Mechanical Measurements and Metrology
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Module -2
Lecture - 27
Measurement of Fluid Velocity

We have been discussing the measurement of velocity of either gases or liquids and we already looked at the pitot tube and pitot static tube. We have also seen the principle behind the operation of these. Here is a simple example of pitot static velocity measurement in incompressible flow. The example 28 will indicate how a pitot static tube can be used for measurement of velocity and then it will also show the effect of compressibility. Compressibility is the effect of change in the density of the medium which will affect the performance of the pitot static tube. In other words the formula we developed earlier have to be modified to suite when the compressibility effect becomes important. As a sequel to this I will also be looking at impact probes for high speed flows. When the speed of the medium is more than the speed of sound in the medium we call it high speed flow.

In fact it could be also very high speed. That means the ratio of the speed of the medium to the speed of sound in the medium is more than one, we call it as supersonic flow and if it is very much large compared to one it will be called hypersonic, supersonic and then subsequently hypersonic flow. These are flows which occur in aeronautics and in mechanical engineering in general. Earlier we discussed the operation of the pitot tube and we mentioned that it is made of based on fluid flow principles. Basically the fluid flow principles involve the equations of fluid mechanics, equation of continuity and equation of Momentum.

Based on the fluid flow principles in lecture 26, we presented the basic relationship between the velocity and the pressure difference measured by the pitot static tube. if you recall the pressure at the entry to the pitot static tube is higher than the static pressure and the difference between the two is measured in terms of a column of length h_m, h_m is the manometric column h_m, rho_m is the density of the manometric fluid, $rho_{infinity}$ in this case is the density of the medium in which the measurement is being made and g is the acceleration due to gravity. We said that the velocity which is inferred from the pitot static measurement equal to a constant C which is to take into account the effect of viscosity and so on.

So we have concept C which is very close to the unity and in most cases we need not worry about it multiplied by square root of the density of the manometric liquid multiplied by g multiplied by h_m this gives you the delta p rho_m gh_m is the delta p in Pascals or kilo Pascals. In other words, SI units divided by the density with medium in which the measurement is being made the square root of that is going to give you the velocity. We assume that the density of the medium fluid is constant. Let us see when this assumption is valid.

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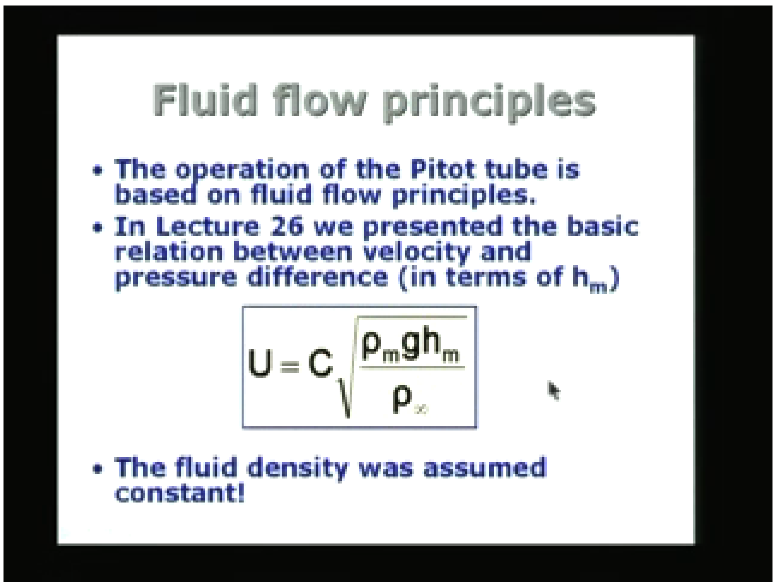

Assumption of constant density is valid strictly in the case of liquid flows. So we say that incompressible flow assumption is valid for most liquid flows. Of course when the temperature does not vary too much within the flow in medium then we can say incompressible or constant density assumption is alright. Also in the case of gases incompressible flow assumption is valid if the flow velocity U, is much smaller than the speed of sound a.

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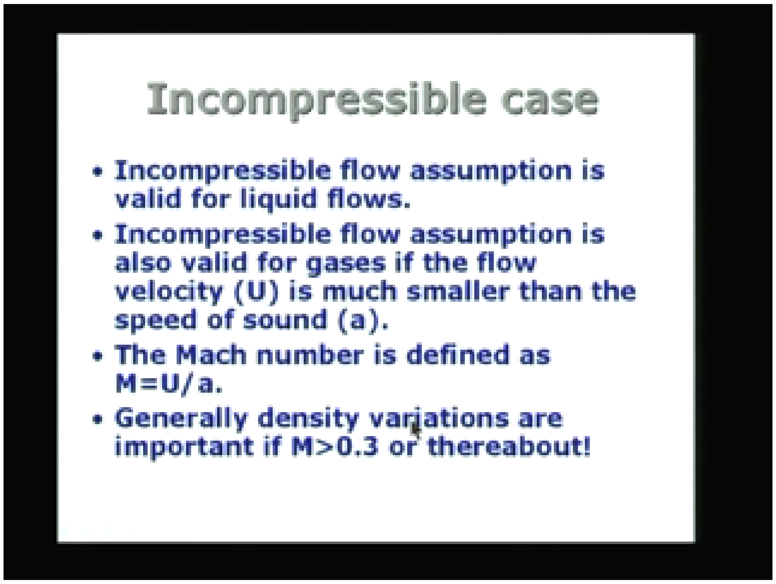

Density assumption is alright. Also in the case of gases incompressible flow assumption is valid if the flow velocity U is much smaller than the speed of sound a. Here a is the speed of sound, U is the speed of the medium, if the value of U is very small compared to a, the ratio M equal to U by a where M is the Mach number will be small. If the Mach number is small then we can say that the incompressible flow assumption is also valid in the case of gases. In practice let us see what kind of flows we have. In the case of liquid flows liquids may be flowing with a few meters per second velocity in most applications and the speed of sound in a liquid is really very high.

It is in fact given by the square root of the bulk modulus of the medium divided by the density, and typically for medium like water the speed of sound will be more than a kilometer per second. And the speed with which the fluid is going to flow in tubes or in ducts which are the kind of situations we meet in practice the velocity may be a few meters per second and therefore the ratio of velocity of the medium to the velocity of sound in a medium is very small. Therefore the liquid flows most of the time we do not even talk about a high speed flows but we only talk about incompressible flow in liquids.

In the case of gases however the speeds can be very high. In aerospace applications for example velocity of the medium related to the velocity of an aircraft could be less than equal to or more than the speed of the sound in the medium and therefore we do have such situations where the Mach number can be large enough so that we cannot make the constant density or incompressible flow assumption cannot be valid.

Generally the density variations are expected to be important and the Mach number is roughly is little more than about 0.3. So the formula which we derived is strictly valid for very low velocity or very small Mach number if you are going to use the formula in the case of gases. With this background let us look at a typical example where a pitot static tube is being used to measure the velocity of the aircraft.

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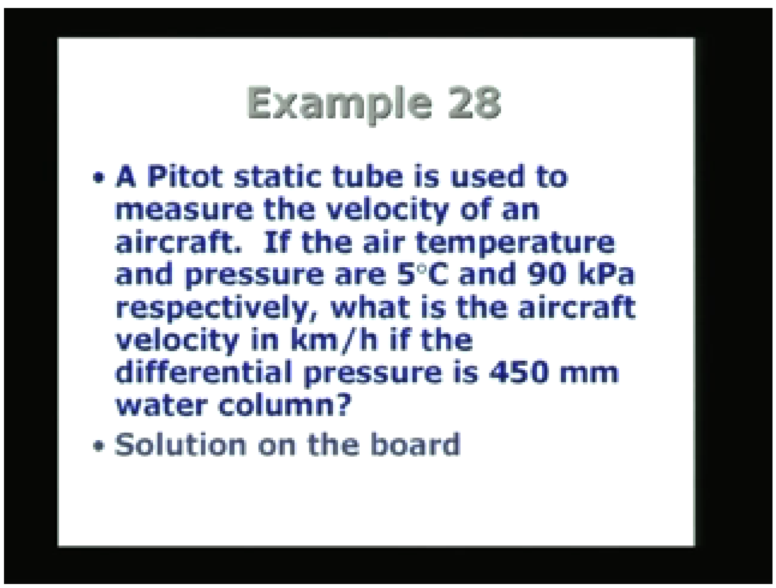

The air steam is at a temperature of five degrees and we have a pressure of 90kPascal's as measured by an instrument carried on the aircraft. And we want to know the aircraft velocity in kilometer per hour if the differential pressure developed by a pitot static tube is 450 mm of water column. We will workout the example and then look at whether the incompressible flow assumption is valid or not and what kind of corrections we have to use and so on. The pressure of the ambient I will call it as p_s as a static pressure as 90 kilo Pascals, 90 into 10 to the power 3 Pascal's. The temperature of the air through which the aircraft is moving is given to be at 5 degree Celsius and I will convert it to Kelvin by adding 273 to that so that it becomes 278 Kelvin.

The pressure difference which is measured by the pitot static tube I will put it here where delta p is 450 mm of water as given and of course I cannot use the pressure in millimeters of water but I have to convert into SI units. So let us assume that the manomertric fluid has a density of 1000 Kg by cubic meter. We can calculate the delta p in SI units by taking it as rho_m g, actually this delta p which is given is h_m so this will be 1000 into 9.81 into 450 mm will be point 45m of water column. So this will be in Pascal's and if we calculate the value comes to 4413 Pascals. That is the pressure developed by the pitot static tube.

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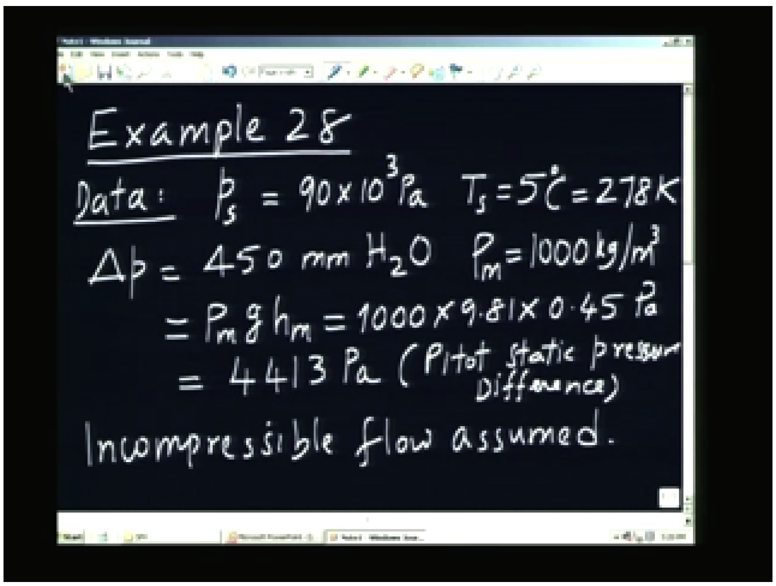

This is the pitot static pressure difference. Now I will calculate flow assumption assuming that the flow is incompressible. Of course we do not know whether it is a good approximation or good assumption but we will make the assumption first and then calculate and find out and then try to recalculate if necessary. So, if you do that let us calculate density $rho_{infinity}$, or if you want you can call it as rho_s.

I will be using the equation of state p_s equal to rhoRT so rho will be p_s by R_gT_s and p_s is 90k Pascals which is 90000 Pascals by R_g is 287 into 278 which will give you the density as 1 point 128 Kg by m power 3. So we have the density now and all I am going to do now is to use the earlier formula. So the velocity of the aircraft under the incompressible flow assumption V is incompressible and I am also going to assume that the factor C equal to unity is very close to 1. Therefore I am going to take it to 1, this will be given by 2 into delta p by $rho_{infinity}$ or rho_s in this case.

All I have to do is plug in the values and the problem says that I must calculate the value the velocity in kms per hour. Here this will give you the quantity in meters per second. So I have to find out what the velocity is in kilometers per hour so we will just make simple calculation meter per second 1 meter is 1 by 1000 kms and 1s is 1 by 3600 Hr and this will go after that and this will give you 1m by s which is the same as 3.6 kms per hour and that is the conversion. So I will just multiply this by 3.6 to get the speed in kms per hour. So, just to indicate the conversion between m by s is kms per hour so it is simply a factor 3 point 6. If I plug in the values this will be 3.6 into 2 into 4413 by 1.128 which comes to 318 point 445 kms per hour. So the speed of the aircraft when the pitot static tube is showing a delta p of 450 mm of water equal to 318 point 445 kms per hour.
Now I want to verify whether the incompressible flow assumption is in deed valid or not. For that I have to calculate the speed of sound in air.

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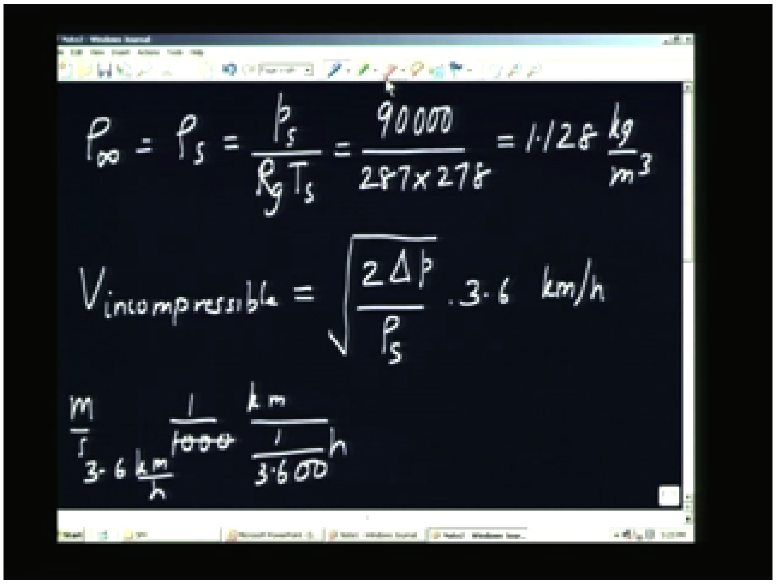

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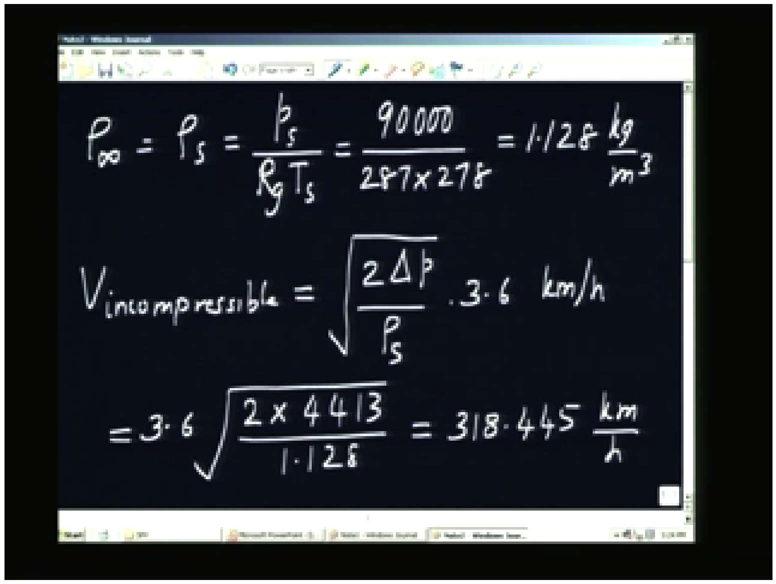

We assume that air is an ideal gas for the present calculation is adequate to you. This assumption with gamma equal to 1 point 4, gamma is nothing but the ratio of the specific heat equal to 1 point 4 for air. And we also know from our thermodynamics that the speed of sound a equal to root of gamma ($R_g T_s$).

You can also write it as gamma p by rho if you want which is one and the same. And in order to calculate again in kms per hour, the speed of sound also if I would like to calculate in kilometers per hour then I will multiply by 3.6 so that consistently everything comes out in the units of kms per hour. So I will take it as 3.6 by square root of 1 point 4 into 287 is the value of R_g, T_s is 278 and this comes out to 1203 kms per hour. So, if I calculate the Mach number the Mach number of the flow assuming that the velocity has been determined by using the incompressible flow assumption so this Mach number is not necessarily the correct Mach number.

The Mach number is based on the incompressible flow assumption for the determination of velocity. Let us call this as U_i by a and the values are 318 point 445 by 1203 and this turns out to be point 265. So the value of the Mach number for the aircraft is point 265. Any thing less than one is called the subsonic so, point 265 of course is subsonic. Sonic means Mach number equal to 1. But this is already .265 but not small because it is close to something like point 3 it does not mean that exactly at point 3 the compressibility is going to come into picture, compressibility will become important gradually.

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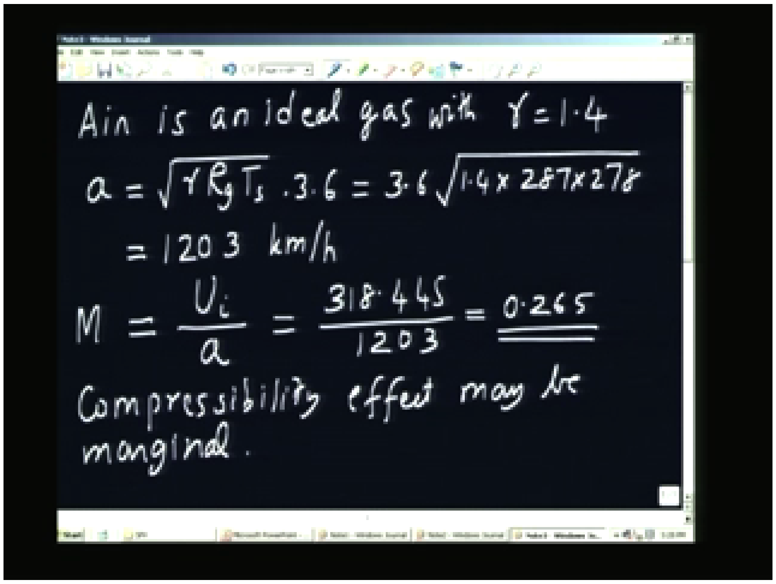

Therefore possibly even at this value of Mach number the compressibility effect may be important. But, we can safely say that, compressibility effect may be marginal. So what I will do is, I will first look at this compressibility calculation or we will give formulae for calculating the velocity using compressible formulae. Therefore the Mach number in example 28 is such that the compressibility effect may be important. So the question now is how we are going to introduce this compressibility effect.

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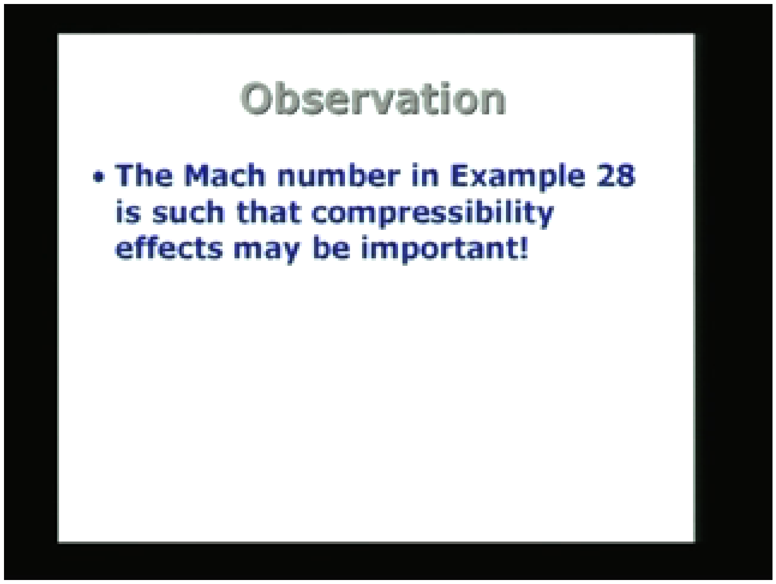

For that let us look at the way we are going to do that. There is a difference between the compressible case and incompressible case. In the case of incompressible flow if you apply the Bernoulli equation as we did the pressure difference you get is going to be related to the velocity. So you see that we were measuring the difference in pressure between the stagnation pressure or the pressure indicated by the pitot tube minus the static pressure indicated by the tap at the side. So in the pitot static tube there is a hole in the very front which is going to sense the stagnation pressure and the hole which is on the side of the tube is going to sense the static pressure. So we were measuring the delta p. However, if I look at the compressible flow case I have to look at the ratio of the pressure.

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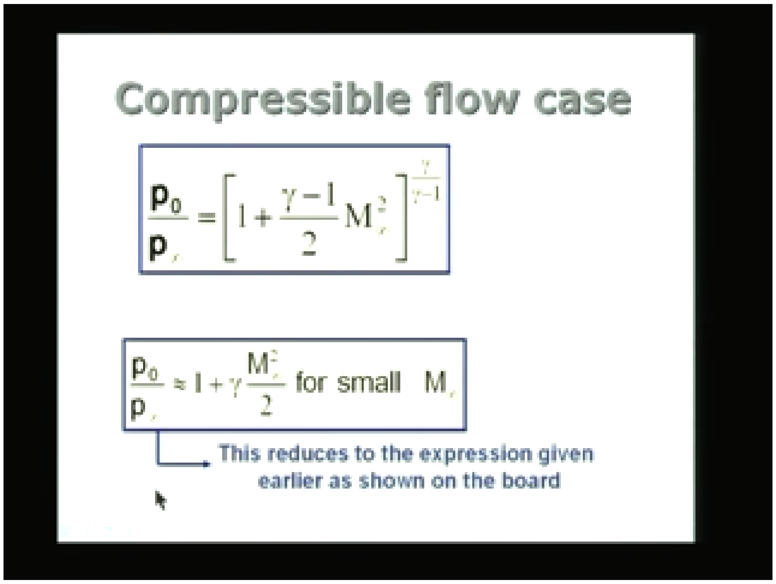

This is the piece of 0 called the stagnation pressure. We always use the 0 as the symbol subscript for stagnation pressure and $p_{infinity}$ is the other static pressure. What does it mean? It means that when the velocity of fluid is brought to 0, how are you going to bring it to 0? If it is brought to 0 by an isentropic process then the pressure equal to the stagnation pressure indicated as p subscript 0.

The ratio of this to the $p_{infinity}$ is given by the formula (1 plus gamma minus 1 by 2 into $M_{infinity}$ square) to the power gamma by gamma minus 1. This expression can actually be derived from gas dynamic principles. The formula is directly given as p_0 by $p_{infinity}$ equal to (1 plus gamma minus ½ into $M_{infinity}$ square) to the power gamma by gamma equal to 1 where gamma is the ratio of specific heats, M is the Mach number and the stagnation pressure to the static pressure ratio is given by this formula. In fact you can do the following:

Suppose the Mach number is very small, that means if this factor gamma minus 1 by 2 into $M_{infinity}$ square is small compared to 1 then this binomial can be approximated by two terms so 1 plus this gamma minus ½ into $M_{infinity}$ square into gamma by gamma minus 1 plus etc. So I am just going to use only two terms in that and if you do that you will get p_0 by p infinity equal to 1 plus gamma $M_{infinity}$ square by 2 for m. If this term is small compared to unity then I can approximate p_0 by $p_{infinity}$ by a two term approximation like this. Actually this expression is this expression when this term is much smaller than 1. Therefore p_0 by $p_{infinity}$ equal to 1 plus gamma $M_{infinity}$ square by two.

So, if you take this 1 to the other side p_0 by $p_{infinity}$ minus 1 equal to 1 plus gamma minus 1 by 2 $M_{infinity}$ power 2 gamma by gamma minus 1 I said if this term is very small compared to 1 I can approximate it as 1 plus gamma $M_{infinity}$ to the power 2 by 2 by taking only two terms in the expansion. Now, if you see p_0 is nothing but $p_{infinity}$ plus some delta p that we are measuring. Therefore I can say p_0 by $p_{infinity}$ minus 1 I am taking one to the other side, this will become p_0 minus $p_{infinity}$ by $p_{infinity}$ and this will be nothing but delta p by p infinity. Therefore I can see that delta p equal to gamma $p_{infinity}$ by 2 into $M_{infinity}$ square which is the square of the Mach number. $M_{infinity}$ is U by $a_{infinity}$. The symbol infinity here is used instead of s in the previous example, $a_{infinity}$ is supposed to represent the approaching stream.

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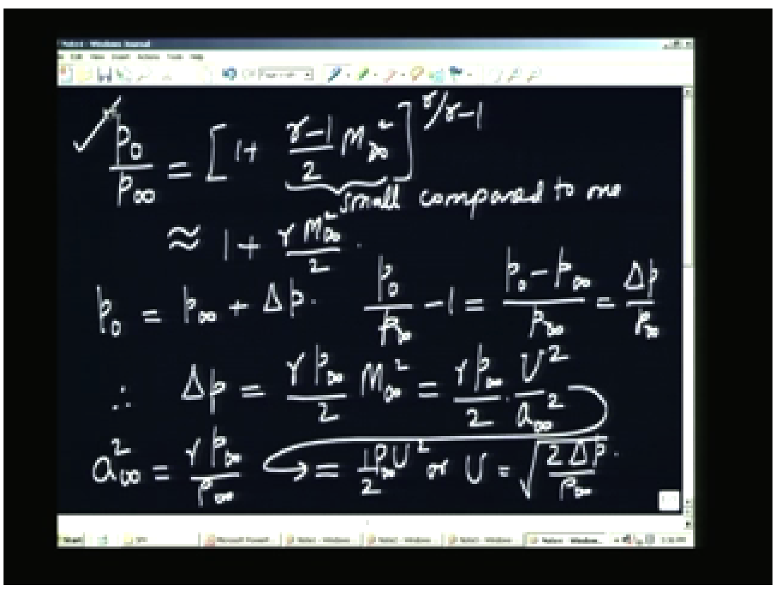

This is the square of this U square by a infinity square. This $a_{infinity}$ square is given by (root of gamma p by rho) whole square or gamma $p_{infinity}$ by $rho_{infinity}$ this is nothing but the square of speed of the sound equal to gamma R_gT infinity or gamma $p_{infinity}$ by $rho_{infinity}$ by using the equation of state I get that. Therefore if I substitute this gamma $p_{infinity}$ by $rho_{infinity}$ in this expression I will go from here to here so I am substituting $a_{infinity}$ square in the denominator, this gamma $p_{infinity}$ will cancel this gamma $p_{infinity}$ therefore

I will get half of U square, gamma is going to go off, $rho_{infinity}$ is going to come here. So you see that 1 by 2 of $rho_{infinity}$ U square equal to delta p or U equal to square root 2 delta p by $p_{infinity}$. What is this formula we got? It is nothing but incompressible formula we had earlier.

Therefore you take this expression which is valid for high speed flow or large values of Mach number compared to point 3 or whatever we talked about. When this term is small it in fact goes to the incompressible formula. So the compressible effect is taken care by this expression. In the incompressible limit it is going to give the value which we had earlier. By looking at the earlier example, the velocity compressible that means when I taken into account the compressible effects I can write in the following form.

It is just the same expression I am writing, I am going to write $M_{infinity}$, $M_{infinity}$ is going to be replaced by U by $a_{infinity}$ and then I will simplify it and what I get is the expression (square root of 2 gamma p by rho) into p_0 by $p_{infinity}$ will become (1 plus delta p) to the power gamma minus 1 divided by gamma) minus 1 into 3.6 to convert the velocity to kms per hour. So all I have to do is to substitute the value gamma is 1 point 4, $p_{infinity}$ is 90kPascals, rho is 1 point 184 and so on. This will come to 315 point 72 kms per hour so you compare with V_i which had a value 318 point 445 so this is the value you get by using the compressible flow formula. Actually there is no particular reason why you should use the approximate formula. You could use this any time the amount of calculation required is very small but when you are measuring delta p you require other quantities also. You have to calculate a whole lot, quantities have to be calculated. The calculation is slightly more involved.

You know all the values just plug it in there and get the value. So the actual velocity assuming that the correct formula is used is 315.72 so you see that you are making 318 point 4 you can see that you are making a small error. So we can calculate the error, this is 72, this is 318 point 4 about 2 point 7 kms per hour is the error. Actually the error is positive. If I use this value I am getting a value of velocity 2 point 7 kms per hour more than the actual velocity of the aircraft. Of course in the case of an aircraft this velocity error may be important because you are assuming that you are traveling faster than you actually are and you may make a mistake. So, in the case of the aircraft this may be an important thing. But if you are doing some laboratory experiments and so on it may still not be a very substantial error.

Actually I can calculate the Mach number based on this velocity. So I will call it as M_c that will be 315 point 72 by 1203 which was the velocity of the sound and this comes to point 262. And if you compare with incompressible one it is .265 so there is a small error in the Mach number. In a typical application we talked about it may be important to consider this. So with this background, now let us look at the use of pitot static tube or equivalent of that in the case of high speed flow. Let us look at what is high speed flows are.

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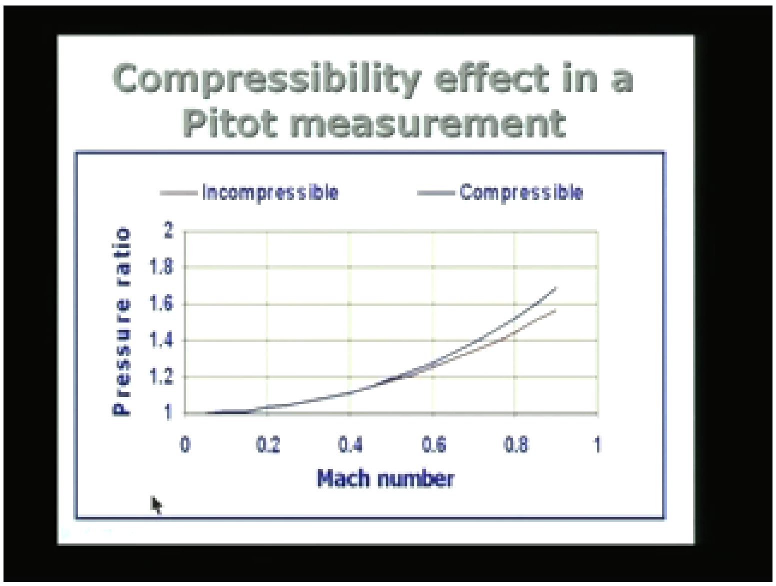

Here is a plot to show the difference between incompressible and compressible assumptions and I have plotted the two results incompressible and compressible as a function of Mach number and you see that the pressure ratio shows the departure between the two starting about somewhere here. It will start branching out somewhere here and you can see that there is a departure from here. The compressible flow case is here and incompressible case is at the bottom.

Another way of plotting that is to amplify the difference between the two. I take the percentage change between the two values and I place the values here. The percentage change is given by this, this difference between the two. So you see that about point 3 it is just taking half it is a small error and it could be as much as about 8% or more than 8% about 10% as we reach a value of Mach number equal to 1. In fact, most modern aircraft fly around .8

or more Mach number. It may be the compressibility effect around that is important. Let us look at the use of an impact probe to measure the velocity of high speed steam of gas.

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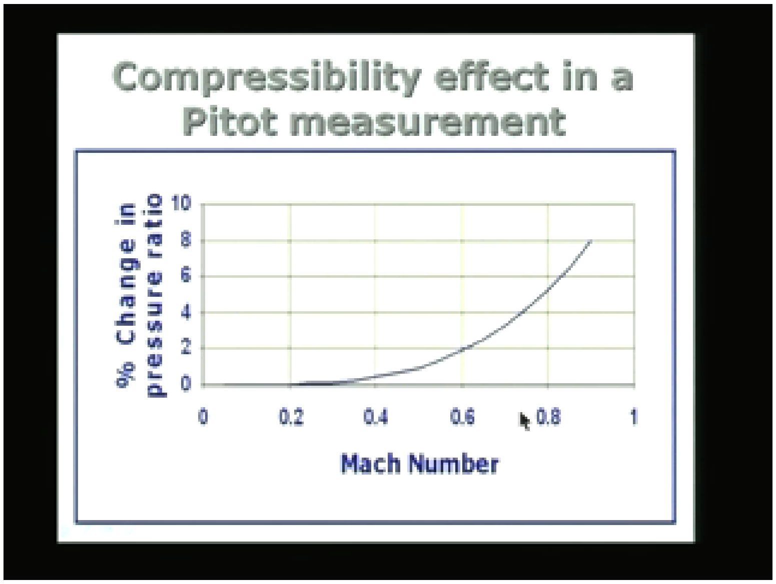

The impact flow is nothing but the pitot static tube. This tube which is shown here has got a hole at the front here and the difference between the subsonic or the low speed case and the high speed case is the change in the structure of the flow in the vicinity of the impact flow. let us look at the steam coming in at a Mach number of M_1 which is greater than 1 because I am talking about supersonic flow now and the pressure is p_s is a static pressure of the stream of air mostly we are going to look at air and this is at pressure p_s and Mach number greater than 1.

And if you recall, at the stagnation point the velocity is actually equal to 0, at this point velocity has to become 0. That means that the Mach number which is greater than 1 has to become Mach number less than 1 and then it continuously reduces from there onwards to the 0 value at the stagnation point. Therefore there are two things: One is Mach number which is supersonic which has to become subsonic M_2 less than 1 and then it has to come to the stagnation here at the stagnation point right in the front in the impact probe.

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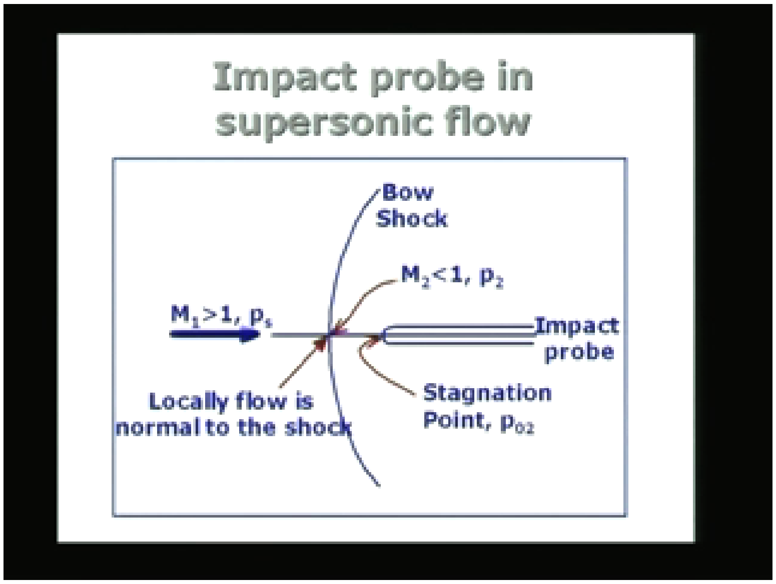

Let us look at some applications from the measurement of high speed. It is observed from the appropriate gas dynamic literature that, in front of the impact probe somewhere ahead of it there is a bow shock a curved shock which is going to be present. In the region ahead of the shock the flow is largely isentropic. The flow subsequently after the shock is also isentropic. But across the shock there is a non isentropic change in the Mach number from a value higher than 1 to a value less than 1. So there is a change from supersonic flow to subsonic flow across the shock and of course the pressure also changes from the value of p_s to p_2. So, if I look at the shock which is shown by this curved line actually it is a surface. If you look at the region very close to this point shown here it appears as though it is normal to the stream which is coming here. So at this location very close to the axis of the impact flow the stream is actually normal to the shock.

So we can use what are called normal shock relations and relate the values of Mach number, pressure, temperature, density, etc upstream of the shock to the values downstream of the shock by using normal shock theory and I will able to actually calculate the values of M_2 given the value of M_1, that is the first calculation. Or I can calculate the p_2 given the value of p_s, if I know the p_s, I know M_1 I will be able to calculate the M_2 and p_2. Now between the point downstream of the shock and the stagnation point I am going to use the isentropic stagnation process which was actually the formula given earlier. Therefore I am going to relate the conditions here to the conditions here through a combination of a shock process and an isentropic process subsequently.

So the sketch clearly indicates that at the stagnation point the pressure p_2 to the upstream of the shock the pressure is p_s, Mach number is M_1. In fact I am not interested in M_2 or even p_2 but I am interested only in finding out whether there is a relationship between the Mach number here and the pressure p_{o2} and the p_s. For achieving that, I have to measure of course the static pressure here, I have to measure the stagnation pressure here p_{02} which is indicated by the impact probe and I should be able to find out the appropriate expression which gives the value.

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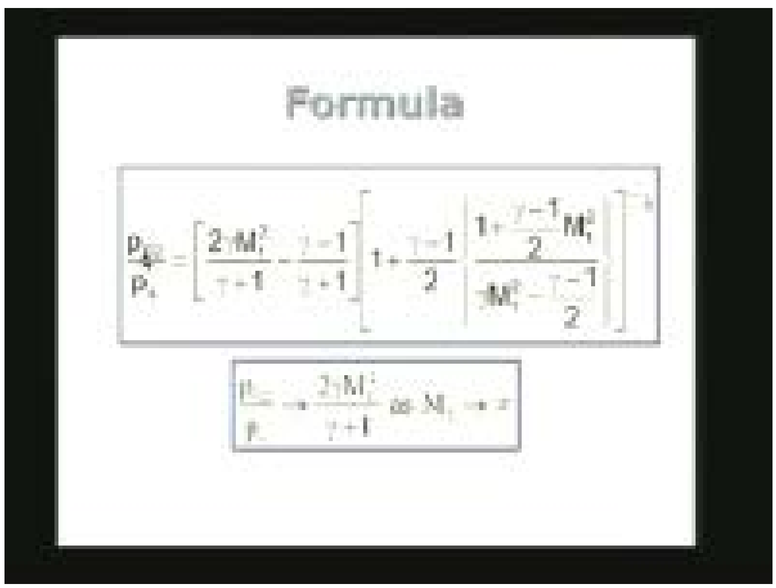

From gas dynamics we can show whether ratio of stagnation pressure p_{02} to the static pressure upstream of the shock or in the stream is coming in at velocities such that the Mach number is M_1 the ratio of this pressure is given by this expression. So it consists of two parts; this is one part and this is second part and in fact this second part is actually due to the isentropic process of stagnation from M_2 to the value 0 at the stagnation point and this is due to the normal shock value.

It is 2 gamma M_1 square by gamma plus 1 minus gamma minus 1 by gamma plus 1 this is first factor and the second factor is (1 plus gamma minus 1 by 2 into M_2 square) to the power gamma by gamma minus 1 where M_2 square is this value so I have a simple combination of these two. And in fact you can see that if M_1 becomes very large all these factors become unimportant and this is only factor which is going to survive and I will get p_{02} by p_s equal to 2gamma M_1 square by gamma plus 1 as M_1 turns to infinity. This is the limiting value. Let me look at the pressure ratio but here I have plotted the pressure ratio not p_{02} by p_s but p_s by p_{02} reciprocal of the pressure ratio which we measure as a function of the Mach number.

I start with a value of 1,2,3,4,5, etc so as we go towards the right here, the stream is becoming a high speed stream a supersonic stream and if you look at this value it will go to 2gamma M_1 square by gamma plus 1 and that will be inverse of that and of that value and asymptotically it will reach that value. And the point is, if I measure this ratio p_s by p_{02} all I have to do is to just read the corresponding Mach number. Actually the impact probe in fact it is called the impact probe because at the front of the probe there is an impact the fluid is going to come to rest at the front so it is called an impact probe because of that.

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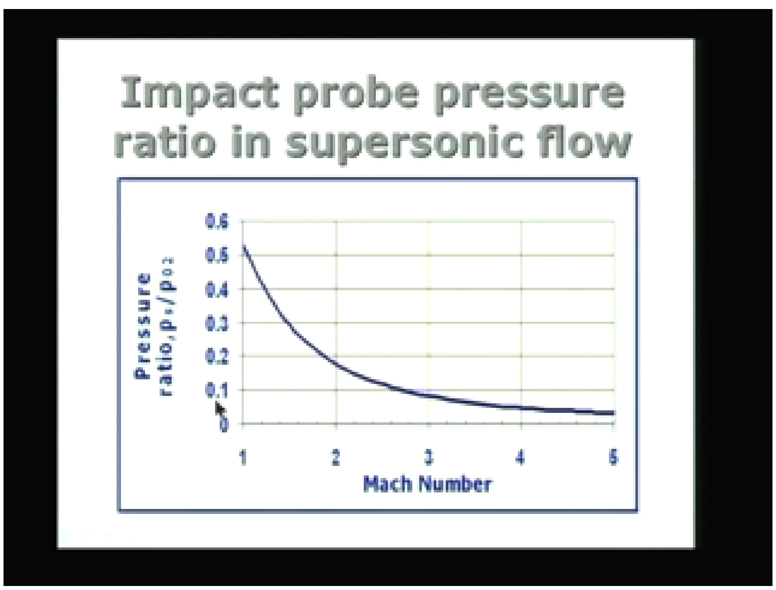

So in the impact probe I am going to measure the static pressure in the mean stream which is coming in, I am going to divide by the pressure indicated by the pitot tube so this ratio is going to be related to the Mach number of the stream. So we can use a graph of this type to measure. For the y axis whatever is indicated is measured, the x axis gives the corresponding Mach number. If you want you can plot the Mach number along the y axis and the pressure is shown along the x axis. But essentially the information is contained in this graph.

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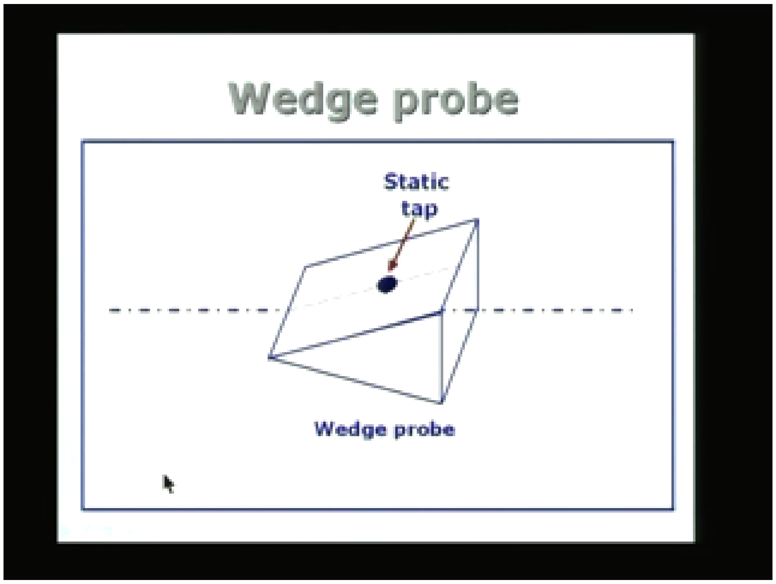

The second type of probe used in supersonic streams measuring the Mach number is a wedge probe. The first one was a pitot static tube. In fact you can tube you there and it will do the job. It does not even to have to any particular shape unlike a pitot tube, it could be even be a simple tube itself. But sometimes we would use like that. What is the difference between the pitot probe and this? It is in the form of a wedge. You can see the cross section it is a triangle, the stream is coming in this direction and the pressure tap is actually on the side on the flat side. On this flat side here I am going to have a pressure tap. Now let us look at the wedge probe and how it is going to work out.

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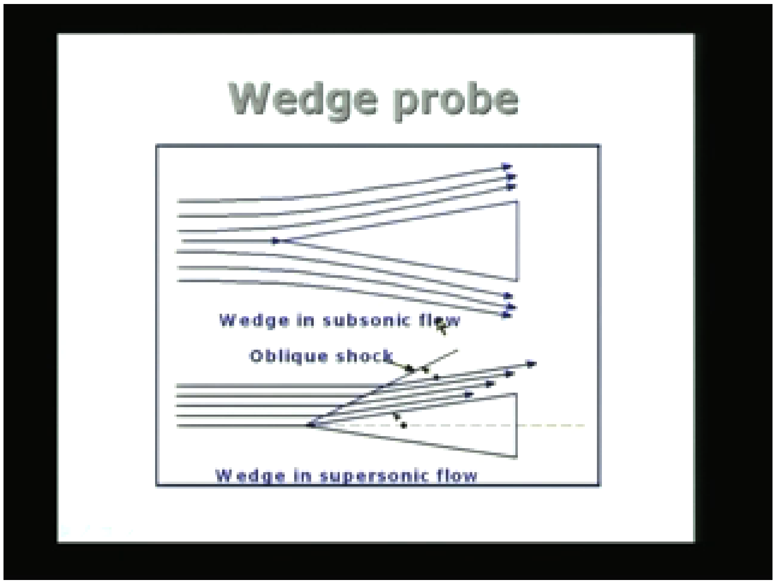

Here there are two sketches; one is a wedge probe in a subsonic flow at the top and the wedge probe in a supersonic flow at the bottom. And what we notice is that in the case of subsonic flow the stream lines are going to be curved like this as shown here, and a nice stream line flow is going to be the result. But in the case of the same wedge kept in supersonic stream, you see that starting from the corner there is a oblique shock, the stream which is coming in and therefore it is called the oblique shock and what happens is that the stream is going to come like here, and just take a change of direction abruptly at the shock wave.

The shock is a very thin region where irreversible effects are taking place like viscosity, heat transfer, and so on they are important within this region and what happens is that the incoming stream which is coming as a nice parallel stream is going to abruptly change its direction by an angle exactly equal to the angle of the wedge so that the stream lines are parallel to the surface of the wedge. Here they are not necessarily parallel to the wedge surface. You can see that it is coming in slightly curved and finally it looks like it is parallel to the wedge. But here it occurs abruptly at the oblique shock. In fact the gas dynamics is going to give us information about how to calculate. I have not shown with the bottom the bottom will be symmetric. I have an oblique shock.

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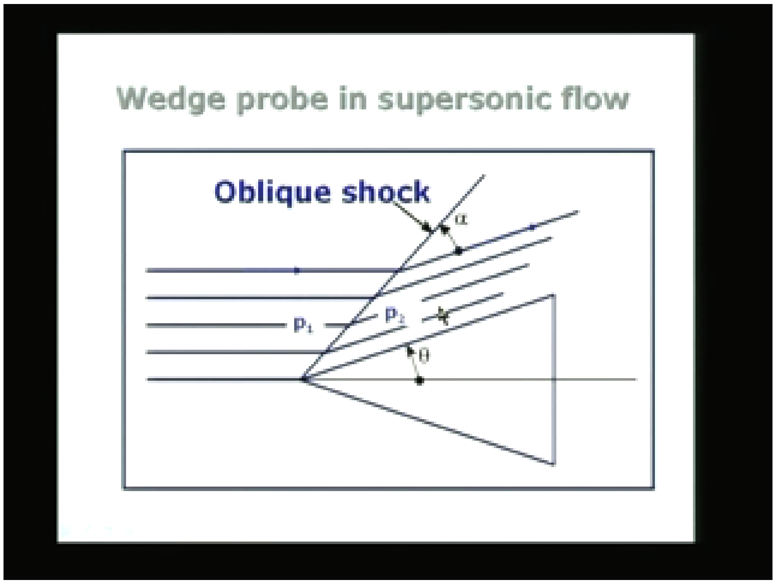

The important factors which we are interested in are the pressure p_1 which is pressure upstream of the oblique shock, p_2 is the pressure downstream of the oblique shock, this pressure is what is measured by putting a hole here. This hole here, is going to measure p_2, this p_2 is measured by the hole there. So let us look at the expression for the relationship which is going to be used for finding out what the Mach number is.p_1 is measured, p_2 is measured, the angle theta is known, because we can choose the correct angle for the wedge may be a few degrees and the oblique shock is going to be located such that there is an angle of alpha between the stream after it turns and the shock.

This angle alpha depends on the Mach number of the stream and the angle theta. So the alpha is a function or the Mach number of the stream and theta and p_2,p_1 are measured so let us look at the formula which gives me the pressure ratio. Again I am measuring a pressure ratio not a pressure difference, contrary to what we were doing in a subsonic low speed flow. The shock angle alpha itself is a function of M_1 and theta, theta is the angle of the wedge. The table available in books on gas dynamics will give you this relationship.

So, if you know what the Mach number of theta is, then alpha will come out as a function of these two. Suppose I know the Mach number the pressure ratio p_2 by p_1 can be shown to be equal to 2gamma by gamma plus 1 M_1 square sin square alpha minus gamma minus 1 by gamma plus 1. So let us look at this formula in some detail. This is in the case of measurement I am going to measure this ratio and I want to find out what is this M_1. So I can rearrange this formula in terms of M_1 equal to some function of alpha and p_2 by p_1 if you want you can do that. However, alpha here is also a function M_1 and theta.

Therefore, because I do not know what the Mach number is, even though I know theta, I do not know what alpha is so some kind of iterative scheme is required for calculation as you can very easily see. So you can assume some alpha or some value of M_1, find out the alpha put it here, and then find out was p_2 by p_1, compare it with what this formula gives for p_2 by p_1 and then if the two are going to agree with each other then you know that it is the correct value of M_1 otherwise you iterate you take a different value and so on and so forth.

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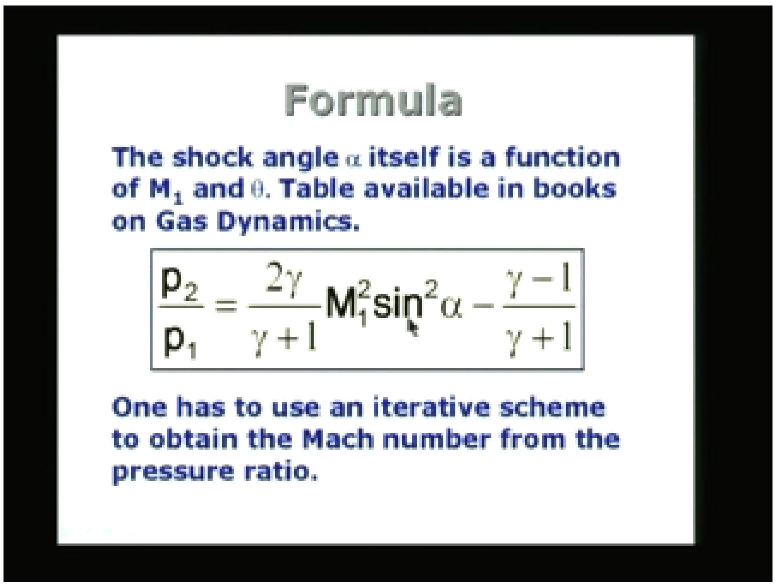

Therefore one has to use an iterative scheme to obtain the Mach number from the pressure ratio. So, pressure ratio is measured, theta is known, we find out what is the Mach number of the stream by using this formula if necessary by iterative sort of relationships. Here is an example, if I have a wedge of angle equal to 8 degrees from the tables in the book on gas dynamics for a Mach number of 1 point 5 alpha turns out to 53 degrees. That is, theta is given, I am also given the Mach number because in this case, I want to find out not the Mach number but the pressure ratio so M_1 equal to 1 point 5, alpha turns out to be 53 degrees from the table then with gamma equal to 1 point 4 we can calculate the ratio of the pressure p_2 by p_1 and that will be given by 2 into 1 point 4 by 1 point 4 plus 1 (into 1 point 5 sine 53 degrees) whole square is M_1 sin square or (M_1 sine alpha) whole square minus gamma minus 1 1 point 4 minus 1 by 1 point 4 plus 1 gamma plus 1 this gives you 1 point 508.

So, if the angle of theta where theta is 8 degrees and it is incoming stream at Mach number 1 point 5 we will get a pressure ratio of 1 point 508. Or working backwards, if you measure a value of 1 point 508 for the pressure ratio and if the angle theta is 8 degrees then the corresponding value of M_1 is equal to 1 point 5. So you can work backwards and find out what is the Mach number.

Direction of the flow is another important thing to look at. We were talking only about the velocity, what about the direction? Of course we know the velocity of the vector and therefore we will be interested in finding out what the velocity magnitude is and also the direction. Here is a simple scheme, suppose I have a tube with a central hole and I have another hole at the top and another hole at the bottom, this is a three holes or three hole probe.

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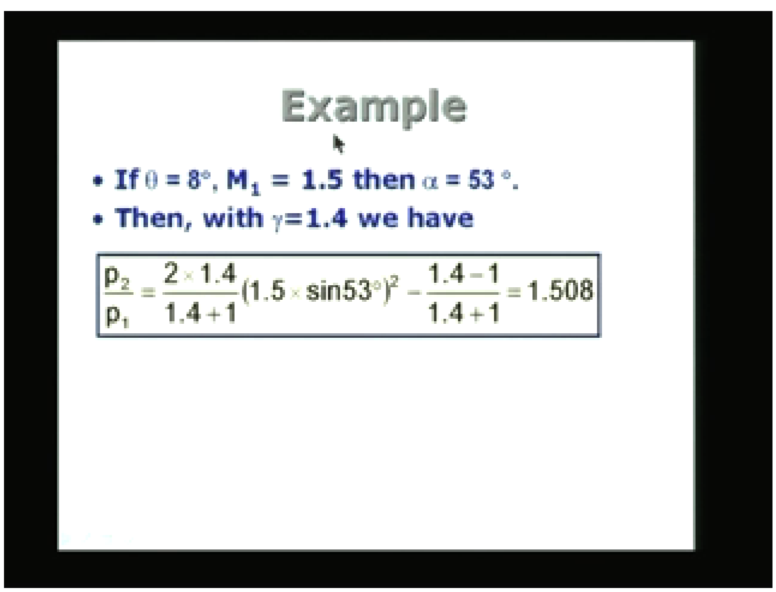

Let us look at the way it is going to behave. Suppose I look at the only low speed flow I am not interested with the high speed flow with the multiple holes probes and so on but only low speed flow, if the velocity vector is in this direction, so suppose I take this as 1, this is 2 and this is 3 delta p between 1 and 2, delta p between 1 and 3 if it is exactly coming normal to the probe these two are going to be equal. So delta p equal to delta p_{13} if V is aligned with hole 1.

Suppose the velocity is not in that direction, suppose the velocity vector is in a different direction if it is coming in this direction, delta p_{12} will be different from delta p_{13}. In this case it is symmetric, both the pressure differences are the same, if it is aligned at an angle let us put it as angle equal to alpha, this is the angle in the plane of the probe, I am assuming that the velocity is coming in the plane, if it is out of plane again we will come out with similar discussion. So let us look at the delta p_{12} is not equal to delta p_{13}. Actually what I can do is, I can experiment with streams at different angles by orienting the pro probe in different directions and finding out how this delta p_{12} and delta p_{13} behaves with respect to angle alpha.

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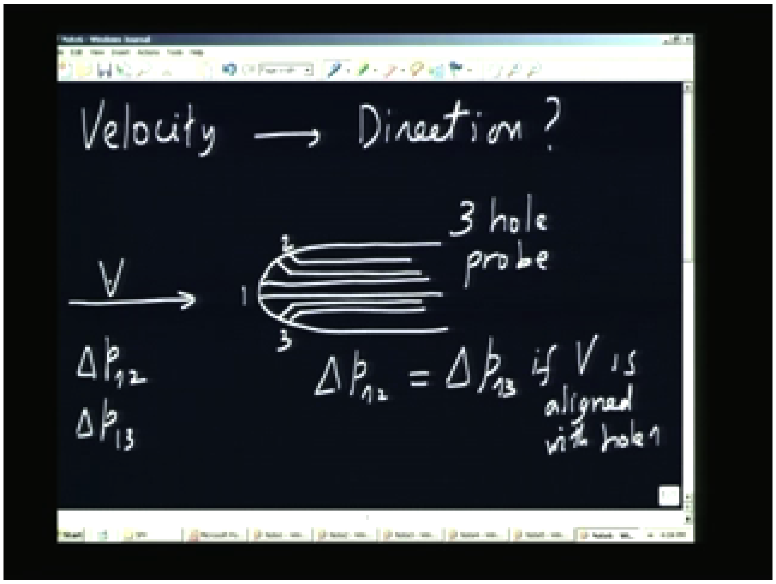

Therefore I will be able to actually use the probe and find out the velocity as well as the direction by using that kind of information. If the velocity vector is in some arbitrary direction, and the probe is oriented again in an arbitrary direction with respect to flow velocity you can in fact think in terms of a multiple hole probe one at the center.

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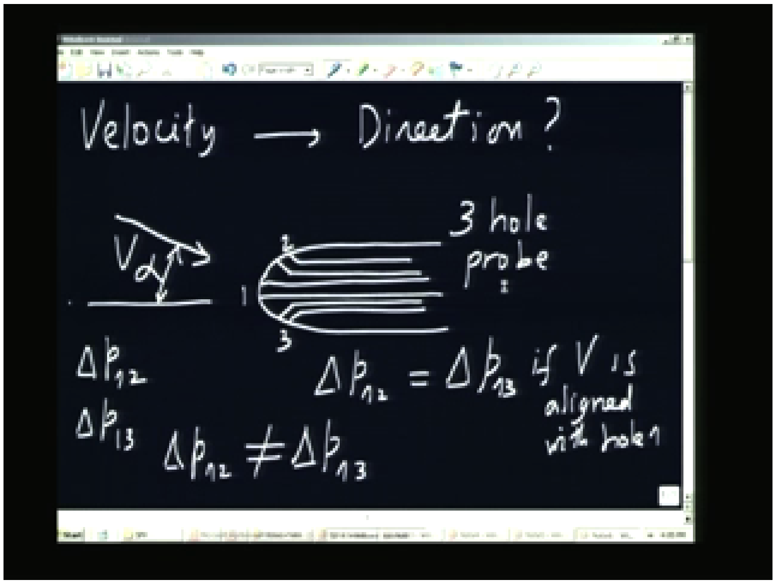

I can have a five hole probe as shown here so we have multiple holes and this can be called 1, 2, 3, 4 ,5. I have four different delta p_{s4} in number. So these will be depending on alpha in this direction this angle phi and you can have angle beta in that direction. That means the probe can be rotated at an angle with respect to V as shown here, it can have at an angle alpha in this plane, beta in this plane and an angle phi like that.

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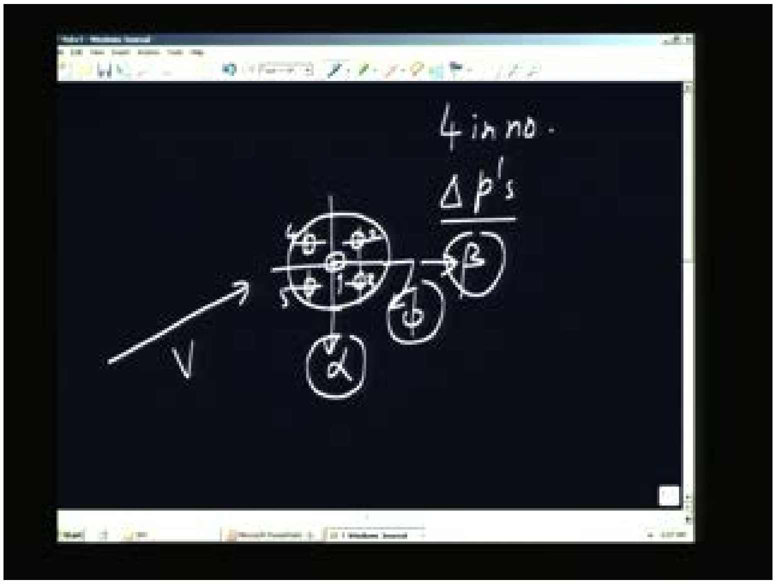

So, the delta p_s are going to be sensitive to alpha, beta and phi and we can again calibrate the probe, so that the velocity as well as the direction information can come from such measurements. This is basically the idea of a multiple hole probe which is used in many important fluid mechanics applications. Thank you.