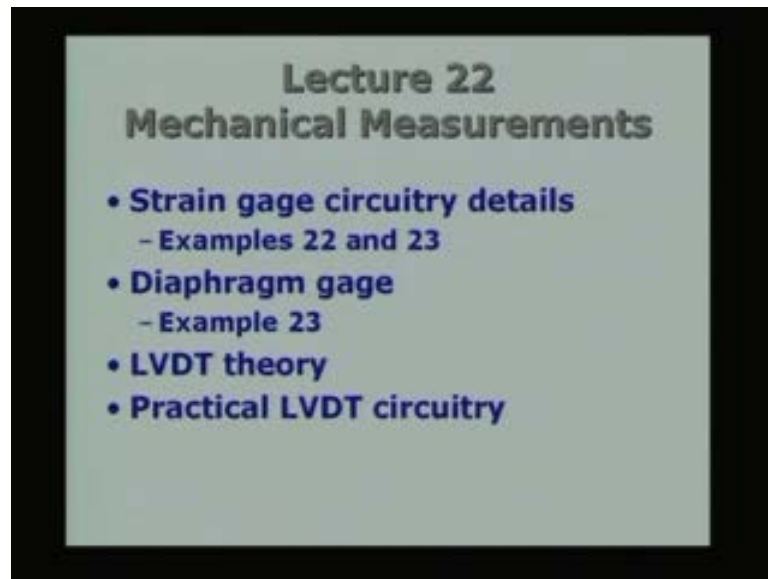


**Mechanical Measurements and Metrology**  
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**Module -2**  
**Lecture - 22**  
**Pressure Measurement**

This will lecture number 22 on the series of Mechanical Measurements. Towards the end of the last lecture we were looking at how a strain gage is used for measurement of displacement or strain. And in fact we were also looking at the elementary circuitry which is the Wheatstone bridge which is used for determining the changes in the resistance in terms of an imbalance voltage.

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So what I planned to do today, is to continue from there, just recapitulate what we did in the last class and then look at one or two simple examples to find out what we expect from strain gage circuitry. Then we will move on to the example of a diaphragm gage for which some of the basic formulae were formulated that was given in the last lecture. So we will take an example to look at what to expect from a diaphragm gage in terms of the thickness of the gage the diameter and so on and so forth. Subsequently we will look at another way of measuring displacement which of course can be used for

measuring displacement in its own right or for measuring displacement consequent to a pressure signal which is converted to a displacement signal by using either a diaphragm gage or a bellows gage.

The use of LVDT or the Linear Variable Differential Transformer as it is known requires the understanding of circuitry, how it is going to function and also some principles of operation of the circuit and how it is used. This will form the later part of the lecture. So in essence, the present lecture looks at the measurement of displacement and relates it to pressure by two different methods; one is by using the strain gage and the second one is by using an LVDT as the transducer which changes the pressure signal through a diaphragm or a bellows to a change in displacement or a movement and then that is stressed by using LVDT. It is essentially an electrical means of looking at the displacement. So let us look at the method of measuring the strain. So we will go back to the strain gage circuitry which we talked about in the last lecture.

Basically a bridge circuit, in fact we called it the quarter bridge we were just about introducing this terminology. So, essentially the quarter bridge circuit consists of the following:

We have four resistances arranged in the form of a bridge which is very familiar to all of you. So we numbered them as  $R_1, R_2, R_3$  and  $R_4$  and we have an input voltage impressed by using a battery so this voltage is  $V$  supply. And we are going to measure the potential difference across these two points and this we call as  $V_0$ . And if you remember, towards the end of the last lecture we said that this will be the strain gage element and the resistance of this particular element is going to vary because of the displacement or because of the straining of the element. We also mentioned something about the values of these resistors.

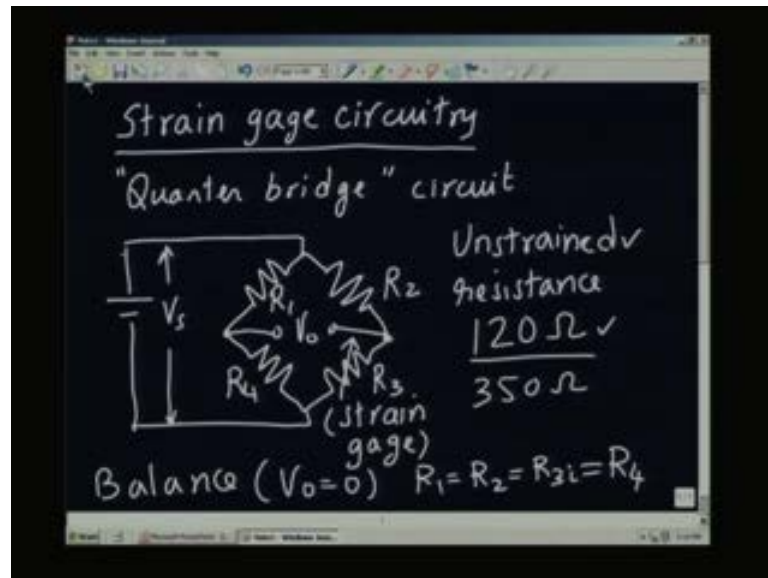
Normally the resistance of a strain gage in the unstrained condition that means that without any strain being impressed, unstrained resistance, or we can call it the base value is either 120 ohms standard or 350 ohms and we were actually talking about one of them, the 120 ohms case. The balance requires, balance means in brackets I will write it  $V_0$  is equal to 0 for any value of  $V_s$ , whatever be the value of  $V_s$  the value of output voltage will be exactly equal to 0 if the bridge is under balance which requires, in this case what we are going to do is, we are going to look at the balance under the unstrained resistance case, in which case we will say that  $R_1$  is equal to  $R_2$  is equal to  $R_{3i}$  standing for the unstrained case, you can call it as the initial

value before a strain is introduced equal to  $R_4$ . Actually we already know that the ratio  $R_1$  to  $R_4$  is equal to  $R_2$   $R_3$  by  $R_1$  by  $R_4$  is equal to  $R_2$  by  $R_3$  is actually the condition for balance. And if all the resistors are equal the balance is of course going to be satisfied.

We also saw in the last lecture, that when the resistance of the strain gage changes here, what will happen is that the voltage across this  $V_o$  which is the output voltage is going to become either positive or negative depending on the direction in which the resistance is going to change, if it compressive or if it is compressive strain or elongation. If it is elongation what happens is the resistance  $R_3$  is going to change to a higher value,  $R_3$  will increase. This is what we have seen from the basic theoretical framework of the strain gage,  $R$  is equal to  $\rho L$  by  $A$ . And we have already discussed the way the length is going to change, the elongation and the area is going to change because of the Poisson's ratio. Therefore if it is under balance with all the resistance being equal when the resistance  $R_3$  which is a the strain gage in the unstrained condition we have a balance and now we would like to what happens when the resistance or the strain gage undergoes a change in length which means there is a strain imposed on that. This is what we are going to look at.

Let us look at the relationship between  $V_o$  and  $V_s$  which I gave in the last lecture. We will recapitulate that and write down the value. We saw that  $V_o$  by  $V_s$ , ratio of the output voltage, the input voltage, is given by  $R_3 R_1$  minus  $R_2 R_4$  in the numerator divided by  $R_2$  plus  $R_3$  into  $R_1$  plus  $R_4$  this is the expression. In fact you see that if the numerator vanishes there is balance and that happens if  $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$  are all equal then it is going to vanish. This is what we saw in the last slide, in the previous expression. Suppose the strain gage undergoes a small change in the resistance because of strain which it experiences, so  $R_3$  is equal to  $R_{3i}$  originally.

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So I will just modify it here;  $R_{3i}$ . It changes by a small amount to  $R_{3i}$  plus  $dR_3$ , a small change is going to take place because of the strain. So all I have to do is, wherever  $R_{3i}$  is there I have to replace it by  $R_{3i}$  plus  $dR_3$  and then we will make the approximation that  $dR_3$  is very small compared to, or the change in the resistance is small compared to  $R_{3i}$ . And therefore in the denominator of this expression, I should replace it by  $R_{3i}$  plus  $dR_3$  but I will ignore the  $R_3$  in that. And also in the previous page, we have indicated that  $R_1$  is equal to  $R_2$  is equal to  $R_{3i}$  is equal to  $R_4$  so we will impose that condition in this case. So if I do that this will be  $R_{3i} R_1$  minus  $R_2 R_4$  so what I am going to do is change  $R_{3i}$  to  $R_{3i}$  plus  $dR_3$ . So what I will do is, write the expression here,  $V_0$  by  $V_s$  is equal to  $R_{3i}$  plus  $dR_3$  into  $R_1$  minus  $R_2 R_4$  by, I will approximately say that equal to  $R_2$  plus  $R_{3i}$  into  $R_1$  plus  $R_4$ .

Actually we had done this exercise earlier and I am just recapitulating. And you must remember that  $R_{3i}$  minus  $R_1$  minus  $R_2 R_4$  must be canceling each other because there is a condition for balance when there is no strain in the strain gage element therefore this portion is going to cancel of with this so you have  $R_1 dR_3$  and now I will assume that all these  $R$ 's are the same because  $R_1$  is equal to  $R_2$  is equal to  $R_{3i}$  is equal to  $R_4$  so let us assume that they are equal to some value  $R$ . I will say this is  $R$  into  $dR_3$  by this will be  $R$  plus  $R$  so two  $R$ , this will be another  $2R$  this becomes  $4R$  square and this  $R$  will cancel of with this so you see that the ratio  $V_0$  by  $V_s$ , will become  $dR_3$  by  $4R$ . So the change in resistance divided by the original resistance divided

by factor of 4 is the value of  $V_0$  by  $V_s$ . And we already know  $dR_3$  by  $R$  is related to the strain by the relation which we gave in the last lecture. So what we can do is we can write this  $dR_3$  by  $R$  as epsilon into 1 plus 2nu, so if you remember  $dR_3$  by  $R$  was related to the strain and it was related through the expression epsilon into 1 plus 2nu so we can rewrite this in terms of the strain which of course can be done anytime one would like to do that. So let us look at this expression and look at a simple example.

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The image shows a handwritten derivation on a blackboard. The main equation is:

$$\frac{V_0}{V_s} = \frac{(R_{3i}R_1 - R_2R_4)}{(R_2 + R_{3i})(R_1 + R_4)}$$

Below this, it states:  $R_3 = R_{3i}$  originally. It changes to  $(R_{3i} + dR_3)$ . A note says  $dR_3 \ll R_{3i}$ . To the right, a boxed equation is shown:  $\frac{V_0}{V_s} = \frac{dR_3}{4R}$ . The final simplified equation is:

$$\frac{V_0}{V_s} \approx \frac{(R_{3i} + dR_3)R_1 - R_2R_4}{(R_2 + R_{3i})(R_1 + R_4)} = \frac{R dR_3}{4R^2}$$

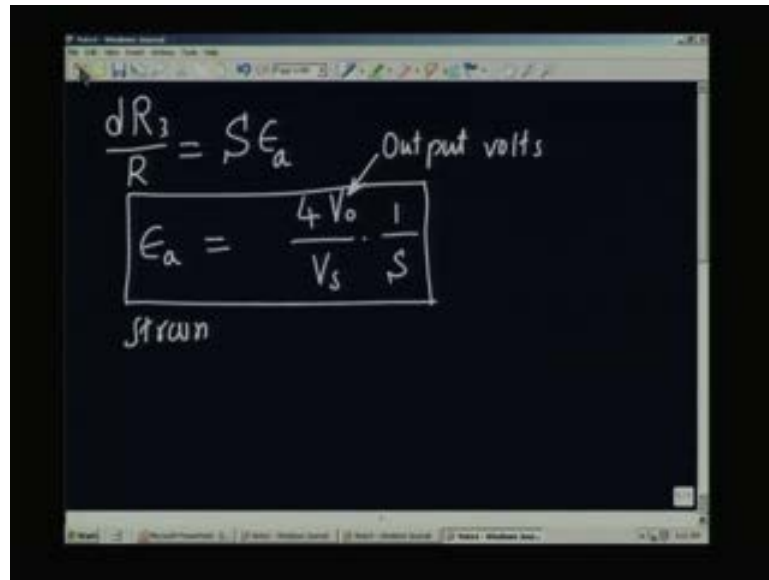
What I will do is I will say  $dR_3$  by  $R$ , here  $R$  is nothing but  $R_{3i}$ . I am writing it as  $R$  because we have made that clear earlier. This will be some  $S$  times epsilon in the axial direction where  $\epsilon_a$  is the strain of the element. So introducing this into the previous expression and rearranging I am going to get the following:

The strain  $\epsilon_a$  is equal to four times  $V_0$  by  $V_s$  into 1 by  $S$ . This is the expression which is going to relate the strain to the output voltage. So, in order to appreciate the types of numbers which are involved let us take a simple example so that we get an idea of the kinds of numbers we are going to have. I will take the example which I will call it as example 22.

We are just keeping track of the number of examples we have used in this set of lectures. Suppose I have a gage factor  $S$ , which is nothing but your gage factor which was roughly equal to 2, we will just make the assumption here as  $S$  is equal to 2 and we will assume that the input voltage  $V_s$  is 5 volts

and all the resistors are equal, all the resistances are equal in the unstrained state for the strain gage and each one equal to  $R$  is equal to 120 ohms. So, we would like to see the kind of strain we are going to get if the output voltage is measured.

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$$\frac{dR}{R} = S \epsilon_a$$

$$\epsilon_a = \frac{4 V_o}{V_s} \cdot \frac{1}{S}$$

Strain

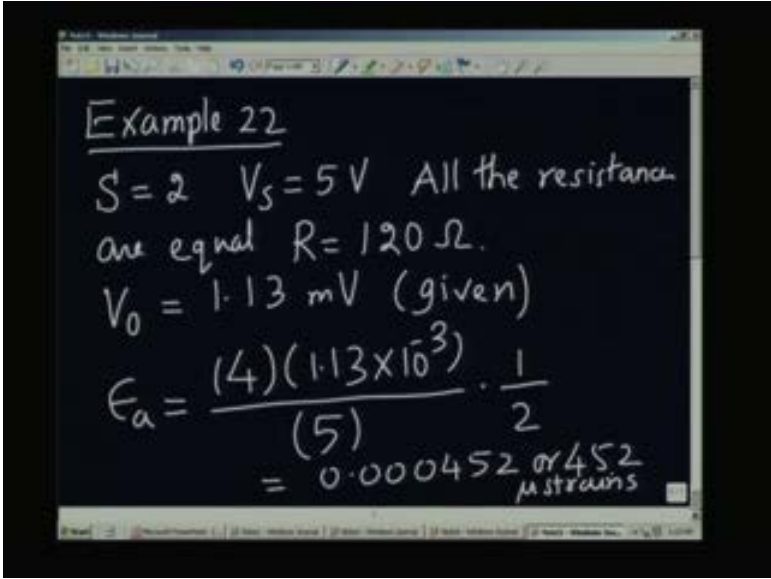
Suppose  $V_o$  has been measured to be equal to 1.13 millivolts, we will later on see how we measure this kind of a small voltage and so on, so this is given. So I would like to know what this corresponds to in terms of the strain of the strain gage, that is what the problem is. So we will just have to use that formula which I wrote in the previous page. So the formula we can recapitulate;  $\epsilon_a$  is equal to  $4V_o$  by  $V_s$  into  $1$  by  $S$  it is a very simple formula so this will be 4 into 1.13 millivolts that will be multiplied by 10 to the power of minus 3 volts by 5 volts into  $(\frac{1}{2})$  and this comes to .000452.

Or if you write it in terms of micro strains it will come to 452 micro strains which is written as  $\mu$  strains. So the strain of the strain gage element which gives an output of 1.13 millivolts when the four resistances are of equal value in the balance condition and each resistance equal to 120 ohms in the balance condition gives me for 1.13 millivolts, I get a strain of 452 micro strains. So we are talking about very small changes in the length as 452 micro strains is nothing but 452 into 10 to the power of minus 6 which is equal to  $\Delta L$  by  $L$ . So you can find out if  $L$  is given  $\Delta L$  will be 452 into 10 to the power of minus 6 of that length  $L$ . So it is a very small strain

which we are talking about. And the assumption that the strain gage is within the elastic limit is very well satisfied with such small strains and therefore whatever we have done seems to be ok.

Now suppose I look at the same problem slightly differently, I will call it as example 23. I would like to look at the change in resistance in this particular case. So the strain is given as the number. Actually what I will do is, I will slightly change the problem a little bit. So the strain is given as 1000 micro strains. So it is easy to work out the numbers.

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Example 22

$S = 2$     $V_s = 5V$    All the resistances are equal  $R = 120 \Omega$ .

$V_0 = 1.13 \text{ mV}$  (given)

$$\epsilon_a = \frac{(4)(1.13 \times 10^{-3})}{(5)} \cdot \frac{1}{2}$$

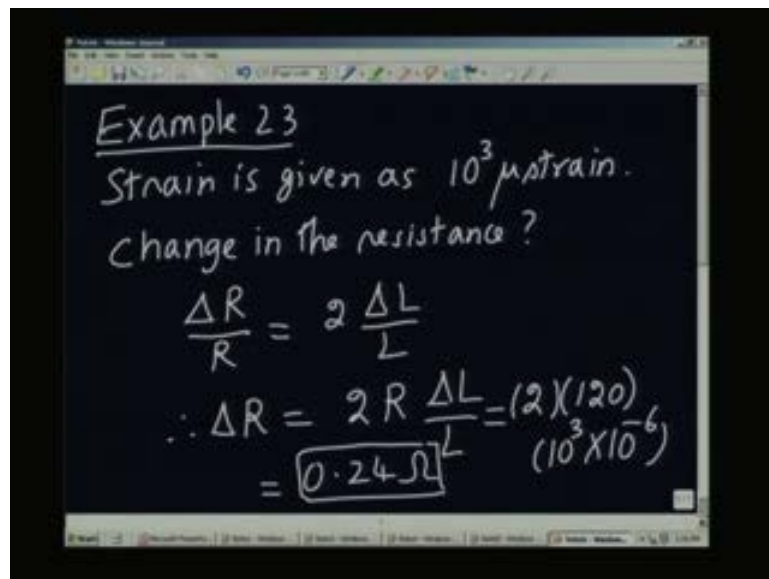
$= 0.000452$  or  $452 \mu\text{strains}$

Now I want to find out if the strain is actually equal to 10 to the power of 3 micro strains what is the change in the resistance? Change in the resistance is needed to be obtained and what is the change in the resistance?

If you remember, we have  $\Delta R$  by  $R$  from the strain gage theory which is nothing but two times  $\Delta L$  by  $L$ , 2 is the gage factor and therefore I can say that  $\Delta R$  is equal to 2 into  $R$   $\Delta L$  by  $L$  2 into 120 ohms,  $\Delta L$  by  $L$  is 10 to the power of 3 micro strains which will be 10 to the power of 3 into 10 to the power of minus 6 and if you work it out it comes to 0.24 ohms. So the change in resistance is very close to being about quarter of an ohm. The 120 ohms is the value of the resistance and because of 1000 micro strains which has been imposed as the strain on the element the resistance has changed by a value of equal to 0.24 ohm.

So the question is, how can we measure such small changes in resistance?  
 So we are going back as it were to describe why we were using a Wheatstone bridge. A Wheatstone bridge is initially balanced. That means all the resistances are equal. And the small change in resistance is going to create an imbalance, this imbalance is going to be available as a small change in voltage and the small change in voltage can be easily measured by using, if necessary an amplifier and then the output can be monitored. The idea is to create, if I take a suitable supply voltage I can get a sizeable value for  $V_0$  microvolt or millivolts is easy to measure either by directly using a millivolt meter or I can use an amplifier and use a regular voltmeter to measure the voltage. This is what one can probably do. Let us look at that part of the problem. So the resistances are changed by a very small value and because of this the voltage which is appearing across the terminals, if you go back, we are talking about this  $V_0$  and  $V_s$  is the input voltage, this is the expression I am talking about, output volts is given.

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Example 23  
 Strain is given as  $10^3 \mu\text{strain}$ .  
 Change in the resistance?  

$$\frac{\Delta R}{R} = 2 \frac{\Delta L}{L}$$

$$\therefore \Delta R = 2 R \frac{\Delta L}{L} = \frac{(2 \times 120)}{(10^3 \times 10^{-6})}$$

$$= 0.24 \Omega$$

In fact output volt is not given but I have been give  $\epsilon_a$ , I can work backwards and find out the output voltage for this particular case. So we will look at the output voltage given by  $V_s$  into  $S$  into  $\epsilon_a$  by 4, this is for a quarter bridge circuit. So we can substitute this and it will be 5 volts,  $S$  is 2,  $\epsilon_a$  is 10 to the power of 3 micro strains so 10 to the power of 3 into 10 to the power of 6 with a negative sign by 4. This gives 2.5 millivolts as the



output. This is either directly read by a millivolt meter if you have access to a millivolt meter or we can use an amplifier of a suitable gain.

For example; if I you use a gain factor of 10 to the power of 3, 1000 gain the output will be like 2.5 volts which can be measured using an voltmeter, so measure using a voltmeter. And we working with DC potentials, all the voltages are at DC( Direct Current) therefore there is no real problem with this measurement. Now we will look at one or two important things which you have not looked at till now. If you remember, when we were talking about temperature measurement using Resistance Temperature Detectors or RTDs we talked about something called self heating.

Self heating is because a current is passing through the resistance and because of dual heating there will be a certain amount of heat produced and depending on the rapidity with which this heat can be lost from the detector, the detector will achieve some temperature, it will attain a temperature which is higher than what it should be. Self heating is going to change the resistance of the detector because of the temperature change. And a strain gage is nothing but a resistance and if you look at the circuit we are using we were talking about using a circuit like this and of course there will be a current passing through this and the current will branch out into two portions; one current will go here and another current will go through that and therefore each one of these resistances are  $R_1$ ,  $R_2$ ,  $R_4$ ,  $R_3$  are going to carry a current.

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The image shows a digital screen with handwritten calculations for the output voltage  $V_o$  of a Wheatstone bridge. The formula used is  $V_o = \frac{V_s S \epsilon_a}{4}$ . The values substituted are  $V_s = 5$ ,  $S = 2$ , and  $\epsilon_a = 10^3 \times 10^{-6}$ . The calculation results in  $2.5 \text{ mV}$ . A note indicates this can be "Directly read by using a millivoltmeter". An arrow points down to "Amplifier of  $10^3$ ", leading to the final "Output =  $2.5 \text{ V}$ ", which is boxed. A note next to the boxed output says "Measure with a voltmeter".

$$V_o = \frac{V_s S \epsilon_a}{4} = \frac{(5)(2)(10^3 \times 10^{-6})}{4}$$
$$= 2.5 \text{ mV} \rightarrow \text{Directly read by using a millivoltmeter}$$

↓  
Amplifier of  $10^3$

$$\text{Output} = \boxed{2.5 \text{ V}} \quad \text{Measure with a voltmeter}$$

And if these currents are all same and if all them have the same resistance, coefficient of resistances for change in temperature probably they will all cancel out, but it is not necessary that it will happen. So temperature compensation will have to be done. And if you remember in the previous lecture we talked about the dummy gage and the dummy gage is going to be in the other arm there are two arms  $R_3$  and  $R_4$ ,  $R_4$  is going to be a dummy gage.

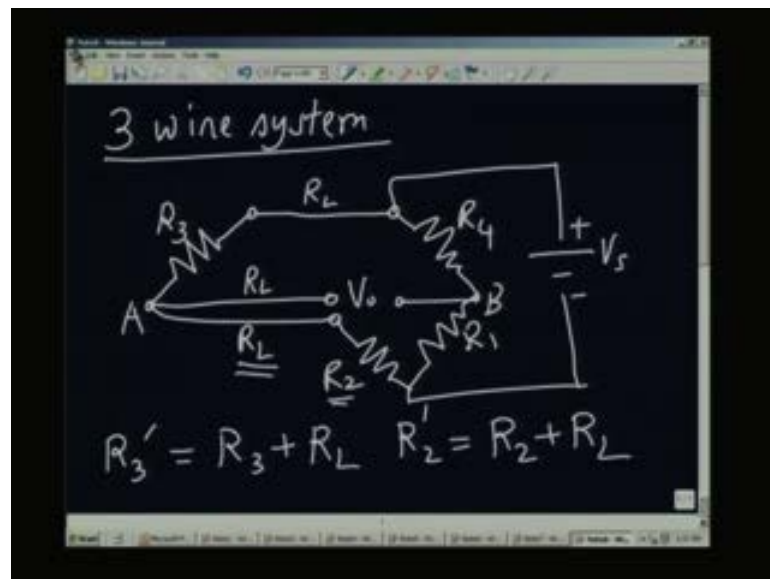
The dummy gage is going to be in the vicinity of the gage which is going to be actually used for measuring the strain. And if these two are going to have the same kind of characteristics the dummy gage is the same gage but it is not going to respond to changes in the strain, there is no strain induced in this one therefore it responds to any temperature change whether it is due to self heating or the changes in the temperature of the ambient. Both of these are going to affect the resistance. So the important thing to remember is, if  $R_4$  is a dummy gage and  $R_3$  is the actual gage, this is the strain gage and this is the dummy gage and whatever changes take place these two will cancel each other that is a very important thing.

You should put it in the arms like this  $R_3$  and  $R_4$ . You put the dummy gage here and the actual gage here, these effects will more or less nullify. But there is another important effect that is of the lead wire resistance. In fact if you recall when we were talking about the RTD, we talked about 3 wire

arrangement and the 4 wire arrangement. So whatever we discussed there should be possibly of use here also because the same principles are involved. We are talking about using a Wheatstone bridge and we are talking about lead wires and therefore it is common to use a 3 wire system. So let us look at the 3 wire system and discuss how it performs. I will draw a simple sketch here.

This is the gage which I will call as  $R_3$ , I will always use  $R_3$  for the symbol gage and it has two leads and these are 3 lead wires which are going to coming out of that. So this is  $R_L$  this is also  $R_L$  this is also  $R_L$ , these are 3 lead wires coming out of the two ends of the strain gage. Now what I am going to connect these two  $R_2$  here and on the other side I am going to connect  $R_4$  and here is  $R_1$ . And I am going to measure the output  $V_o$  here and I am going to impose the supply voltage here, this is  $V_s$ . This is positive and negative terms of the battery and I am going to monitor this. By an arrangement like this what is happened is, if you look at this  $R_L$  this  $R_L$  is added to  $R_2$  because I am measuring the imbalance voltage between this point and this point, if you want you can call it AB, imbalance voltage is measured across AB.

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So if you look at this arm actually it is  $R_2$  plus  $R_L$ . And if you look at this arm this is  $R_3$  plus  $R_L$ . Therefore I am adding a common value of resistance to these two sides and therefore if I look at the balance equation let us see

what happens.  $R_3$  must be replaced by  $R_3$  prime equal to  $R_3$  plus  $R_L$  in the expression which we used earlier. I am going to use  $R_2$ , I am going to replace  $R_2$  prime is equal to  $R_2$  plus  $R_L$  and let us look at the expression which we had earlier. So the expression for  $V_0$  by  $V_s$  which I wrote down earlier will have to be modified. This will be  $R_3$  prime  $R_1$  minus  $R_2$  prime  $R_4$  by  $R_2$  prime plus  $R_3$  prime into  $R_1$  plus  $R_4$ .  $R_1$  and  $R_4$  do not change. The lead wire resistances are going to affect only  $R_2$  and  $R_3$ . And now I will substitute the values  $R_3$  prime is  $R_3$  plus  $R_L$  into  $R_L$  minus  $R_2$  plus  $R_L$  into  $R_4$  and we have  $R_2$  plus  $R_3$  plus  $2R_L$  into  $R_1$  plus  $R_4$ .

Remember,  $R_1$  to  $R_4$  are all the same. That is how we have built up the bridge. Therefore this  $R_1$  into  $R_3$  and  $R_4$  into  $R_2$  will cancel each other and  $R_1$  into  $R_L$  is  $R_2$  into  $R_L$  will also cancel each other under normal circumstance. That means, the balance is not going to be affected by the presence of the lead wire resistances. So you can say that under the unstrained conditions, under no strain condition balance is not affected by the lead wire resistances, this is a very important observation we are making. So the lead wire resistances are going to compensate each other and therefore the balance condition under no strain is not disturbed.

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$$\frac{V_0}{V_s} = \frac{R_3' R_1 - R_2' R_4}{(R_2' + R_3')(R_1 + R_4)} = \frac{(R_3 + R_L) R_1 - (R_2 + R_L) R_4}{(R_2 + R_3 + 2R_L)(R_1 + R_4)}$$

$R_1 = R_2 = R_3 = R_4 = R$  (say)

Under no strain condition balance is not affected by the lead wire resistances

Unbalanced case:  $R_{3i} \rightarrow R_{3i} + \Delta R_3$

$$R_2 + R_{3i} + 2R_L = 2R + 2R_L = 2R(1 + \frac{R_L}{R})$$

Now let us see what happens when the balance is not there and the resistance  $R_3$  has changed by a small amount. So in fact if you would like to look at it I can make it  $R_{3i}$  here and  $R_{3i}$  here to be consistent with the earlier notation.

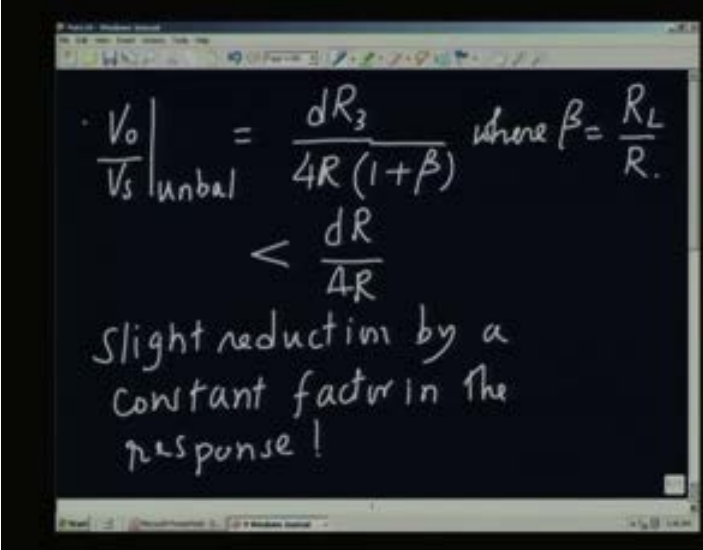
Now I am going to perturb this  $R_3$  by a small amount, and remember that this  $R_{3i}$  is equal to  $R_1$  is equal to  $R_2$  so here also I can make that equal to  $R_4$  is equal to 120 ohms or whatever equal to some  $R$  let us say, whatever may be the value. So I can in fact simplify this expression by putting all these resistances equal to  $R$  common value and before we do that let us just look at what happens in the unbalanced condition. So, in the unbalanced case  $R_{3i}$  becomes  $R_{3i}$  plus  $dR_3$ . So due to the straining of the strain gage there is a small change in the resistance and therefore I can substitute into that and write the expression for that.

So we will say, unbalanced due to the change in resistance  $R_3$  by a small amount. This will give you  $\Delta R_3$  or  $dR_3$  by  $4R$  into  $(1 + \beta)$ , where  $\beta$  is equal to  $R_L$  by  $R$ . You can see how it comes out. You have  $R_2$ , in this case here  $R_2$  plus  $R_3$  this is here plus  $2R_L$ . And  $R_2$  is equal to  $R_{3i}$  so this becomes  $2R$ . So I am taking the factor 2 outside and  $R$  also I am taking so this 2 you can see that  $R_2$  plus  $R_{3i}$  plus  $2R_L$  is nothing but  $2R$  plus  $2R_L$  is nothing but  $2R$  into  $1 + 2R_L$  by  $R$ . So this  $2R$  into  $1 + 2R_L$  by  $R$  I am calling it as  $1 + \beta$  and that it is what I wrote down in the next page.

Here you see that the balanced condition is not disturbed when the strain is not there in the strain gage. That means the lead wires are compensated. And the equation for the output voltage as a fraction of input voltage is the same as the earlier  $dR_3$  by  $4R$  but with a small factor in the denominator which is a constant factor. That means because  $1 + \beta$  is greater than 1 this is less than  $dR$  by  $4R$  which we got earlier. Therefore there is a slight reduction by a constant factor in the response.

So a constant factor is not a problem, but if it varies only we have to worry. That means there is degradation in the performance of the circuit because the output is slightly smaller than earlier but it is independent of the size of the change in the resistance and therefore independent of the size of the strain and that is what is desirable. So the lead wire compensation by having a 3 wire arrangements for the gage does the following two things; one, it does not change the null condition under the balance condition when there is no strain in the strain gage. And secondly it reduces by a constant factor the output when the strain gage has undergone a change in its length. This is a very important thing to know.

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$$\left. \frac{V_o}{V_s} \right|_{\text{unbal}} = \frac{dR_3}{4R(1+\beta)} \quad \text{where } \beta = \frac{R_L}{R}$$
$$< \frac{dR}{4R}$$

Slight reduction by a constant factor in the response!

Let us just take a simple example, the same kind of example we had earlier. For example, earlier we had all  $R$ 's is equal to 120 ohms. And suppose we assume that  $R_L$  is equal to 10 ohms just an example it is possible, it could be even larger. Suppose for some reason you require a long lead wire and the lead wire has got a large resistance equal to 10 ohms, then an important thing to remember is that these two lead wires are going to run very close to each other may be in the same sheet together, and both of them are going to respond to the same changes in temperature of the surroundings if at all there is some change and because they are in the two opposite adjacent arms of the bridge these effects will cancel. This is something very important to remember.

The lead wires are made to go close to each other, they are taken in the same sheet, if they are subject to any temperature variation they are going to undergo same changes in temperature and if  $R_L$  changes by a small amount both the  $R_L$ 's will change and therefore it is not going to affect the measurement. So the temperature compensation is because there are at the two adjacent arms of the bridge. So we can determine the value of beta which will be nothing but  $R_L$  by  $R$  which will be 10 by 120 which will be 1 by 12.

Now I can determine  $V_o$  by  $V_s$  for the same case we studied first, that is 10 to the power of 3 micro strains, here  $\epsilon_a$  is given. And also the voltage

given  $V_s$  is 5 volts, I am taking the same example, so  $V_0$  by  $V_s$  now is going to become  $S$  by 4 into  $\epsilon_a$  by 1 plus  $\beta$  and this will be  $S$  where  $S$  is 2 by 4 into  $\epsilon_a$  which is 10 to the power of minus 3 because 10 to the power of 3 and 10 to the power of minus 6 by 1 plus 1 by 12 and this comes to 0.462 millivolts per volt. If I substitute  $V_s$  is equal to 5 volts, then  $V_0$  will be equal to five times .462 millivolts. And you compare with the previous value when  $\beta$  is 0 this will be nothing but 2 by 4 into 10 to the power of minus 3 this is 0.5 millivolt per volt if  $\beta$  is equal to 0. That is no effect of the lead wire resistance or the lead wire resistance is very small then you will have 0.5 millivolts per volt. So the change in the output or change in the response in percentages is nothing but a decrease so we will indicate by an arrow going down 0.462 minus 0.5 by 0.5 into 100 and this is about minus 7.6%. So a constant value of minus 7.6% is the reduction in the response of the strain gage because of the lead wire resistances.

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$$\begin{aligned}
 R &= 120 \Omega & R_L &= 10 \Omega \text{ (say)} \\
 \beta &= \frac{R_L}{R} = \frac{10}{120} = \frac{1}{12} & \epsilon_a &= 10^{-3} \text{ mst} \\
 & & V_s &= 5 \text{ V} \\
 \frac{V_0}{V_s} &= \frac{S}{4} \frac{\epsilon_a}{1+\beta} = \frac{2}{4} \cdot \frac{10^{-3}}{1+\frac{1}{12}} \\
 &= 0.462 \text{ mV} & & (0.5 \text{ mV if } \beta=0) \\
 \downarrow \text{change} &= \frac{0.462 - 0.5}{0.5} \times 100 = -7.6\%
 \end{aligned}$$

In fact in practice the equipment or the experimental set up in which we are going to measure the pressure in this case, or in the general case it may be the strain of a member which is going to be loaded it may be a big equipment in which we are trying to measurement of the strain.

So the wires have to necessarily go a long distance from the place where the strain gage is mounted to the meter which is going to probably several meters away that is quite far away lengthwise so you need lead wires. So this

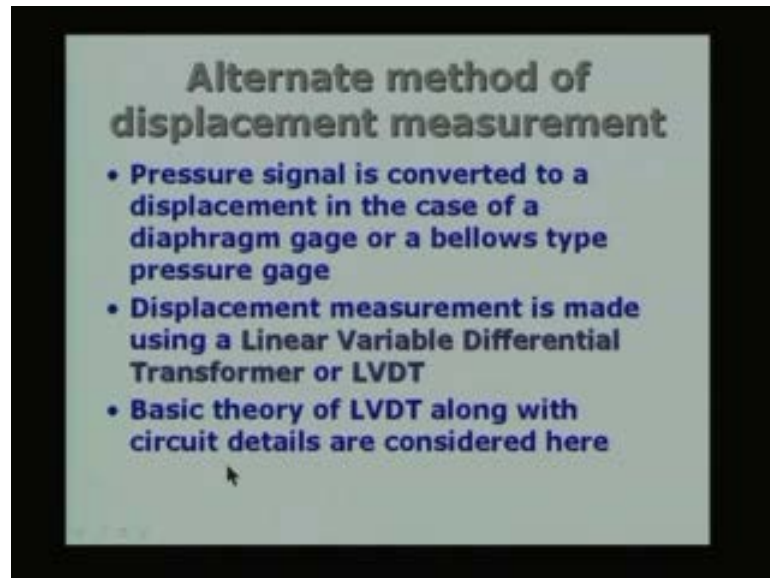
lead wire resistance cannot be avoided, and if the length is large enough between the place where we are making the arrangement and the electronic equipment which is sitting somewhere away from it then the lead wire resistances may be in this case 10 ohms or it may be even more. So the lead wire resistance cannot be avoided because of the placement of the equipment in the laboratory space.

In the case of process measurement the process pressure which is being measured may be several tens of meters way from the control room. The control room is where we are going to monitor the pressure and temperature and other process parameter and the pressure gage itself may be located very far away from the control room in which case again, you are having a large length of wire running from the gage to the electronic equipment. So the  $R_L$  could be large, 10 ohms or 20 ohms or even 50 ohms. But if the 3 wire arrangement is used the effect of that will be eliminated excepting that it is going to degrade the performance of the  $V_o$  by  $V_s$  by a constant factor and in this case it is going to be  $1$  by  $1 + \beta$  where  $\beta$  is the ratio of the resistance of the lead wire to the resistance of the gage or the resistance which is used in the Wheatstone bridge circuit. So with this background we have basically covered a measurement of pressure using a strain gage.

Now let us take a look at the alternate methods of displacement measurement. As I mentioned in the last lecture displacement measurement is also an important activity under the mechanical measurements category. And instead of describing the measurement of length or displacement separately or as a separate set of lectures I have sort of introduced or doubt tailed it to the present discussion because it is very convenient to do so. So it is very clear that in its own right displacement may have to be measured. And in fact there are many other methods which will be probably covered in a metrology part of the course.

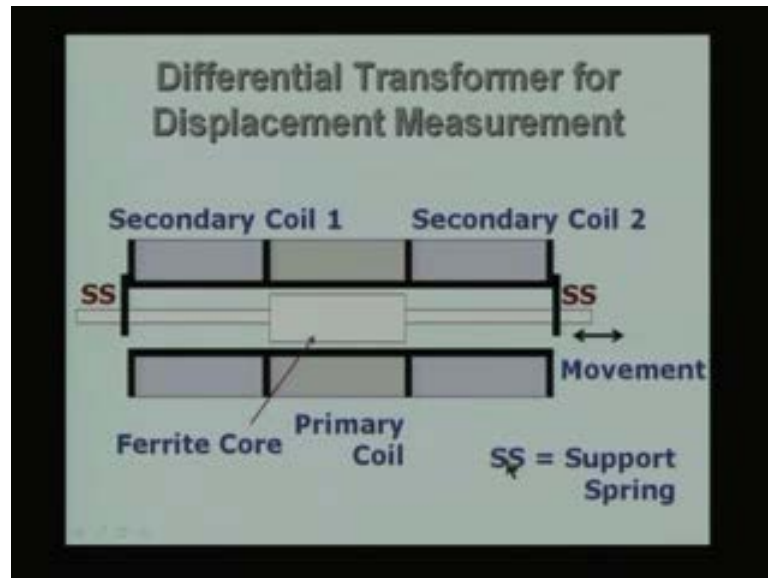


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So what I am going to do is to look at the alternate method. one of the alternate methods is the use of a linear variable differential transformer which I referred to earlier in passing. So the pressure signal is converted to a displacement in the case of a diaphragm gage or a bellows type gage. So in the next slide I will show how it is done. Displacement measurement is made using a Linear Variable Differential Transformer or LVDT. And what we are going to do is look at the basic theory of LVDT along with some circuitry details in the present lecture. So let us look at the possible ways of doing it but before that let us just recapitulate the LVDT construction.

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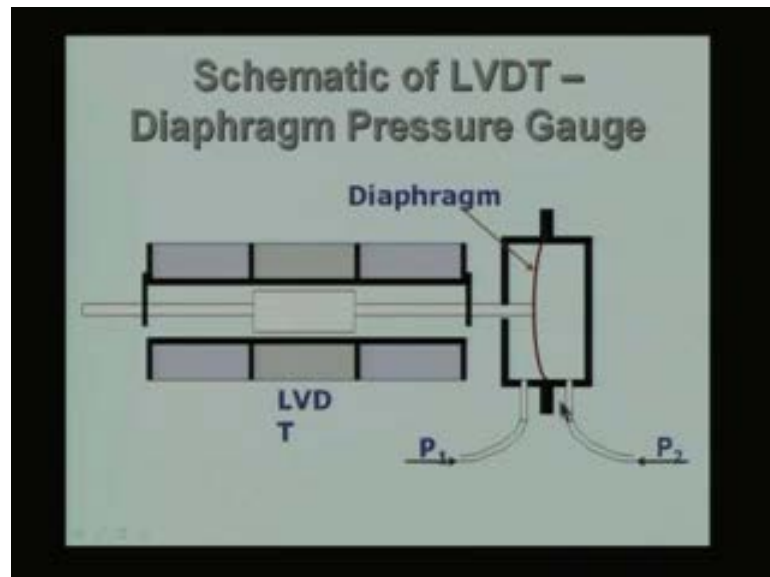
It consists of a primary coil, you see here a primary coil, this is actually cylindrical thing and I have taken a cross section, there is a core which is made of a ferrite material and it is capable of moving in the longitudinal direction it can move in and out like this. It can either move this way or this way so that is what is indicated by this movement. There are two secondary coils. Secondary coil 1 and Secondary coil 2 which are coaxial with the primary coil. And if you remember your basic electrical engineering course you will realize that the transformer is an AC instrument. That means if the coil one carries an alternate current then the flux lines will link through the ferrite core to the two secondary coils.

So, if the alternate current is impressed or alternating voltage is impressed on coil one or the primary coil the Secondary coil 1 and the Secondary coil 2 both are going to respond to this, and produce their own outputs. These outputs are also going to be alternating current and what we are going to do is look at the output of these two coils. If the ferrite core is exactly centered, that means symmetrically placed with respect to the Secondary coil 1 and Secondary coil 2 then the two outputs of Secondary coils 1 and two are exactly equal to each other. So if you subtract one from the other, that means taking into account the proper phase relation between the two if we subtract the two I am going to get a null voltage. In the next lecture let us see exactly how it is done.

Secondary coil 1 and Secondary coil 2 are going to produce the same voltage and therefore if these are subtracted I am going to get the null which means that under the null condition the ferrite core is symmetric with respect to the two coils namely the Secondary coil 1 and the Secondary coil 2. Suppose now, we impress a small displacement either to the right or to the left, let us see what happens. If it is displaced to this right in this case the Secondary coil 2 will be linked more closely, magnetically because the ferrite core is a magnetic material and this shaft shown here is a non magnetic material may be aluminum for example. So the primary coil and the Secondary coil 2 will have more coupling as compared to the primary and Secondary coil 1. And therefore the Secondary coil 2 is going to produce a larger voltage by a transformer action. Secondary coil 1 is going to produce a smaller voltage and therefore if you take the output of the two minus the output of one it will be a positive value.

Suppose now it moves to the left in which case the Secondary coil 1 is going to be magnetically linked and therefore this is going to produce a higher voltage and this is going to produce a lower voltage output and therefore the difference will be a negative value. One of them will be negative with respect to other.

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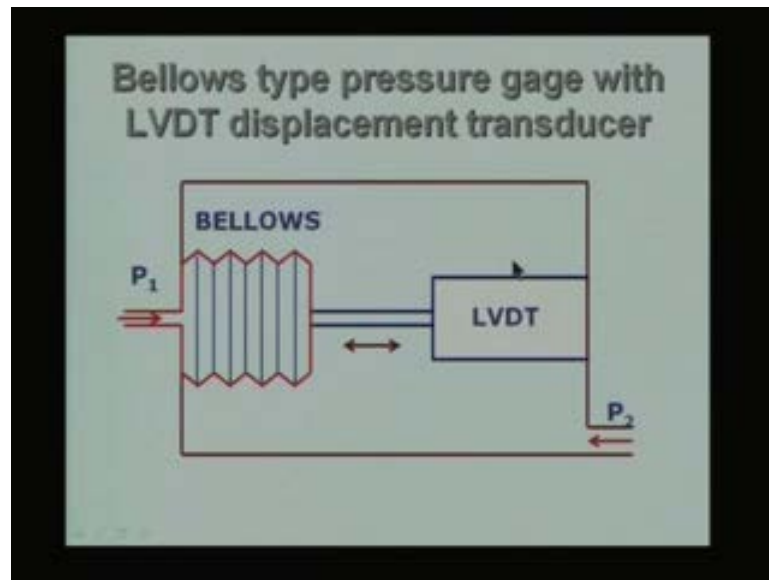


Actually both the outputs are AC outputs and therefore when I say positive or negative I think we will have to look at it more carefully a little later on.

So let us see how I can use this LVDT for measuring pressure and that is our idea. I have shown here a diaphragm gage where a diaphragm is stretched between a supports like this, it could be also a strain measurement in a mechanical experiment involving a straining of a material.

Here we are talking about pressure, and therefore the gage will be in the form of a diaphragm, the diaphragm is like what we discussed earlier. I will come back to the case of a diaphragm a little later on. I will take a specific example. I am applying a pressure  $P_1$  on one side  $P_2$  on the other side, if  $P_2$  is greater than  $P_1$  as it is shown here, the diaphragm is going to bend like this and this rod is attached to the diaphragm and therefore the diaphragm will move towards the left. So, when the diaphragm gets a strain like this or it changes shape like this, then this is going to move to the left and therefore it is going to throw out of balance the LVDT transducer which is shown here. This will be more closely linked to the primary and this will be less closely linked to the primary and therefore the voltage produced by this coil will be more than the voltage produced by this coil therefore you have an unbalanced voltage which is what is a measure of the displacement of the diaphragm. And here you see that I am measuring the displacement at the center that is the maximum displacement point. Of course, it could be kept anywhere else and suitable calibration can be done. In this case I have shown it to be exactly at the center and therefore it is going to measure the maximum deflection of the diaphragm which occurs at the center of the diaphragm. And we have a higher pressure on this side and the lower pressure on this side.

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Another way of doing it is, let me just look at the bellows type pressure gage with a LVDT transducer. Here what I have done is, I have shown the bellows element. The pressure  $P_1$  is applied to the inside of the bellows and the pressure  $P_2$  is applied to outside of the bellows and the shaft of the LVDT is connected to the bellows like this. And you can see what is going to happen. If  $P_1$  is more than  $P_2$ , the bellows will expand in this direction, it will move in this direction and therefore this rod will be pushed to the right side and therefore LVDT will give you one type of output.

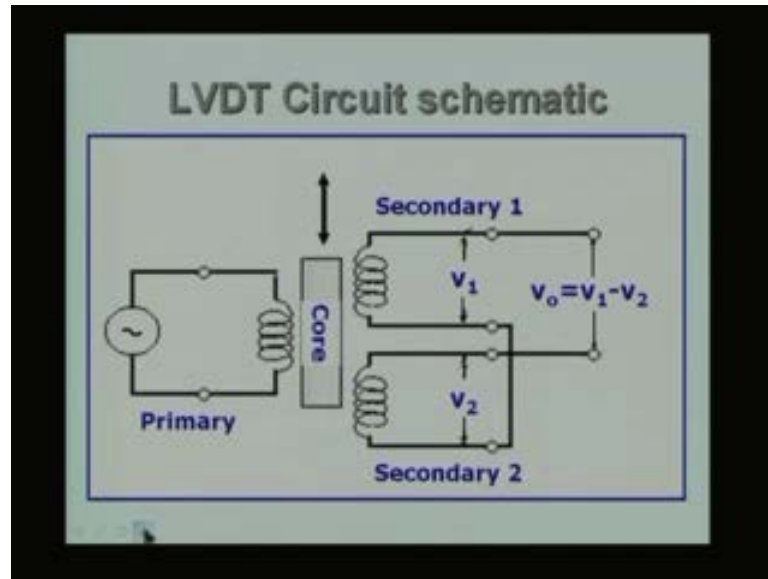
However, if  $P_1$  is less than  $P_2$  it will move in the other direction. That means, whenever there is an imbalance in the pressure between inside the bellows and outside the bellows there will be a net movement either to the plus side that is this side or to this side and therefore we get an output from the LVDT which is going to be dependent on the difference  $P_1$  minus  $P_2$ . So let us look at the circuit schematic. We will have to resume from here a little later on, in the next lecture.

So how does LVDT work?

We can just show it schematically. We have a primary coil and this is driven by an AC voltage which is usually several kilo hertz and may be a few volts 2 to 3 volts peak to peak and may be 4 to 5 kHz, a frequency. It is a sine wave which is going to drive the primary and the core is shown here as moving up and down and there are two coils which we indicated earlier, the

Secondary coil 1 and this is Secondary coil 2. And the two are going to generate voltages proportional to the input voltage.

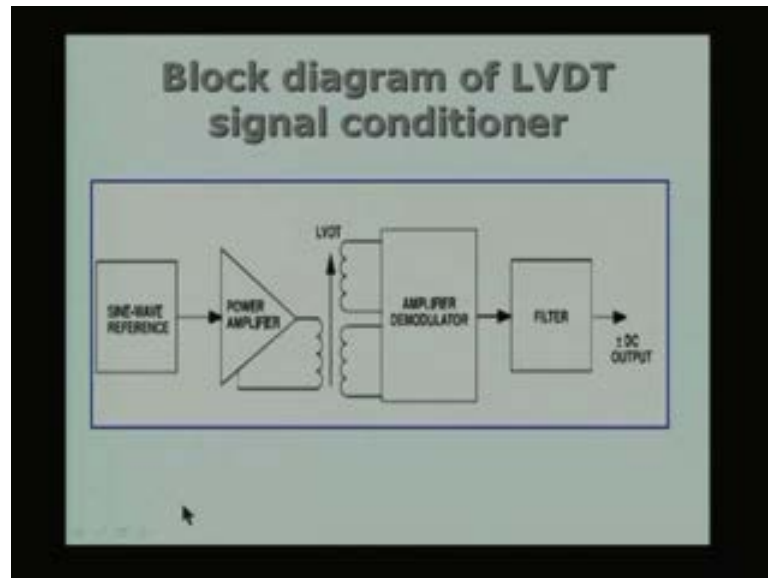
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Of course this voltage depends on the turns ratio, this turns ratio of this to this and this to this and so on. So this will produce a voltage equal to  $V_1$ , this is also a AC voltage, this is also equal to  $V_2$  and this is  $V_2$  which is also equal to a AC voltage. I have schematically shown this as the difference between two AC voltages but how it is done is slightly different which will become clear later on. So let us look at the typical output the block diagram of an LVDT signal conditioner.

We have a sin wave reference generated which is going to drive the primary coil of the LVDT. It may require a power amplifier to drive the current through this coil then the LVDT itself has got two coils and we are going to amplify and also demodulate this signal from the AC signals which are going to come out from the two signals  $V_1$  and  $V_2$  which are the AC signals and I am going to demodulate. We will see how demodulation is done. I will just give one or two simple ways of doing it. The demodulated voltages are filtered to remove AC component or the ripple and what you get is simply a DC output.

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The block diagram of the LVDT signal conditioner simply consists of a sine wave generator which is going to drive the primaries through a power amplifier. The two secondary voltages are going to be amplified and demodulated to remove the ripples or the AC part through a filter and then we get the DC output. This DC output is proportional to the change in the position of the core of this LVDT. So, if the core of the LVDT is collected, for example, for the diaphragm gage or to the bellows gage this gives you the movement.

And if you remember when we talked about the diaphragm gage we said that the diaphragm displacement at the center of the diaphragm is proportional to the pressure difference across the diaphragm. Therefore this displacement which appears as a DC output here is actually proportional to the pressure difference across the diaphragm and therefore we can link the DC output of the LVDT transducer to the pressure difference across the diaphragm. So in the next lecture we will look at more details of the LVDT signal conditioning and we will give one or two examples. And also I will consider a problem involving a diaphragm gage which I thought I would do in this lecture but I think we will have to do in the next lecture, thank you.