**Mechanical Measurements and Metrology**
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**Module - 2**
**Lecture - 29**
**Laser Doppler Velocimetry and Ultrasonic Methods**

This will be lecture number twenty nine on mechanical measurements. We will continue our discussion regarding velocity measurement. In fact this lecture will bring to a close the second module. So today we are going to discuss further the Laser Doppler velocimetry for understanding the methods which use the reference beam system. We need to understand a little bit about the Doppler effect.

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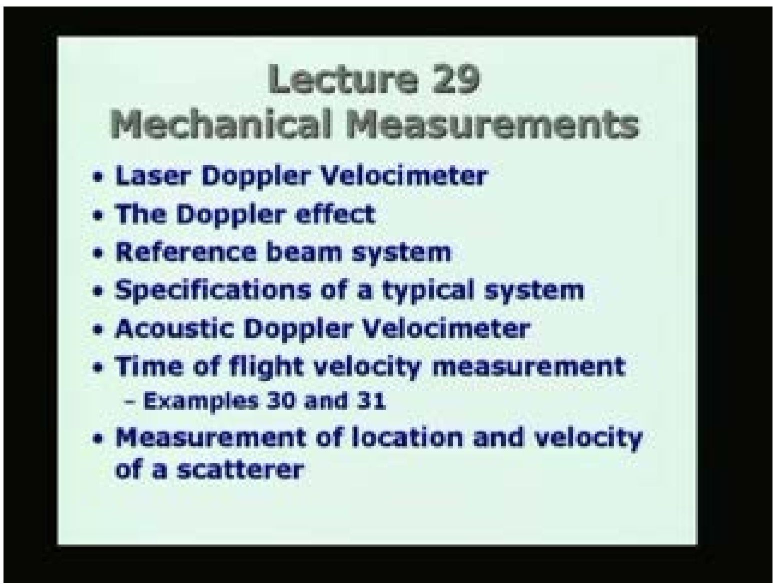


In fact the Doppler effect is common not only to optical waves, light waves, but it can also be used for the ultrasonic waves. Therefore I am going to generalize the discussion on the Doppler effect so that we can also look at what is called acoustic Doppler Velocimeter. The common feature is the theoretical base same for the Doppler Velocimeter, Laser Doppler as well as the acoustic Doppler Velocimeter. I will also discuss in some detail about the time of flight velocity measurement and we will also look at what is called the cross correlation method of measuring velocity. Let us introduce the Doppler effect and try to understand what this Doppler effect is.

Suppose we consider a wave, in this case to start with, I am considering waves which are optical waves or light waves. They are actually electromagnetic waves which mean that the waves are both in the electric field and the magnetic field varying with respect to space and time. The waves propagate with a velocity equal to C, that is the speed of light, and as you know the speed of light is very large which is equal to 3 into 10 power 8 m by s. The fundamental relationship which we are going to make use of is with the relationship between the frequency, the wave length and the speed of light. So we know that the speed of light C is the product of the wave length and the frequency.

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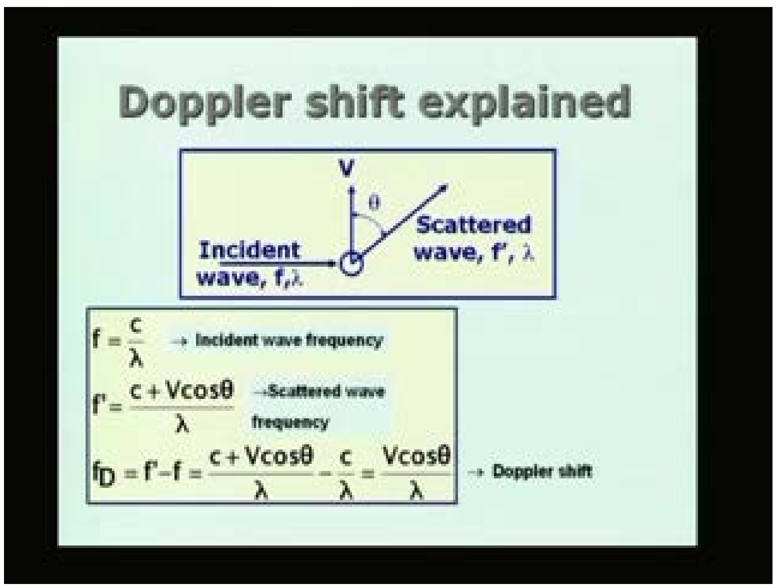


Frequency represents variation with respect to time while the wave length represents variation with respect to space direction. So the electro magnetic radiation will be characterized by, here the incident wave has a frequency f and a wave length lambda. Now what I am going to do is to look at what happens when a wave is incident on a particle which is moving at a velocity V. In this case I have deliberately taken a particle which is moving in a direction perpendicular to the incident wave. We can identify two things; the particle because of its electrical properties, in general it is going to scatter the incoming radiation into all directions. So we can take a look at what happens in a particular direction. The scattered wave is being considered in a direction with respect to the velocity vector of the particle.

The particle is the scatterer, the incident wave is coming with a frequency f and a wave length equal to lambda and the scatterer is moving with a velocity equal to v in a direction perpendicular to the incident wave and I am looking at what is happening to the scattered radiation in the direction making an angle theta with respect to V the velocity vector. So in the equation it is the incident wave f the frequency of the incident wave is the velocity of light divided by the lambda.

In case I am applying this equation to a wave of different type for example, thick wave the C will be the speed of the acoustic waves. This is the general expression but right now, I am just looking at the optical waves. So C by lambda equal to f is for the incident wave, if f is the incident wave frequency given by C by lambda. So let us look at what happens to the wave which is scattered in this direction. In other words it has got a velocity v and it has got a component in the direction of the scattered wave. So V(cosine) of this angle theta is the velocity of component in the direction of scattered wave. Therefore the scattered wave is going to move with a velocity greater than the speed of light C because a certain velocity due to this component is going to be added to that.

So we will generally say that the speed has changed from C to C plus V cosine theta in this case. Of course if it is in other direction it would be negative. So the speed can be either higher or lower depending on the angle theta. In this case angle theta is less than ninety degrees therefore cos theta is positive so I am just going to get that one. The fundamental observation which we call as the Doppler effect is the following. The lambda the wave length of the radiation does not change. When the speed has changed from C to C plus V cosine theta the frequency adjusts itself such that lambda remains constant. So the frequency changes in tune with the velocity. So we will say that f prime is the frequency of the scattered wave which is shown here which is still at the same wave length lambda given by C plus V cosine theta that is the velocity in the direction of the scattered radiation because of the additional V cosine theta term divided by lambda.

So we will generally say that the difference between the f prime is the scattered frequency and at the incident frequency is called the Doppler shift
f prime minus f equal to C plus V cosine theta by lambda minus C by lambda so it gives you V cosine theta by lambda. So the Doppler shift in this

case is due to the movement of the scatterer and it has got a component in the directional scattered radiation or I am looking at it in a direction where this scattered direction has got a component of the velocity of the scatterer in that particular direction.

Suppose I look at the wave in the forward direction, this is the incident direction and this is the forward direction where theta will be 90 degrees and therefore V cos theta will be 0. Therefore there will be no change in the frequency of the radiation as it gets scattered in the forward direction. If it is scattered in a direction for which theta is not equal to 90 degrees that means forward direction is actually theta equal to 90 degrees. In that case there is no shift in the frequency but the shift is for all other angles. So, as long as theta is not equal to pi by 2 in this particular case what happens in the general case? In the general case we have a scattering particle and the incident radiation may be incident on it at an angle theta $_i$.

Actually the entire thing need not be in the same plane. The incident wave, the scattered wave etc need not be in the same plane but we can generalize it. The incident wave is incident on the particle or the scattering particle at an angle equal to theta $_i$ this is the incident angle and the scattered wave I am looking at in this particular direction.

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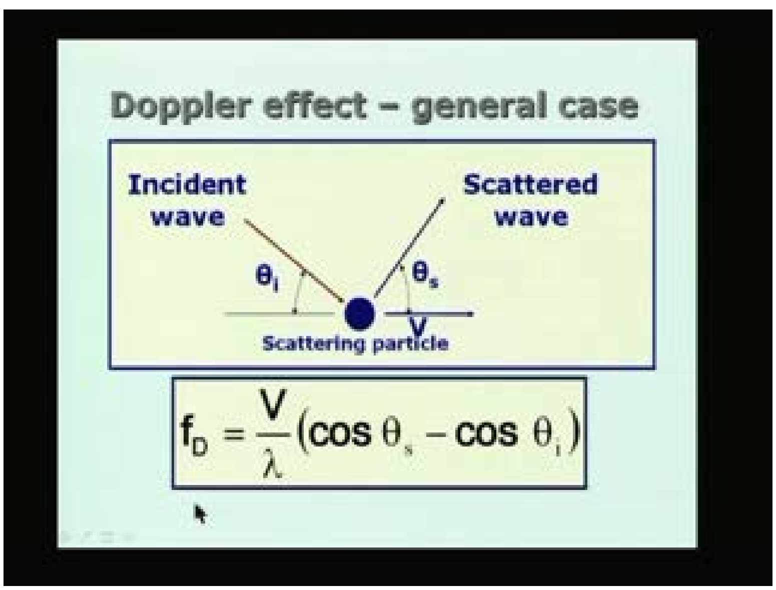


I can choose whatever direction theta is and the velocity of the particle is in this direction. So it has got two components now, V cosine theta is the component in the scattered wave direction minus V cos $theta_i$ is the component along the incident direction. Therefore there are two components of shift or two Doppler shifts one for the incident wave and the other for the scattered. Therefore the net effect will be a net Doppler effect of V by lambda into minus cosine theta $_i$ is the fundamental equation for the Doppler effect. Now we are going to look at a Laser Doppler Velocimeter and see how it functions, how it uses this Doppler effect to measure the velocity of a disturbance or a particle which may be present in the flow.

In the case of fringe system, the two beams of laser which are going to intersect at a focal point of the lens are going to interfere and form a set of interference fringes and the particle which is present in the fluid is going to go across this fringe field system and gives you a burst signal from which we are able to get the velocity. However, in the case of the Doppler system which uses the reference beam system we are going to determine the velocity based on the Doppler shift. So the frequency shift is going to give us the information.

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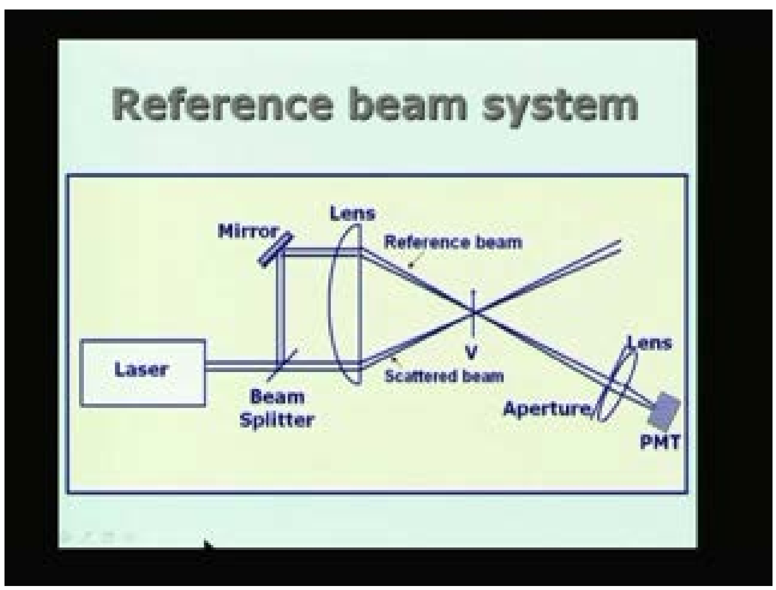


What we have is similar to the fringe system. I have got the laser, a beam splitter then the mirror so that I can create two beams of light from a single laser source, this is one beam and this is the other beam, I call this the reference beam. And in this case what I am going to do is, I am going to do reference beam very weak compared to the scattering beam. The scattered beam is much stronger than the reference beam. That means the mirror can have a reflectivity which is small so that the amount of reflected light here is very small.

In other words, I can have a small value for the intensity of this beam which is going through and this is called the reference beam and the reference beam is actually caught by the lens through an aperture and it is communicated with the Photo Multiplier Tube. Now let us look at the other beam the scattered beam. The scattered beam is going to come to a focus at the same point as the reference beam. In other words, these two beams are going to come to focus at this point. And when a particle which is moving with a velocity moves through this field the radiation which is coming from this beam gets scattered in this direction and the scattered wave is going to be at a frequency shifted with respect to the frequency of the reference beam. Therefore what the Photo Multiplier Tube is going to see is the reference beam and super posed on that the scattered beam with a slight difference in the frequency. So we have two beams; the reference beam and the scattered beam with a small shift in the frequency. Now we are going to locate what happens to this light at the PMT.

Now let us work out on what is going to be the net effect on the PMT. The PMT or the Photo Multiplier Tube is nothing but an instrument which is used to record the intensity of the radiation which is falling on it. Just to recapitulate, the electro magnetic radiation light is characterized by an electric field and a magnetic field. This is the electric field, this is the magnetic field. If radiation is passing in this direction, this is the direction of propagation. Suppose that is the direction of propagation the E and H fields are perpendicular to that. I am just looking at the electric field vector or the H any one of these. These two are actually not independent, both E and H have a certain relationship between the two. Therefore in the electric field the energy which is carried by the field, so energy is proportional to the square of the electric field, and if I calculate the square of the electric field and average over a length of time it gives the average energy which is going to be carried by the wave. And the Photo Multiplier Tube responds to the energy which is falling on it in the form of intensity. Intensity is nothing but energy per unit area per unit solid angle and so on. The energy is what the Photo Multiplier Tube is going to respond to.

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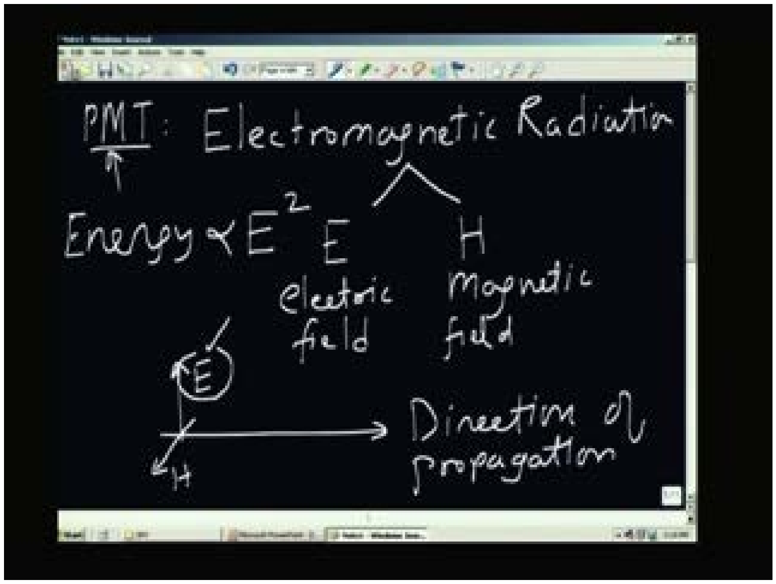


Now let us look at what is happening in this particular scheme. We have the reference beam, the frequency is f or corresponding to that let us say, $f_1$ or $Omega_1$ is the circular frequency which is nothing but 2pi(f). And the scattered beam is at a slightly different frequency because it is scattered by the particle which is moving and therefore it is shifted with respect to $f_1$ we will say it is equal to $f_2$. And correspondingly I will say $Omega_2$ is the circular frequency. Here this is a weak beam, we have chosen this to be a weak beam, the scattered beam is also weak because even though I have large intensity of beam passing through that focal point only a small amount is going to be scattered because it depends on the number of scattering elements present there. Therefore the scattered intensity is usually weak.

In fact that is the reason why I want to make the reference beam weak so that I am going to combine two beams which are relatively of the same intensity or same magnitude. So the total energy indicated by the Photo Multiplier Tube consists of the sum of two things. One is the energy due to the incident beam, so I am going to add two vectors so I will say this is [$E_1$ cos $Omega_1$t plus $E_2$ cos $Omega_2$t] whole square of this, this is the instantaneous value and I am just squaring the value and this will consist of three terms $E_1$ square cos square plus,2$E_1$ $E_2$ cos $Omega_1$t cos $Omega_2$t. In other words it has got three terms. And if you look at what is happening with the Photo Multiplier Tube it is not going to respond to the instantaneous value but it is going to respond to the average value.

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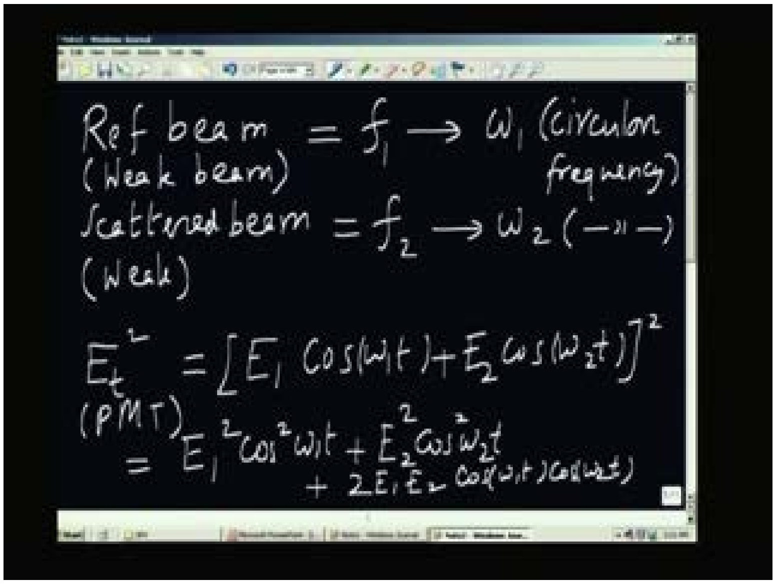


Therefore if you average this quantity here $E_1$ square cos omega t it will give you some constant value. Similarly, this will also give you some constant value. Actually the values are going to be the average value so the PMT responds to the mean value which will be given by $E_1$ squared by 2,$E_2$ squared by 2, and the third quantity is proportional to $E_1$ $E_2$ cos Omega$_1$ t cos Omega$_2$ t this can be re written as $E_1$ $E_2$ cos by using trigonometric relation (Omega$_1$ plus Omega$_2$)t plus cos(Omega$_1$ minus Omega$_2$)t by factor that 2 will go off with that so this is what is going to happen. Omega$_1$ and Omega$_2$ are close to each other because these are optical frequencies, the frequencies of the wave. Anyway I have got a term Omega$_1$ plus Omega$_2$ and I have another term Omega$_1$ minus Omega$_2$. This Omega$_1$ plus Omega$_2$ is very large and therefore if you take the mean value this is going to give 0.

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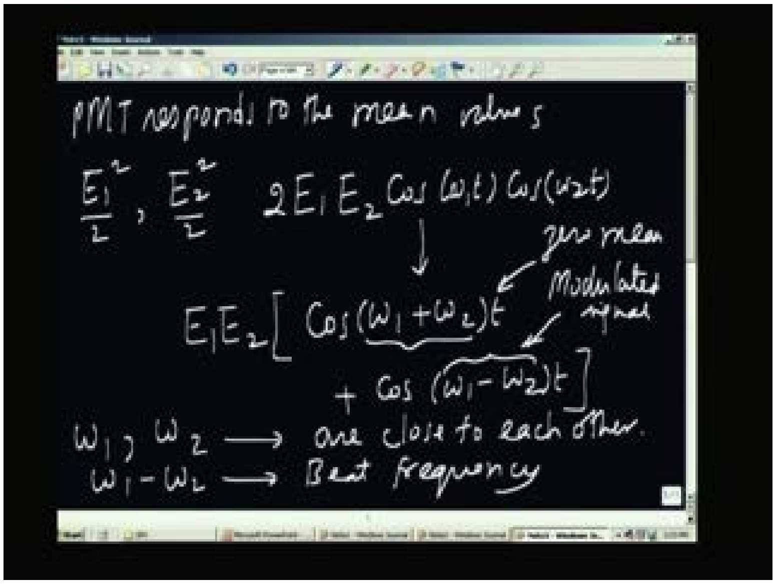


However, this is going to be a modulated signal. In fact this is called the beat frequency Omega$_1$ minus Omega$_2$ is called the beat frequency. Therefore what I will get by looking at the output of the Photo Multiplier Tube is whenever a scattering particle or a set of scattering particle moves through the focal region a small a burst of signal will appear and this burst of signal will show you the beat frequency. So the modulation will be the beat frequency. Suppose a set of particles are moving in the focal region then the scattered wave will contain a burst signal, the burst signal will have Omega$_1$ minus Omega$_2$ will be the signal it is going to have.

Therefore you will get something like this. This is the Omega$_1$ minus Omega$_2$. Remember Omega$_1$ and Omega$_2$ are very close to each other. Therefore the difference is going to be a small frequency. And this will actually appear as a variation in the signal which is given by the PMT. The condition required for this is that, the particles should be small, large particles will not give you a good signal so the particles should be small and usually submicron diameter in size. Let us look at the specifications of a typical Laser Doppler instrument.

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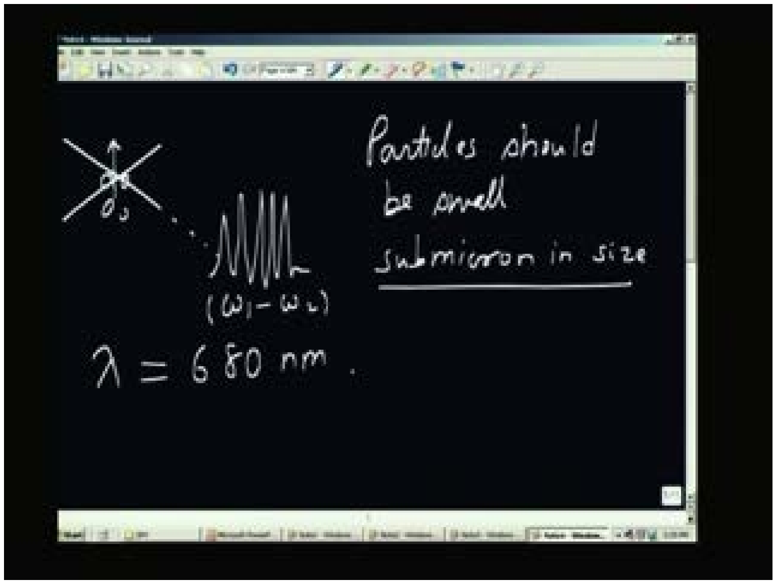


There are two models available namely Canon LV - 20Z and LV - 50Z. You can look at one of them. The measurement range is minus 200 to plus 2000 mm by s so 2 m by s is the maximum velocity you can measure. Focal length of the optics is 40 mm, the depth of focus is plus or minus 5 mm, the laser spot size is 2.4 by 0.1 mm.

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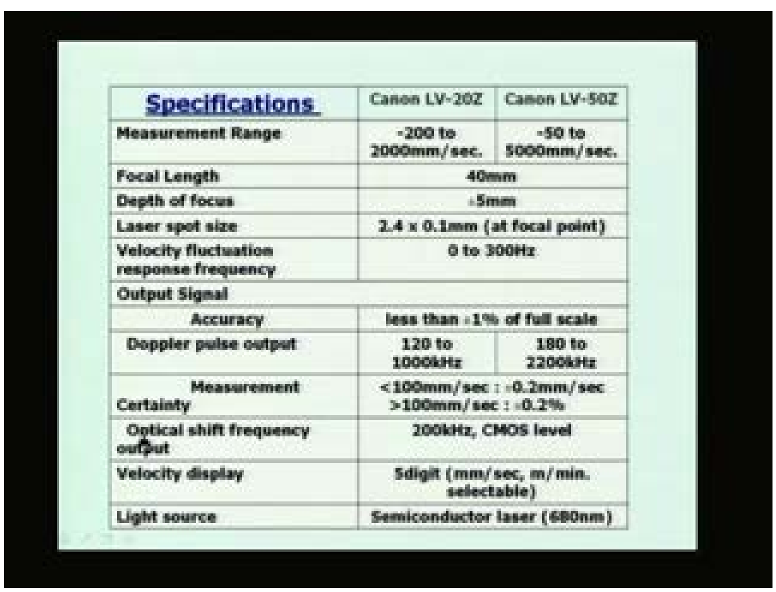

| Specifications | Canon LV-20Z | Canon LV-50Z |
|---|---|---|
| Measurement Range | -200 to 2000mm/sec. | -50 to 5000mm/sec. |
| Focal Length | 40mm | |
| Depth of focus | ±5mm | |
| Laser spot size | 2.4 x 0.1mm (at focal point) | |
| Velocity fluctuation response frequency | 0 to 300Hz | |
| Output Signal | | |
| Accuracy | less than ±1% of full scale | |
| Doppler pulse output | 120 to 1000kHz | 180 to 2200kHz |
| Measurement Certainty | <100mm/sec : ±0.2mm/sec >100mm/sec : ±0.2% | |
| Optical shift frequency output | 200kHz, CMOS level | |
| Velocity display | 5digit (mm/sec, m/min. selectable) | |
| Light source | Semiconductor laser (680nm) | |

It is a very small region which is going to come to a spot and the volume over measurement is going to be done is proportional to this. The velocity fluctuation responds that means that it responds to 0 to 300 Hz. Let us look at the output signal. The Doppler pulse output is anywhere between 120 to 1000kHz and the upper limit about 1MHz. So you see that the beat frequency is one way or giving here 120 to 1000kHz. The measurement uncertainty is given by less than 100 mm by s is 0.2 and above that it is 0.2%. And then the velocity display is a 5 digit display and the light source you can see here is semi conductor laser working at the wave length of 680 nanometers. And if you work out the characteristics of the laser at 680 nanometers, so the typical numbers for a Laser Doppler instrument would be, laser wave length is 680 nanometers, the velocity of light is C is 3 into 10 to the power 8 m by s the universal constant. Actually I have taken the vacuum velocity, the frequency can be calculated as the ratio of C by lambda where C is 3 into 10 to the power 8 by 680 nanometer, 1 nanometer is 10 power minus 9 nanometers so it gives you 4.41 into 10 power 14 Hz.

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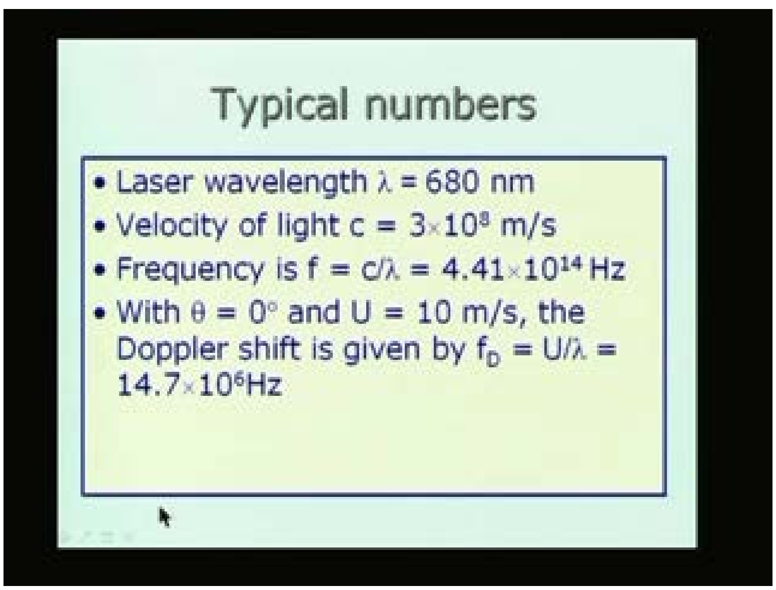


The frequency is very high and no instrument and no Photo Multiplier Tube will certainly not respond to this hertz. It is too large a frequency to be resolved by the Photo Multiplier Tube, it only gives the average value. Suppose I take theta equal to 0 degrees then this is the scattered angle is 0 degrees that means that the velocity is parallel to the direction of scatter 10 m by s, the Doppler shift is given by $f_D$ equal to U by lambda and U equal to 10 m by s, lambda equal to 680 nanometers and when you just do that you will get 14.7 into 10 power 6 Hz. So the frequency of the Doppler signal is going to be 14.7 MHz for a velocity of 10 m by s.

So, in the instrument actually the upper limit was 2m by s so it will be five times less than this and that is roughly the value as shown here. So the principle of operation of the Laser Doppler instrument is that you are going to have two laser beams of light crossing at the focal point of a lens and then one of them acts as a reference beam which is the weaker beam and the stronger beam is going to be scattered. In fact you can see that the weaker beam is also going to be scattered, and the idea is that this amount of scattered light because of the weaker beam is so small that the PMT is not going to respond to that. So the stronger beam is going to be scattered in sufficient strength so that you get the Doppler signal riding over the mean value. And if you subtract the mean value and look at only the alternating value which is the Doppler value that will show you a frequency which is directly proportional to the velocity. As you can see here the Doppler shift is directly proportional to the velocity. And you also notice that I do not need any calibration of this instrument. The velocity is Doppler frequency multiplied by lambda.

If the lambda is known for the laser beam then you measure the frequency of the scattered the Doppler shifted radiation and all you do is to U equal to $f_D$ times lambda. It is a very simple linear relationship that exists between U and $f_D$ and it is truly a linear instrument. So, with this background let us look at the case of, suppose I do not want to use a laser radiation suppose I want use acoustic radiation, acoustic radiation can be from any low frequency to high frequency, instead of being transverse waves as in the case of electro magnetic waves these acoustic waves are longitudinal waves. Acoustic waves are usually in the form of pressure pulses. The pressure changes as the wave passes through the medium. Of course it requires a medium to be present.

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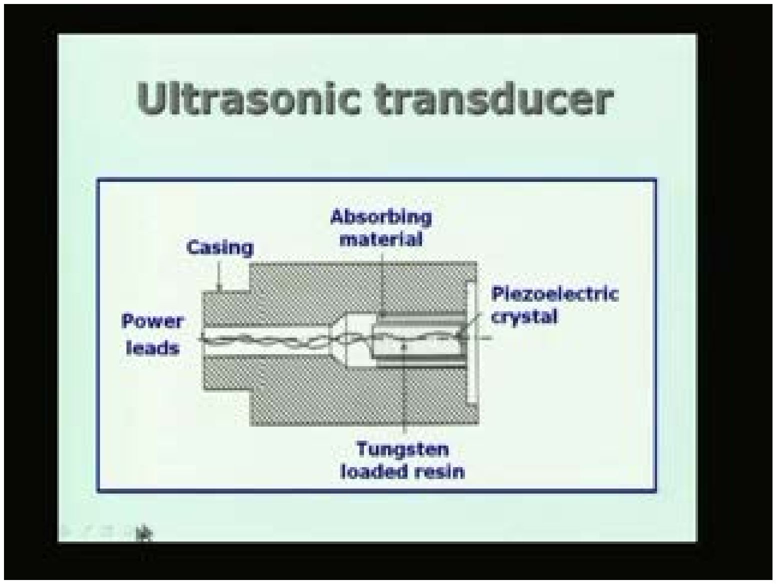


And in the case of laser instruments there is no need for any medium to be present. But in the case of acoustic Doppler instrument you will need to have some kind of a medium present. It may be either a liquid or a gas. And the acoustic waves have to propagate through that and then we can use either the time of flight or the Doppler effect. The reason why we can use the time of flight in this case is, because the speed of the wave in the medium is usually not very large. In the case of electro magnetic radiation it is 3 into 10 power 8 m by s is too large, I cannot measure the time it takes for it to propagate from one place to another and then from the speed of propagation find out the distance traveled etc which is not possible. However, in the case of ultrasound wave, ultrasonic waves are usually above 20 kHz a typical figure would be 100 kHz. We produce such a wave using a transducer.

It is a piezoelectric crystal which is shown here and there is an absorbing material and for that the waves are confined to the forward direction, power is supplied to that and the piezoelectric crystal if you subject it to AC signal it gives you pressure pulses and undergoes displacement and the displacement is confined. In fact the displacement is going to be communicated to the medium which is next to the piezoelectric crystal and it gets excited. So when you oscillate the electric field to which the piezoelectric crystal is subjected to you get ultrasonic waves which are going to travel in this direction. And in order to make it go only in the forward direction we have a tungsten loaded resin material here which is going to absorb the radiation which will be otherwise passed in this particular direction.

So only the radiation passing in this direction is going to be used for the subsequent measurement. The ultrasonic transducer can be used both as a transmitter and as a receiver. What do I mean by that? Suppose, I impress an alternative electric field on the piezoelectric system it will get excited and it will give ultrasonic waves. However, if I subject it to incoming acoustic wave, for example some ultrasonic wave is coming and is impinging on that it will give rise to an electrical signal. That means in the reverse direction when the ultrasonic signal is incident on the piezoelectric crystal it will give rise to an alternating current here or if you supply alternating current here it is going to give rise to ultrasonic waves. So the same transducer can be used either as the transmitter or as a receiver.

In most of the experiments we are going to excite the piezoelectric crystal over a very small finite period of time. So we will generate a pulse of ultra sound wave and this pulse will be used for the determination of the velocity either by using the Doppler shift method,or by using the time of flight method. Let us look at the time of flight method. I have just indicated a schematic here. So I have a tube which is carrying a fluid moving with a velocity V, this velocity V, I can take it as a mean velocity and in order to have a sufficiently long length of travel for the wave I have a long length like this and the inlet fluid is coming here and it travels along this length of this cell and then gets out here. So this is the place where I am going to make the measurement. I have got a transmitter here and a receiver here and I will call this as station number one and station number two. The fluid is always coming in here and going out here.

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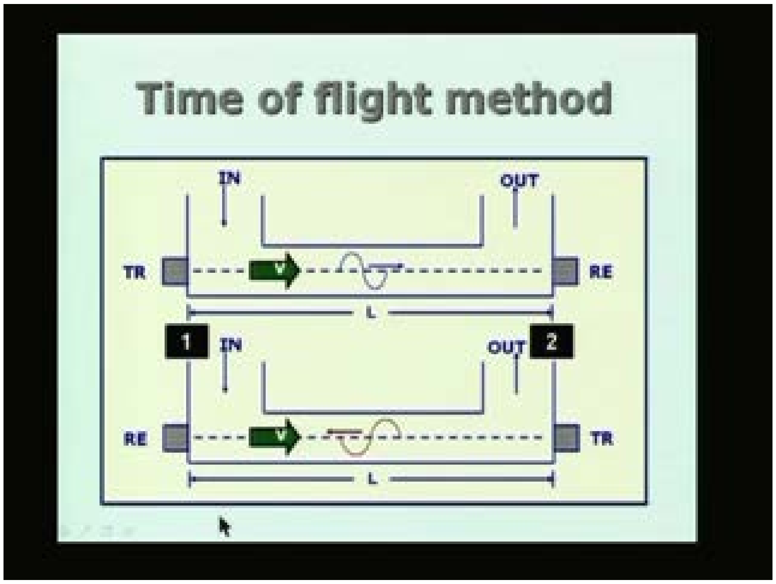


In the second part, this is the receiver and this is the transmitter. I can change the role of the transmitter and receiver by just driving this and receiving the signal here or driving this and receiving the signal at this way. So essentially I have the transmitted beam which is going to go through this medium and I have shown a wave which is passing through. This wave is shown with respect to time; it is not with respect to space. So the frequency is some value f in the forward direction. And in the backward direction you can see that the frequency has slightly changed because there is a velocity component in the direction of the propagation of the wave and the value of the velocity is moving in this direction is V plus c for the propagation speed. In the second case when it is moving against the stream it is V minus c.

So let us look at the time of flight the time taken by the wave to travel from the transmitter to the receiver. In the first case path 1 to 2,1 is the left point, 2 is the right point the wave travels at a speed of c minus V. Therefore time of flight is given by this where the time is nothing but the distance traveled divided by the velocity so L by c minus V this is for the forward path. If I look at the path from 2 to 1 the wave travels at a speed of $V_B$ equal to c plus V, it is traveling faster now. Therefore the time of flight equal to $T_{21}$ equal to L by c plus V.

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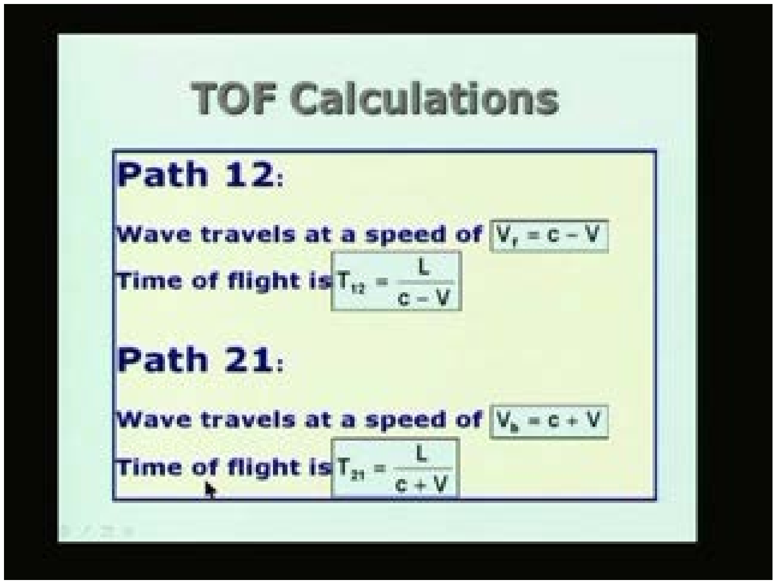


So, if I now take the difference between the two times of travel or the times of flight $T_{12}$ minus $T_{21}$ will be given by L by c minus V minus L by c plus V it comes to 2 LV by c square minus V square, V is the velocity of the medium, c is the speed of the wave. And usually in practice, for example, if I am having water as a flowing fluid in the tube the velocity may be a few meters per second. And the velocity of the ultrasound wave is the same as speed of sound which may be about more than a kilometer per second it is almost like 1.5 kms.

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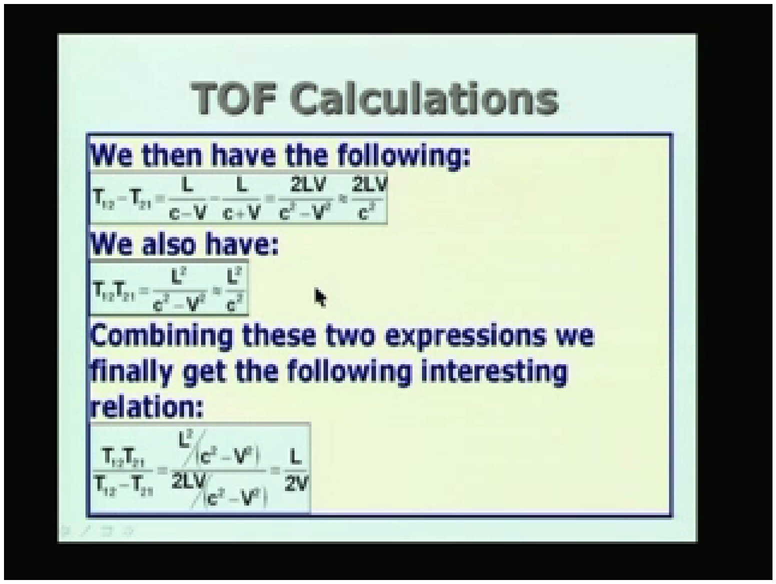


Therefore you can see that I can ignore this portion with respect to c square and therefore approximately I can say that the difference $T_{12}$ minus $T_{21}$ the difference is times of flight is 2L into V by c square. In fact, I can also multiply $T_{12}$ and $T_{21}$ and you can see that I am going to get L square by c square minus V square which is roughly equal to L square by c square for the same reason because the velocity is usually very small compared to c square. Now you see that $T_{12}$, $T_{21}$ if I now combine these two equations, if I divide $T_{12}$, $T_{21}$ by $T_{12}$ minus $T_{21}$ you see that I am going to get $T_{12}$ $T_{21}$ by $T_{12}$ minus $T_{21}$ equal to L by 2V as I have done here. Therefore you can re write it in terms of the velocity as L by 2 into $T_{12}$ minus $T_{21}$ by $T_{12}$ $T_{21}$.

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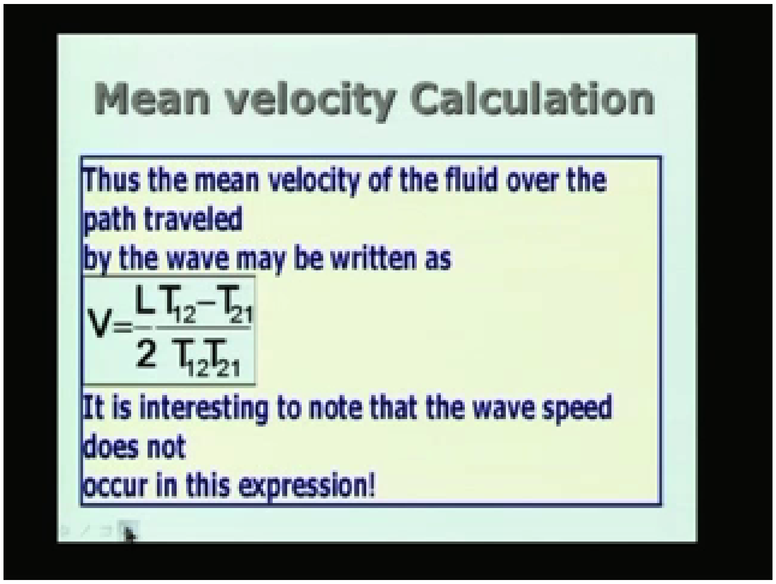


So if I measure the times of flight $T_{12}$ $T_{21}$ then I can take the product of these two and can get the velocity of the fluid simply by taking L by 2 into this particular quantity. What is interesting in this particular expression is that the speed of the sound or the wave speed has completely disappeared. So I need not worry about the speed of the wave in the medium which is absent in the picture. So, if I manipulate such that I measure these quantities $T_{12}$ $T_{21}$ and I calculate this ratio $T_{12}$ minus $T_{21}$ to $T_{12}$ $T_{21}$ I can directly measure the velocity. Again you see that V is a linear function of this, L is of course the distance between the two receivers and the transmitter which is already known. We can measure it by using a scale. Therefore there is no need for any calibration here and all you do is measure this quantity and then get the velocity.

So let us look at the situation where such a thing can be used. One arrangement is what is called the clamp on type ultrasonic mean velocity meter. So I have a transmitter and a receiver. Of course this can become a transmitter or receiver depending on whether I drive it or I use it in the receiver mode and then the flow is taking place here. The acoustic wave is going to travel across like this and when it travels in one direction it is going to be slowed down and in the other direction it is going to be speeded up. Therefore the times of flight are different for the two directions and can be done for this case also.

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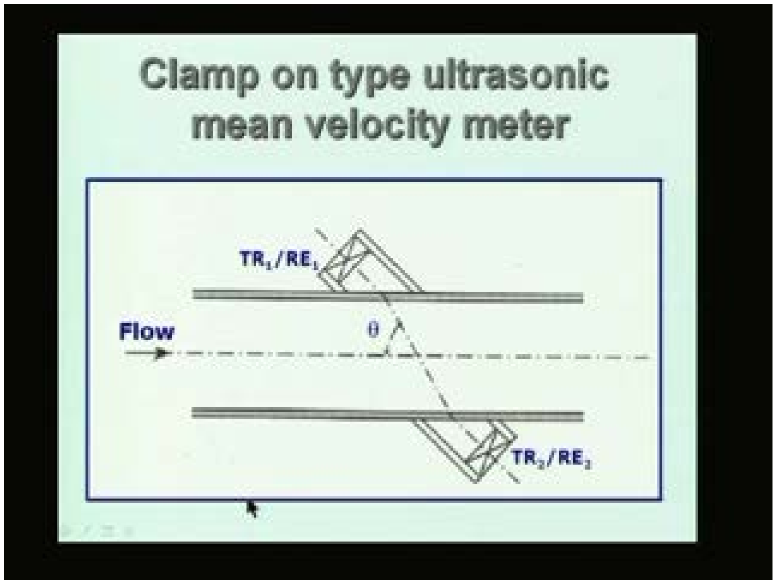


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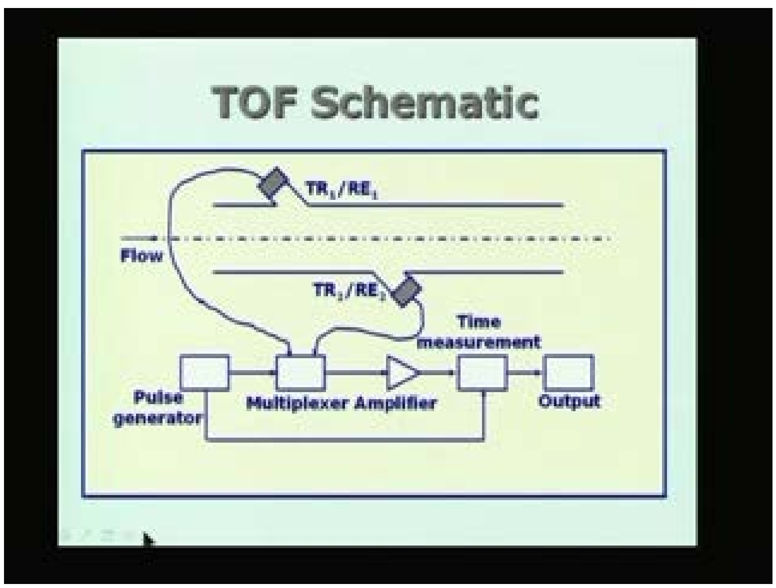


So a schematic of an instrument which uses time of flight measurement is shown here. I have the pipe carrying the fluid, I have the transmitter receiver here, another transmitter receiver here and then I drive it by using a pulse generator. The pulse generator will first drive this and after a gap it will drive this. You will drive this and this alternatively. So first this becomes the transmitter and after sometime this is going to receive a signal and this multiplexer will look at two signals one which is transmitted from here and received here and the second one transmitted from here and received here and then with respect to the pulse generated signal it will calculate the measurement time. All these are done electronically, and if necessary, there can be an amplifier here so that the amplified signal is going to be used for the time measurement and from the time measurement I can calculate the quantity $T_{12}$ $T_{21}$ by $T_{12}$ minus $T_{21}$ or the inverse of that and that is your output and this output is directly proportional to the velocity V. So the time of flight schematic which is shown here requires some complicated electronics but it is possible to directly measure the velocity and it does not require any calibration.

The other advantage of the ultrasonic measurement is that, the medium need not be very clean, it can have even particles, it can be dusty, for example, air bubbles can be present, slurries etc. We can also measure the velocity of slurries and so on. These are some of the advantages. However, the quantities which you are measuring are really very small and therefore the time measurement becomes very critical. We will be measuring times, scales of the order of microsecond or less. Therefore the instrument can be quite expensive because of that. Let us look at another way of doing this.

Suppose, I have a scatterer present in a medium, and the scatterer is moving with velocity U so as what we did in the case of Laser Doppler I can also use acoustic Doppler or ultrasonic Doppler to measure the two quantities. In this case I am going to do the following. I am going to measure the position of the scattering particle by using the time of flight and I am going to measure the velocity by looking at the change in the frequency, f is going to change to f prime so the back scattered wave is going to be at a different frequency because it is scattered by the scatterer and the scatterer is moving in this direction. Therefore I can measure by this particular method both the locations of the scatterer as well as the velocity of the scatterer. At least I can measure the velocity component in the direction of the scattered wave by using this.

We have a transmitter receiver here and somewhere along the this direction there is a particle and a scatterer which is moving with a velocity equal to U and I would like to the measure the velocity U and also I would like to measure the distance between these two that is the distance $X_s$.

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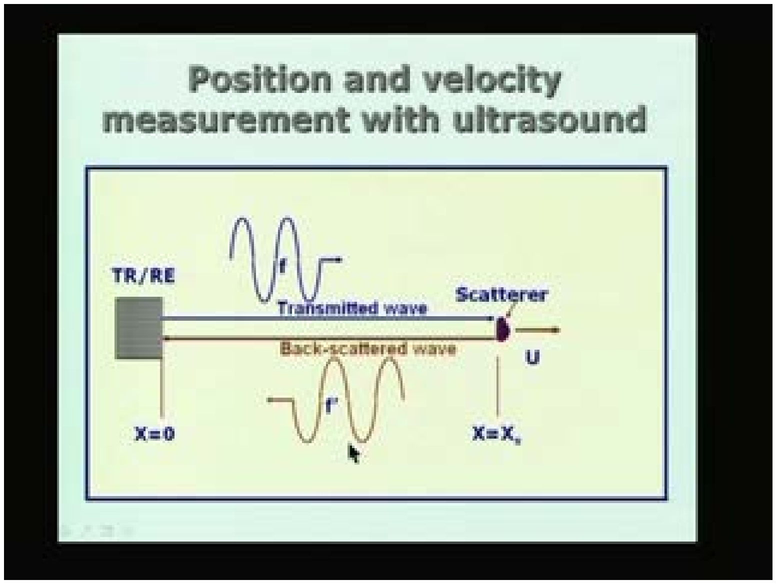


So at t equal to 0 suppose I say that we have an acoustic signal after sometime I will receive a signal like this. So this is the transmitted signal and this is the received signal. What is the difference between these two signals? For this of course the frequency is f and this is f prime. The frequency content of this signal is going to be different from the frequency content here. So here I am getting double information. I am getting one measurement of time so I will say time of flight total because the wave has c so it is actually traveling twice the distance.

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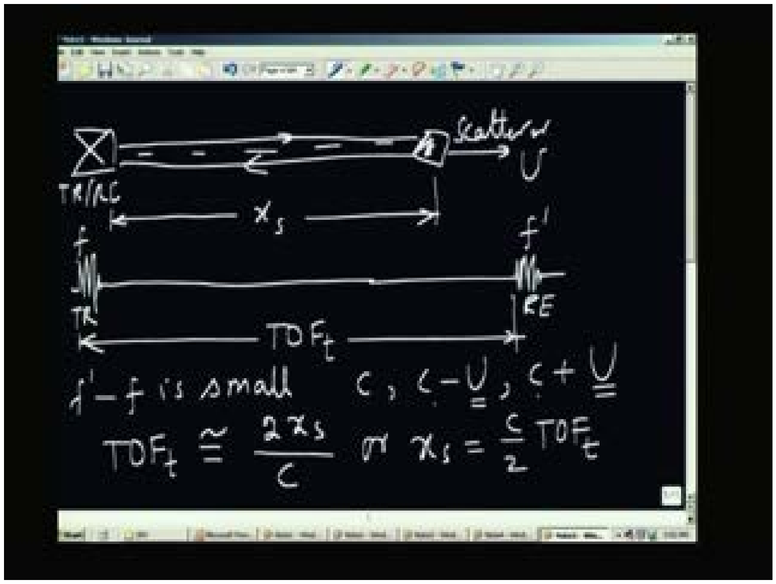


And if you remember what I said earlier f prime minus f is small that is number one, and number two is, if you look at the speed c, c minus U or c plus U any of these things this is small compared to this normally. Therefore the total time taken will be simply given by $2(x_s)$ by c even though part of the time it has traveled a little faster and a little slower depending on the velocity of the scatterer. It is not going to be important from the total time of flight itself as it is simply given by $2x_s$ so I can show it as approximately equal to this or $x_s$ equal to c by 2(TOF) total that is I just measured the time duration from the transmitted signal to the received signal and I get that. And now you see that the f prime and f are different.

So if I measure the f prime minus f I need to do modulus value, it can be either positive or negative and this will be nothing but multiplied by lambda will give you the velocity of the particle. This is very easy to see. This is just like what we had in the case of the Laser Doppler instrument also. So simultaneously I can measure the velocity and also I can measure the location. So whenever such a situation occurs it is possible to use both of them and they can be measured at the same time.

Here is example 30: The situation is like this; I have got a tube carrying a fluid whose velocity I want to measure and the transmitter receiver is kept at an angle of 45 degrees. So there is a transmitter receiver pair, so I am going to make the measurement of the time of flight this is d equal to 0.1m and we will assume that fluid velocity is given by 2 m by s. So I am going to measure $T_{12}$ that is this is one and this is two, I am going to measure $T_{21}$ and I am also going to measure $T_{12}$ $T_{21}$ these are the products and also I am going to get the ratio.

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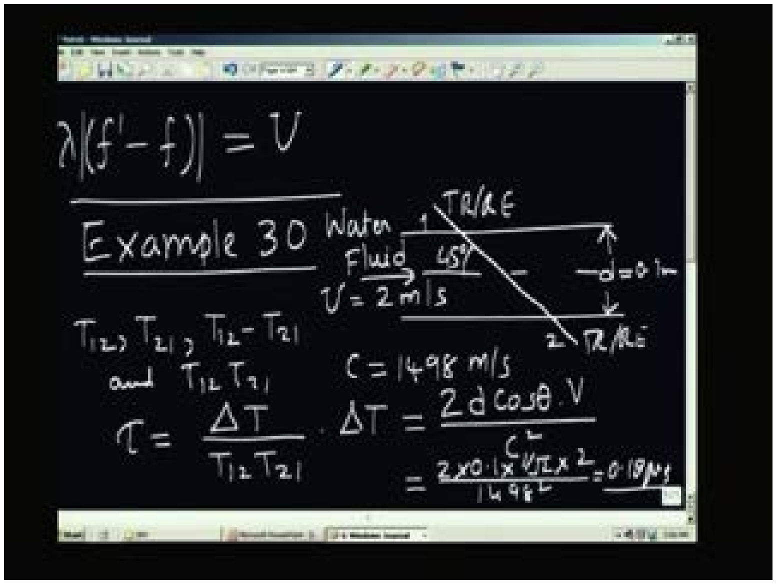


So we will call the ratio tau as delta t by $T_{12}$ $T_{21}$,suppose I have water as the fluid the speed of the acoustic wave is given by 1.5kms almost 1498 km by s and you see that the velocity U is 2 m by s very small compared to that it is very small compared to that and in fact I can show that the delta t with theta equal to 45 degrees. You remember I already gave the general the Doppler shift formula where cos theta $_i$ minus cos theta $_s$ is there so all I have to do is to use that. It will come out to be 2d cos theta V by c square the difference in $T_{12}$ and $T_{21}$ 2d cos theta V by c square and theta is 45 degrees. All I have to do is to substitute that value so this will be 2 into 0.1 into cos theta is 1 by square root of 2 into 2m by s by $(1498)^2$ and this comes to about 0.18 microsecond the difference in time between the forward path and the reverse path are the time taken by the acoustic wave to propagate in the forward direction minus that in the backward direction is about .18 micro second that means our measurement of time must be very accurate.

Suppose I calculate the tau for the same case then tau can be shown all you have to do is to do the multiplication and so on this will be nothing but V sin 2 theta by d for this particular case. You can put that values of $T_{12}$ $T_{21}$ and so on. And then get this, this will be sine 2 theta because theta is 45 degrees sin 2 theta is sine 90 is 1. This becomes simply equal to V by d for theta equal to 45 degrees. In fact that is the reason why we choose the 45 degrees angle in this case and you will see that V by d V is 2m by s d is 0.1 it comes to 20 by s. So tau is a very good indicator because its a large value and if you can calculate that directly by using electronics to do that you will get a number like this and let us take one more example, in order to calculate the Doppler shift so the shift in the Doppler frequency is given by $f_D$ equal to f by c into V cos theta the Doppler shift is given by f by c into V cos theta and all I have to do is to substitute the values.

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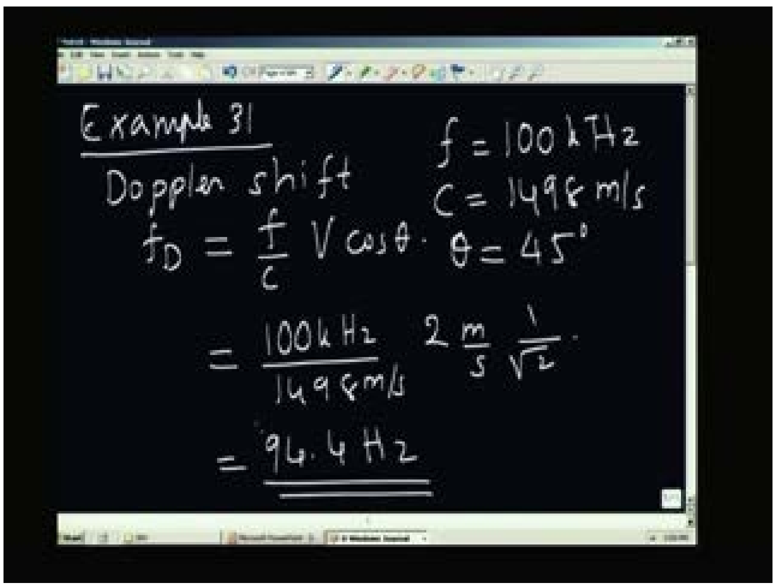


Let us assume that the frequency f is 100 kilo hertz, and the speed is again 1498 m by s. Theta is again in this case I have taken pi by 4 or 45 degrees if take that we will see that the Doppler shift is going to be assuming $V_s$ 2 m by s as we did earlier this will be 100 kHz by 1498 m by s and V is actually 2m by s, cos theta is 1 by square root of 2 and this comes to about 94.4 Hz. It is a very small frequency the Doppler frequency is about 94.4 Hz. And if you look at the output wave form you will see the following. You will see this, something like this is 100 kHz. If I take the other one it will be like this, the other one will be like this, this is 100 Hz and this is 100 kHz so the signal will consist of this where this is superimposed on this. Therefore what you see will be like this with lot of high frequency. All I have to do is to remove the high frequency filter out and then I will get the Doppler frequency. So with this we have looked at two ways of using the acoustic velocity meter. One is using the time of flight and the other one is using the Doppler shift. Here is another simple method called the cross correlation type velocity meter a simple in operation.

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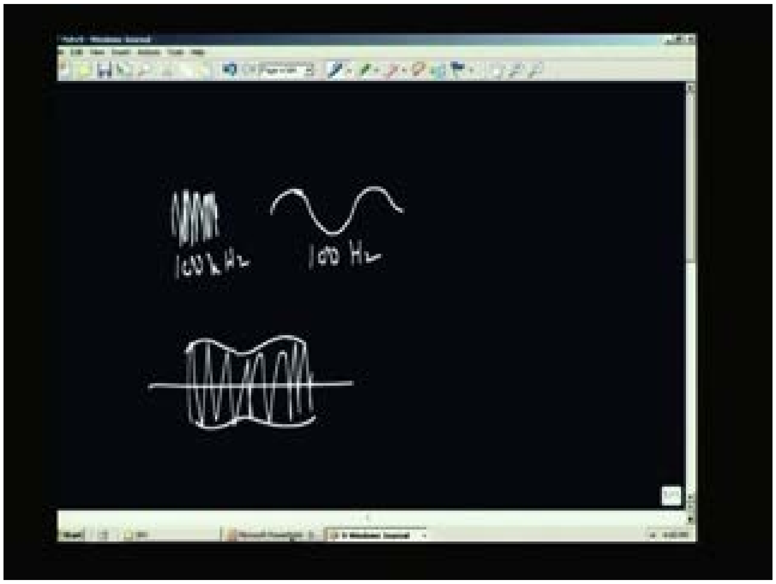


So we have the flow taking place through the tube as usual and we have two transmitters and two receivers arranged at two locations, one here and the other one some where down stream. Suppose a small disturbance is there in the flowing fluid for example, it could be air bubbles or some particles the particles will go through this region and after sometime it will cross this region. So, when suppose a transmitter and receiver are always on, so I have a acoustic beam which is going through from the top to the bottom when the transmitter when the disturbance passes the beam it will give rise to a small change in the signal which is received. Therefore $S_1$ will show the presence of the disturbance at some particular time and after certain time the same disturbance will pass through this region and I will get the disturbance at $S_2$. Therefore the disturbance will propagate presumably at the same speed as the same speed of the flowing fluid.

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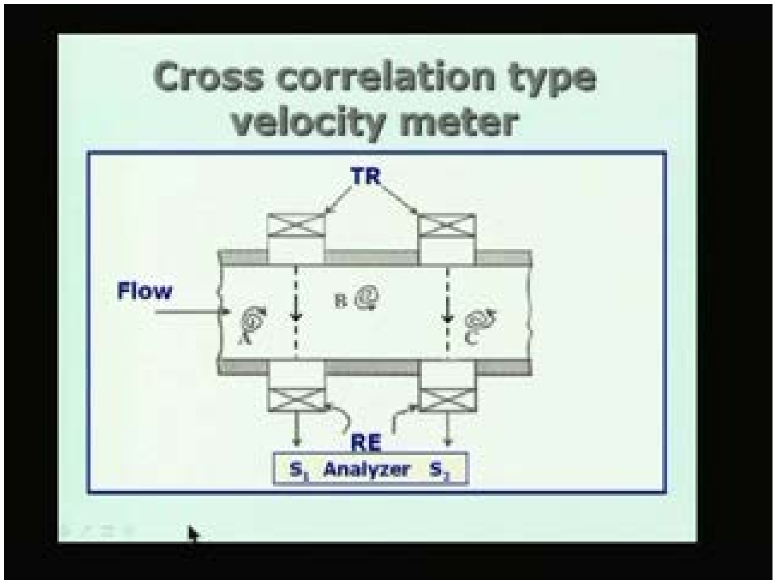


Therefore if I look at the $S_1$ and $S_2$ ,I will get the following $S_1$ is the signal with respect to time I get a small blip here the value of the intensity has come down because scatterer has passed through this region and I get a blip like this and after a certain time delay tau time delayed tau I get the second one which is going to show the presence of the blip.

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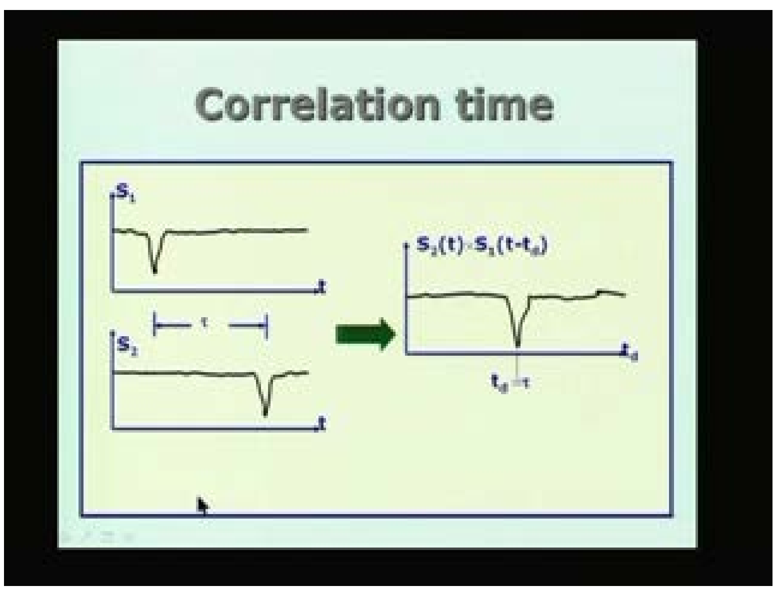


So from here to here I take the product of $S_2$ into $S_1$ with t minus $t_d$ so the product of these two are with different time delays. If I take with different time delays you will see that when the time delay is exactly equal to tau which is called the correlation time or the cross correlation time what will happen is that you are going to get a big signal here big blip here. Therefore by looking at the time td this time td must be equal to the time I took for the disturbance to move from the first location to the second location. Therefore I am measuring directly the time taken by the disturbance to move and therefore it is very easy to see that the velocity must be equal to the distance between the two stations divided by the correlation time tau. So the correlation time measurement is a very simple way of doing it. Only thing is you require two receivers and two transmitters and a certain length of the pipe in which the flow is allowed to take place and it is possible to measure the velocity. Thank you.