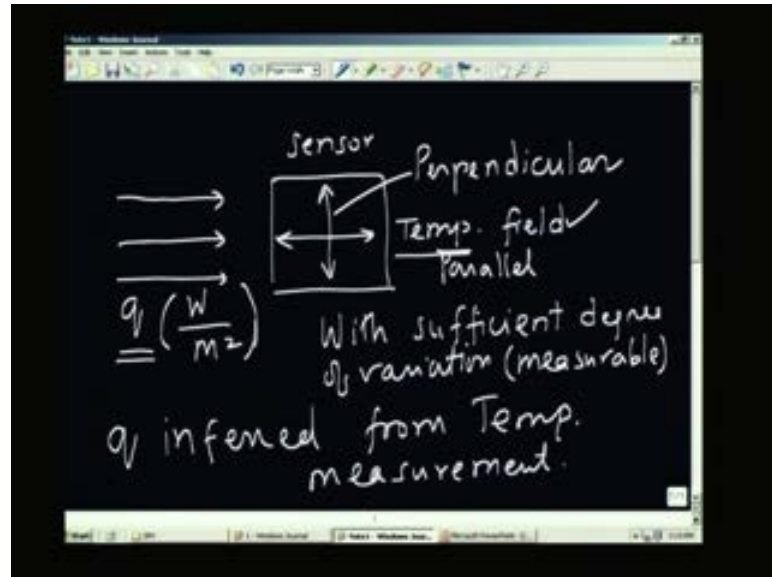
So the medium will be the sensor at the transient temperature field within sensor will be used to find out either the average or the instantaneous value of the heat flux as the case may be. Another way of dividing or looking at

the heat flux measurement is to look at what is the effect of the heat flux. Whether it is going to give rise to a temperature field in the direction of the heat flux or in a direction which is not along the heat flux which must be in a different direction. For example, I can have the temperature field set up in the sensor perpendicular to the direction of the heat flux.

The example will be a foil gage or Gardon gauge. Gardon is the name of the person who invented this particular heat flux gauge or it could be parallel to the direction of the heat flux as in the case of a conduction type heat flux gage. Therefore here you have a heat flux which is shown by arrows like this  $q$ , this is usually measured in watts per square meter the rate at which energy is crossing per unit area perpendicular to the direction of the heat flux. And suppose I have a sensor it is schematically shown as a box here, this is the sensor, I can have a temperature field in this direction that is the direction flux, this is the incident heat flux and this is the temperature field or it could be perpendicular. That means I could have a temperature variation in this direction, this is the perpendicular direction and this is parallel direction so these are basically the two different types.

We may also be actually measuring heat flux in a system of coordinates other than Cartesian or heat transfer in a radial direction or radial heat flux in a radial direction and so on. But, basically we are talking about the effect of the  $q$  is to setup a temperature field with sufficient degree of variation so that it is measurable with whatever instruments we can think of.

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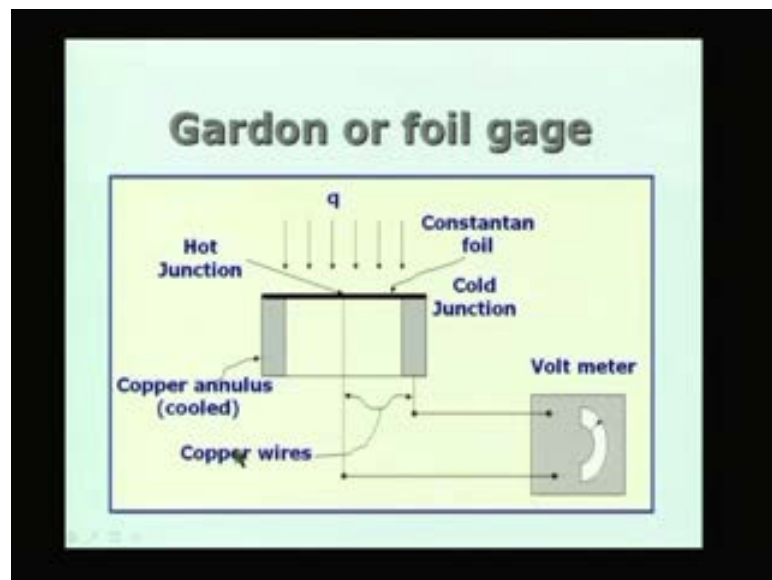
Actually you will now realize that, even though I am not specifically mentioning, the heat flux is going to setup a temperature field. Therefore, I am measuring the heat flux by measuring the temperature. Therefore  $q$  is inferred from temperature measurement. As we already discussed, the measurement of temperatures by various methods we are simply transferring or changing the quantitative to be measured that is the heat flux into a corresponding temperature or a temperature difference or temperature variation.

It may be either linear, non linear or whatever. Therefore we are going to infer  $q$  by measuring the effect of  $q$  on a sensor which is to setup a temperature field. The temperature field can be either steady state. That means it is not going to vary with respect to time or it could be a transient temperature field. It means that, it is going to vary with respect to time. Of course in the case of transient measurement, it may vary very mildly with respect to position but very strongly with respect to time that is also possible. Here is a simple Gardon or foil type gage and I have indicated schematically the basic idea of the sensor.

Here I have a foil and called the foil gage. Foil means a thin material in the form of a very thin sheet. Here I have taken a constantan foil or in fact the Gardon the person who suggested to use of this particular material constantan foil. Your temperature measurement constantan was a material

which was used in the case of a thermocouple. The constantan copper thermocouple is one of the thermocouples we discussed about earlier. Therefore I am making the foil in the form of a constantan foil, it is very thin and it is stretched tightly over the annular cylinder like this. This is the cylindrical block or copper the annular cylinder copper and the constantan foil is tightly stretched over this. That means it is intimately connected at the periphery. And you will also see that I have a small thin copper wire attached at the geometric center of the foil and another copper wire attached to the copper cylinder.

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So, if you look at carefully you will see that there is a junction between the constantan foil and the copper at this point. And at this point again there is a constantan copper junction. There are two junctions in other words. The material of the foil is now stretched over this and it forms two junctions. One junction is at the geometric center of the foil with a copper wire and the other one at the periphery with the copper cylinder or copper annulus which itself is connected to a copper wire. These junctions indicate the temperature difference between the center point and the junction at the periphery. So, if I connect a millivoltmeter across these two copper wires or if these two copper wires are connected to the voltmeter the output which I am going to measure then it is an indication of the temperature difference between the center and the periphery.

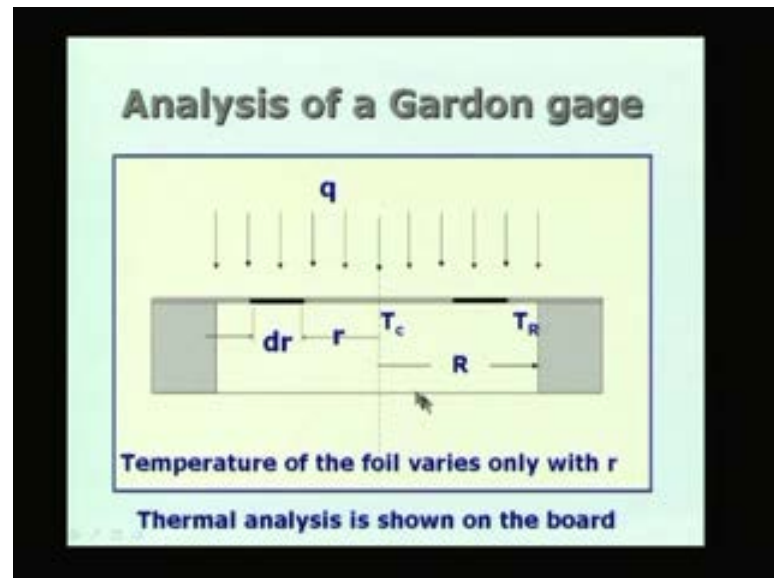
Now we should ask the question why there is a temperature difference. If there is no heat flux falling on this foil and if I am cooling this copper annulus by either having water steam going through this and inside this and then removing the heat or it could be air cooled so the temperature of the copper annulus is going to be maintained at a low value by removing the heat it gathers from the constantan heat foil. So the constantan foil gathers the heat from the outside here normal to the constantan foil and the heat is removed from the periphery by a cooling arrangement. Therefore under the steady condition, I am able to maintain the amount of heat which is absorbed by the plate or received by the plate is exactly compensated by the amount which is removed by the cooling medium at the periphery.

So what happens is, we have a thin foil and the heat transfer has to take place by conduction radially in the foil, so the heat is received transversely or perpendicular to the direction of the temperature field which is going to setup and therefore this corresponds to the case where a sensor temperature distribution is in a direction perpendicular to the direction of the heat flux. So what I have is a temperature variation along the radial direction here.

By a simple analysis, let us also find out the variation, how it varies and what the relationship is with  $q$  and the various dimensions of the sensor and so on. So the point is that, the  $q$  which is to be measured is now converted to a  $\Delta t$  between the center of the foil and the periphery of the foil and that is measured by a millivolt which is the conversion of temperature now to an electrical signal by thermoelectric phenomena. So what I am doing essentially is converting  $q$  or transducing  $q$  to a potential difference suppose  $q$  is proportional to the potential difference. In fact we will see later that there is a direct linear relationship between  $q$  and the  $\Delta t$ , I am going to measure, or the voltage I am going to measure.

The analysis is very simple. The  $q$  is indicated here which is incident from the transverse direction. What I have to do is take a small annular ring consisting of the foil I take a small ring and make energy balance for this. I find out the energy coming in and the energy leaving and then make an energy balance which will give me the equation which governs the temperature field within the foil. And of course at the center I have to use a suitable boundary condition.

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At the periphery I know that I am going to maintain the temperature of the copper cylinder at a value because it is going to be cooled by some medium. So this is going to be maintained at some temperature. Now I am going to find out how energy is transferred from the heat flux which is falling perpendicular to the foil and how it is removed by the coolant here and how it sets up a temperature field. Here this is 0 the center of the foil, the  $R$  is the periphery of the foil the radius of the foil is  $R$ , I will assume that the foil has got some thickness called as  $\delta$ . Here I am showing only one portion one half it is symmetrical you rotate this small rectangle with respect to this axis and you will get the foil.

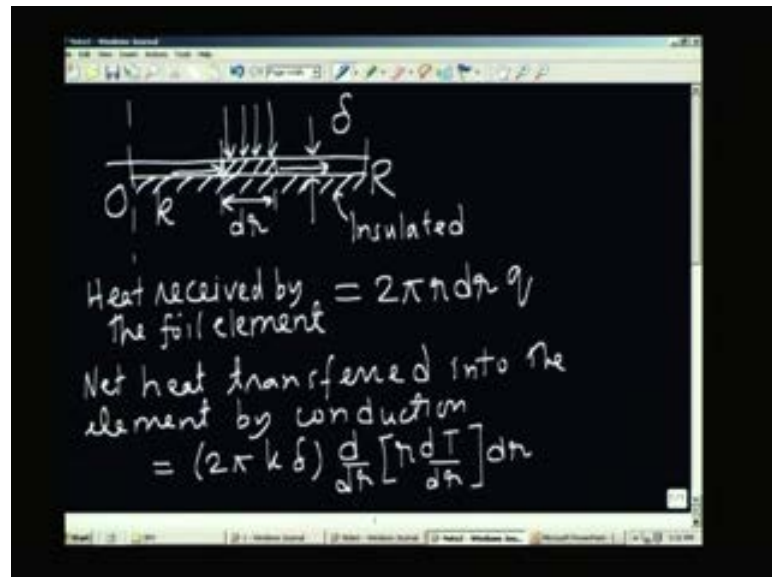
I am going to look at a small piece of material and we will assume that the thermal conductivity of the material of the foil is  $k$ , over this small area a certain amount of heat is being received. So, if you take this as  $dr$  the heat coming in or heat received by the foil element will be equal to the area of the exposed portion. For the analysis we are going to assume that there is no heat transfer on this side. That means it is almost like it is insulated.

Of course in actual practice there may be some small amount of heat lost from the back and it can be accounted for if we know the conditions on the other side. But to make the analysis simple we will assume that it is insulated in the bottom. So, the heat received by the foil will be nothing but the area of the element. Area of the element is nothing but  $2\pi r dr$ , area

times the heat flux  $q$  is the amount of energy received. Of course per unit time is actually the power received by the element. This is also having heat transfer by conduction in this direction and heat transfer by conduction in this direction within the foil. Heat is conducted because there is a temperature variation in the radial direction.

Therefore the net heat transferred into the element by conduction will be this minus this is what is coming and what is leaving. That is, what is leaving minus what is coming or what is coming what is leaving is going to give you the difference between the two. So the difference can be written by using Fourier law of heat conduction and assuming that the conduction is now in the radial direction the temperature is varying radially and it is not varying across the thickness of the foil because the foil is very thin. So we have one unidirectional temperature field and by using a simple Taylor series expansion this can be shown to be equal to  $2\pi$  into  $k$  into  $\delta$  by  $dr(rdT$  by  $dr)$  into  $dr$ , this  $dr$  comes from the Taylor expansion, this  $d$  by  $dr(rdT$  by  $dr)$  is the net conduction flux and  $r$  is coming inside because as the radius changes the area available for heat transfer also changes. In fact  $2\pi$  into  $\delta$  into  $r$  is the area at the left side here and  $2\pi$  into  $\delta$  into  $r$  plus  $dr$  will be the area of this side.

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So the area is continuously changing in the radial direction. Now the heat received by the foil element must be exactly equal to the net heat transferred

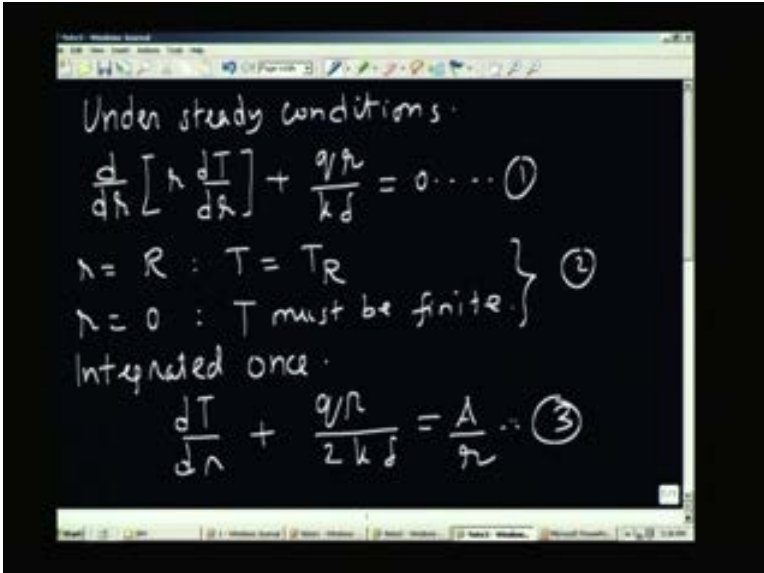


into the element because under steady conditions these two must be equal. So, under steady or steady conditions these two must be equal, we equate these two. Therefore I will get the following equation:  $d$  by  $dr(rdT \text{ by } dr)$ , I am just going to simplify it by removing some common factors and so on. So it is  $q \text{ r by } k \Delta$  equal to 0 the governing equation. So the temperature field is governed by a simple equation of this particular type where  $q$  is the constant heat flux which is being applied on one side of the gage, the other side being assumed to be insulated.

What are the boundary conditions I have to imply?

At  $r$  equal to  $R$ ,  $T$  equal to  $T_R$ , whatever is specified as the temperature of the coolant as the temperature of the cylindrical annulus that is  $T_R$ . At  $r$  equal to 0, all we can do is to say that  $T$  must be finite. So, if you take this equation number 1 and these are the boundary conditions this can be integrated twice and I am going to just indicate that. If you integrate it once you will get  $rdT \text{ by } dr$ , you have to integrate term by term plus  $q \text{ r square by } 2k \Delta$  equal to 0. So this  $r$  can be removed and that  $r$  will cancel of with one of the  $r$  here and that is the equation we are going to get. The right hand side of course will not be 0 but will give a constant of integration, we will call it  $A$ . And when you divide by  $r$  that  $r$  will come here so it is  $A \text{ by } r$ .

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Under steady conditions:

$$\frac{d}{dr} \left[ h \frac{dT}{dr} \right] + \frac{q/r}{k\delta} = 0 \dots \textcircled{1}$$

$$\left. \begin{array}{l} r = R : T = T_R \\ r = 0 : T \text{ must be finite.} \end{array} \right\} \textcircled{2}$$

Integrated once:

$$\frac{dT}{dr} + \frac{q/r}{2k\delta} = \frac{A}{r} \dots \textcircled{3}$$

So, if I integrate once more, this is one integration and this will give you this equation. The second integration will give you a straight forward integration

$T$  plus  $dT$  by  $dr$  will give you  $T$ , plus  $q$  by  $4k \Delta$  into  $r$  square equal to  $A \ln r$  plus  $B$  and  $rq$  by  $2k \Delta$  was there,  $r$  is integrated and that gives  $r$  square by 2. Therefore this becomes  $4$  by  $4k \Delta$  here and this we can call as equation number 4. Now you see that this becomes infinity  $r$  equal to  $0$  this becomes infinite. Therefore this has to be equal to  $0$ . That is why I said the temperature must be finite at the origin. At  $r$  equal to  $0$  it cannot become infinite and therefore the system goes to  $0$ . Therefore  $T$  plus  $qr$  square by  $4k \Delta$  equal to  $B$  is the solution to the problem. I can also write the following: if you put  $r$  equal to  $R$  I know that it is  $T_R$  plus  $qr$  square by  $4k \Delta$  equal to  $B$ .

So, from these two by subtraction you see that you get  $T$  minus  $T_R$  plus this minus this I will take to the right hand side that will become  $q$  by  $4k \Delta$  into  $R$  square minus  $r$  square. So we will call this equation 5. And what does it mean? The temperature difference between any radial location and the periphery  $T$  minus  $T_R$  is given by  $q$  by  $4k \Delta$  into  $R$  square minus  $r$  square and this is always positive, for any  $R$  less than  $r$  it is positive. Therefore we expect the temperature to be high in the middle of the foil and as we go towards the periphery the temperature is going to come down to the value which is specified by the cooling arrangement we have provided. So there is a temperature difference which is going to be generated by the application of the heat on the foil and this temperature difference is in a direction perpendicular to the direction in which heat flux is applied. That is the basic idea in this particular gage.

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Second integration will give

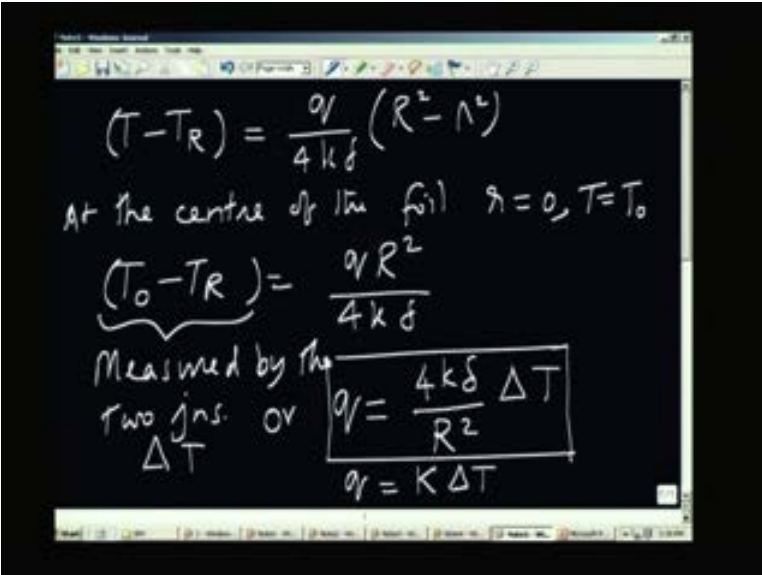
$$T + \frac{q}{4k\delta} r^2 = \int_0^r \frac{A}{r} dr + B \dots (4)$$

$r=0 \rightarrow \infty$

$$\left. \begin{aligned} T + \frac{q}{4k\delta} r^2 &= B \\ T_R + \frac{q}{4k\delta} R^2 &= B \end{aligned} \right\} \begin{aligned} (T - T_R) &= \frac{q}{4k\delta} (R^2 - r^2) \end{aligned} \dots (5)$$

So it is  $T$  minus  $T_R$  equal to  $q$  by  $4k\delta$  into  $R$  square minus  $r$  square. If I put  $r$  equal to 0 at the center of the foil. So we have  $r$  equal to 0,  $T$  equal to  $T_0$  and therefore  $T_0$  minus  $T_R$  equal to  $q$   $R$  square by  $4k\delta$  and this is nothing but measured by the two junctions. Whatever output we mentioned when we are discussing the gage it is actually this temperature difference. So what I am doing is I am setting up a temperature between the center of the foil and the circumference of the foil at the radius equal to  $R$  equal to  $q$   $R$  square by  $4k\delta$  or I can rearrange this equation as  $q$  equal to  $4k\delta$  by  $R$  square delta  $T$  where  $T_0$  minus  $T_R$  I am calling this as delta  $T$ . So the delta  $T$  is the differential temperature indicated by the differential thermocouple. So we are using a differential thermocouple which directly measures the temperature difference between the center of the foil and the circumference. Therefore the measured quantity is this inferred quantity.

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The image shows a blackboard with handwritten mathematical equations. The first equation is  $(T - T_R) = \frac{q}{4k\delta} (R^2 - r^2)$ . Below it, a note says "At the centre of the foil  $r = 0, T = T_0$ ". The second equation is  $(T_0 - T_R) = \frac{q R^2}{4k\delta}$ . Below this, it says "Measured by the two jns.  $\Delta T$ " and "OR". To the right, there is a boxed equation  $q = \frac{4k\delta \Delta T}{R^2}$ . Below the box, it says  $q = K \Delta T$ .

So I am measuring a temperature difference and therefore I can also write this equation in the form  $q$  equal to  $K$  delta  $T$  where  $K$  is the gage constant. So  $q$  equal to  $K$  delta  $T$  where  $K$  is nothing but  $4k$  delta by  $R$  square, the gage constant. This is the fundamental basis for the Gardon gage. This is the Gardon gage characteristic. So, Gardon gage characteristic is given by  $q$  equal to  $K(\Delta T)$  where  $\Delta T$  is the temperature difference set up between the center of the foil and the periphery and  $K$  is the gage constant which is related to, if you can see here the thermal conductivity of the material is coming, the diameter or the radius of the foil and this thickness of the foil so it is the geometric and the thermal properties of the foil which are going to come into the picture.

So by the suitable choice of delta  $R$  square and the material, material of course is the constantan foil and once you choose the constantan foil the thermal conductivity is already known, there is no change in that, it is going to be about 20 watts by m degree Celsius. So the thermal conductivity is fixed. So what I can do in practice is to choose the delta and  $R$  combination such that I get the desired value of  $K$  and the desired value of the measurable value for delta  $T$ . So you see that if  $K$  is very large, delta  $T$  will become small and so on. So it is a question of having some kind of balance between these two. What is measurable in delta  $T$  should be measurable quantity. If you have a copper constantan thermocouple so copper constantan is also called the  $T$  type thermocouple the 1 degree Celsius difference will

correspond to about 40 micro volts this is the Seebeck coefficient.

So, if I am able to maintain or get about 5 degree Celsius difference between the center and the periphery it will correspond to only 5 into 40 about 200 microvolts it will be 0.2 millivolts, 200 microvolts is only 0.2 millivolts. Of course I can use an amplifier and amplify the signal and so on so that is the part of the electronics which is going to come with the sensor. But as far as the sensor is concerned this is the fundamental equation.

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Handwritten notes on a blackboard:

$$q = K \Delta T \quad K = 4 \frac{k \delta^2}{R^2}$$

Ganton gage characteristic.

Copper - Constantan (T)

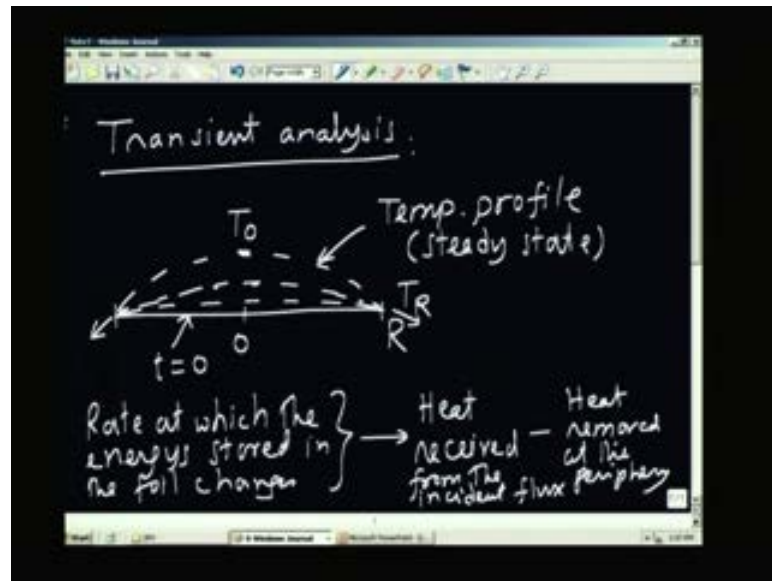
$1^\circ\text{C} \rightarrow 40 \mu\text{V}$  (Seebeck coefficient)

Let us discuss about the transient behavior of this particular foil type gage. Suppose you have the foil, this is the origin, this is the R, this is the circle so the temperature at the middle is  $T_0$  and this is  $T_R$  so it is a quadratic relationship so this is your temperature profile. Suppose the gage is exposed to a heat flux all of a sudden, what is going to happen? Initially the entire gage is at the temperature equal to  $T_R$  so this is at  $t$  equal to 0. At  $t$  equal to 0 the entire gage is at the temperature at the periphery and after sometime the temperature is going to become this and it is going to grow like this and finally it is going to come to this particular steady state.

So when you subject the gage to a transient by applying a step input in the form of a heat flux turned on at  $t$  equal to 0 initially the temperature is uniform and slowly the temperature profile builds up. And you can see the following. At any instant for example, if I take a look at this particular

temperature profile, the amount of heat flux which is being received by the foil partly is utilized to heat it up and partly it is removed here because there is a coolant which is removing the heat. So the rate at which the energy is stored in the foil changes. This should be made up of two terms. One is the heat received from the incident heat flux minus heat removed at the periphery. Basically this is the idea.

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So if I can get suitable expressions for each one of these I will be able to model the transient behavior of the foil and then obtain the response time of the system. Let us use the known equation  $T$  minus  $T_R$  equal to  $q$  by  $4k$  delta  $R$  square minus  $r$  square and we also know that  $T_0$  minus  $T_R$  equal to  $q$   $R$  square by  $4k$  delta.

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$$\begin{aligned}
 (T - T_R) &= \frac{q}{4k\delta} (R^2 - r^2) \\
 (T_0 - T_R) &= \frac{q R^2}{4k\delta} \\
 \hline
 \frac{(T - T_R)}{(T_0 - T_R)} &= \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \\
 \therefore (T - T_R) &= (T_0 - T_R) \left( 1 - \frac{r^2}{R^2} \right)
 \end{aligned}$$

In fact from these two I can get the following:  $T$  minus  $T_R$  by  $T_0$  minus  $T_R$  dividing one by the other  $q$  by  $4k\delta$  will go off so I will get  $(1$  minus  $r$  by  $R)$  square. That is,  $T$  minus  $T_R$  the temperature at any location in the foil with respect to the periphery temperature is the given by  $T_0$  minus  $T_R$  into  $1$  minus  $r$  square by  $R$  square, it is a parabolic distribution. In fact, if I want to find out the energy contained within the foil I have to find out the following. So the energy contained in the foil is nothing but, density times the specific heat times the volume of the element  $2\pi r dr \delta$  this is for an element into  $(T$  minus  $T_R)$  this is the amount of energy within the foil. Therefore I have to integrate between  $0$  and  $R$ . So, for this  $T$  minus  $T_R$  let us use the previous expression (Refer Slide Time 37:46)  $T$  minus  $T_R$  equal to  $T_0$  minus  $T_R$  into  $1$  minus  $r$  square by  $R$  square and then integrate it.

So I will use  $(T$  minus  $T_R)$  and integrate that. Here  $\rho$  is a constant,  $C_p$  is constant they are assumed to be independent of temperature so I can remove all the terms which are independent of time. Therefore I can perform integration. And you can show that this is equal to,  $\rho C_p \delta \pi R^2 (T_0$  minus  $T_R)$  by  $2$ . So  $dE$  by  $dt$  is what I want. The time rate of change of the energy contained within the foil will be given by  $\rho C_p \pi \delta R^2$  by  $2$   $d$  of  $(T_0$  minus  $T_R)$  by  $dt$  where  $2$  is already divided by  $dt$ . This is a part of the expression which corresponds to the rate at which energy is being stored within the material.



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$$E = \int_0^R \rho C_p 2\pi r \delta r \frac{dT}{dt} (T - T_R)$$

$$(\text{show}) = \frac{\rho C_p \delta \pi R^2 (T_0 - T_R)}{2}$$

$$\frac{dE}{dt} = \frac{\rho C_p \pi \delta R^2}{2} \frac{d(T_0 - T_R)}{dt}$$


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$$\text{Removal rate at } r = R = -2\pi R k \delta \left. \frac{dT}{dr} \right|_{r=R}$$

This must be equal to the energy which is coming in within that amount of time  $dt$  some amount of heat flux is there and that heat flux is going to come in minus what is removed at the periphery. So I can find out what is removed at the periphery. So the removal rate at  $r$  equal to  $R$  is nothing but the heat transfer by conduction from the foil to the annular cylinder. So this will be minus  $2\pi R k \delta$  so you look at the  $2\pi R$  is the periphery multiplied by  $\delta$ . This is the area of cross section  $2\pi R k \delta$  into  $k(dT/dr)$  at  $r$  equal to  $R$ .

Here I am going to use the same expression which I had earlier and then obtain this. At any instant of time there is a sudden instantaneous temperature gradient at  $r$  equal to  $R$ . And what I will do is I will manipulate the equations. So this is equal to  $4\pi k \delta$  multiplied by  $T_0$  minus  $T_R$ .  $T_0$  is already a function of time. Instantaneously there is some temperature. In other words at any instant of time the temperature comes to a local steady state value and it goes from one steady state to another steady state locally. That means the shape of the curve is not changing and under the condition of that you are going to get this value. Now I can write the final equation. So,  $d$  of  $(T_0 - T_R)$  by  $dt$  into  $\rho C_p \delta \pi R^2$  by 2 this is the rate at which the energy is changing plus this quantity  $4\pi k \delta$  into  $T_0 - T_R$ . So this is the heat transfer at the periphery and minus  $\pi R^2$  is the area of the foil multiplied by  $q$  is the heat which is collected equal to 0.



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$$= 4\pi k \delta (T_0 - T_R) \quad \frac{dT}{dt} + \frac{T}{\tau} = \frac{T_a}{\tau}$$

Finally.

$$\boxed{\frac{d(T_0 - T_R)}{dt} \cdot \frac{\rho C_p \delta \pi R^2}{2} + 4\pi k \delta (T_0 - T_R) - \pi R^2 q = 0}$$

first order eqn.  
We can divide by  $\frac{\rho C_p \delta \pi R^2}{2}$  to get

So the transient is governed by this expression and is a first order equation. If you remember we had a first order system which was in this form  $\frac{dT}{dt} + \frac{T}{\tau} = \frac{T_{\infty}}{\tau}$  so this can go to the right hand side and it becomes like this and  $\frac{T}{\tau}$ . So I have to do is to bring it to this form and therefore by dividing throughout by this coefficient here I will be able to bring it to this form, so if you do that we can divide by  $\rho C_p \delta \pi R^2$  by 2 to get the final form which is what we are interested in. And that will be  $\frac{d}{dt} (T_0 - T_R) + \frac{8k}{\rho C_p R^2} (T_0 - T_R) = \frac{2q}{\rho C_p \delta}$  this is the final equation. Therefore you can see that this is nothing but  $1$  by time constant.

Therefore the time constant for the foil type sensor is given by  $\tau = \frac{\rho C_p R^2}{8k}$  and this  $k$  by  $\rho C_p$  is nothing but the thermal diffusivity of the material. So we see that  $\tau = \frac{R^2}{8\alpha}$ . This more or less completes the simple analysis of a transient in a foil type element. Here are the results of this particular calculation. I have written transient response also  $\frac{R^2}{8\alpha}$  what we derived there by a simple analysis.

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$$\frac{d}{dt}(T_0 - T_k) + \underbrace{\frac{8k}{\rho C_p R^2}}_{1/\tau} (T_0 - T_k) = \frac{2q}{\rho C_p \delta}$$

$$\therefore \text{Time constant } \tau = \frac{R^2 \rho C_p}{8k}$$

$$k/\rho C_p = \alpha \text{ Thermal diffusivity}$$

$$\therefore \tau = \frac{R^2}{8\alpha}$$

Let us take a typical example of a gage. This is example 32. It may be constructed using a 6 mm diameter foil of 50 micrometers thickness. The foil is very thin 50 micrometers and the diameter is 6 mm. The thermal conductivity of foil material is typically equal to 20 watts by m degree Celsius. So it is a copper constantan foil. So copper constantan thermocouple pair gives an output about 40 microvolts by degree Celsius.

So the given data corresponds to delta equal to 50 micrometers which is 50 into 10 power minus 6m square, R equal to 3 mm which is nothing but 0.003m, k is 20. The gage constant turns out to be K equal to 4k delta by R square 4 into 20k delta is 5 into 10 to the power minus 3 by 0.003 square which will be 444. 4 watts by m square degree Celsius. You will notice that the gage constant is in the units of the heat transfer coefficient.

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### Transient response

- The transient response of the foil gage is sketched out on the board.
- The gage behaves as a first order system with the time constant given by

$$\tau = \frac{R^2}{8\alpha}$$

$\alpha$  is the thermal diffusivity of the foil material

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### Typical gage Example 32

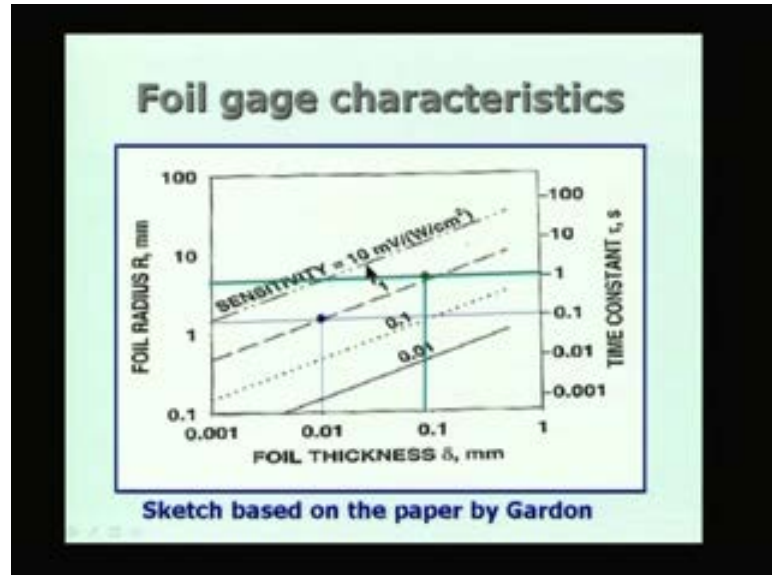
A typical gage may be constructed using a 6 mm diameter foil of 50  $\mu\text{m}$  thickness. The thermal conductivity of the foil material is typically  $k = 20 \text{ W/m}^\circ\text{C}$

Copper Constantan thermocouple pair gives an output of about  $40 \mu\text{V}/^\circ\text{C}$

In fact now I can rewrite this in terms of the thermocouple output. And you remember 1 square Celsius correspond to about 40 micro volt so instead of degree Celsius I put 40 micro volts here so I will get 444 by 40 equal to 11.1 watts by m square microvolt. So this is the performance index or the gage constant for the foil type sensors with delta equal to 50 micrometers and R equal to 3 mm. In fact I have made a plot. Actually the plot is taken from the paper by Gardon. The paper appeared in ASME journal of heat transfer in

1962.

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He has made a plot showing the foil thickness on the x axis the foil radius on the y axis it is a log plot and on the right hand side he has given the time constant corresponding to the R and delta values. Several lines are given here corresponding to different sensitivities. Sensitivity is nothing but the reciprocal of K so sensitivity is so many millivolts per watts per centimeter squared. This is a more usual way of representing the sensitivity.

Instead of microvolts and watts per meter squared used in terms of millivolts and watts per centimeter squared and watts per centimeter squared will correspond to about 10 to the power 4 watts by sq. m and 1 millivolt will correspond to 10 power 3 microvolts. The point corresponds to 1.11 if you take the reciprocal of what I gave there accounts to 1.11 and that is very close to the value shown by Gardon. He has got one case where sensitivity equal to 1 but my sensitivity is slightly more and this is the corresponding value of the thickness of the foil and this is the corresponding value of the thickness of the radius of the foil 3 mm and 50 micrometers.

I have also taken another example here. If I take a foil of 0.01 mm thickness and incidentally, this is in millimeters and this is also in millimeters both are in millimeters. So you see that a foil with 0.01 mm thickness with a radius of slightly more than 1 mm also gives the same sensitivity. So, sensitivity can

be obtained for different radius delta combinations. Let us see why we should have different radii and different thicknesses. If you have a very high heat flux to be measured, you require a thick foil otherwise the foil will get damaged it will get over heated and possibly a larger radius for that. So the radius delta combinations or the radius  $r$  delta combinations are because you may want to design different heat flux gages for different heat flux ranges. Sensitivity is one thing, this range is the second thing one has to worry about I have taken a typical example of factory assembled water cooled foil gage.

Here you can see the gage. The gage consists of a cylindrical copper and there is a foil and you are taking the thermocouple out here, the two leads one from the center one from the periphery is coming and these two are the two tubes which carry water in and out of the gage. There is also another gage which is not water cool it is a very small gage or small heat flux values as we can see here. We have a very small gage without any water cooling but the diameter is much smaller and you will also see that if you remember  $R$  square by  $8\alpha$  it depends on the square of the radius. If you increase the radius of the foil the time constant becomes much larger so it goes at the square of the radius.

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Therefore if you want to have a highly sensitive gage with a low time constant it is going to follow the heat transfer really fast and whatever heat flux changes takes place it will indicate immediately so you must have a

small radius because it is  $R^2$  by  $\alpha$  and I do not have any control over  $\alpha$  because the material of the foil is fixed,  $\alpha$  is fixed therefore I can only reduce the  $R$  value. So, the smaller the gage the faster is the response and that is in fact one of the reasons why we have a different radii and so on.

For example, in this case for full scale output of 10 millivolts this gage works with 0 to 5 watts per square centimeter or from 0 to 5000 there are several ranges available. The gages are made in different ranges so 0 to 5 watts per square centimeter up to 0 to 5000 watts by  $\text{cm}^2$  it is a very high heat flux. In the case high heat flux gages you may have to have good cooling arrangement. For very low heat flux gages cooling may not be necessary or air cooling may be sufficient.

Response time which is what we talked about just a little while ago, you can have 0 to 5000 watts by  $\text{cm}^2$  which has got response time of 3 milliseconds. And for 0 to 5 watts by  $\text{cm}^2$  it requires one second why because you require a large radius otherwise you are not going to get enough  $\Delta T$ . When the heat flux is large the  $\Delta T$  is going to be proportional to that. Therefore you get a large enough measurable  $\Delta T$  with a small radius. And when you have very low heat flux you require a larger catchment. So, transducer calibration accuracy is plus or minus 3%, repeatability 1%, and sensitivity can be as high as 2 millivolt by watts by  $\text{cm}^2$ . If you go back to the figure we had earlier (Refer Slide Time 47:37) we are talking about the region between 1 and 10, sensitivity is 1 here and is 10 and somewhere in between 1 to 2 that is very easily obtained.

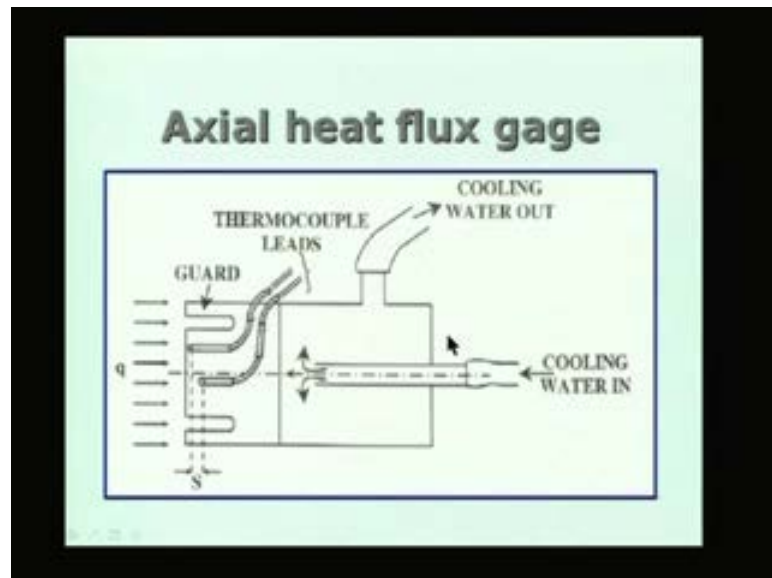
So the heat flux gage that is the foil gage we considered till now, the heat transfer or the heat flux and the direction in which the temperature gradient is setup is perpendicular to it. The second case where the heat flux and the temperature gradient or the temperature variation is setup in the same direction or parallel to it is considered here.

Essentially what is the principle involved?

In principle we want  $q$  parallel to  $\Delta T$ . The earlier method was complicated where we want  $q$  and  $\Delta T$  perpendicular to each other and  $q$  parallel to  $\Delta T$ . We would take a slab of material and allow the  $q$  to impinge on one side and maintain the temperature here to cool this surface. When the heat flux is coming here and you cool the surface automatically that  $\Delta T$  will come. We will say this is  $T_{\text{hot}}$ , this is  $T_{\text{cool}}$  so this is very

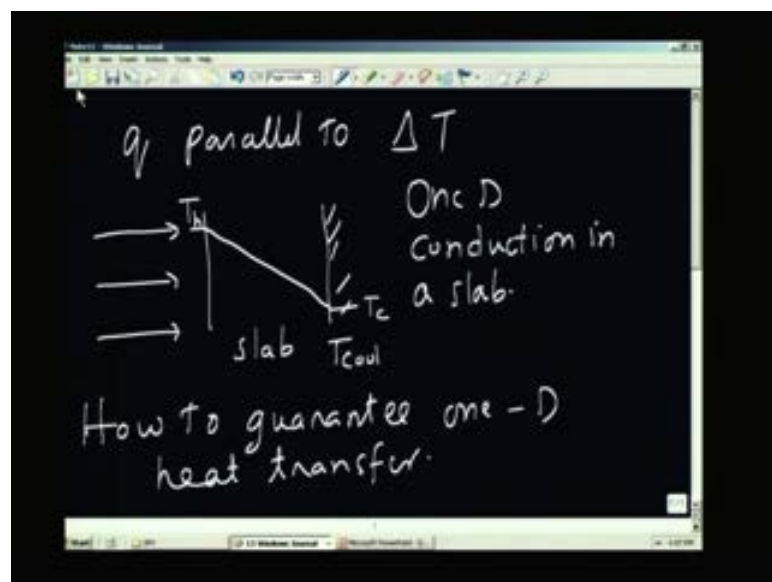
simple. It is one dimensional conduction in a slab.

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It is not a real sweat here it is a very simple idea. But the main problem is how you make sure that the heat transfer is going to be one dimensional.

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How to guarantee one dimensional heat transfer? Why is it so difficult? Suppose I have a finite size material  $L$  and this is the diameter of a block of



material. Suppose I have heat flux coming here there may be a loss here. However, if I want to set up one dimensional temperature field that means I must have adiabatic sides. So, if the periphery or the side is adiabatic then I can guarantee one dimensional. How do we guarantee? How do we obtain adiabatic condition or adiabatic boundary condition is the major problem. Of course you can insulate it on the side. Of course insulation will have some thermal conductivity. So what will happen is if you have  $q$  here some amount of heat is lost here and the temperature gradient here  $T_h$  and  $T_c$  will be smaller than if no heat loss is present. This is  $T_h$  minus  $T_c$ , one dimensional is greater than  $T_h$  minus  $T_c$ , two dimensional where you have heat loss in this side. How to make it one dimensional? We use what is called a guard arrangement. Thank you.

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