

Mechanical Measurements and Metrology

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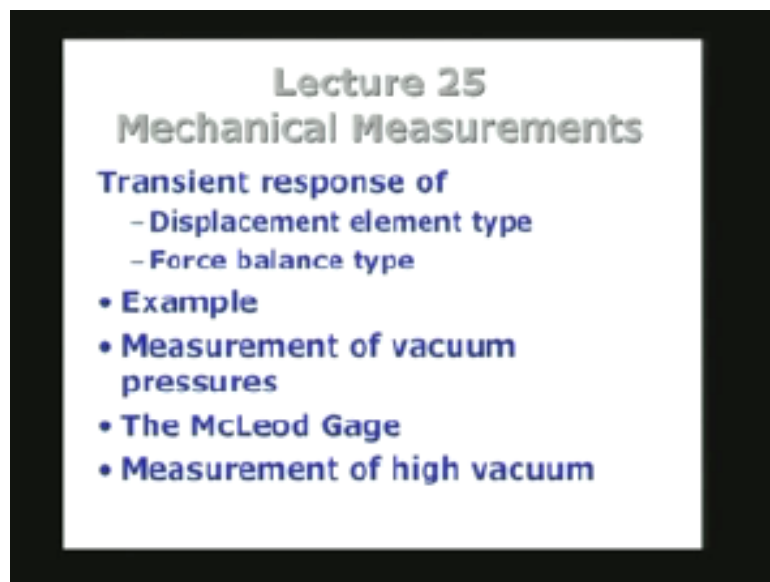
Module -2

Lecture - 25

Transient Response of Pressure Transducers (continued)

This will be lecture number 25 on Mechanical Measurements. We will recall that in the previous lecture we were looking at the transient response of a u-tube manometer. We were also planning to take a look at transient response of other types of transducers namely the displacement element type and the force balance type.

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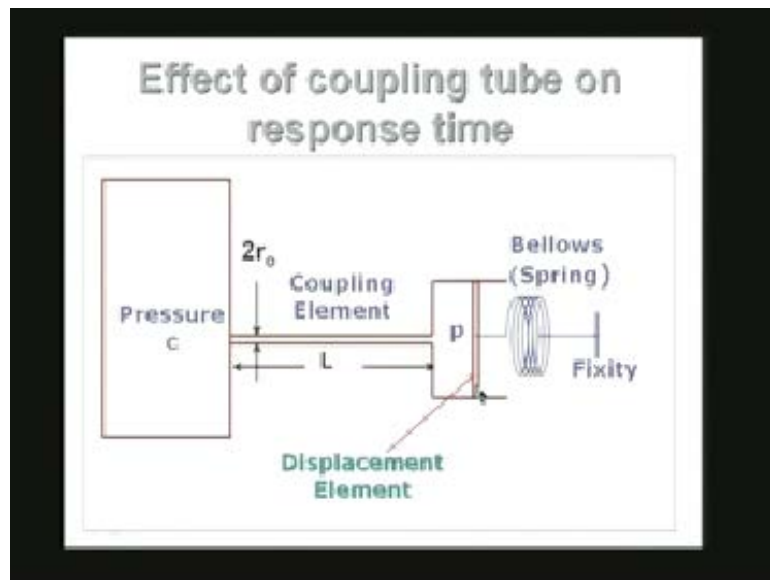
Let us look at it with an example and then at the measurement of vacuum pressures below the atmosphere. And typical vacuum gages will be the McLeod Gage and then high vacuum measurement requires special gages as we will look into it. So let us look at the types of pressure gages. As far as transient response is concerned we recall the u-tube manometer or u-tube pressure measuring instrument as a typical second order response.

Let us discuss about the displacement element type with an example namely the pressure gauge with a diaphragm or a piston and a spring which actually

is like the bellows type gauge. It is idealized as the piston with a spring. The spring element is actually the bellows itself and the bellows itself during expansion and contraction acts like a displacement element, it is like a piston. So, the generalized indication of the bellows type gage is to indicate it as the piston moving in a cylinder with a spring element being bellows itself and the bellows is restrained at one end, and it is allowed to move freely at the other end.

That is indicated by the fixity here. We have a pressure in the chamber C which is to be measured using a gauge which is located some distance away in this case connected by a tubular element or coupling element which is in the form of a tube of diameter $2r_0$ or radius is equal to r_0 and length is equal to L .

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So, we have a coupling between the pressure to be measured and the pressure which is indicated by the bellows gauge shown by C and P respectively. If C is varying with respect to time P will also vary with respect to time and we want to know the relationship between C and P and the displacement which is going to be shown by the spring element. The displacement is measured from some datum and the movement of the piston or one end of the bellows element which is not fixed with respect to fixed end is not going to move and that is the displacement we are going to measure. The nature of this measurement is shown in a simple sketch.

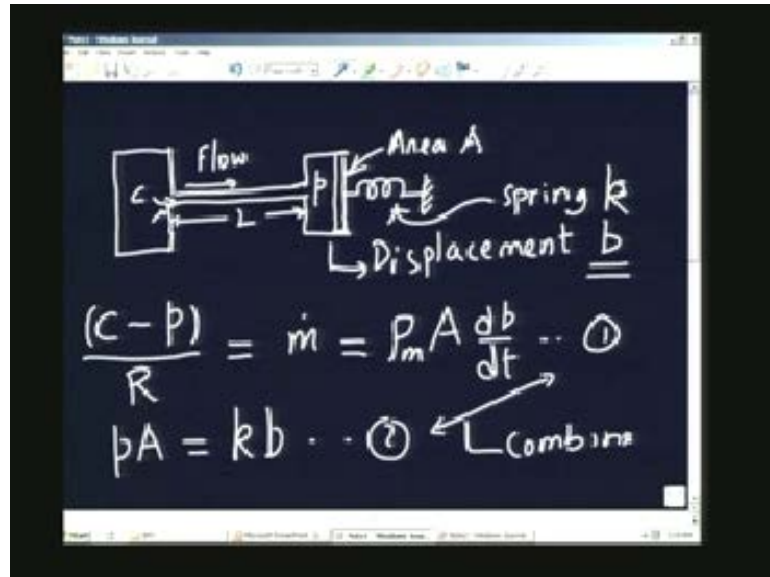
I have indicated a chamber whose pressure is C which may vary with respect to time, there is a coupling element of length L and a cross sectional diameter $2r_0$, connected to a spring loaded piston, the area of the piston is A and then the displacement is measured with respect to some datum may be the point here and I am measuring in this direction so the displacement is measured as b . Now what happens is, if the pressure C and the pressure P are different that is if C is greater than P then some amount fluid has to move in this direction because there is a pressure gradient along the tube and therefore a flow will take place in this direction.

What is the consequence of this flow?

This flow is going to push the piston to the right and therefore the consequence will be displacement b . So we can see that C minus P divided by the resistance offered by the coupling element C minus P by R this must be equal to the mass flow rate because this is by the definition of the resistance. If you remember, the resistance were defined as the potential difference divided by the current C minus P is the potential difference $m \dot{}$ is the current so the ratio was R therefore I have now cross multiplied I written C minus P by R is equal to $m \dot{}$ and let us see what this $m \dot{}$ is? $m \dot{}$ is nothing but the volume change in the piston because the piston is moving to the right the rate of change of volume multiplied by the density is going to give m so it will be ρ_m of the manometric liquid multiplied by the area of cross section, A multiplied by db by dt . So you can see that the equation has become db by dt into ρ_m into A is equal to $m \dot{}$ is equal to C minus P by R this can be re written slightly in a different fashion let us call this equation 1.

Suppose this spring has a constant equal to k we also know that the force opposing the motion is given by PA equal to kb these are the two equations which govern the problem. PA equal to $k(b)$ and C minus P by R is equal to $\rho_m A db$ by dt . So now I can combine these two together and I can write the equation as; C minus..... if you see here instead of P , I will write kb by A by R is equal to $\rho_m A db$ by dt .

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Or I can further rewrite in the following form: $R \rho_m A$ this A also I will multiply and that will become $\rho_m R$ into A square by $k db$ by dt . So I have multiplied by R and multiplied by A by k so this will give you plus b is equal to CA by k this is the governing equation. If the pressure is different between the chamber whose pressure I want to measure and the pressure in the piston is different, there will be a net flow of the fluid from the chamber to the piston.

Of course if the pressure difference is otherwise, some amount of fluid will go from the piston and it will go back to the chamber. So whatever I am saying is for C greater than P and for C less than P the flow will be in opposite direction. When the flow is in the opposite direction the volume is going to decrease. When the flow is taking place into the piston it is going to increase. Suppose the pressure is changing sinusoidally or in a periodic fashion sometimes the flow will be into the piston and sometimes the flow will be from the piston side to the chamber side. So the governing equation is a first order equation, ordinary differential equation. And you can also see this is b in meters, db by dt is in m by s , this whole thing must be in seconds and we can identify it with the time constant.

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$$\frac{\left[c - \frac{kb}{A}\right]}{R} = \rho_m A \frac{db}{dt}$$

$$\boxed{\frac{\rho_m R A^2}{k} \frac{db}{dt} + b = \frac{cA}{k}}$$

Governing equation (1st order)

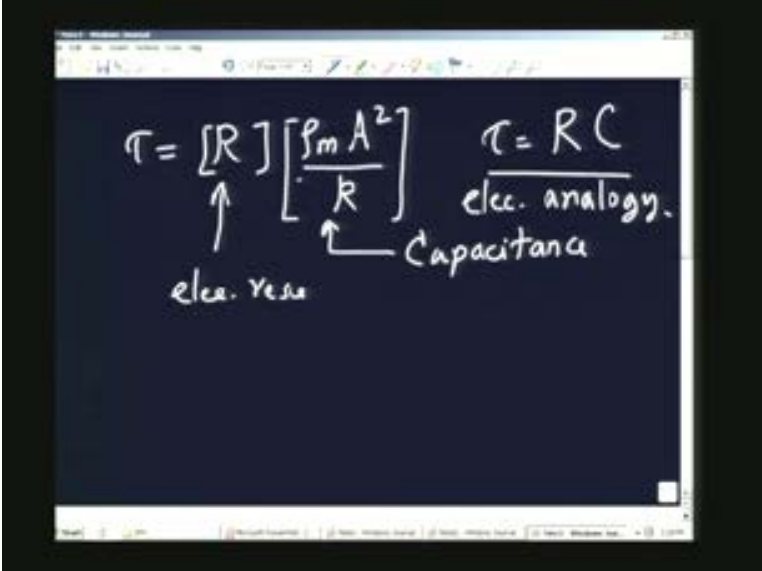
Time constant $\tau = \left[\frac{\rho_m R A^2}{k}\right]$

So time constants in seconds I will call it as tau is equal to $\rho_m R A^2$ square by R this is your time constant. So we can split into two parts. So tau I can write as, $R \rho_m A^2$ square by k and this is like an electrical resistance and this is like the capacitance because a first order system which involves a circuit consisting of a resistance and capacitance is exactly like this. The fluid flow resistance due to the coupling element is identified with electrical resistance, and the other factor which involves the area of cross section of the pressure gauge of the area of the diaphragm is coming here, the spring constant is coming here, and the density is coming here this is like a capacitance of the gauge, the electrical capacitance tau is equal to RC well known from electrical analogy. So we will say this is electrical analogy.

Therefore the way we are looking at it is that, when there is a transient pressure applied on the system some amount of fluid has to move either into the pressure gauge or to pressure transducer volume or it has to move out of the pressure transducer volume. This is where the capacitance of the gauge is going to come into picture which is a composite variable depending on the density. Then we get the area of cross section. If the density increases more mass flow has to take place, it is proportional to mass flow and mass flow is proportional to the density. Higher the density higher the mass flow rate and higher the mass flow rate, higher the velocity and so on and so forth.

The area of cross section becomes larger that means more volume of the fluid has to move into the gauge to bring out a change and therefore it is a higher capacity. It actually goes at the sphere of the area and if the spring constant is high it will reduce the capacitors and if the spring constant is low it will increase the capacitors. So you see that the k is in the denominator, it is a very weak spring and it will move it will move by a considerable amount for a given pressure change. If the k is very large it will move very small amount. So the capacitance is determined by the density of the fluid then the area of cross section and also the spring constant. So you see that the physical quantities which come into the picture or as shown here. Let us look at the typical example.

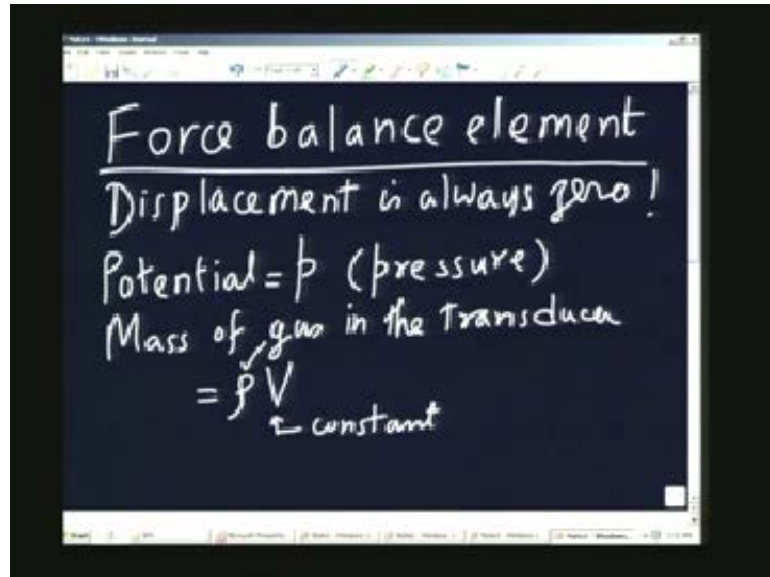
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The image shows a digital screen with handwritten mathematical expressions. On the left, the equation $\tau = [R] \left[\frac{\rho_m A^2}{k} \right]$ is written. An upward arrow points from the text "elec. res." to the $[R]$ term. Another upward arrow points from the text "Capacitance" to the $\left[\frac{\rho_m A^2}{k} \right]$ term. On the right, the equation $\tau = RC$ is written, with "elec. analogy." written below it.

We have a pressure signal which is fed through a line having an inside diameter of two millimeters and a length of 2.5m this is a typical application. The line is connected to a pressure transducer having a volume of 5 mm this is given, assume that during measurement this volume remains constant.

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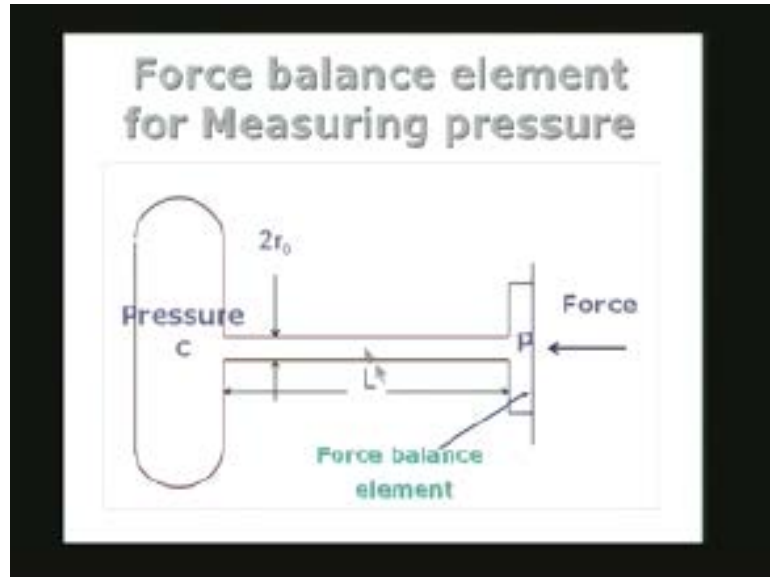


The line is connected to the pressure transducer having a volume of five millimeters. Assume that during measurement this volume remains constant so this is a different type of an application. Air is at 1.2 bar and 30 degree Celsius, this is the transmitting fluid and calculate the time constant of the system in this particular case. Then the second part says, is this arrangement suitable for measuring a transient which is cycling at 100Hz and how you come to your conclusion regarding the answer to the particular part. The arrangement is slightly different. We had a pressure C coupling arrangement, a spring and a piston arrangement. The next arrangement I am looking at is a force balance element. This is slightly different from that.

In this case what I am doing is I am connecting the pressure which is to be measured C through a coupling element to a gauge which also has a diaphragm element. But what I do is, I make the diaphragm have null movement. That means any movement of the diaphragm is compensated by applying a force from the other side such that it goes back to... The force required here is directly related to the pressure and the area of cross section as you can see. When the pressure and the area multiplied by the area of the cross section is exactly equal to the force applied there will be no movement of the diaphragm. In this case I am using what is called a force balance element in which I am applying a force in the opposite direction such that the displacement is completely null.

Let us take a look at the case where the fluid being a compressible fluid. For example, air inside the system, the pressure chamber is here and the coupling tube is full of that air. And now if you see what is happening there is no change in the volume because I am nullifying whatever displacement that may take place.

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When the pressure changes immediately I am putting a force such that the pressure is nullified. That means there is no change in the volume. But you remember from the equation of state for a gas that p is equal to ρ, R, T because pressure, density and temperature are all related, if the pressure changes the density of fluid has to change. So, if the density of the fluid changes some mass must go from the chamber through the pipe into the pressure gauge such that the density increases.

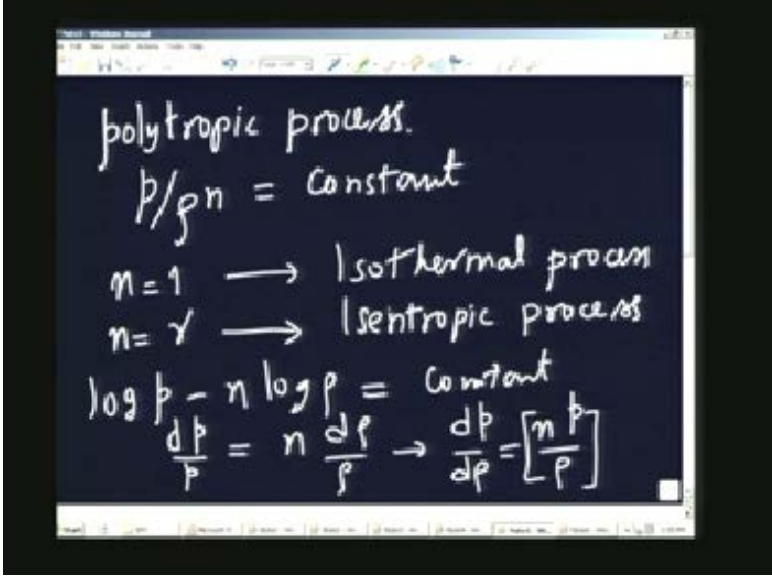
Let us see what is going to happen in the case of a force balance element. This is another way of using the pressure transducers where we are going to make the displacement of the spring element which is always 0. Instead of allowing it to displace I am going to keep it. Let us identify the potential for the change in the density that is pressure. And the mass of gas in the transducer is equal to ρV , and V is a constant, there is no change in the volume. Therefore ρ is going to change. When you have a different pressure, when the pressure changes in the chamber the density of the fluid

has to change. That means to make the density more some amount of mass has to transfer just like in the previous case.

Let us assume that this change in density takes place through a polytropic process. What is this polytropic process?

This is a concept from thermodynamics. We will assume that the pressure and density change according to the relationship, P by ρ power n is equal to constant, n is equal to 1 corresponds to isothermal, the temperature remains fixed throughout so we can assume n is equal to 1, n is equal to γ , the ratio of specific heat if it is a isentropic process and in general it may be anywhere between these two values n is equal to 1 and n is equal to γ . We are interested in finding out the transient response of the system or estimate time constant for the system assuming that is going to be a first order system. This is no different from the previous case.

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polytropic process.
$$P/\rho^n = \text{constant}$$
$$n=1 \longrightarrow \text{Isothermal process}$$
$$n=\gamma \longrightarrow \text{Isentropic process}$$
$$\log P - n \log \rho = \text{constant}$$
$$\frac{dP}{P} = n \frac{d\rho}{\rho} \rightarrow \frac{dP}{d\rho} = \left[\frac{n P}{\rho} \right]$$

We expect it to be a first order system, and we want to estimate the time constant and for estimation process it does not matter what value of n you are going to assume. The typical value will be of the order of the magnitude which we are going to determine from this. Therefore purely from the utility point of view I can take either n is equal to 1 or γ or any value in between and it is not going to make much of a difference. That means the actual process undergone by the gas as it flows from one side to the other side it is not very important for this particular purpose.

Let us look at this p by ρ power n is equal to constant. I can differentiate logarithmically, so what I do is $\log p$ minus $n \log \rho$ is equal to constant. Constant is now logarithm of that. I am just taking the log of that. The logarithmic definition will give you dp by p is equal to $n d\rho$ by ρ . We calculate this because the mass inside is ρV and again I can logarithmically differentiate dm by m is equal to $d\rho$ by ρ since V is equal to constant. dm by m the change in the mass to the mass which is already there is simply related to the change in density as a ratio of the density ρ itself which gives rise to it.

Now dp by p is equal to $n d\rho$ by ρ I can also write it as dp by $d\rho$ is equal to np by ρ . So the capacity or the capacitance I can take as the ratio dm by dp . This is the capacitance of the gauge. So I can write dm by dp using the previous expression as dm by $d\rho$ ($d\rho$ by dp) I am just using the definition of C . Here, we already have dp by $d\rho$ here is equal to np by ρ so here I am going to substitute dp by $d\rho$ which is dm by $d\rho$ into dp by $d\rho$ is np by ρ so this goes like this. Therefore the capacitance is determined by this expression. You also remember that p is equal to $\rho R_g T$ where R_g is the gas constant, this is the equation of state. So, using the equation of state p is equal to $\rho R_g T$ so p by ρ is equal to $R_g T$ so we can see that dp by $d\rho$, C will be equal to dm which is $V d\rho$.

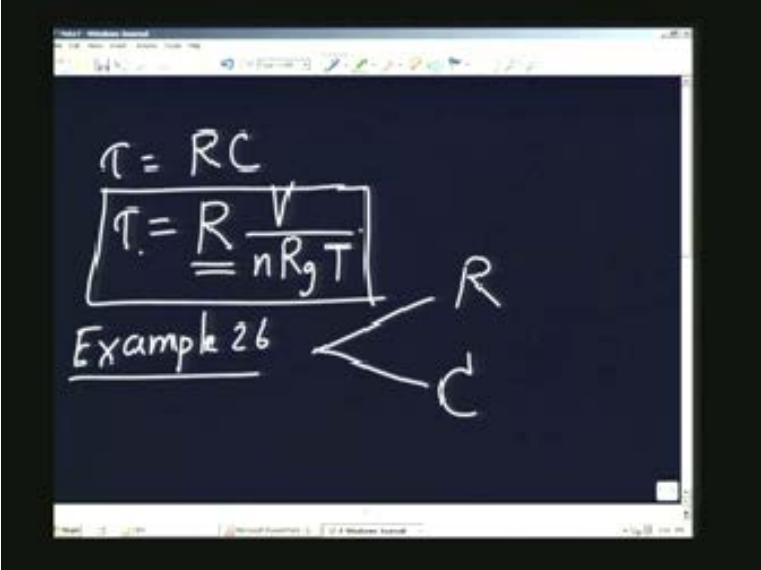
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$$\begin{aligned}
 m &= \rho V \quad \frac{dm}{m} = \frac{d\rho}{\rho} \quad \because V = \text{const.} \\
 C &= \frac{dm}{dp} = \frac{dm}{d\rho} \cdot \frac{d\rho}{dp} = \frac{dm}{d\rho} \cdot \frac{\rho}{np} \\
 p &= \rho R_g T \quad (\text{eqn. of state}) \\
 C &= V \frac{d\rho}{dp} = \left[\frac{V}{n R_g T} \right]
 \end{aligned}$$

So $V \frac{d\rho}{dt}$ by $d\rho$ this becomes V and $\frac{dm}{d\rho}$ is nothing but V because m is equal to ρV so this becomes V , $\frac{d\rho}{dp}$ $\frac{dm}{d\rho}$, m is equal to ρV . Therefore this becomes ρV where V is constant I take it out and $\frac{dm}{d\rho}$ becomes V because ρ is constant. So this becomes $V \frac{d\rho}{dp}$ is nothing but $V \frac{d\rho}{dp}$ will give you $R V \frac{d\rho}{dp}$ will give you 1 by $n R_g T$. This is your expression for C .

So I can write the time constant without going through the intermediate steps the time constant of τ is the product of R and C . R is already known to us. Once the length of the tube is given, the diameter of the tube and the fluid is given we calculate the resistance of the coupling element and that multiplied by C is what is determined. So $R V$ by n into gas constant times T so this is your τ .

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$$\tau = RC$$

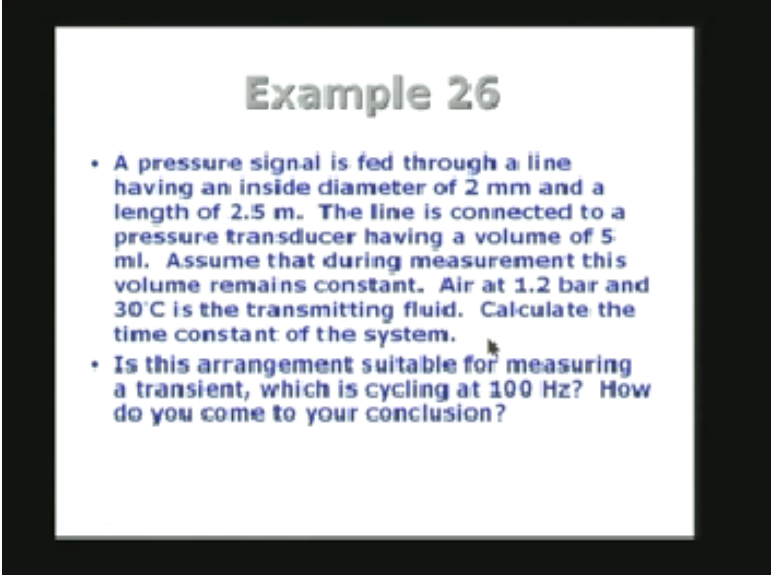
$$\tau = R \frac{V}{n R_g T}$$

Example 26

$\left\{ \begin{array}{l} R \\ C \end{array} \right.$

Here in this problem we have a null element or the force balance element and the diameter of the tube is 2 mm, length is 2.5 meters and then we have a volume of 5ml which is a constant in this case and then the air is at 1.2 bar 30 degree Celsius and I want to find out the time constant system.

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Example 26

- A pressure signal is fed through a line having an inside diameter of 2 mm and a length of 2.5 m. The line is connected to a pressure transducer having a volume of 5 ml. Assume that during measurement this volume remains constant. Air at 1.2 bar and 30°C is the transmitting fluid. Calculate the time constant of the system.
- Is this arrangement suitable for measuring a transient, which is cycling at 100 Hz? How do you come to your conclusion?

Exactly the calculation which was indicated now has to be performed for this. This is example 26 which requires the calculation of R which goes here and we require the capacitance. The steps are as follows:

The volume of transducer given is 5 mm, 5 into 10 power minus 6 cubic meter and everything is given in SI units. We have the volume given, the pressure is specified as 1.2 bar 1.2 into 10 power 5 Pascal and that one bar is exactly equal to 10 to the power 5 Pascal. The temperature is given as 30 degree so I have to add 273 to make it to equal the absolute temperature in Kelvin because everything requires use of absolute temperature because we are using thermodynamics equations and they need to be done this way. The gas constant for air is two 287 joules by kg Kelvin.

Here we can make use of a polytrophic process. In the previous expression, we are having n in the denominator. So, if I use a value of n equal to 1, I get the over estimate for τ , because τ will be largest for n is equal to 1 and τ will be smallest if the process happen to be isentropic. Therefore I will be getting an over estimate or the upper bound for the time constant. So let us assume that the upper bound is adequate for us.

I will take n is equal to 1 which is a polytrophic process and n is equal to 1 is nothing but an isothermal process which is also possible. If the entire thing

is well exposed to the outside ambient the temperature more or less is going to remain constant, so we can justify this to some extent.

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Handwritten calculations on a digital screen:

$$\begin{aligned}
 V &= 5 \times 10^{-6} \text{ m}^3 \quad (\text{SI units}) \\
 p &= 1.2 \times 10^5 \text{ Pa} \\
 T &= 30 + 273 = 303 \text{ K} \\
 R_g &= 287 \text{ J/kg K} \quad n=1 \\
 &\quad (\text{isothermal}) \\
 C &= \frac{V}{R_g T} = \frac{5 \times 10^{-6}}{287 \times 303} = 5.75 \times 10^{-11} \text{ (m-s}^2\text{)}
 \end{aligned}$$

Therefore it is also possible to say that the time constant will be largest possible value because probably isothermal conditions are going to be applied to this particular case. So with this we can calculate the capacitance C which is nothing but V by $n R_g T = R_g(T)$ (n is equal to 1) I have taken so V is 5 into 10 to the power minus 6 by 287 into 303 this will give you of value of 5.75 into 10 to the power minus 11 m-s square. This is unit of the capacitance.

Of course electrical engineering or electronics or in electrical circuit it will be farad or micro farad but here it is a mechanical unit because it is m - s square C is equal to 5.75 into 10 to the power minus 11 m - s square. The calculation of the resistance is similar to what we did in the example 25. R is calculated as in example 25. All you have to do is to calculate the viscosity and density for the air at 1.2 bar and 30 degree Celsius. So you have to look up at the tables giving the properties and then evaluate the value of R . We can calculate R as 8.535 into 10 power 7 units will be 1 by m - s. Therefore the time constant τ is RC . The product here will come out to be 4.907 into 10 power minus 3 s or 4.907 millisecond or roughly equal to 5 milliseconds. So the pressure transducer for the first order system with a time constant is given by 5 milliseconds.

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Handwritten calculations on a digital screen:

R is calculated as in example 25.
(student should do this)

$$R = 8.535 \times 10^7 \text{ } 1/(\text{m-s})$$
$$\tau = RC = 4.907 \times 10^3 \text{ s}$$
$$= 4.907 \text{ ms} \sim \boxed{5 \text{ ms}}$$

$\boxed{100 \text{ Hz}}$ Period 0.01 s
It will not be useful $\rightarrow 10 \text{ ms}$

This is the first part of the question which is answered by determining the time constant. Now we want to find out if it will be useful for a signal which is varying at 100 Hz. So 100 Hz means it is hundred cycles of variations are going to take place per second and therefore the period is equal to 1 by 100 or 0.01s or 10 milliseconds.

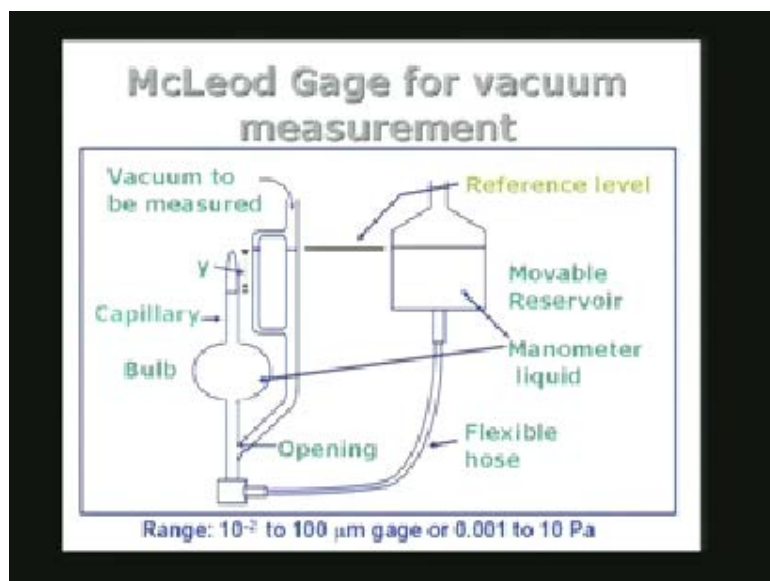
When we discussed about the first order system and its response we were talking about temperature sensors. We mentioned that the time constant of the system, the ratio of the characteristic time of the signal compared to the thing must be at least of the factor 5 or 10, but here it is not satisfying that requirement. Therefore when you have a 100 Hz signal it is not going to be all right to use this so it will not be useful. We cannot use this transducer for the dimensions which are given. It can be due to two things, the coupling element, the resistance is one thing and the second one is the capacitance. By reducing either one of them we can improve the response of the system.

For example, for a 100 Hz signal which is going to input I require a smaller diameter or the smaller volume for the transducer. The volume is 5 mm which is very large but if we can reduce that it is going to improve the situation or you can reduce the length of the coupling element, or increase the diameter of the coupling element. But the most important here will be the volume. If you can decrease the volume of the pressure transducer which

is held constant for whatever value we have given, then it is going to certainly improve the matter.

Now let us look at measurement of pressures which are below the atmospheric pressure. In many occasions and many applications, we use vacuum pressures. In many chemical instruments which are used in Chemistry, chemical analysis or physical analysis and so on, it is necessary to remove or reduce the gases which are present and that can be done by pulling a vacuum and you would like to measure the vacuum pressure which will be of course below the atmospheric pressure.

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If we use a mercury in u-tube manometer to measure vacuum it goes to 760 mm of mercury or whatever the atmospheric pressure is it will go and get stuck there. So we cannot measure very small values. We can only measure a rough value for vacuum. If you want to improve measurement how are you going to do that? The measurement of vacuum is an independent of that activity which is required in many applications and we would like to have instruments which can be used specifically for measuring vacuum pressures.

Again, there are different types of vacuum pressure gauges. One of them is a McLeod gauge and there are also somewhat specialized gauges. The McLeod gauge is actually a manometer and it is also a compressor coupled into one. This is how the McLeod gauge is constructed. So vacuum to be

measured is going to be communicated. This whole thing is the McLeod gauge along with the movable reservoir. For a moment imagine that this movable reservoir has moved to a lower level so that this meniscus which is here is going to come to this level of the opening. That is, this reservoir is going to be moved down such that the level of the liquid in the reservoir as well as this portion of the gauge is at the level of the opening. That means the vacuum to be measured which is coupled here is going to occupy this whole thing which is shown here. This whole thing completely encloses the medium at the vacuum pressure which is to be measured.

Suppose we assume that the volume of this bulb, this capillary and this side tube everything put together is some volume V_B , when the level of reservoir is brought down such that it is just at the opening some amount of the residual gas in the vacuum chamber equal to some volume V_B is going to be trapped in this one. And now slowly I move the reservoir up then what happens is that the liquid which is here at the opening level slowly raises and when it does so it traps a certain amount of gas in this portion with the original volume V_B and slowly as I raise it that volume is going to compress to the volume above the meniscus here.

This is the volume above the meniscus here. This volume is given by the area of cross section of the capillary multiplied by the height y here which is shown here. And what I do is, after bringing down to this level and then allowing the gas to occupy the entire region in this side I am going to move this reservoir slowly up such that the level comes to the standard reference level. I have got a reference mark here so I will move it up such that the level is at this reference level so this meniscus, this meniscus and this meniscus are all at the same level equal to the reference level. And when such a thing happen the amount of gas which was inside is now compressed to higher pressure. Therefore this region above the capillary is full of a high pressure gas.

The pressure of the gas originally was equal to the vacuum pressure and after this process it has become a higher pressure which is equal to this. And if I use mercury as the liquid which is contained in the moving reservoir which is here and here the difference y directly gives you the pressure of the gas which is compressed now placed above the meniscus in the capillary tube. Therefore this pressure will be equal to y mm of mercury.

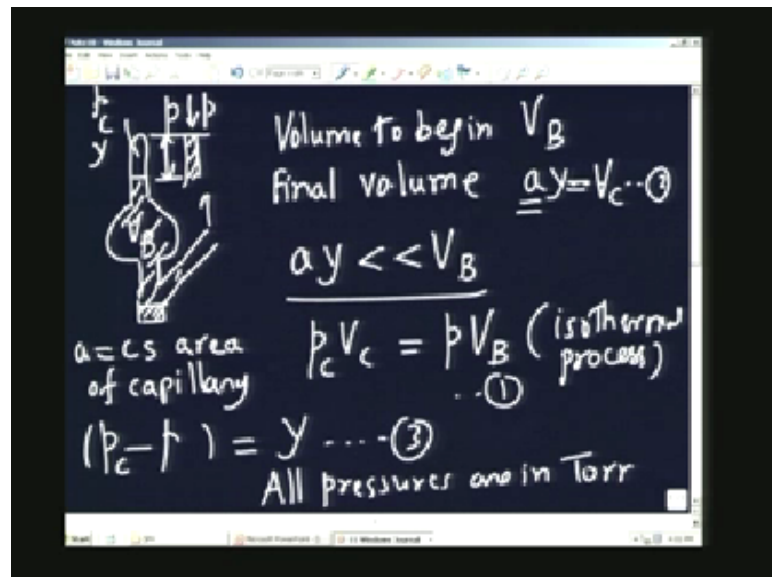
Now the point is, I am using the McLeod gauge to sample a certain volume of gas equal to V_B and I am going to perform an operation of compression to

a higher pressure which is given by this height of the column here between here and here. If I assume that the temperature of the entire instrument is constant during this operation, the process of compression is going to follow an isothermal process, that is the crux of the matter.

The range of the instrument is 10^{-2} to 200 micrometer, micrometer is micrometer of mercury or 0.001 to 10 Pascal's this is the range of pressure which we can measure using this instrument. We have this and here is the opening, and this is connected to the side tubes and so on. So initially the level of the mercury is here and this whole thing is, this is V_B , the volume is V_B at pressure equal to p , the initial pressure. Finally what is going to happen is it is going to be up to this. And if you remember what we did the meniscus level will be here and this is what it is and this is your y . So the volume to begin with is V_B . The final volume is y multiplied by area of cross section.

We will assume that the area of cross section of the capillary tube is $a(y)$ so a is the cross sectional area of the capillary. The dimensions we are going to use are as follows: the volume of the bulb may be 100 cc that is 100 milliliters, the diameter of the capillary may be $\frac{1}{2}$ millimeter. Therefore we are talking about a very small value of a and therefore this final volume is going to be very small compared to this. So, ay is very small compared to V_B .

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Now you see that this pressure is p here and we will say this is the pressure in capillary at the end of the process. So during the isothermal process we have pV is equal to constant so $p_c V_c$ is equal to $p V_B$ this is the isothermal process and this is one equation. You also know that the final volume is ay this is equal to nothing but V_c this is one expression we have. Thirdly, the difference between p and p_c is nothing but so many millimeters of mercury y .

Now I am going to use everything in terms of millimeters of mercury. So, all pressures are in millimeters of mercury which is also called the Torr. These are the three equations we have. So, p minus p_c is equal to y or p is equal to y plus p_c and then what I have to do is p_c is equal to $p V_B$ by V_c . So I can eliminate p_c from here by using the second equation here and then manipulate. Therefore it is p minus p_c is equal to p minus $p_c V_c$ is equal to $p V_B$ so p_c minus p is equal to minus p plus $p V_B$ by V_c because p_c into V_c is equal to p into V_B . Therefore p_c is equal to $p V_B$ by V_c so I can take p (minus 1 plus V_B by V_c) and V_c is nothing but ay and that comes to p (minus 1 plus V_B by ay) and p minus p_c is also equal to y that is from the manometer equation. So p (minus 1 plus V_B by ay) is equal to y or the pressure in the vacuum chamber is given by p is equal to y by V_B by ay minus 1.

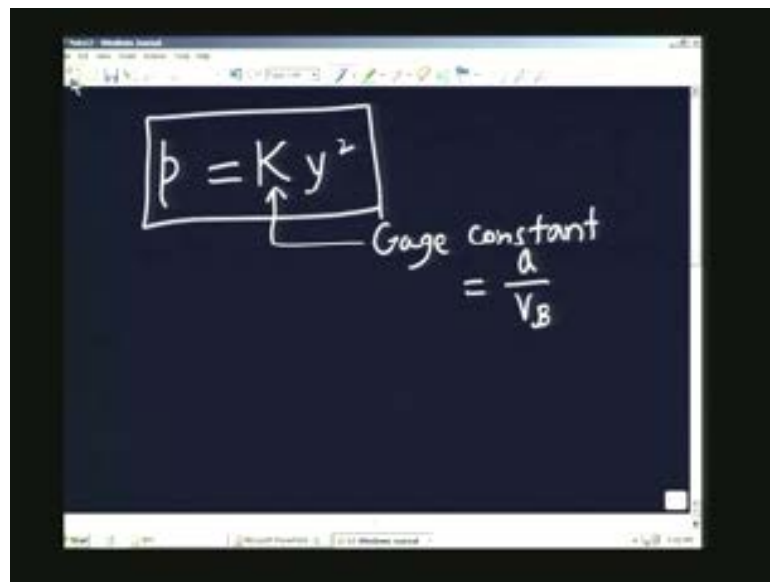
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$$\begin{aligned}
 (p - p_c) &= -p + \frac{p V_B}{V_c} & p_c V_c &= p V_B \\
 & & p_c &= \frac{p V_B}{V_c} \\
 &= p \left[-1 + \frac{V_B}{V_c} \right] = p \left[-1 + \frac{V_B}{ay} \right] = y \\
 \text{or } p &= \frac{y}{\frac{V_B}{ay} - 1} = \frac{ay^2}{(V_B - ay)} \\
 &\approx \left(\frac{a}{V_B} \right) y^2 = K y^2
 \end{aligned}$$

This entire thing we can multiply by ay and so it becomes ay square by V_B minus ay . If you remember, I said this is very large compared to this volume,

I can simply say it is equal to $a y^2$ by V_B or if I call this a by V_B as the gauge constant K some K constant $K y^2$ square. So the pressure of the vacuum chamber which is what we are trying to measure is p is equal to $K y^2$ square. It is a non linear relationship between p and y and K is the gauge constant is equal to a by V_B . Here a is the area of cross section of the capillary, V_B is the volume of the bulb and the capillary, and the capillary is of course a very small volume we can ignore that so it is nothing but the area of cross section of capillary into the ratio of the capillary to the volume of the bulb.

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$$p = K y^2$$

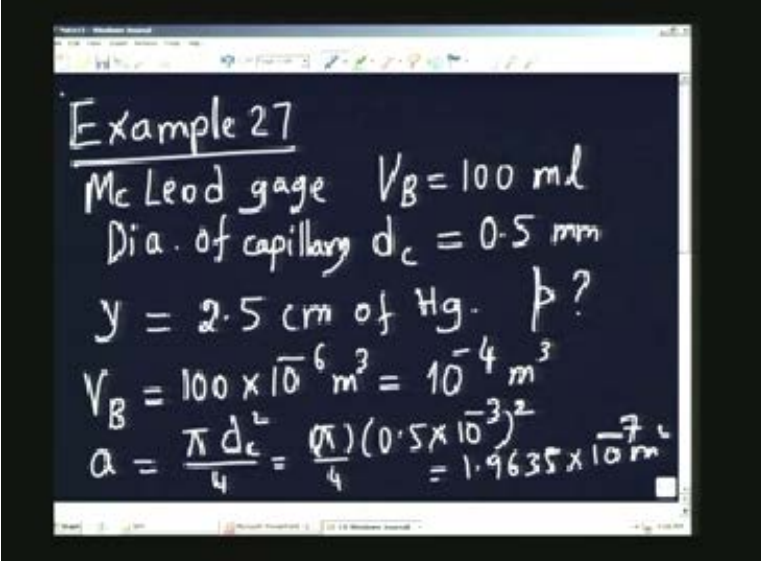
Gage constant
 $= \frac{a}{V_B}$

Here is example 27 to work out: We have a McLeod gage, the McLeod gage is named after the inventor of the instrument, so McLeod is the name of the person. V_B is given as 100 milliliters or cubic centimeters and capillary diameter we will call it as diameter of the capillary, d_c .,0.5 mm and we are also given the value of y in a certain measurement which is 2.5 cm of mercury. This is a particular case where we have made the measurement. So I want to find out the corresponding p for this particular case for the vacuum pressure. So all I have to do, is to substitute the values into that. Everything is put in SI units. So V_B is equal to 100 into 10 power minus 6 cubic meters is equal to 10 power minus 4 cubic meters and the area of cross section of the tube is π into d_c square by 4 π by 4(0.5 into 10 power minus 3) whole square, this will give you 1.9635 into 10 power minus 7 m square. And y also you must put it in meters that is 2.5 cm which is divided by 100 to get the meters, 2.5 by 100.

Let us substitute into the equation or you can calculate the gauge constant if you want and then calculate it. It is p is equal to ay square by V_B this is approximate because I am ignoring the minus ay in the denominator. If you want you can taken into account this will be $(1.9635 \text{ into } 10 \text{ power minus } 7 \text{ into } .025)$ whole square by $10 \text{ power minus } 4$ which is V_B and I want p in millimeters of mercury so I will multiply it by $10 \text{ power } 3$ because everything is in meters, that is so many Torr. So this will come to $1.227 \text{ into } 10 \text{ power minus } 3$ Torr or 1.227 milli torr . So the pressure is 1.227 milli torr . And if you remember the exact formula would have been p is equal to ay square by V_B minus ay . So the error can be calculated by calculating this quantity and comparing with these two quantities where you will be able to find out the error.

The error comes out to be in percentages $4.9 \text{ into } 10 \text{ power minus } 3\%$ a very small error, so it is quite negligible. That means this is a good representation but we need not use this instead we can use this directly p is equal to ay square by V_B .

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Example 27

McLeod gage $V_B = 100 \text{ ml}$
 Dia. of capillary $d_c = 0.5 \text{ mm}$
 $y = 2.5 \text{ cm of Hg. } p?$
 $V_B = 100 \times 10^{-6} \text{ m}^3 = 10^{-4} \text{ m}^3$
 $a = \frac{\pi d_c^2}{4} = \frac{\pi (0.5 \times 10^{-3})^2}{4} = 1.9635 \times 10^{-7} \text{ m}^2$

This is a good enough approximation for calculating the pressure of the vacuum chamber. With this we have looked at one way of measuring the pressure in a vacuum chamber.

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$$p \approx \frac{ay^{2v}}{V_B} = \frac{(1.9635 \times 10^{-7}) (0.025)^2}{10^{-4}} \times 10^3 \text{ Torr}$$

$$= 1.227 \times 10^{-3} \text{ Torr}$$

$$\text{or } 1.227 \text{ mTorr}$$

$$p = \frac{ay^2}{\sqrt{a-ay}} \quad \text{Error } 4.9 \times 10^{-3} \%$$

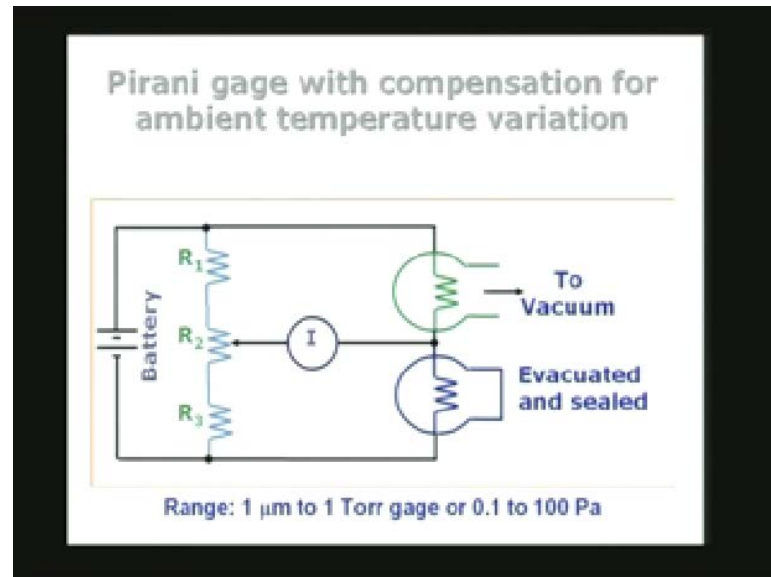
There are also other gages. We can classify vacuum into low vacuum then high vacuum then we have ultra high vacuum. At low vacuum we are talking about pressures below the atmosphere and on the negative side from below the atmosphere it could be anywhere up to minus 760 mm of mercury that is 76 cm of mercury below this thing. That means the pressure we are talking about is not very low, then if we go even lower we get what is called high vacuum.

In fact in practice the creation of vacuum requires different types of mechanical arrangements. We require different types of vacuum pumps to create different levels of vacuum. Rough or low vacuum requires rotary vacuum pumps or reciprocating vacuum pumps and then if you want to go to higher vacuum pressures you require something like a diffusion pump which is basically a method by which condensing oil is going to trap the molecules and take it out of the chamber. For ultra high vacuum you may require using a cryogenic pump or cryo pump. Or you may even use gutter pumps which are special pumps which evaporate a metal and after evaporation it is going to trap the gas and take it away.

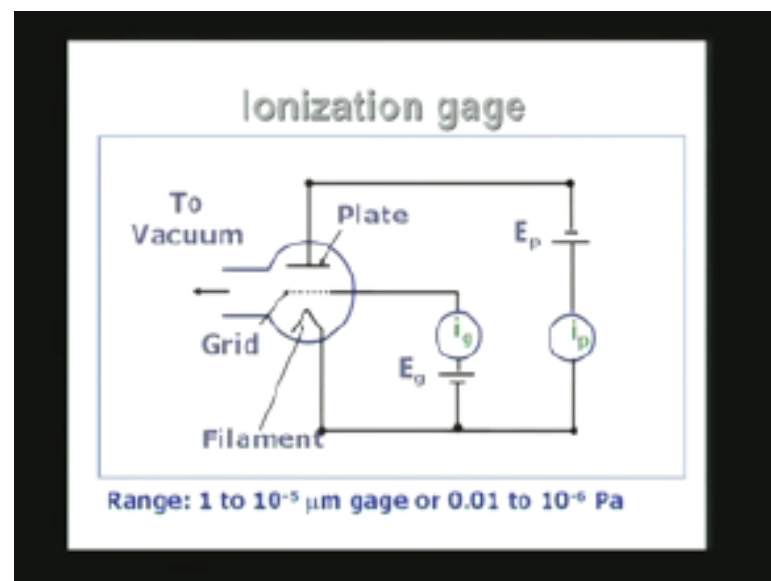
The vacuum to be measured comes in different ranges and to cover different ranges of course we require different methods. The pirani gage which uses essentially the resistance thermometer idea is for not very high vacuum 0.1

to 100 Pascal's and then we have what is called an ionization gage for much higher values or we can use an alphanatron gage etc. Thank you.

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