

Mechanical Measurements and Metrology

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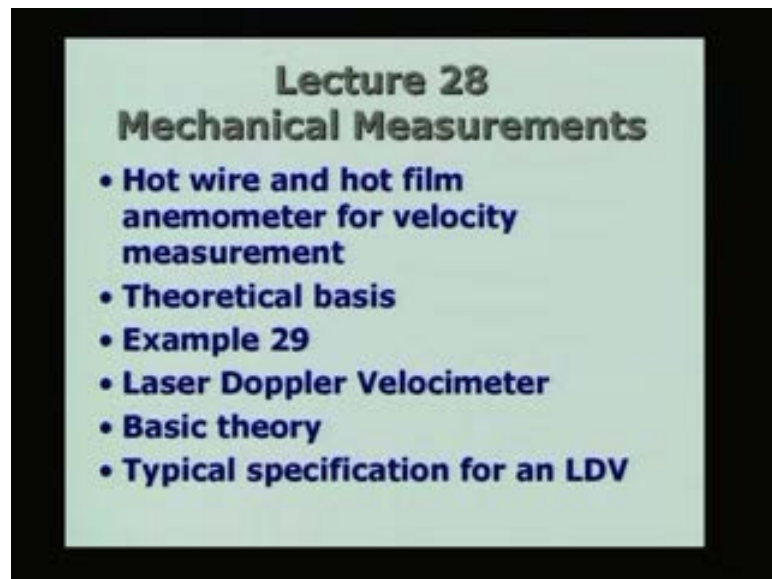
Module - 2

Lecture - 28

Hot Wire Anemometry and Laser Doppler Velocimetry

This is lecture number 28 Mechanical Measurements. We have been looking at the measurement of velocity of fluids. We will consider two more techniques which are useful for measurement of velocities of basically liquids and gases. And most suitably we are going to talk about measurement of velocity of a fluid like air which is very common in most applications.

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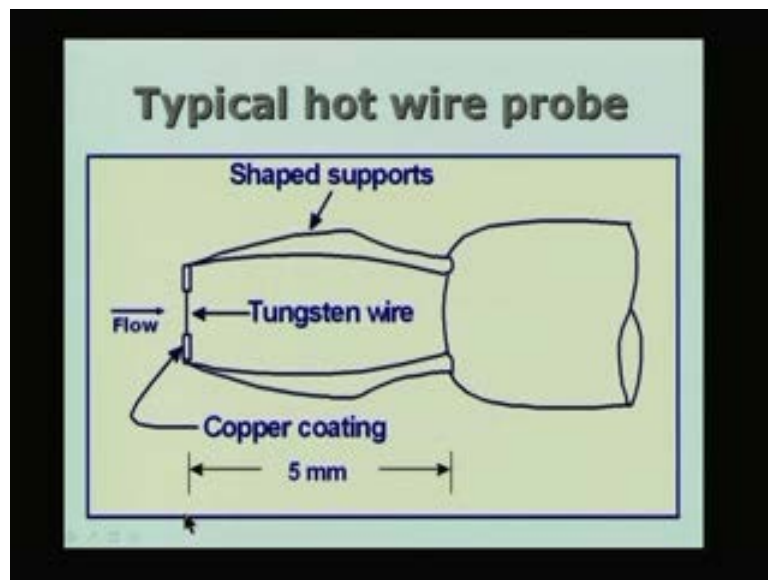


The first instrument I am going to take a look at is the hot wire or the hot film anemometer for velocity measurement. I will go through the theoretical basis for the hot wire, and then I will give an example which describes how it functions and what kind of numbers we can expect. The second method we are going to look at is the Laser Doppler Velocimeter. The difference between the hot wire and the Laser Doppler is that the hot wire or the hot film requires a probe to be inserted into the flow while the Laser Doppler Velocimeter does not require a probe to be introduced into the flow.

Therefore it is a non intrusive method or a non invasive technique for measuring the velocity.

However, in the case of Laser Doppler measurement we require some scattering particles to be present in the flow field and that will be one of the deficiencies for this method again with respect to Laser Doppler measurement. We will look at the basic theory and we will try to look at the typical specifications from LDV instrument. The first thing I am going to look at is the typical hot wire probe.

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There are two types of hot wire probes. One is the hot wire and the other one is the hot film, and let us just look at the way they are constructed. Basically, it consists of a very thin wire which is shown here between two supports. There are two supports which are going to hold in place the wire which is usually made of tungsten or platinum. And the fluid which is flowing whose velocity if you want to measure flows across the tungsten wire and the wire itself is heated by passing a current through that. You can very well see that if there is a flow or if there is a flowing fluid which is passing the tungsten wire the temperature of the wire or the heat transfer from the wire is going to be dependent on the flow velocity.

Therefore if the flow velocity changes the tungsten wire experiences a different cooling rate. Based on this we can measure the velocity of the

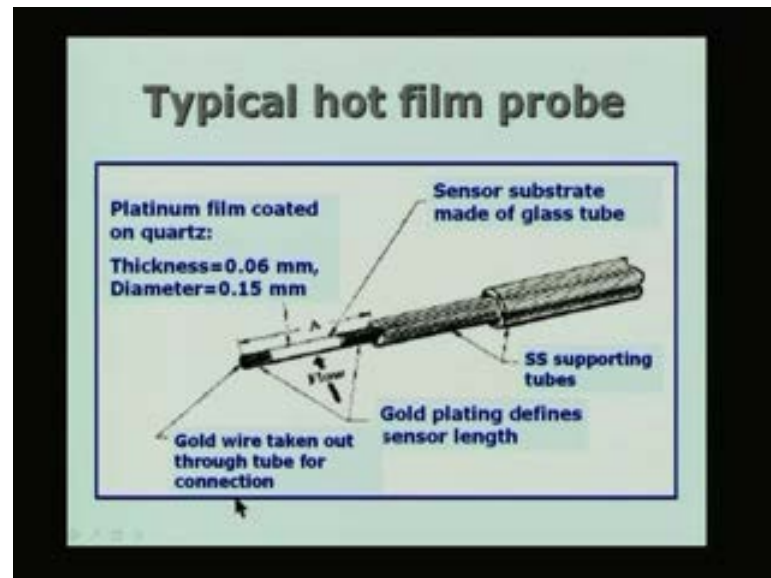
flowing fluid. The length of the tungsten wire is very small, short as shown here. The distance between this point and this point could be a few millimeters may be 4 or 5 mm and you can see that this will be less than or close to a millimeter or 2 mm is the length of the tungsten wire. Usually it is made of a very thin wire. And in practice, when we look at it, you will not be able to see with your ordinary eye you will require looking at it through a microscope to verify that the wire is certainly there.

Of course another way of finding out whether the wire is there or not is to find out whether it is an electrical continuity must be there. So, the reason why we have a support which is shaped in this fashion is to prevent heat transfer through the supports by conduction or to reduce the heat transfer conduction through the supports because any heat transfer other than from the flowing fluid to the tungsten wire should be as small as possible so that we have a specific relationship between the flow velocity and the heat transfer rate which is going to determine the velocity of the flowing fluid.

So, in this case the supports are having some portion which is copper coated and the length between these two is actually the active length of the hot wire probe. And of course there is a handle through which the wires are going to be connected to the tungsten wire are taken out. Basically the wire is a resistive element. The tungsten wire here or the platinum wire or whichever wire that is going to be used is the resistive element. The resistance of the wire changes with temperature. So it is similar to the resistance thermometer.

We already discussed about thermometry. We also talked about the resistance element being used for measurement of pressure. In the present case we are looking at the measurement of velocity during the same effect, the thermal effect on a wire whose resistance depends on the temperature.

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You have also seen a typical hot film probe. The difference between the hot film and the hot wire is that, the wire is in the form of a thin wire which is cylindrical in shape whereas in the case of a hot film the material whose resistance is going to vary with temperature is actually coated on a substrate. In this case you can see that we have a platinum film coated on quartz. The quartz tube is here.

This is the quartz tube whose diameter is about 0.15 mm, the thickness of the platinum film is 0.06 mm and there is a certain length here, beyond which there is gold plating. The distance between these two points here actually defines the sensor length. The connection to the sensor is taken. One connection is taken through the inside of the tube and the second connection is taken here and these supporting tubes are going to make it possible to introduce the hot film into the stream whose velocity I want to measure.

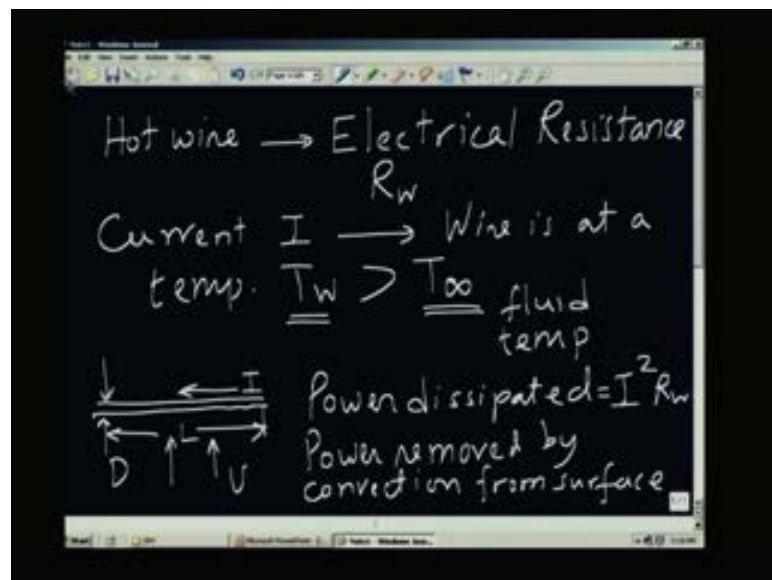
You can see that the stream is moving perpendicular or normal to the probe and the flow is in this direction. And the characteristic, or the resistance of the probe is dependent on the flow velocity and the typical hot film probe is going to measure or infer the velocity based on some output which is going to be an electrical output. We will look at how the electrical output is going to be the relative velocity. Between the hot wire and the hot film there is no difference in terms of the operating principle. The hot wire probe is very delicate because the wire is very small in diameter. Therefore it is

susceptible to damage. Even if you have a very high velocity stream it can blow off the wire.

However, in the case of the hot film because the film is actually deposited on a substrate and the substrate itself is a strong material in this case we have a glass tube or a tube made of quartz. The hot film is more rugged and therefore it is one of the major reasons why you go for a hot film probe. The second point is that the hot film can be made very thin as you see here. And because it is a very thin film the frequency response of the film also can be made very rapid. It can respond to very rapid fluctuations in the velocity.

Of course the hot wire also will be able to respond to very rapid fluctuations in the velocity. And in fact both the hot film as well as the hot wire probe are used for measurement of turbulent velocities where the velocity fluctuates with respect to time and the measurements can be made because the hot wire or the hot film will be able to follow these variations with respect to time. If you recall, when we talked about the pitot tube they are very slow instruments. One may think they will not be able to rapidly adjust to the change in the velocities and Therefore they are basically steady state instruments, instruments useful for measuring steady flow while the hot film and the hot wire.

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Also, the Laser Doppler Velocimeter is useful especially when you want to

measure rapid changes in the velocity. Let us look at the theoretical basis of the hot wire anemometer. The hot wire essentially consists of an electrical resistance which is nothing but, the resistance of the wire. Let me indicate it as R_w that is the resistance and we pass a current I , so that the wire is heated up at a temperature T_w greater than the fluid temperature which we term as T_{∞} . So, this is the wire temperature and this is the fluid temperature. Suppose I assume that the wire is in the form of a long cylinder this is the diameter and we will say this is the length of the probe or the effective length.

As we have indicated earlier the flow is taking place perpendicular with a velocity equal to U . And we can in fact see that because the current is passing through this suppose the current is I , the power dissipated by the wire due to this current is $I^2 R_w$, where R_w is the power dissipated, this must be equal to power removed by convection. If we assume that all other modes of heat transfer are important or very negligible the power removed by convection from the surface in fact under the steady state condition.

So, the power dissipated by convection removed by the flow is nothing but the area of the surface, which will be $\pi D L$ into the heat transfer coefficient h into the temperature difference between the wire and the gas are the flowing fluids. In this case of course we are talking about anemometer which is used in a gas stream. So $\pi D L$ into h into T_w minus T_{∞} . So, under steady state we will equate the two; $I^2 R_w$ must be equal to $\pi D L$ into T_w minus T_{∞} into h so the heat transfer coefficient h is a function of U the velocity.

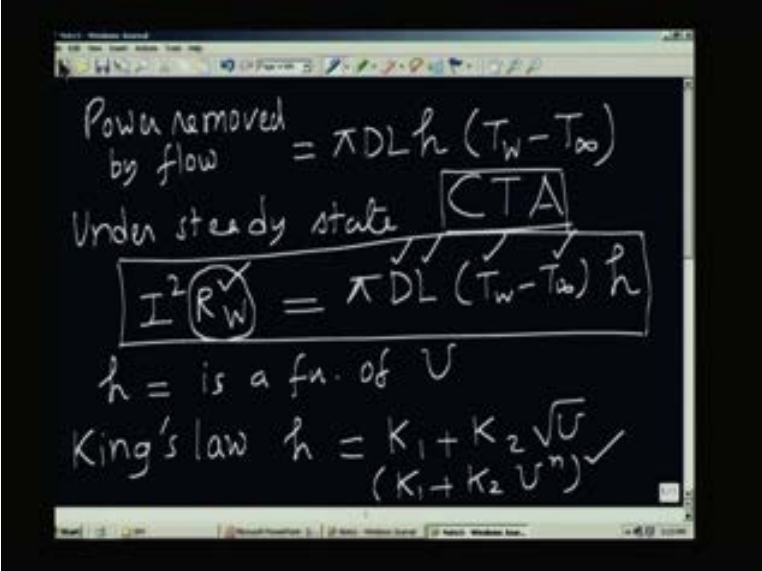
Normally, we invoke what is called the king's law which is an empirical relation between the heat transfer and the velocity U . So the heat transfer coefficient is proportional to is equal to some K_1 plus K_2 times square root of U . In fact in general square root of U is a specific relationship. In general I can write it as K_1 plus K_2 times U to the power of n where the exponent n is very close to half. Of course depending on the nature of the flow, the size of the wire and so on the value of n is going to in fact vary. So, for the moment let us assume that the relationship is of the form K_1 plus K_2 into U to the power of n .

Now look at this equation; what I have written down is, $I^2 R_w$ is equal to $\pi D L$ into T_w minus T_{∞} into h . So we will see that this is the constant

determined by the diameter of the wire, this is the length of the wire which is already given. Now on the left hand side we have the wire resistance which may be either constant or vary. So, if I actually use the instrument such that the wire temperature remains constant, this is called the constant temperature anemometer, CTA and that is the constant temperature anemometer. In the case of constant anemometer temperature, T_w is constant.

Of course the fluid temperature is given and it is also a constant. So you see that in the case of constant temperature operation, R_w is also a constant. Therefore $I^2 R_w$ is a constant equal to the product of all these constant (h).

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Power removed by flow = $\pi D L h (T_w - T_\infty)$

Under steady state CTA

$I^2 \overset{\checkmark}{R_w} = \pi D L (T_w - T_\infty) h$

h = is a fn. of U

King's law $h = K_1 + K_2 \sqrt{U}$
 $(K_1 + K_2 U^n) \checkmark$

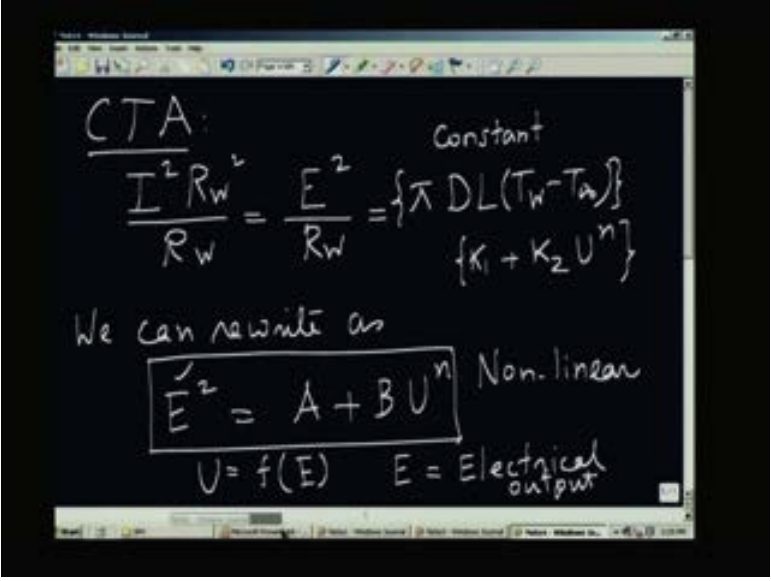
Therefore I can say that I^2 is proportional to the heat transfer coefficient that I want to write. And you also realize that if R_w is constant I can do the following. In the case of constant temperature anemometer I will write as CTA for constant temperature operation, $I^2 R_w$ is in the left hand side and I can write it as $I^2 R_w$ square by R_w which is nothing but the square of the voltage across the resistance divided by R_w on the right hand side.

I have $\pi D L$ into T_w minus T_{∞} these are all constants because T_w is maintained constant into h . How it is maintained constant? I am going to write it as K_1 plus $K_2 U^n$, so I will cross multiply by R_w so $R_w \pi D L$

T_w minus these are the constants into K_1 which is another constant. Therefore we can rewrite it as E^2 is equal to A plus BU^n , a very interesting relationship between the velocity and the output. The E is nothing but the voltage across the resistance which is considered as the electrical output of the constant temperature anemometer, obviously the relationship is non-linear. And once I measure the value of E under the constant temperature operation of the wire I can find out the corresponding U . In fact, you can use an inverse relationship U is a function proportional to E or U equal to some function of E . This is the general expression we are going to get, where E is the electrical output.

Let us look at the corresponding circuitry of the constant temperature operation and what I am going to do is to use a bridge circuit. The hot wire bridge circuit consists of the following: We have the hot wire sensor which is exposed to the fluids, flow velocity, I want to measure, I am calling it R_2 here in this sketch, R_1 is a standard resistance, R_3 and R_4 are also standard resistances.

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CTA:

$$\frac{I^2 R_w}{R_w} = \frac{E^2}{R_w} = \left\{ \pi D L (T_w - T_a) \right\} \left\{ K_1 + K_2 U^n \right\}$$

We can rewrite as

$$\boxed{E^2 = A + B U^n} \quad \text{Non-linear}$$

$U = f(E) \quad E = \text{Electrical output}$

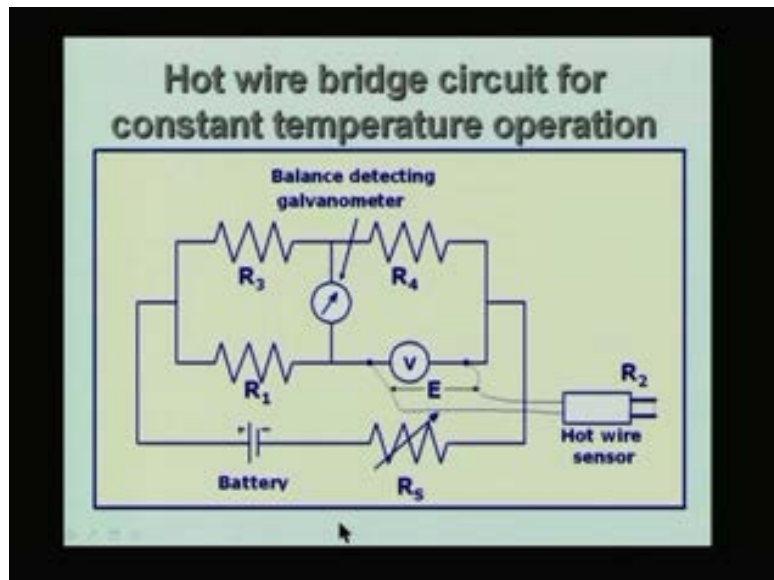
Now R_1, R_2, R_3, R_4 are connected in the form of a Wheatstone bridge. Then I have got a balance detecting galvanometer connected across the bridge here. And if the balance detecting galvanometer shows in 0 it means that the bridge is under balanced condition. Just imagine what is happening; I have connected the hot wire sensor in this arm of the bridge and if I maintain the

current by maintaining the temperature constant whatever may be the velocity here. So, if the velocity goes up the heat transfer coefficient is going to go up.

Therefore it tends to cool the wire or the temperature difference between the wire and a gas is tending to decrease. So, in order to maintain the temperature I have to supply more heat so that the temperature remains constant. So, if I want to supply more heat I have to change the current which is passing through the hot wire and that is done by the series resistance R_s , which is actually operated by a feedback circuit which detects the out of balance here and appropriately takes the action to change the arrows driven back to the balance. It is done electronically by taking a feedback across this balanced in the galvanometer and that voltage is actually used to control the R_s here such that this is driven to balance condition. So, if it is under balance condition always, that means the R_3, R_4 are fixed resistors, R_1 is also fixed therefore R_2 is always of the same value as long as the balance is maintained.

Therefore the current which I am going to vary by R_s , which is also flowing through the hot wire, current is going to change. When the current changes the resistance is the same, the current times resistance is the voltage across the hot wire sensor and that is what is measured by this voltmeter. This voltmeter is going to give you the voltage across. That is what I have shown here, E is the electro motive force or the voltage potential difference across the voltmeter. Actually you have connected the hot wire sensors.

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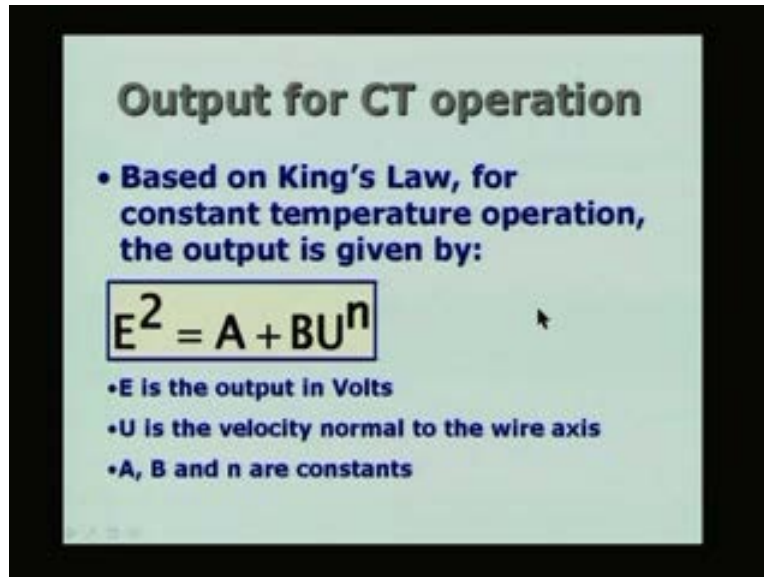
So in the hot wire bridge circuit for constant temperature operation, I am taking the out of balance voltage and using it to control the R_s by a feedback circuit so that the current is varied such that the balance is always maintained.

When the balance is always maintained, the four resistances have the same fixed value. Therefore the hot wire is operating under constant resistance or constant temperature condition, because the resistance and temperature are related to each other. Therefore if the resistance is constant that means the temperature must be constant. Therefore the hot wire under constant temperature operation requires a bridge circuit to vary the current through the R_2 such that we are going to maintain the balance. And the voltage appearing across the hot wire is actually a measure of the velocity E square is equal to A plus B U power n where A and B are constants, and E is the voltage appearing across the hot wire.

It is easy to see that the hot wire can be calibrated because A and B are two constants to be determined. In fact, n is another constant and usually n is roughly equal to half for very thin wires. So, if n is fixed at half A and B can be determined by making two measurements with two different known velocities being impressed on the hot wire, and then finding out what the output is and then you can find the values of A and B . So, calibrating the hot wire anemometer is a very simple procedure. King's law is an empirical

relationship obtained by Mister King long long ago, for constant temperature operation. The output is given by E^2 is equal to $A + BU^n$. Of course n is close to $\frac{1}{2}$ as we have seen, E is the output in volts. In this case U is the velocity normal to the wire axis is very important. The velocity should be directed normal to the axis of the wire.

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Output for CT operation

- Based on King's Law, for constant temperature operation, the output is given by:

$$E^2 = A + BU^n$$

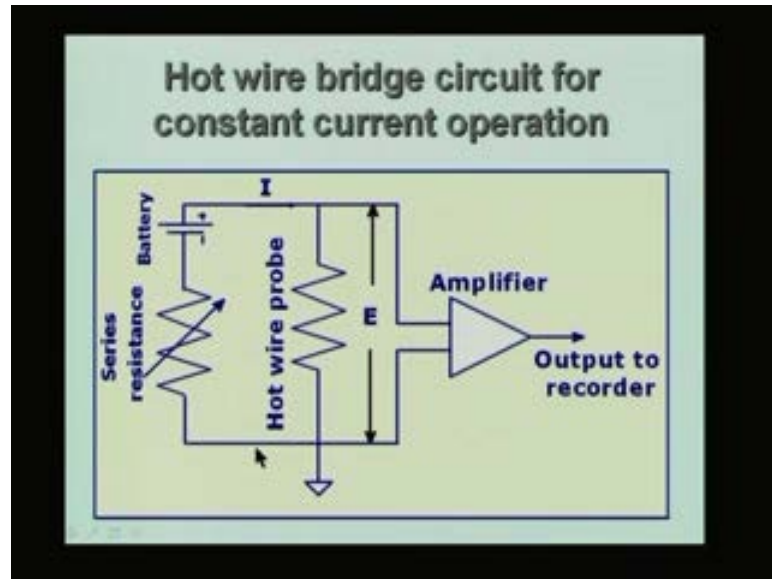
- E is the output in Volts
- U is the velocity normal to the wire axis
- A , B and n are constants

If it is not only that component which is normal to the wire axis is going to give rise to sufficient amount of heat transfer whereas the parallel component is not going to significantly give rise to any cooling. Therefore it is going to respond to the velocity which is normal to it. A , B and n are constants which can be determined of course as indicated by suitable calibration technique. Let us look at the hot wire bridge circuit for constant current operation. So, if I go back to the equation here; $I^2 R_w$ is equal to $\pi DL (T_w - T_{\infty}) h$, if I keep I constant we can see that first time, I kept the R_w constant but now I am going to consider that as a variable. This is a constant and you can also see that this is not a constant any more. Therefore both the current is kept constant. Here this is not constant, this is going to vary, if this varies this also varies let us indicate it by a plus sign there or a multiplication sign there which means that this is going to vary and this also is going to vary.

And of course h is going to vary with velocity given by the same relationship the King's law. In fact we can show that if this is the case based

on the same King's law for constant current operation the output is given in the form of an equation E_0 minus E is equal to AU^n by $1 + BU^n$ where A and B are constants, n is an index which is also a constant may be of different value, E_0 is the value of the output when the velocity is equal to 0.

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When there is no flow whatever value you are going to get is the E_0 and the E is the value you are going to get when there is a stream of velocity U flowing perpendicular to the axis of the wire. Therefore it is E_0 minus E is equal to AU^n by $1 + BU^n$.

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Power removed by flow = $\pi D L h (T_w - T_\infty)$

Under steady state **CTA**

$$I^2 R_w^x = \pi D L (T_w^x - T_\infty) h$$

h is a fn. of U

King's law $h = K_1 + K_2 \sqrt{U}$
 $(K_1 + K_2 U^n)$

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Output for CC operation

- Based on King's Law, for constant current operation, the output is given by:

$$E_0^2 - E^2 = \frac{AU^n}{1 + BU^n}$$

- E is the output in Volts
- E_0 is the output in volts when $U=0$
- U is the velocity normal to the wire axis
- A, B and n are constants

Here is the equation: E^2 is equal to $A + BU^n$, in this case the temperature is constant. What is the advantage of constant temperature operation?

Of course the hot wire itself is going to be maintained more or less at a constant temperature throughout its operation. Therefore it is not subjected to any temperature cycling and because of that it will be possible to have a long life for the hot wire because the hot wire or the hot film is not going to

be subjected to any thermal cycling because the temperature remains fixed. Therefore that is a plus point as far as the temperature operation is concerned. In the case of constant current operation of course the relationship looks more complicated.

In this case you will see that the value of E increases as U increases. You can see that E will come to the right hand side. If you take this left hand side E actually reduces. It reduces because I am keeping a constant current. If I go back to the figure here (Refer Slide Time 25:29) I am going to maintain a constant current through the resistance. When the current is constant if the hot wire is going to be subjected to a higher velocity fluid flowing across it then its temperature will come down because it is going to be cooled by a higher heat transfer coefficient.

Therefore the hot wire is going to reduce in temperature and the hot wire material has a resistance coefficient such that the resistance will reduce when the temperature reduces. So, when the R_w , or the wire resistance decreases the voltage across this is going to decrease as indicated by the relationship which is given here E_0 minus E is equal to AU power n by 1 plus BU power n . It indicates clearly that as U increases, the output E actually comes down. So, before we take a look at the performance we will take a look at a typical case of hot wire anemometer for which the specifications are given here.

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AF210 series Specifications: Hot Wire Anemometer	
Range Flow 0.1 m/s 10.00 m/s	Accuracy $\pm 0.12 \text{ m/s} \pm 1\% \text{ reading}$ ($+5^\circ \text{ to } +45^\circ \text{C}$) $\pm 0.20 \text{ m/s} \pm 2\% \text{ reading}$ $-20^\circ \text{C to } +5^\circ \text{C \& } +45^\circ \text{C to } +70^\circ \text{C}$
Operating temperature	0 to $+50^\circ \text{C}$
Operating temperature probe tip	-20 to $+80^\circ \text{C}$
Probe length	300 mm plus handle
Probe Diameter	13 mm
Dimensions	140 x 79 x 46 mm
Weight	250 gm

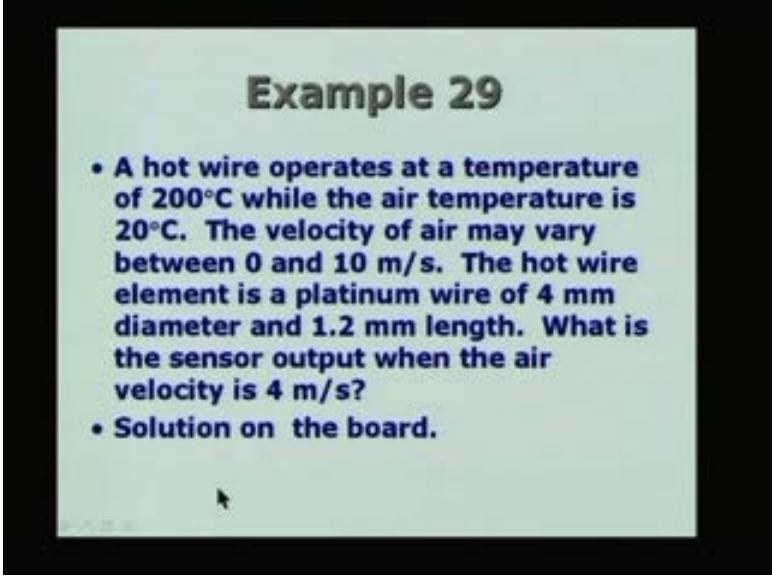
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Here the flow range is 0.1 to 10 m by s that is the range of the operation of this instrument for 0.1m by s is the lowest and 10 m by s. You cannot measure temperature velocity much lower than this. The accuracy is plus or minus 0.12m by s plus minus 1% reading from plus 5 to 45. This is the temperature of the fluid whose velocity is measured within the range 5 to 45. It has got plus or minus 0.12m by s plus or minus 1% of the reading. However, it goes up to plus or minus 0.2 to plus minus 2% reading minus 20 to plus 5 as well as plus 45 to 70. So the range of operation of the instrument is of course minus 20 to 70 degrees for the temperature of the fluid which is flowing. So, the operating temperature of the hot wire itself is 0 to 50 degrees. The operating temperature generally is 0 to 50 degrees centigrade. The probe tip itself is minus 20 to plus 80 that is the temperature range in this particular case.

Some hot wire anemometers or many hot wire anemometers which use either platinum element or a tungsten element operate at around 200 degree Celsius which is the normal value of this. In this case it is using minus 20 to plus 80 is the probe tip temperature variation, the probe length is 300 mm so that it can be introduced into the flow probe diameter, or the diameter of the probe is 13 mm dimension. The weight is about 250g. It is a very small hand held instrument it can be introduced into the flow and this is a very easy way of measuring the velocity. In practice what we do is, when we are not sure about the direction of the velocity we can change the direction of the probe and find out when you get maximum value when the output is maximum. That happens when the velocity of the fluid is exactly normal to the direction of the probe.

Therefore you can roughly find out the direction of the flow and also by maximizing the output. Now let us take a look at a typical case of a hot wire anemometer. The hot wire operates at a temperature of 200 degree Celsius and the air temperature is 20 degrees. The velocity of air may vary between 0 and 10 m by s. The hot wire element is platinum wire of 4 mm diameter this 4 micrometer diameter and 1.2 mm length we have to correct these four micrometers. And what is the sensor output when the air velocity is four meter per second that is what we want to find out. This is example 29:

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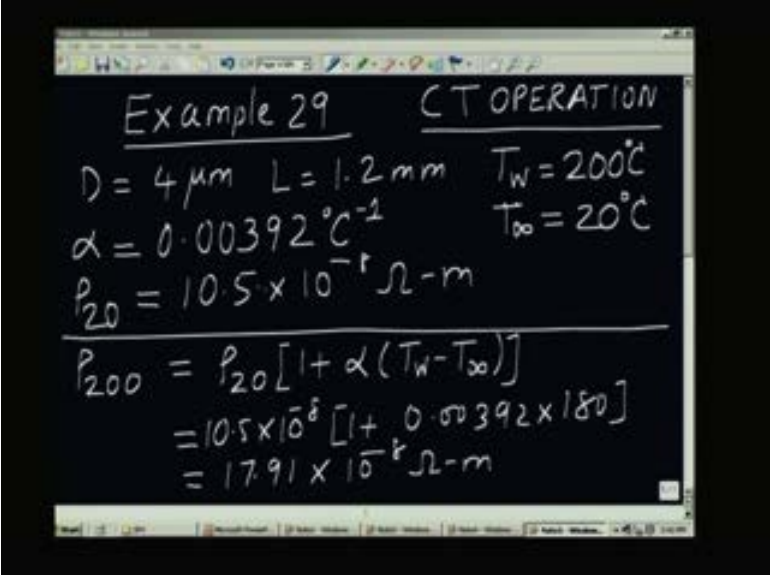
Example 29

- A hot wire operates at a temperature of 200°C while the air temperature is 20°C. The velocity of air may vary between 0 and 10 m/s. The hot wire element is a platinum wire of 4 mm diameter and 1.2 mm length. What is the sensor output when the air velocity is 4 m/s?
- Solution on the board.

If the wire diameter D is 4 micrometer, length of the probe is L is equal to 1.2 mm and its operating temperature which is T_w in our earlier nomenclature is 200 degree Celsius the temperature of the air whose velocity we are measuring is 20 degree Celsius, so T_w and T_{∞} are specified. And we are assuming constant temperature operation, CT operation which is more common, because of the obvious reason that the hot wire is not going to be subjected to too much of thermal cycling and the thermal coefficient. The α is the coefficient of resistance temperature, coefficient of resistance of platinum is 0.00392 then the resistivity of platinum at 20 degrees is given as 10.5×10^{-8} ohm meter.

Now we can actually calculate the resistance of the wire just to find out the typical values you meet within practice. For that I require the resistivity at 200 degrees which is given by $\rho_{20} [1 + \alpha (T_w - T_{20})]$ which happens to be also T_{∞} so this is the same. So all I have to do is substitute the values 10.5×10^{-8} into $1 + 0.00392 (T_w - T_{\infty})$ is $200 - 20$ which is for 20 and this comes out to be 17.91×10^{-8} ohm meter.

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Example 29 CT OPERATION

$$D = 4 \mu\text{m} \quad L = 1.2 \text{ mm} \quad T_w = 200^\circ\text{C}$$
$$\alpha = 0.00392^\circ\text{C}^{-1} \quad T_\infty = 20^\circ\text{C}$$
$$\rho_{20} = 10.5 \times 10^{-8} \Omega\text{-m}$$

$$\rho_{200} = \rho_{20} [1 + \alpha (T_w - T_\infty)]$$
$$= 10.5 \times 10^{-8} [1 + 0.00392 \times 180]$$
$$= 17.91 \times 10^{-8} \Omega\text{-m}$$

Therefore based on the resistivity of the material, I can calculate the resistance of the wire which is nothing but ρ at 200 into L by A and ρ at 200 into L by πD^2 by 4. So we can just substitute all the values. So ρ 200 is equal to 17.91×10^{-8} and L is 1.2 mm. So I will convert it to meters by π by 4 into D is equal to 4 micrometers. So $(4 \times 10^{-6})^2$ whole square comes to 17.1 ohms. So you see that such a very thin wire has got a resistance of 17.1 ohms.

Now I can use the King's law and empirically it has been determined that the Nusselt number is given by a relationship of C_1 plus C_2 (square root of Re) based on the diameter of the wire, where C_1 for operation in air C_1 is equal to 0.3 and C_2 is equal to 0.362 so this information is empirical relationship of heat transfer theory. So I am going to use this nusselt number and from the nusselt number I can calculate the heat transfer coefficient and that is the idea. But for this calculation I require to calculate the Reynolds number and the Reynolds number can be calculated at a mean temperature of air equal to the mean between the temperature of the wire and temperature of the fluid will be T_w plus T_∞ by 2 it will come to 110 degree Celsius 200 plus 20 by 2 will give you 110 degree Celsius and from air tables the properties required may be written down.

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Handwritten calculations on a blackboard:

$$R_w = P_{200} \frac{L}{A} = P_{200} \frac{L}{\pi D^2/4}$$

$$= \frac{17.91 \times 10^8 \times 1.2 \times 10^{-3}}{\left(\frac{\pi}{4}\right)(4 \times 10^{-6})^2} = 17.1 \Omega$$

Use King's law

$$Nu = C_1 + C_2 \sqrt{Re_D} \quad \begin{matrix} C_1 = 0.3 \\ C_2 = 0.362 \end{matrix}$$

What are the properties required? I require the kinematics viscosity for calculating Reynolds number $24.15 \times 10^{-6} \text{ m}^2 \text{ by s}$. then I require the μ subscript, μ is just to say it is the gas property or the air property, k_g is the thermal conductivity which comes to $0.032 \text{ watts per meter Kelvin}$ and the prompt number is 0.7 . But the way I have given the King's law prandtl number is not going to explicitly come, it is already built into it. Therefore this value is not important from the point of your calculations which we are going to give out. So, if I calculate the Reynolds number based on the diameter I am going to take U is equal to 4 m by s . If you remember the problem find out what is the output when the velocity is 4 m by s ? So Re_D will be nothing but UD by ν_g which will be $4 \text{ into } 4 \text{ into } 10^{-6}$ to the power minus 6, this is $4 \text{ micrometers by } 24.15 \text{ into } 10^{-6}$ to the power minus 6 and this value is 0.667 Reynolds number which is very small.

This information has to be used in determining the nusselt number and the heat transfer coefficient. If you remember nusselt number based on diameter is h into D by k_g and Nu_D is equal to C_1 plus C_2 square root of 0.667 this is Reynolds number. So if I use this, this gives you 0.596 and I have to get the heat transfer coefficient 0.596 into 0.032 is the gas conductivity divided by diameters $4 \text{ into } 10^{-6}$ to the power minus 6. The heat transfer coefficient turns out to be $4768 \text{ watts by sq m Kelvin}$, the heat transfer coefficient turns out to be 4768 .

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Handwritten calculations on a blackboard:

$$\text{Mean temp of air} = T_w + T_\infty = 110^\circ\text{C}$$

$$\text{Air tables: } \nu_g = 24.15 \times 10^{-6} \text{ m}^2/\text{s}$$

$$U = 4 \frac{\text{m}}{\text{s}} \quad k_g = 0.032 \text{ W/m-K}$$

$$Pr = 0.71$$

$$Re_D = \frac{UD}{\nu_g} = \frac{(4)(4 \times 10^{-6})}{(24.15 \times 10^{-6})} = 0.667$$

$$Nu_D = \frac{hD}{k_g} = 0.3 + \frac{0.62 \sqrt{0.667}}{1 + \frac{0.62 \sqrt{0.667}}{4 \times 10^{-1}}} \sqrt{0.667} = 0.596$$

$$h = \frac{0.596 \times 0.032}{4 \times 10^{-3}} = 4.768 \frac{\text{W}}{\text{m}^2\text{K}}$$

Now that I know all the quantities namely the heat transfer coefficient, the diameter, the length and the temperature which is also given, I can find out the heat transferred from the wire when it is exposed to a stream at 4 m by s. You have $\pi D L h$ into T_w minus T_g or T_{infinity} . This will be π into 4 into 10 to the power minus 6 into 1.2 into 10 to the power minus 3 into 180, this will be 0.013 watt. So 13 milliwatt is what I am going to be dissipating at 4m by s.

When the velocity of the fluid is 4m by s I am going to have about 13 milliwatts of heat which is being rejected and this must be equal to the electrical input. Therefore I can immediately find out the potential difference that is required. So I can also calculate the corresponding current and in fact I will directly calculate the output E will be nothing but the Q by R_w . Q is nothing but the electrical dissipation divided by R_w and you can just substitute the values and this will give you a value of 0.47 volts. So the voltage output is given by 0.47 volts.

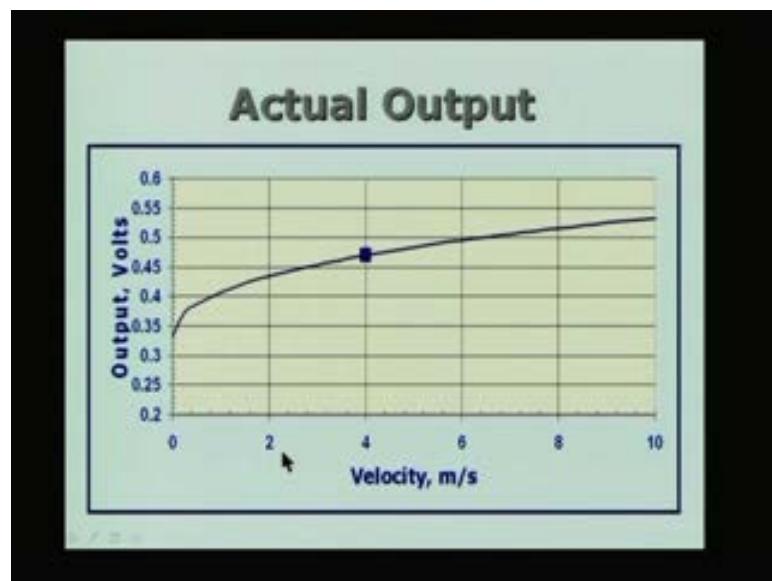
If I vary the velocity of the liquid from the lowest possible to 10 m by s the range given in the problem I can in fact calculate the value of E for each velocity and make a sketch. It has been done here.

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$$Q = \pi D L h (T_w - T_\infty) = (\pi)(4 \times 10^{-3})(1.2 \times 10^3) = 0.013 \text{ W} \quad \times 180 \quad \underline{13 \text{ mW}}$$
$$E = \frac{Q}{R_w} = \boxed{0.47 \text{ V}}$$

You can see that the output volts have been plotted on the y axis and on the x axis, I have got the velocity in m by s. You can see that for 4 m by s the value is 0.47 and if I vary the velocity from 0 to 10 m by s, I get a relationship which is actually a non-linear relationship. And normally what we would like to do is, if it is possible to make it a linear relationship it will be good and that can be done by the following method:

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How do you linearize?

If you remember we had E^2 is equal to $A + BU^n$ and A is nothing but E_0^2 when U is equal to 0, A will be simply nothing but the output which is going to be E_0^2 so we can say that E^2 minus E_0^2 is equal to BU^n . I will divide throughout by E_0^2 . Therefore I will get the following, so E^2 by E_0^2 minus 1 is equal to B by E_0^2 U^n , so I will just call it as $K U^n$ because B by E_0^2 is another constant.

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The image shows a digital blackboard with handwritten mathematical derivations. The first equation calculates heat transfer Q using the formula $Q = \pi D L h (T_w - T_\infty)$ with values $(\pi)(4 \times 10^{-3})(1.2 \times 10^3)$, resulting in 0.013 W and 13 mW . The second equation calculates voltage $E = \frac{Q}{R_w} = 0.47 \text{ V}$. The third part, labeled 'Linearise', shows the equation $E^2 = A + BU^n$ being rearranged to $E^2 - E_0^2 = BU^n$ by dividing both sides by E_0^2 .

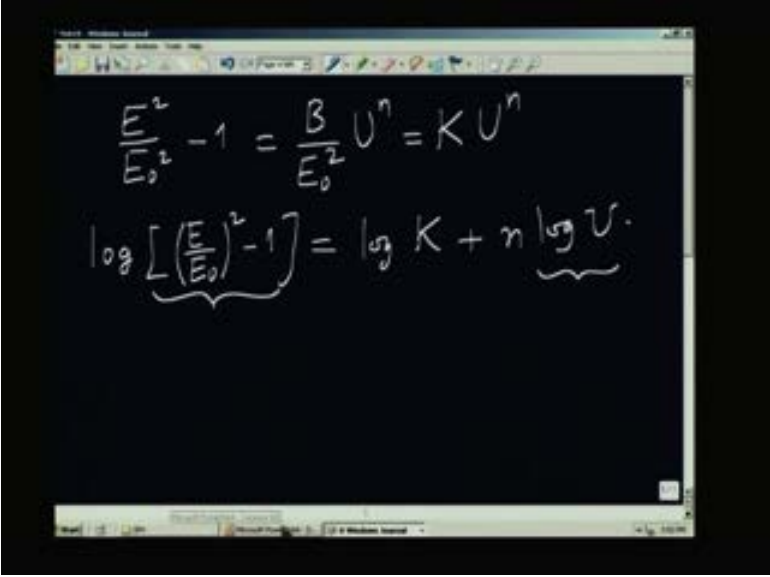
$$Q = \pi D L h (T_w - T_\infty) = (\pi)(4 \times 10^{-3})(1.2 \times 10^3) = 0.013 \text{ W} \quad 13 \text{ mW}$$
$$E = \frac{Q}{R_w} = 0.47 \text{ V}$$

Linearise: $E^2 = A + BU^n$

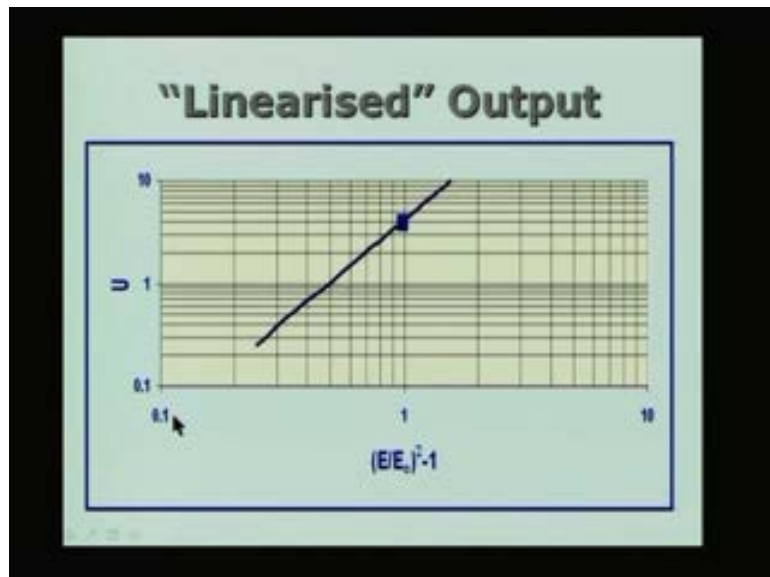
$$E^2 - E_0^2 = BU^n$$

So, if I take the logarithms of this, \log of $(E \text{ by } E_0)$ whole square minus 1 will be logarithm of K some constant plus $n \log U$. So, if I plot the logarithm of this the quantity works as the logarithm of U . I will get a straight line and that is in fact what is done on the slide. Here is how the linearization can be done.

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$$\frac{E^2}{E_0^2} - 1 = \frac{B}{E_0^2} U^n = K U^n$$
$$\log \left[\left(\frac{E}{E_0} \right)^2 - 1 \right] = \log K + n \log U$$

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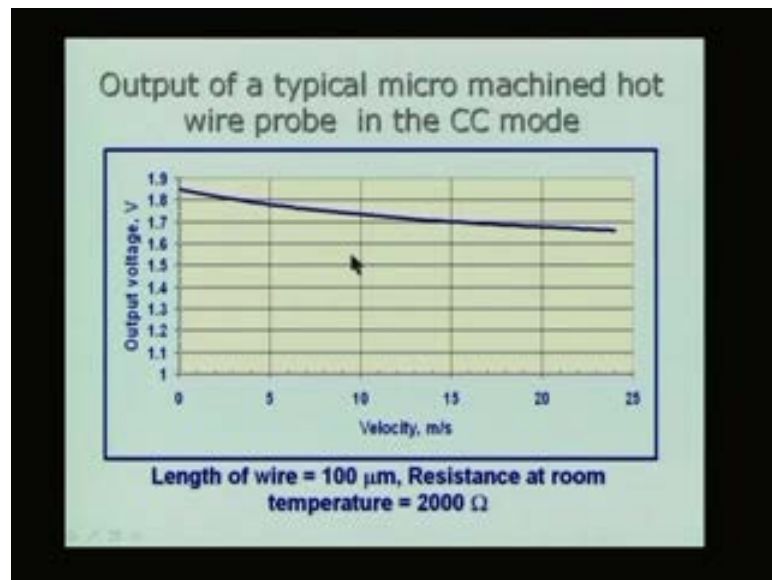


Now, instead of plotting x axis I have put the velocity on the y axis. And if you see here I have plotted the logarithm, I have used the logarithm scale on the x axis and the y axis. Therefore the value of velocity goes from, of course I cannot go to 0 in the logarithmic scale, 0 will become minus infinity, it is not possible to show it here. Therefore I am showing it from some value close enough to 0.

You can see that the relationship is actually linear. The advantage of this is that you can simply read from here. Of course it is linearized but it is not really a linear relationship. It is linearized on the log graph. In fact you will be able to notice that the slope of this curve is actually related to n . In this case of course n is equal to $1/2$ if it is not equal to $1/2$ by making a plot like this one must be able to find out the value of n in this case.

If we make a plot of CC mode operation for a typical case the length of wire is 100 micrometer a very short wire and resistance room temperature is 2000 ohms a very thin wire, and if you plot the variation of output voltage V as a function of velocity that will be maximum at 0 velocity because it is at 0 velocity the resistance is high, and the current is constant. Therefore it is going to give you a large output and as the velocity increases the temperature of the probe is going to come down, it is going to reduce. Therefore resistance is going to reduce and what I get is a reduced output and the output is again non-linear. This is given by $1 - n$ if you remember AU power n by $1 + BU$ power n .

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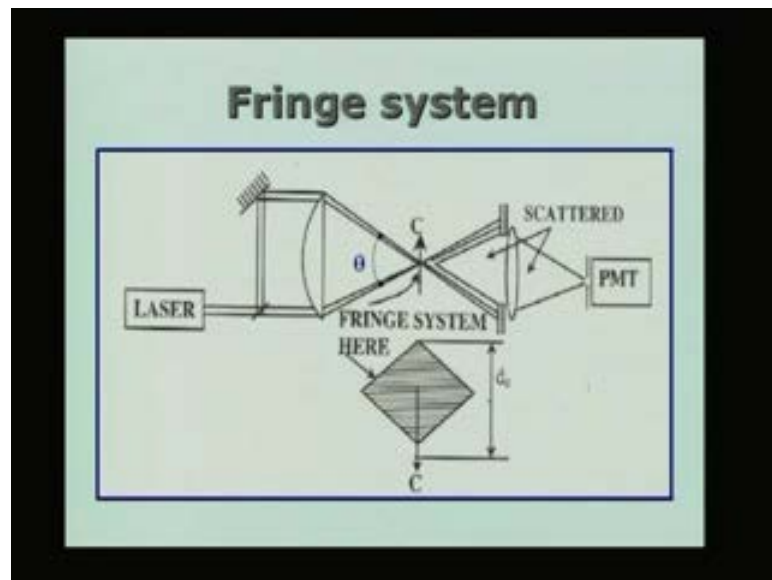
The hot wire anemometer can be used for determining the velocities and also the fluctuating velocities in a turbulent flow field, the direction and so on. Let us look at the use of Laser Doppler Velocimetry or the measurement of velocity using lasers. Here is a system used for Laser Doppler measurement. Let us look at the basic operating principle. So what I have here is what a

fringe system is.

Again there are two ways of using the Laser Doppler Anemometer. In a Doppler Velocimeter there are two different ways of arrangement. One is called the fringe system and the other one is called the reference beam system. So, here we have a laser source and a beam splitter. What is a beam splitter? From a single laser beam it produces two beams one moving in this direction and the other one reflected half and this is going to hit a mirror and it is going to be made into a beam parallel to the old beam but slightly away from the beam which started from a given laser beam.

I am creating two laser beams one here, and the other one here. The idea is to split it into two beams and these two beams are going to be condensed by a lens to a spot at this point. So these two beams are going to make a cross at this point. There are two beams; one here and the other one here. These two beams are from the same source. Originally they come from the same laser source. Therefore they have a coherent relationship between each other.

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So the phase relationship between the laser beam 1 and laser beam 2 is such that they start with the same phase. Therefore these two beams are going to be in the cross. When they cross at the spot here it can be brought to very small spot. By focusing at this spot it is going to form a set of fringes and the fringes have been schematically shown here in the form of a diamond

shaped region. It is a very small narrow region.

A large number of fringes are being formed because of the different path lengths of each way of light which is here and here because of that and they are coming at an angle θ with respect to each other. And because of the phase relationship, they form a set of dark and bright fringes. What I am going to do is, I am going to have some kind of a particle in the flow. I am going to introduce some particle into the flow so that when it moves through this fringe field it is going to scatter the radiation and the scattered radiation whenever it is in the brighter part of the fringe field it will give you a bright image and when it moves into the shadow or the dark part of the fringe it will give you no radiation. So I must have particles which are smaller than the spacing between the fringes because, if they are big particles they will not be able to do anything as they will just go through and some part of the particle or the others will be reflecting light all the time or scattering the light and therefore it is useless. So the particles must be really small so that they are smaller than the fringe spacing.

We have a very small diameter particles moving across this fringe field. and if these particles are present in the flowing medium and if you assume that these particles are so small that they are moving almost with the same velocity as the velocity of the gas in which it is going to be present, then the pattern of reflection when it passes through this contains the information about the velocity of the particle C . It is the particle velocity, and we are assuming that the particle velocity is same as the gas velocity. Therefore the velocity of the gas is inferred from the pattern of reflected light or the light scattered by the particles which cause the fringe field.

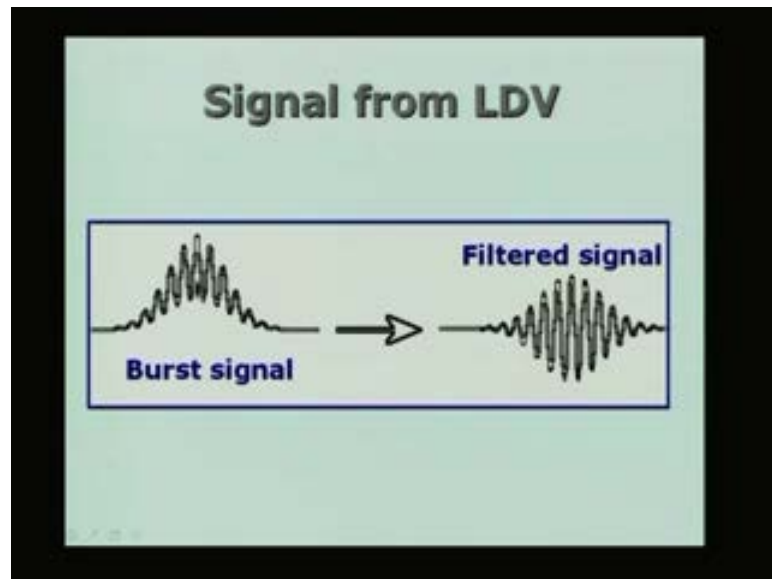
You can imagine the fringe field to be something like a set of fence post. You have a parallel post and suppose a person is walking behind the fence you will see him sometimes and sometimes he may not be visible. There is a periodicity to the image you obtain. So, if he walks very fast, this periodicity will become very small, period will become small and if he walks very slowly it will become larger. Therefore the variation frequency with which this scattered image varies depends on the velocity. It is directly related to the velocity given that the fringe spacing remains constant.

Like in the case of fences the fence posts are at constant intervals. So the principle is very simple. You just collect the light which is scattered, you can see here that I have scattered the light alone, I am collecting the main beam,

the two beams which are going to cause are now stopped here they are not going to fall on to the detector. The photo multiplier tube here is going to give you a burst signal.

The signal is going to be like this. When the particle moves across the fringe field it is going to give you a burst signal like this. And when there are no particles there is no signal. And if I filter this signal I will get a wave form like this and for this wave form I have to find out the frequency contained in it and that frequency is related to the velocity of the particle which is moving through the fringe field.

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So, if I look at the formula, I can write it in the following way: λ is the wave length of the laser radiation, the frequency obtained from the burst signal is f , and the velocity V is the velocity component normal to the fringes. You can see here that the fringe field is shown as a set of parallel lines and the particle is moving at an angle, only this component is going to be important because you will not get information about the velocity in this direction.


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Formula

λ = wavelength of the laser radiation
 f = burst signal frequency
 v = velocity component normal to the fringes

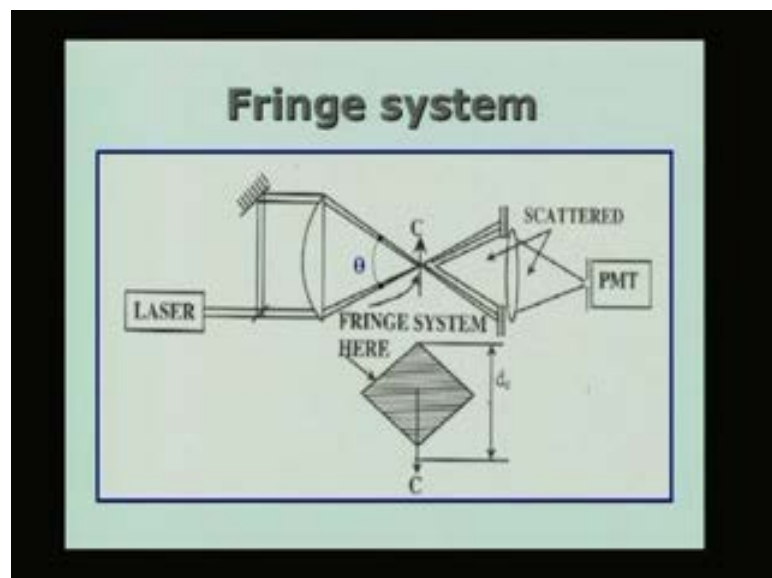
$$v = \frac{\lambda f}{2 \sin\left(\frac{\theta}{2}\right)}$$

Fringes



You will get the component in this direction and you can see that the velocity is nothing but the frequency of the burst signal into the distance between the fringes and the distance between the fringes can be obtained from the theory of optical theory related to interference of light given by λ by $2 \sin \theta$ by 2.

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Theta is the full angle and $\theta/2$ is actually the half angle the angle between these two half angle and the fringe spacing is given by $\lambda/2 \sin(\theta/2)$ and this is the velocity. So the velocity can be inferred by measuring the frequency of the burst signal so the burst signal frequency has to be determined by using electronic signal processing so that you can get the velocity of the particle which is moving across the fringe field. This is one way of using the Laser Doppler Velocimeter that is to use the fringe. In this case, we have used what is called the fringe system by crossing two beams analyzing them to interfere and form a set of fringes. The subsequent one the second one uses the reference beam technique. Thank you.