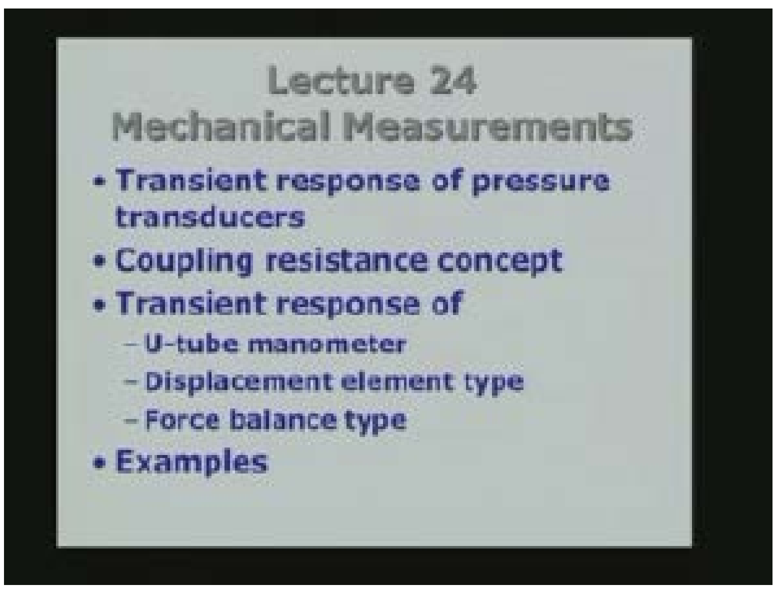
Mechanical Measurements and Metrology
Prof. S. P. Venkateshan
Department of Mechanical Engineering
Indian Institute of Technology, Madras
Module - 2
Lecture - 24
Transient Response of Pressure Transducers

This will be lecture number 24 mechanical measurements. We have been looking at measurement of pressure. Now let us look at the transient response of pressure measuring devices. Therefore let us look at how the pressure transducers respond to the pressures which are varying with respect to time.

(Refer Slide Time 1:25)



We will use the concept of coupling resistance to characterize the connection between the pressure point or the point at which pressure has to be measured and the location of a transducer. Normally the transducer is far away from the place where the pressure is being fed into it. So the resistance of the tube through which the pressure has to be propagate or the pressure information has to propagate is going to come into picture.

When we were talking about measurement of temperature we talked about the resistance to heat transfer between the system and the surroundings and these have something to do with the connective environment in which the system is located. Analogous to that in that case, we had a thermal resistance or flow of heat between system and surrounding and that decided what was going to be the response time of the system.

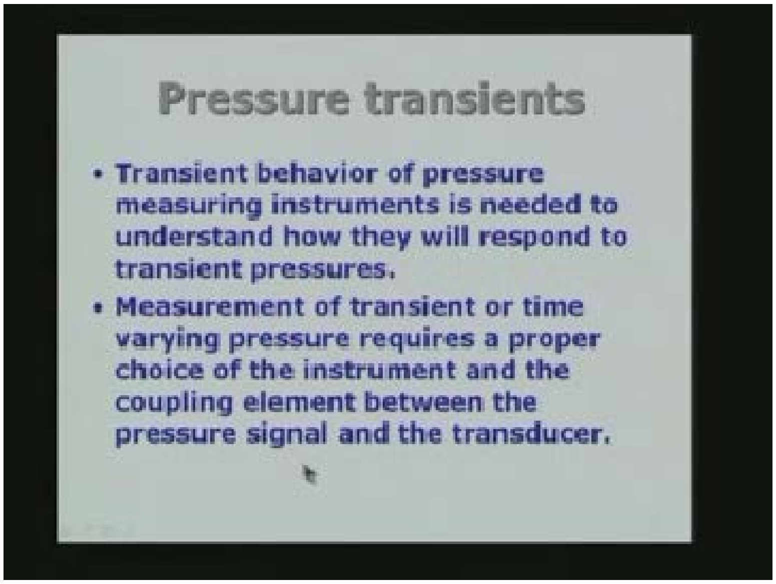
In an analogous fashion in the case of pressure transducer the connecting line between the point of measurement of pressure, and the location of the pressure transducer is going to be an important factor. The shorter the length, the faster is going to be the response. So the coupling resistance is like the convective resistance or the film resistance in the case of heat transfer from a system to surroundings and then of course there is inherent inertia of the system like the mass specific heat product or a thermal system the pressure transducer also has got inertia.

Let us find out how to characterize the time response of various systems. Let us take a few examples: One is the u-tube manometer which is a very simple system. But from the point of your transient, it is more complicated that the other two. Then we are also going to take a look at displacement element type and thirdly we are going to take the force balance type. These have been already looked at purely from the point of view of operation earlier. We are going to look at the transient behavior of these systems also.

Pressure transients occur in practice in many measurements. The pressure which we are going to measure is going to fluctuate in time or vary with respect to time. An example would be the pressure inside a cylinder of an internal combustion engine. The pressure is continuously going to vary as the piston undergoes the various strokes. And if you pick up the pressure within the cylinder using a transducer and plot it as a function of the location or the position of the piston you will get the pressure displacement diagram which is like the PV diagram which is familiar to us in thermodynamics. That is an example where I will be interested in finding out or measure a pressure which is varying with respect to time.

Depending on the speed of the engine this variation could be very fast or if it is idling for example it could be a slow variation, when it is running at higher speed the variation is going to be more rapid. There may be many other examples where pressure variations are going to be present and we want to measure them. Or for example, you take the simplest case which is the u-tube manometer and you are going to connect it to the pressure which I want to measure and suppose the pressure fluctuates or varies from a low level to a high level.

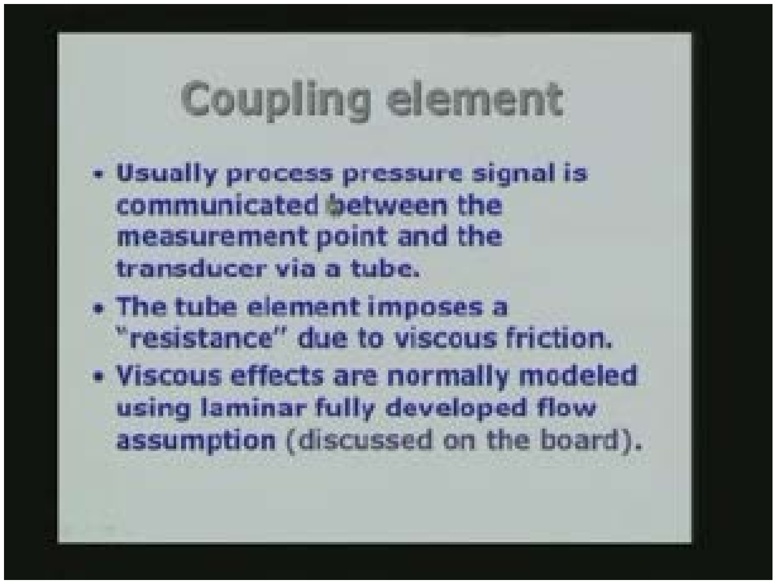
(Refer Slide Time 6:33)



So the pressure is going to vary form a low level and suddenly we change the pressure to high level, then the fluid within the u-tube is going to follow the pressure which is imposed on it and possibly goes into oscillations. Therefore the nature of this response depends on, in the case of u-tube manometer we are going to look at the various forces which are going to be coming into picture and, how the balance is going to be achieved. So the important thing to remember is that the coupling element and the resistance of the coupling element is one of the important factors which are going to come into picture.

In the case of u-tube manometer there is no coupling resistance by itself but there is a resistance to the motion of the fluid within the u-tube itself. So, it is an internal resistance we are talking about. The methodology of looking at the resistance is similar in all the cases. Therefore let us look at the coupling element to start with. As indicated here usually process pressure signal is communicated between the measurement point and the transducer via tube, it may be a tube which is a metallic tube or it could be a tube made of some plastic material like tygon, the tube may be very small, very short, very large and it can have different diameters and so on and so forth.

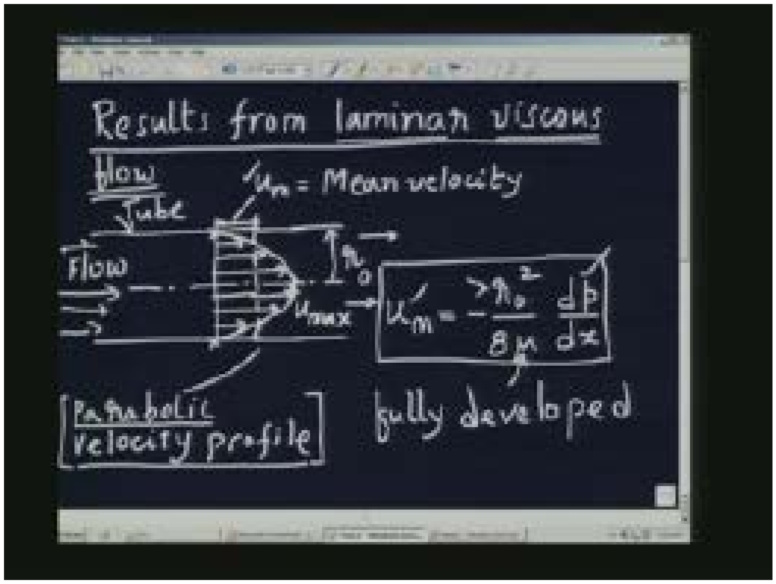
(Refer Slide Time 7:08)



Therefore the question to be answered here is how we are going to model the flow within the tube. And what we notice is that the tube element as you see in the slide imposes a resistance due to viscous friction, all fluids have viscosity and when the fluid moves through the tube it is going to experience a resistance to motion which is because of viscosity of the fluid. So these are normally modeled using laminar fully developed flow assumption which is justified by noting that the fluid motion within the tube is relatively at low velocity. That is most applications are going to be of low velocity and if we were to calculate the Reynolds number based on the mean velocity of the fluid inside the tube it will be in the laminar region. So, that is the assumption we are going to make. Hence let us look at the way we are going to model this.

We are going to borrow some results from laminar viscous flow theory. Let us take the important results form the laminar viscous flow theory and then use it for our purpose. Here is a tube which is a circular cross section tube and the fluid flow is taking place from left to right, it enters here and goes out from the right side. If the tube is long enough, the velocity profile inside the tube or the variation velocity within the tube, because of laminar viscous flow will be what is called fully developed. That means the shape of the velocity profile is not going to change along the length of the tube but it is going to remain the same. And of course the same amount of fluid is moving across from left to right so the velocity profile totally is unchanged as the fluid progresses along the length of the tube.

(Refer Slide Time 11:21)

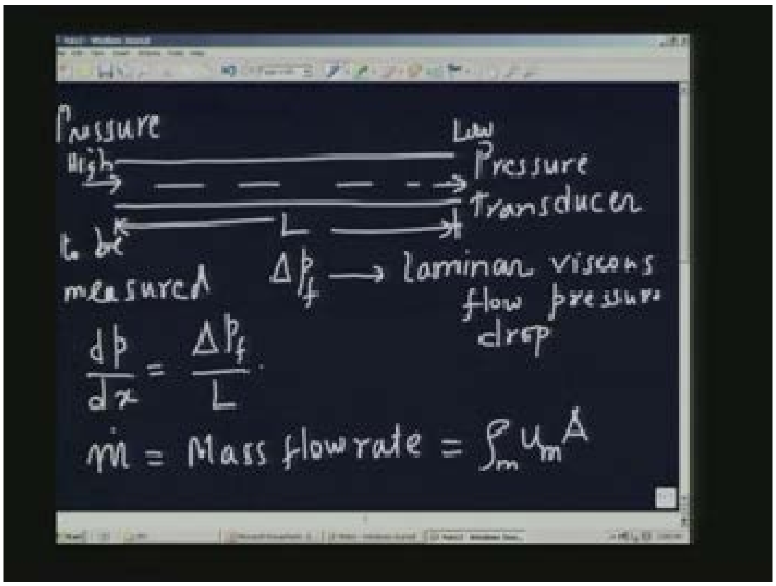


Now the flow takes place from left to right. The flow velocity variation with respect to radius in the real direction remains the same. And this velocity profile is given by what is called a parabolic velocity profile which is indicated by a parabola here, the velocity is 0 at the walls and a maximum at the center line or the axis of the tube. I can in fact, calculate based on the mass flow rate I can replace the velocity profile which is parabolic by a mean velocity which will be a rectangular profile. And we can in fact show that the velocity, mu_m which is given here, is related to the radius r_o square and in the denominator, I have the viscosity mu and then I have the pressure gradient dp by dx. So this is the relationship I am going to borrow from laminar viscous flow theory.

Now with this background let us look at what is going to happen in the case of tube flow. So let me just take a long tube,and I am going to have the pressure transducer here at one end and the pressure to be measured is at the left end. So, if I take a certain length L of the tube because of friction when the flow takes place from right to left from this side to this side, there is going to be a friction in the tube, and therefore there is going to be a pressure drop which I call as the pressure drop due to friction delta P subscript f. This is identified with the pressure drop due to viscous flow and this will be due to laminar viscous flow pressure drop.

Actually, the pressure drop will be negative for the pressure here is higher and the pressure here will be lower. This is high and it is low. Of course finally the two pressures are going to be the same. Initially when there is a flow there will be pressure drop so the pressure at induct will be higher than the pressure here. And we can show that dp by dx will be nothing but delta P due to friction divided by L, and this is going to go along with the expression between u_m. So you can see that the mass flow rate is nothing but the density of the fluid rho_m multiplied by the velocity of the mean velocity multiplied by the area of cross section of the tube rho_m into u_m into A so this will be m.

(Refer Slide Time 14:20)

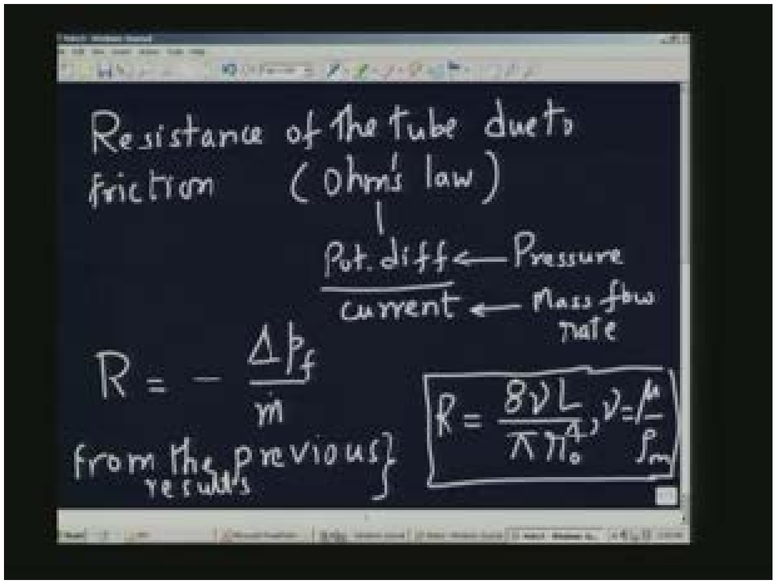


So we will introduce an important definition here. We will say resistance of the tube due to friction. The analogy will be with ohms law. In electricity we have the ohms law. The ohms law says that the current is potential difference divided by the resistance. So we will identify the potential difference here with the pressure difference and we will identify the current and instead of the current I have the mass flow rate. So we will say that this is potential difference divided by current. So, if this is in electricity, in fluid flow problem the potential difference is pressure drop pressure and current will be mass flow rate. Actually current is also a rate. It tells us how many charges per second are flowing through the conductor. The potential difference is the voltage across the conductor from one end to the other end and that is how you get the ohms law.

So I am just getting to know the analogy with respect to ohms law. Therefore I will say that resistance R will be minus delta p_f by m. This minus is to just take care of the fact that the pressure is actually decreasing in the direction of mass flow rate. I want the resistance to be a positive quantity therefore I will say minus delta pf by m. All I have to do now is to combine the expression. If you remember, I have got u_m is equal to rho square by 8nu dp by dx and in fact this dp by dx is nothing but delta p_f by L. So I am going to take this expression and the second expression I have got is rho_m u_m into A equal to the mass flow rate so I am going to substitute here and then number three I am going to substitute delta p_f m dot and obtain the resistance.

Therefore from the previous results after some simplification we can show that R will be nothing but 8nuL by pi rho power 4 where nu is equal to nu by rho_m. This is the concept of the resistance due to the flow in a tube which is in analogy with the ohm’s law of electricity where the potential difference is divided by current with resistance where the potential difference is the pressure drop and the current is the mass flow rate. So the ratio of these two of course with a negative sign is because I want the resistance to be of positive quantity when the pressure is actually decreasing in the direction of flow.

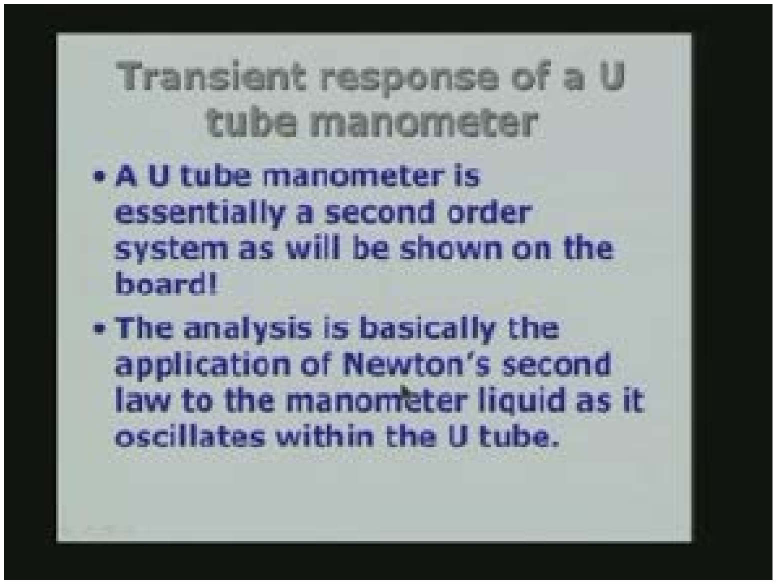
(Refer Slide Time 18:42)



This is the important result I am looking for. This result will be used in various applications. Let us look at one of them which is the case of transient response of a u-tube manometer. The u-tube manometer consists of a heavy liquid or a liquid of suitable density which is taken in a u shaped tube held vertical in orientation and one limb is connected to the pressure which is to be measured. The other limb is connected to a pressure which is a reference, may be atmospheric pressure or may be some other pressure.

Essentially the head or the difference in the height of the manometric liquid in the two limbs multiplied by the gravitational constant and the density of the fluid is going to give you the pressure difference across the two sides and that is how you measure the pressure. So the head is developed proportional to delta p and that is the manometer head. Essentially we are going to show that it is a second order system. The analysis is basically the application of Newton’s second law of motion to the manometer liquid as it oscillates within the tube. I am already foreseeing that the liquid is going to oscillate in u-tube. The u-tube manometer is going to oscillate when you suddenly connect the pressure to be measured and then settle down. Finally of course it has to come to a steady state when the head will become a steady value. Before it goes into that particular value it is going through go through oscillations. This is a schematic of u-tube.

(Refer Slide Time 20:03)



At any time the fluid is like this a manometric liquid is occupying this side. So at any time this is a function of time because I am talking about transient. So I am applying a pressure here and I am applying another pressure so delta P is the pressure difference between these two. And initially what I expect is that it is at the datum like this, this is before we apply the pressure the initial stage. And then, once you apply the pressure it starts going through motion and I am capturing what is happening at a particular instant of time. The terminology is the following:

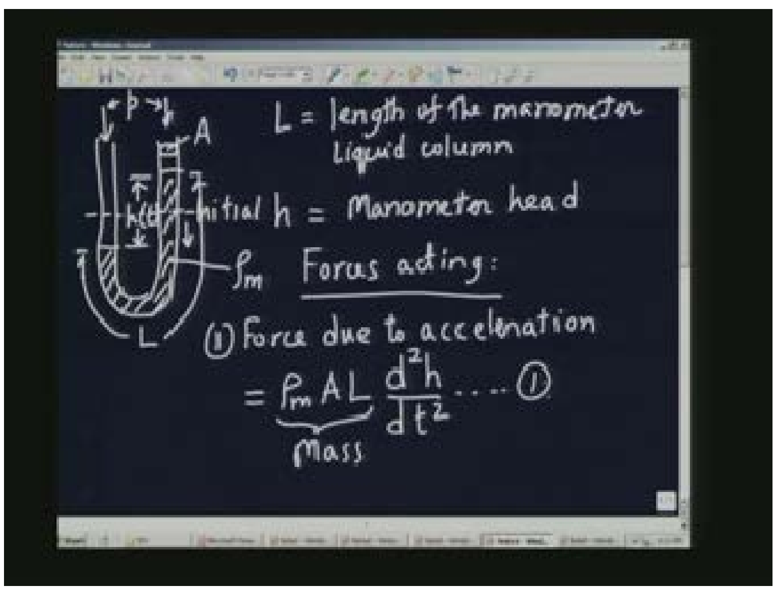
This is the length of the column, L is the length of the manometer liquid column. This is the total length and h is the manometer head which we have already talked about earlier. The area of cross section is you can have a circular tube, the area of cross section is A, and the liquid has a density of rho_m and it has a viscosity which will be mu and so on. So I want to find out what are the forces which are acting on this fluid and the liquid at any instant t. There are actually four forces.

When you want to write the second law of motion for the liquid, I have to identify all the forces which are going to be coming into the picture and then I have to balance the forces. That is what the second law of motion is all about. So, one is the force due to acceleration because the fluid which is oscillating it has got a certain velocity and certain acceleration, force due to acceleration, this will be given by the mass of the fluid or mass of the liquid that will be rho_m A into L, A into L is volume, rho_m is the density, this is your mass. And we will assume that the height h is changing with respect to time and the derivative which gives the acceleration is d square h by dt square. So this is the force due to acceleration.

Let us look at the other forces. Second force is the force which is giving rise to the change. The manometer is changing, the liquid is oscillating only because there is an external stimulus to do it. This is given by the force due to pressure and that can be written as p into A. This is the pressure which is helping h to increase or change. Then we have the force opposing the motion.

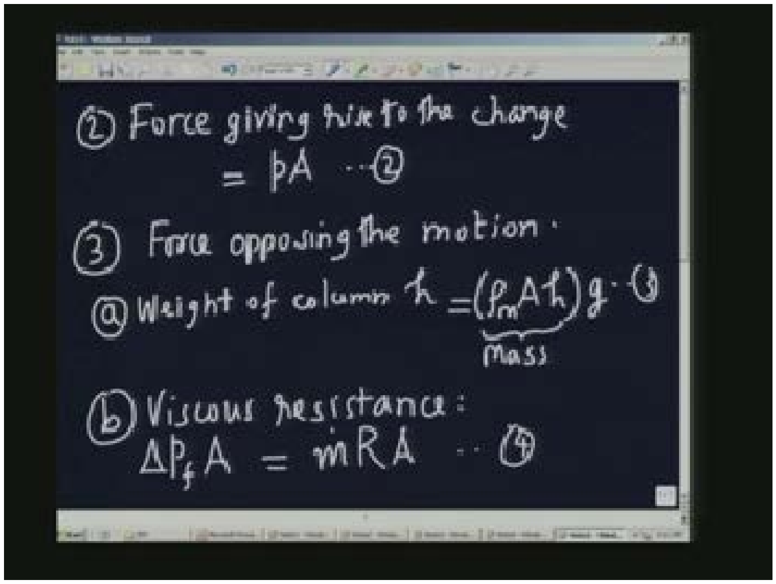
Actually there are two forces opposing the motion. Therefore I will say weight of the column h. We can see here that there is a height of the column it is applying the force in this direction and h is trying to go up whereas this is going to come in the lower direction so there is a resistance to motion because of the weight of the column h. This is given by the density rho_m and A h, this is your mass into acceleration due to gravity.

(Refer Slide Time 24:39)



This is not the total mass, the total mass would have been rho_m A into L which is for the entire column. Here it is only the extra height which you are having on the right side of the u-tube manometer. Then there is the viscous resistance.

(Refer Slide Time 26:59)



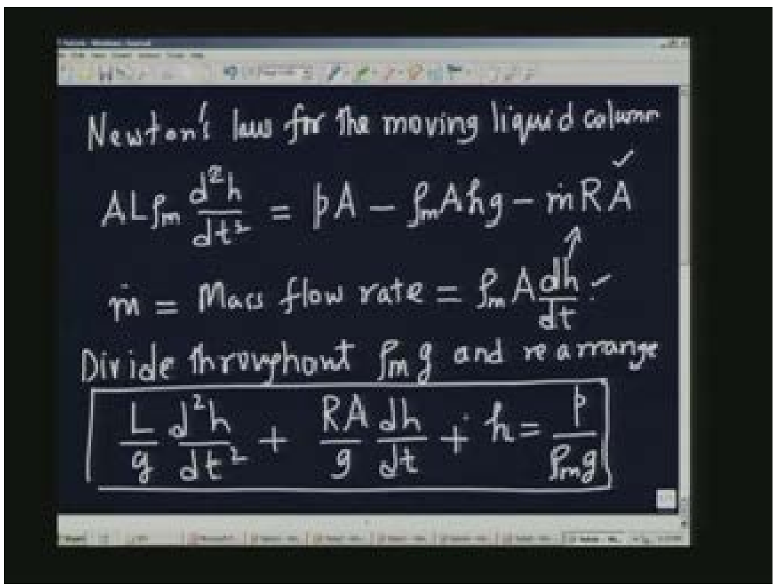
I am assuming that the entire liquid in the manometer is moving over to one side. So one would assume that each point in the liquid is moving up with a velocity equal to dh by dt, dh by dt is the velocity of the liquid and I am assuming that it is incompressible therefore the velocity is dh by dt. Now how does the viscous friction come on top of that?
This is the main problem.

I am assuming that when it is moving up it immediately goes into the fully developed flow so that actually it is moving up it is a parabolic velocity profile. But I am looking at two things the mean velocity dh by dt, I will call dh by dt as the mean velocity and there is a velocity profile in the tube which is due to this viscous effect because in the central line of the tube the velocity will be higher and the velocity at the sides where it is in contact with the surface of the tube is 0.

Let us look at it as a simple modeling, because everything ultimately comes into how we look at it. So with this background, let us look at the resistance due to viscous resistance which will be given by the definition of resistance. By the definition of resistance we know that resistance is equal to the pressure drop divided by the mass flow rate. So the pressure drop multiplied by area is a force. So what I want is delta p_f multiplied by the area of cross section. The viscous resistance is nothing but delta p_f multiplied by the area of cross section. This is what I want delta p_f into area of cross section is

what is required and this will be nothing but delta p_f is m dot into R, by definition of the resistance multiplied by the area of cross section. I can say this is 1, this is 2, this is 3 and this is 4.
So there are four forces. The second law of motion will tell us that the net force will be the net positive minus the net negative. We have to take into account the forces which are helping the motion to come about and the ones which are going to prevent the motion from taking place. So I will write the Newton's law for the moving liquid column. This is what force balance is. I have AL rho_m d square h by dt square which is the net acceleration force, this must be equal to positive minus the negatives.

(Refer Slide Time 34:34)

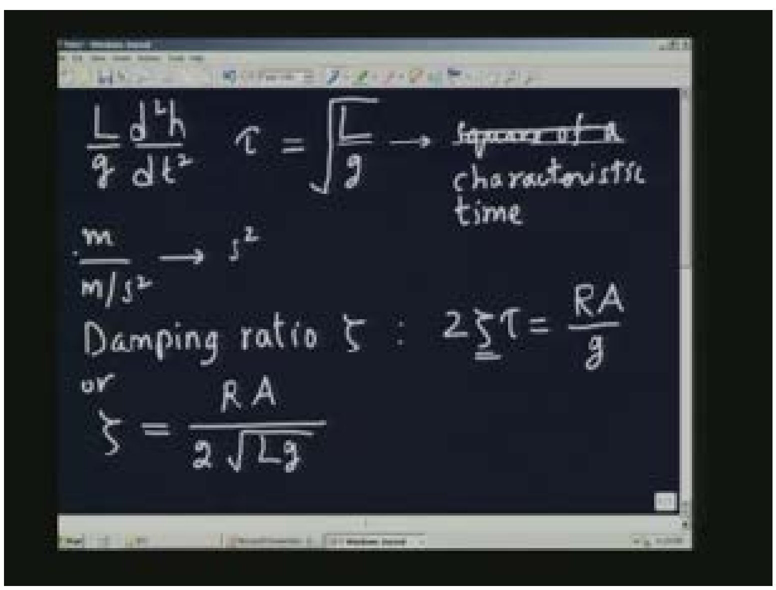


So we have pA minus rho_mAhg minus the viscous force m dot into RA and m dot itself is the mass flow rate is given by rho_m A into velocity dh by dt again by using our idea of modeling. So I am going to substitute this here. I am also going to do the following. I am going to divide throughout by rho_mg and rearrange so I will get the following equation L by g d square h by dt square plus RA by g dh by dt which is this term. This term is going to come here after I introduce this term plus h that is this term equal to p by rho_mg. This is the equation which governs the variation of h. So what we notice is that it is a second order differential equation.

And in the case of a thermal system we had a first order system there. First order system is governed by a first order differential equation. Here because the equation is governing the problem is a second order equation here because the equation is which is governing the problem is second order equation the u-tube manometer liquid is a second order system. Its motion is governed by a second order differential equation because this is nothing but the application of the Newton's laws of motion and Newton's law will automatically bring in the secondary derivative as what is happening here. It is the height which is varying with respect to time and the equation is given in this form.

Usually equation will be written slightly in a different form which we call as the characteristic form so what we will do is we will define some new quantities. For example, we have L by g d square h by dt square, L here has units of meter, g has got m by s square, this has units of s square so L by g is a square of a characteristic time. If you remember in the case of a first order system we had a characteristic time we called it the time constant of the first order system. Here we have a characteristic time, of course L by g is square of a characteristic time or if I take square root of L by g it should be a characteristic time. I remove the square it is simply the characteristic time. So I will call this as tau. Then we will also introduce what is called the damping ratio.

(Refer Slide Time 38:11)

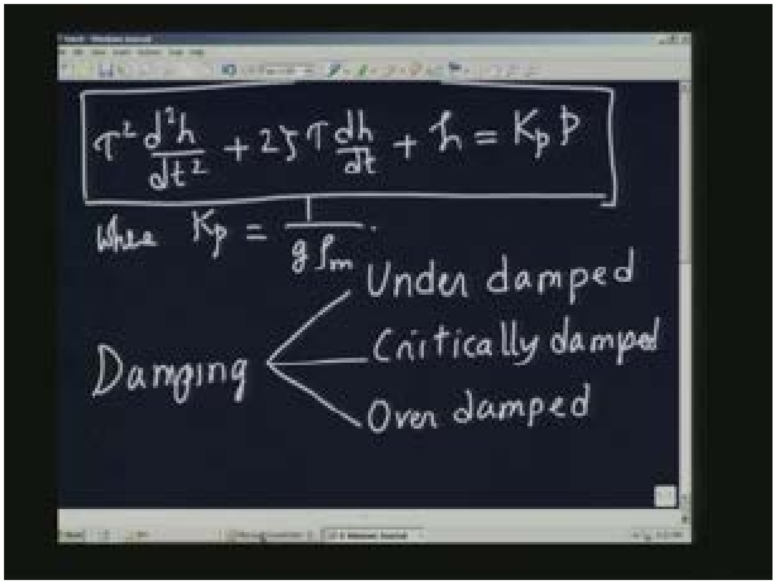


In Applied Mechanics you would have come across the vibration problem, you would come across an equation similar to this. This is actually the vibration of a spring mass system with damping. The first term is the acceleration term, the second term which is proportional to velocity dh by dt is the damping term and the damping comes because of the viscosity in this particular case, laminar viscous flow will give rise to a linear damping and then the third term here is the displacement and the spring constant is going to come here.

Of course the spring constant is not appearing here, because in this case I have L by g which has gone into this particular thing. So you would have a spring constant coming into the term corresponding to the displacement term. So there are three terms here; this will be the acceleration term, this is the damping term and this is the spring term. And on the right hand side you have the forcing function. The analogy is very complete and what I am doing is I am going to just introduce the damping ratio which is usually given the symbol zeta.

We will say 2 zeta tau is equal to RA by g. This is what I am introducing but I am introducing in terms of the other quantity. Actually if I substitute the values you can see that zeta will come out to be RA by 2 square root Lg. R is the viscous resistance, A is the area of the cross section of the tube divided by 2 square root Lg where L is the column length of the liquid and g is the acceleration due to gravity. So, if I introduce all these things into the equation governing the problem I will get the following: L by g will be tau square I am going to remove that and finally what happens is the equation governing the problem tau square d square h by dt square plus 2zeta tau dh by dt plus h is equal to P by rho_gm. I will write it as Kp P where Kp 1 by g rho_m.

(Refer Slide Time 40:40)

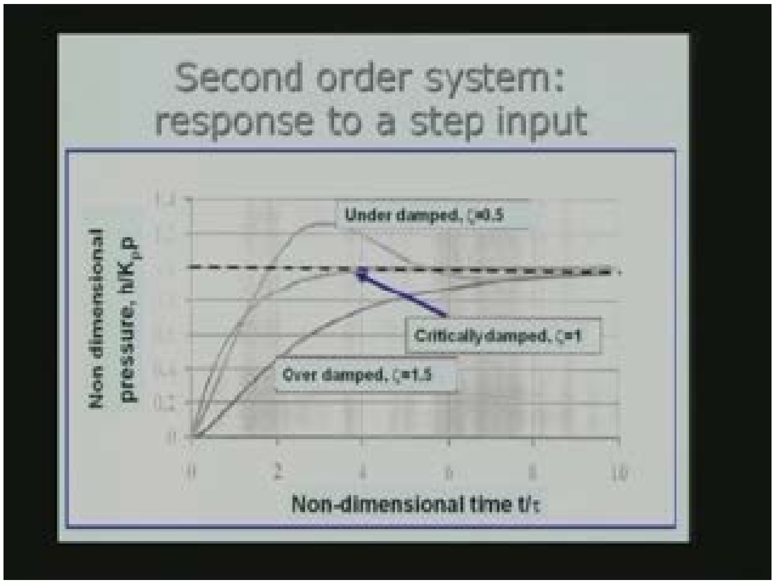


This is the final form of the equation and this is second order equation. And depending on damping I can have different cases. We can have what is called under damping or under damped. We can have what is called critically damped or we can have the over damped case. So, three cases are possible.

Now what are the differences between these cases?
In the case of under damped system the liquid is going to undergo periodic oscillations, the flow is periodic. That means it will go up and down, up and down and then finally settle down. In the case of critically damped system it will just approach the steady state value from one side. And over damped case also it is similar. The over damped case it will take a long time for the value to settle down to the steady state value.

We have a second order system. We are talking about what is called a response to a step input. This is just like what we had in the case of first order system thermal system when we suddenly change the temperature of the surroundings.

(Refer Slide Time 40:43)



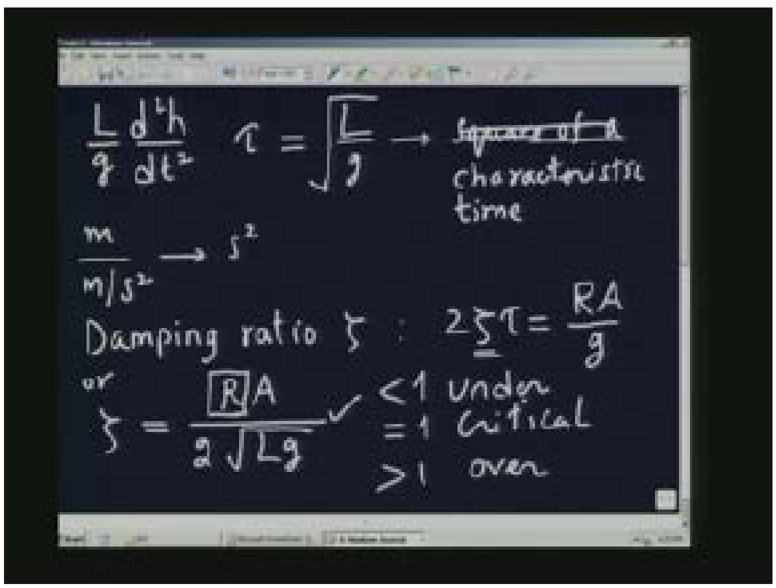
Here for example initially the pressure is the same on the two sides of the u-tube. Suddenly the pressure on one side is taken to a value is equal to 1 this is the value. This dashed line shows the input or the step input in this case. On the x axis, on the time axis what I have done is I have taken the ratio of t to the characteristic time t by tau. I have plotted t by tau along this axis up to about 10, I have taken 0 to 10. On the non dimensional pressure which I call as h by KpP, this is non dimensionalized with respect to the forcing function KpP is the value which is input, h is the height of the column and if the system were to have a very fast response it would immediately go here and show this value.

Of course the system is damped whether it is under damped or over damped but certainly there is a damping. So what happens is, in under damped case I have taken zeta is equal to 0.5 which corresponds to zeta less than 1 in turn corresponds to under damp case. As the function goes up the pressure goes up the height overshoots and goes to about 1.2 a little more than 1.2 you see that about 25% or more then it is slowly comes down and it goes below and again after ten time constants it is just about to settle down to the steady state value.

Critically damped has a very fast response faster than the under damped case but it never goes to the positive side. It just goes here and at about a value equal to 4 or 5. It has already become very close to the steady state. In the case of over damped it is going to take a long time. Even here it has not reached the value. It will take a long time for it to reach the steady state. So what does this particular figure indicate?

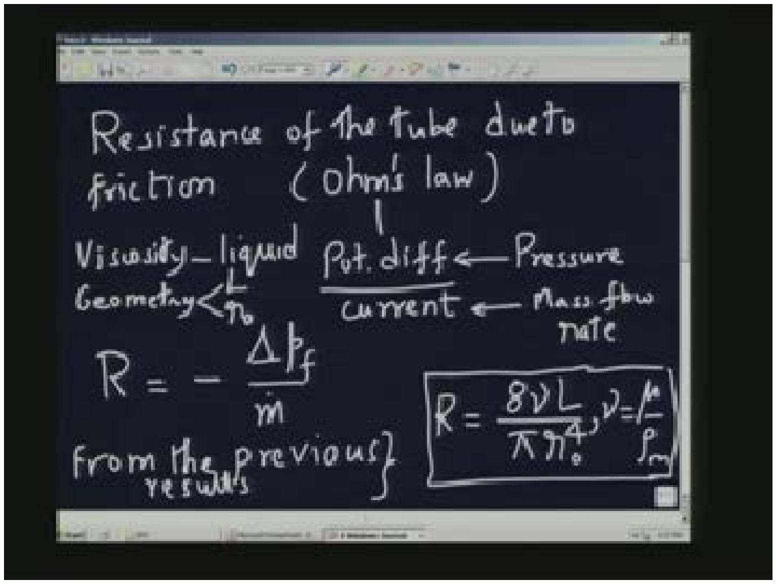
It indicates that the damping of the system is going to play a very significant role in determining the behavior of the system. To understand this further let us look (Refer Slide Time 40:40) at the parameter zeta where this parameter is going to determine the damping. And if you look here (Refer Slide Time 38:11) this is what is going to determine the damping. The damping parameter is this, if it is less than 1 it is under damped, equal to 1 is critical or critically damped and greater than 1 is over damped. So the important parameter which is going to determine that is this R a fluid resistance.

(Refer Slide Time 44:32)



The fluid resistance is 8nu L by pi into r_o power 4 so it depends on the viscosity, viscosity depends on the liquid so the liquid you take inside the tube is going to be an important parameter that is its property. Then it depends on the geometry, geometry depends on the length of the column and the radius of the tube of the area of cross section. And you see there is a r_o power 4.

(Refer Slide Time 46:10)



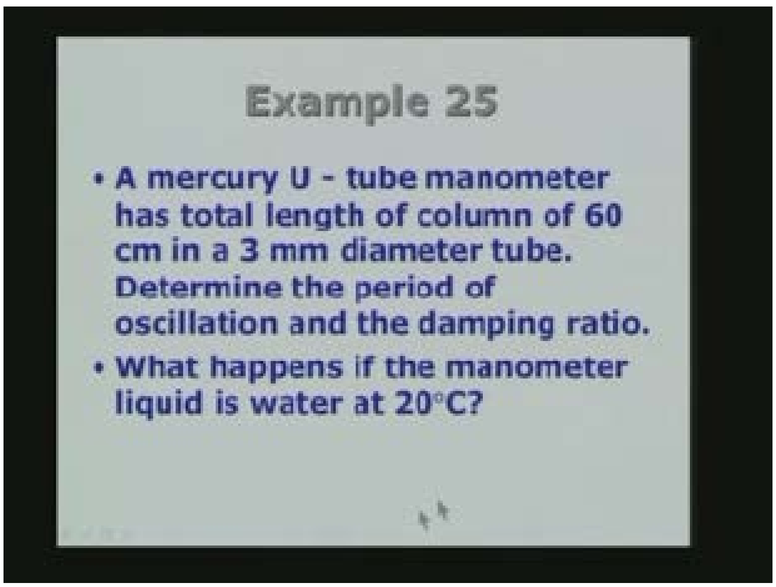
If you take smaller and smaller radius the resistance becomes larger and larger and therefore if you take a u-tube manometer with very small diameter for the u-tube you will have possibly over damped condition. Again, if the length is very long, if you have a very large column for the liquid in a manometer you are going to having a large resistance, because L is occurring in the R here. The long length and small diameter are going to drive it towards over damped condition. Of course, viscosity also going to play a role may not be a very major role but the major role is going to be due to the length and the radius of the tube.

Normally in practice, we take a manometer tube may be a few millimeters diameter 3 to 5 millimeters diameter sometimes a little more so the diameter is going to be about 3 to 4 millimeters. The length of the column is determined by the density of the material of the liquid and the largest pressure you would like to measure.

So if you remember the largest pressure you may measure is determined by the head difference between the two sides of the u-tube column. So if you want to measure a certain pressure that determines the length of the column because the height which you want to measure is related to length of column. So you require a long length if you want to measure a large pressure difference. Therefore the design of the manometer will require the choice of the material or the medium which is used as the manometer liquid may be mercury, may be water or may be some other liquid. Then the length of the column and the diameter is going to be determined in terms of a few millimeters.

Hence, you see that already the characteristic of the u-tube manometer is decided once we have decided to have these particular parameters chosen to have certain values to meet with the requirements. We will take the example of a u-tube manometer and see how to look at the transient behavior. This is example 25. This is a mercury u-tube manometer. It has a total length of 60 cm which is the column length and if it is 60 cm of the liquid height it gives you the idea of the maximum pressure we can measure.
It will be probably little less than this and it has a 3 mm diameter tube.

(Refer Slide Time: 49:26)

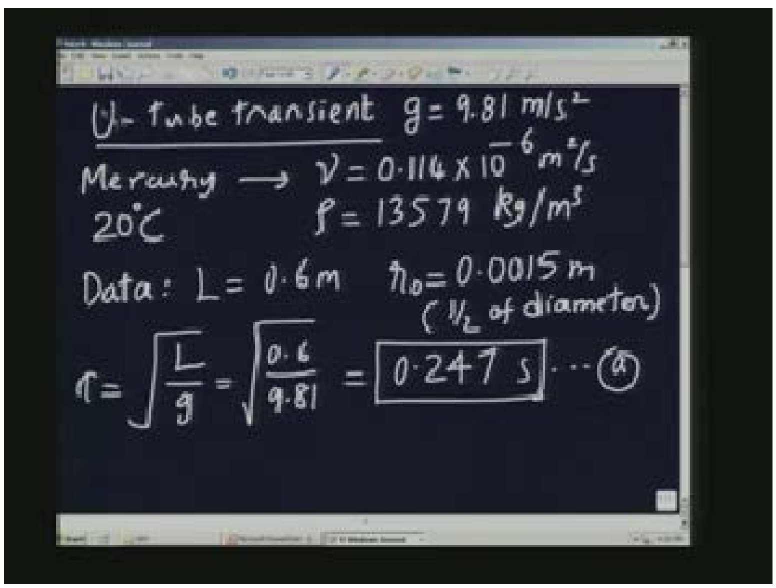


We want to determine the period of the oscillation and the damping ratio. The period of oscillation is nothing but the characteristic time and damping ratio we wanted to determine. In this case the liquid is mercury. And I also want to find out what happens if the manometer liquid is water at twenty degrees Celsius instead of mercury. If I change from mercury to water what is going to happen to the time constant and also to the damping in this particular case?
Of course time constant is square root of L by g does not depend on the nature of the liquid. It depends only on the length of column and the gravitational constant. Therefore it is not depend on the material or the liquid

medium which is used for the manometer liquid.

This is the u-tube transient. For example, we can get the properties of mercury from here, nu is 0.114 into 10 power minus 6 m square by s. I am taking everything at 20 degrees Celsius. And the density is 13579 kg by m cube. We are already given the data. L is equal to 60 cm and we are converting it into meters 0.6m and r_o I am given the diameter of the tube, r_o is diameter by 2 that will be 3 mm by 2 so it is 1.5 mm and I will write it as 0.0015m. So this is half of the diameter because if you remember the resistance is based on the radius of the tube and the length of the column. And of course we will take g is equal to 9.81m by s square which may vary slightly between different places so it may not be a major parameter to worry about, the variation may not be very important.

(Refer Slide Time 52:56)

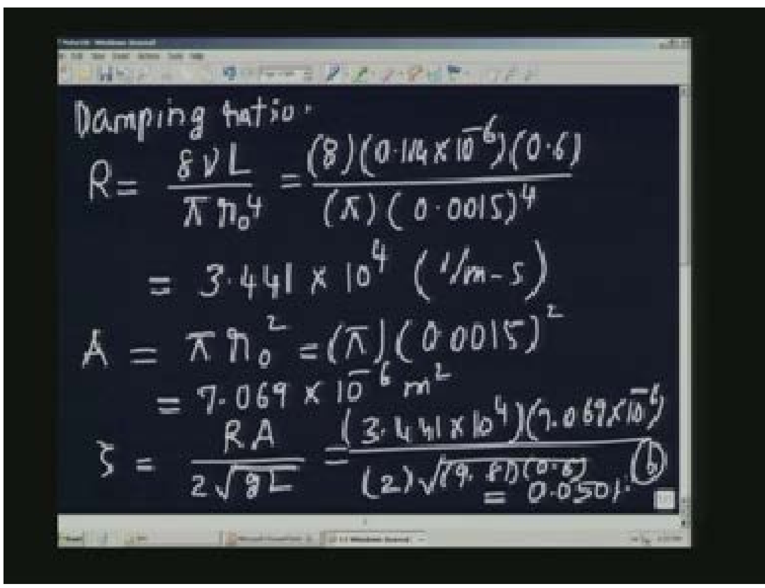


So immediately I can calculate the characteristic time is square root of L by g square root of 0.6 by 9.81 which comes to 0.247s. So the length of the column is 0.6, and the characteristic time for the system is 0.247 by s so this is one answer we wanted and we will call it as (a). The next answer we are interested in is finding out the damping ratio which is the other parameter which is going to characterize the system, the time constant is here, the characteristic time is here. So, to calculate the damping ratio I need to calculate the various things which are going into that expression. Therefore the liquid resistance R is given by 8 nu L by pi r_o power four 8 nu is (0.114

into 10 power minus 6), length is 0.6m by pi (0.0015) whole power 4 given by (3.441) (10 power 4) and units is 1 by m-s. So the area of the cross section of the tube is pi r_o square. This will be (7.069) (10 power minus 6) ms. All these are required in calculating the value of zeta. So the damping ratio zeta is nothing but RA by 2 square root of gL and I just plug in the values R into by 2 square root of 9.81 into 0.6 this comes to 0.0501. So the damping ratio comes out to be 0.0501.

What does it mean?
It means that it is under damped because it is less than 1 therefore we have under damped system. If you observe mercury in u-tube manometer invariantly you will see that it is going to be an under damped system and you can subject it to a sudden application inside, It will go through lot of oscillations. In fact that is the answer (b) which was required in our problem.

(Refer Slide Time 55:56)



i

The next one is, if the liquid is water instead of mercury. The only thing which changes is nu and of course we want rho also. These are the only things that change. Of course the change in density is important because the u-tube now will measure much small pressure difference across the u-tube. The head corresponds to much smaller pressure. So all I have to do is to recalculate the values using new for water at 20 degree Celsius which is 1.006 into 10 power minus 6 m square by s. Of course the density is also smaller. It is about thirteen times smaller than the density of the other one.

In fact I can calculate R which is proportional to nu. Therefore this will come to a value of 3.036 into 10 power 5, the resistance has changed and this is again 1 by m-s and the parameter zeta becomes 0.4424 less than 1 it is still under damped but it is much more damped than the case of mercury which you saw earlier.Thank you.

(Refer Slide Time 58:15)