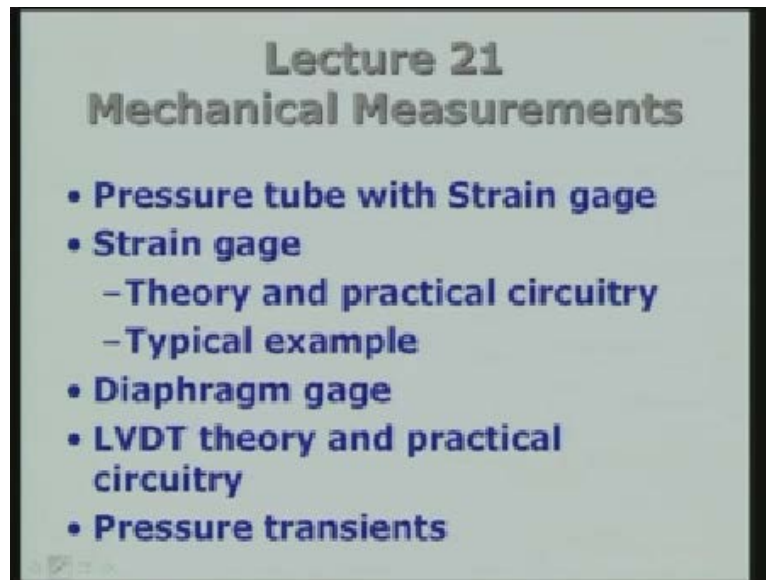


Mechanical Measurements and Metrology
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Module -2
Lecture - 21
Pressure Measurement

So we will be starting our lecture number 21 on Mechanical Measurements. In the previous two lectures we have been discussing some aspects of measurement of pressure. In fact we looked at manometry or the use of liquid or height of a liquid as a measure of the pressure using different types of manometers. We talked about the u tube and the well type with the straight tube and the inclined tube. We also worked out an example to demonstrate the working of such a manometer. After that we started looking at other types of measurements of pressure, different types of gages which can be used, we looked at the Bourdon tube as one of the cases, it is a mechanical instrument.

And then we started talking about other types of gages like diaphragm gage and the use of a strain gage for measuring the deflection of a diaphragm and we introduced some elementary ideas regarding this including LVDT. We discussed nature of LVDT in brief and how it is used to measure a deflection or a distance which is the deflection due to action of the pressure. So in other words, pressure signal is converted into a deflection or a length and this length is measured using either the LVDT or a strain gage.

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So what I will be doing in this lecture is to confine myself more to discussion on the strain gage type of measurement of deflection. In fact strain gage can also be looked upon as an instrument which is used for measuring very small lengths. So from that point of view, it has got very important metrological application in the measurement of length. And measurement of length is somewhat simpler to understand because lengths are basically a comparison.

Length is defined as a standard length of an object which is kept under stated conditions of pressure and temperature and this length becomes the unit with which we compare other lengths or lengths of other objects. So the meter scale is actually international standard which is used actually and the meter scales and its sub multiples and multiples are used for measurement of length. So metrology will of course be considered in more detail by another professor later on. Therefore I will not go into details about metrological aspects of measurement of length. Briefly I will touch upon the measurement of length as it is relevant to the measurement of pressure in this particular lecture.

So to look at what we are going to do, we are going to look at pressure tube with strain gage and then we will look at this strain gage its theoretical and practical aspects something about circuitry and then if time permits we will take a look at the diaphragm gage and then LVDT and so on. But I will think

we will have just enough time to get more carefully or in more detail the aspects of strain gage as a instrument for measuring length and hence the pressure. So, if we look at the next slide, I have in fact shown a strain gage element. It consists of, as you can see here the wavy line here, the wavy line happens to be a material which is deposited or etched on a metal foil, etched means the rest of the material is removed only this amount of material shown here in the form of a zig zag is going to be present. And you see that it consists of sectors which are straight and then there is a small bridge between the two straight segments again another straight segment and so on.

So the idea is to make a large length for the conducting material, this is a conducting material which is the metal foil going to have a large length equal to the number of such u multiplied by twice the length of each segment. In other words, to obtain a certain required resistance we are going to have the etched metal in the form of a length equal to l which consists of these units. Each unit is something like one stripe, one small bridge here and another stripe, this is one unit.

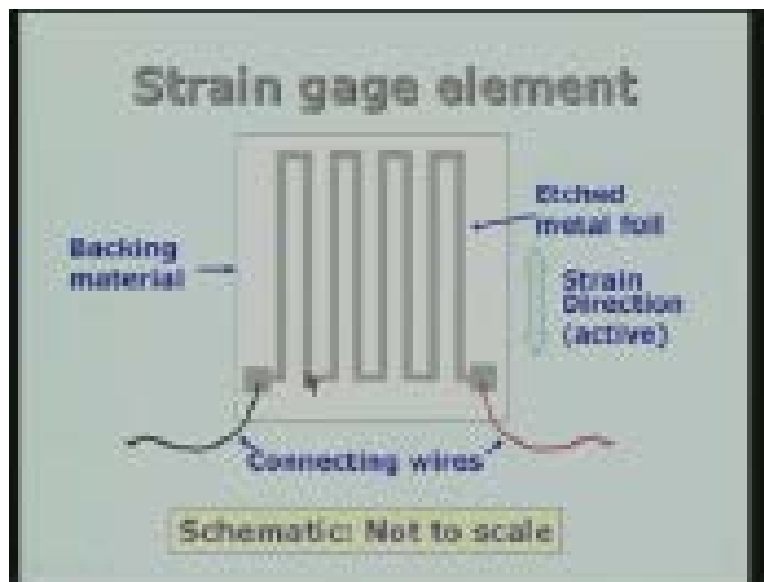
Therefore, we have four such units here and therefore the length of this will be equal to eight times the length of each one of the vertical units. And the strain gage element in the form this wavy distribution of material is actually held by a backing material and this backing material may be made of plastic or paper or some such material and this is going to usually bonded to the surface whose strain we want to measure. The strain gage is used measure the strain of a surface on which this strain gage is mounted. The assumption is that the wire which is shown here or the conductor which is shown here is going to undergo a strain which is identical to the strain undergone by the material on which it is bonded. And in fact, we can recognize two directions here; one we called as strain direction and I call it active that means if you subject this strain gage element to an extension or a stress or a strain in the direction shown here by the vertical arrow here it will respond that means it is going to be active. We will later see why it is so.

If however you were to apply a strain in the direction perpendicular to the straight segments, that means in the other direction normal direction your strain gage is not going to respond and therefore that will be called as the inactive direction.

Basically the strain has to be in a direction which is coincident with the arrows shown here. It may be either extension or contraction. That means it

may be either extension of the length that means positive strain or it could be a compression which means it is going to be a negative strain. And the strain itself is defined as the original length the ratio of the length increased change in the length divided by original length so it is a ratio and it has no units. And because the strains which we measure in practice are very small sometimes we call work we refer to the micro strain that means it might be one micro meter per meter of the original length, so in terms of micro strain we can measure.

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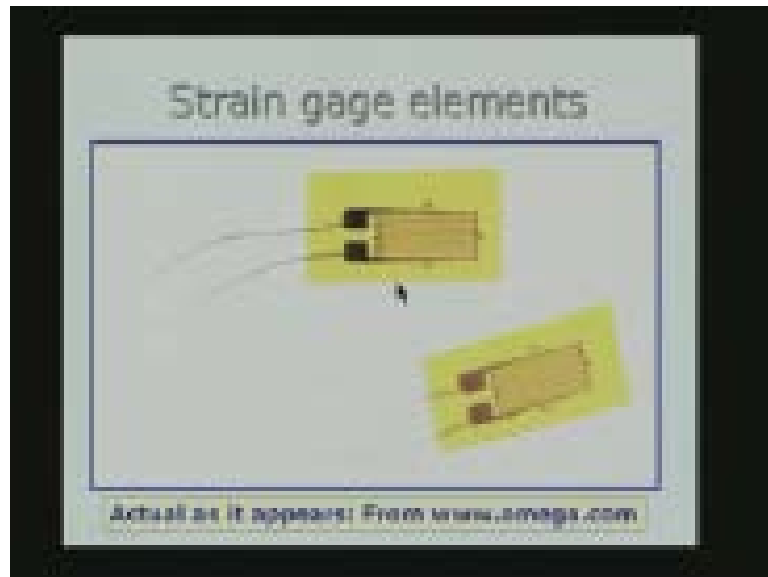


In most of the applications, we will be measuring very small deflections and the strain undergone by this etched material is going to be very small. Later we will look into the different materials which are used and so on. This is just schematic it is not to scale, so let me just show how a typical strain gage looks like. So I have taken it from the website of omega.com. Omega is a manufacturer of transducers of various types they also manufacture strain gage elements and what I have shown here is a typical strain gage element which consists of the two pads from which connection is taken to the outside of electronic circuitry. These are the segments which we talked about, the number is determined by the length required for the strain gage and in fact it is determined by the resistance of the element.

If there is no strain in the element it will have a certain resistance. This resistance is the resistance of the strain gage which is going to be coated.

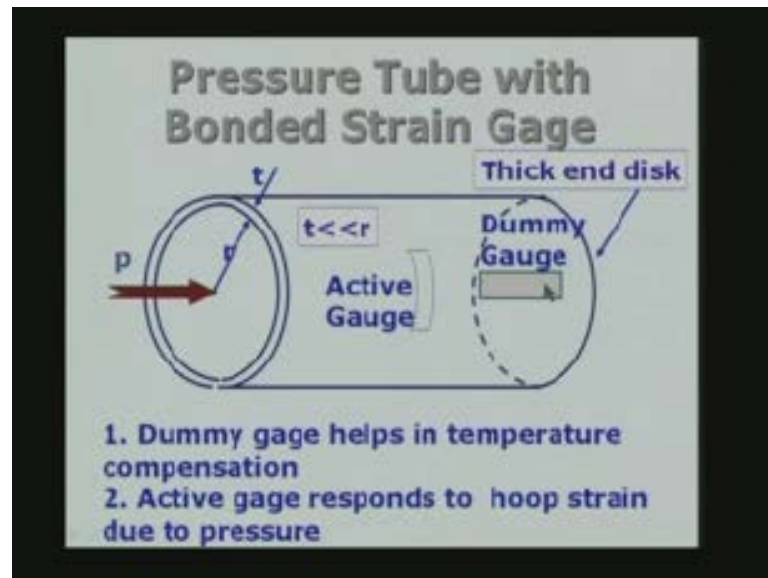
There are two values which are usually used in practice; one is 120 ohms which is standard, the next one is 350 ohms these are two standard values. So 350 ohms is larger value and 120 ohms is the smaller value, so two values are normally used in practice. and you can see as it is shown here, this material is deposited on a substrate which is in the form of a foil and the foil will have a backing which is aggressive backing so all you have to do is to peel out the paper and then the aggressive backing is going to be available to be stuck on to the surface whose strain we want to measure. And the adhesive gives the enough bonding or expected to give enough bonding so that the strain of the strain gage element is equal to the strain of the parent element which is subject to the stress and therefore it is experiencing a strain.

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So what I will do now is to look at the possible way of arranging the strain gage. Of course after looking at this arrangement we will try to see how to characterize the strain gage and how it is used for measurement and so on that will come later.

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One arrangement is to have a pressure tube. Pressure tube is simply a short length of tube as shown here of a thin walled construction. so you have a radius of tube equal to r and the thickness equal to t as you can see here, the radius is r and thickness is t and normally the thickness is very small compared to the radius. The radius may be about 10 mm or 12 mm or some such value and the thickness could be a few tenths of a millimeter that means it could be a one tenth or two tenths of millimeters and so on. So you see that the thickness is going to be very small compared to the radius and therefore this cylinder is essentially a thin shell, the assumption is that the cylinder acts as a thin shell. And the active gage is going to be bonded on to the circumference as shown here so the longer dimension here for this rectangular piece of element is at active direction. That means it is going to measure the strain in the circumferential direction as we can say, that is also called the hoop's stress or hoop's strain.

The direction along the circumference is the hoop's strain and we also have a dummy gage which means it is going to be connected in such a way that it not going to undergo any strain, and how can we do that? We can do the following; the end of the tube is closed via very thick disk, the shell is very thin and the disk is very thick. For example, I can have a disk of three or four millimeters or more closing the tube on the right hand side. When that happens the strain undergone by the thick disk is going to be very very small and negligibly small and therefore if I go back to the sketch the strain

indicated by the dummy gage will be essentially 0. That means the dummy gage resistance does not change during the operation of the instrument.

Why do we have a dummy gage?

Suppose I have the active gage here, and suppose the pressure is introduced inside the tube, it will of course undergo a strain because of the hoop's strain and therefore, it is going to undergo a change in resistance because change in length as we shall see in a little while from now and therefore resistance of gage is going to change. But the resistance will also change if the temperature of the system changes. For example, if the instrument is located in a laboratory space and the temperature of the laboratory fluctuates during the day time the active gage is going to indicate a change in resistance even when there is no stress because the temperature is changed and because of temperature there is an expansion and because of expansion it is going to show a change in the resistance. Therefore if I locate a dummy gage very close to the active gage that means these two are going essentially experience the same temperature variations in the laboratory.

Both the gages are going to undergo similar changes in resistance because of temperature and therefore the dummy gage acts as a compensation for the temperature change. That means the dummy gage and active gage both undergo similar changes in resistance due to temperature changes and therefore they are going to cancel each other depending on how we are going to connect it. Later on we will see the circuitry and at that time we will clearly learn how to connect it. So the pressure tube with bonded strain gage is a very simple idea. The active gage is going to respond to the hoop's strain and the dummy gage is going to respond only to the temperature variations which the active gage will also do and the compensation is obtained by having the active gage and the dummy gage very close to each other and the dummy gage is going to be mounted on a very thick plain disk which undergoes very little strain because of the pressure acting inside.

Of course when I say pressure acting inside it is Δp and outside it is the atmospheric pressure I will be measuring p minus p_a and therefore what I get is the gage pressure. And a pressure tube bonded strain gage can be designed for very large range of pressures from a very small to very large pressures because it is simply determined by the strain of the tube element and how it depends on the thickness and the radius and that is all what it is. So let us look at the way it is going to work out.

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Hoop stress in a thin cylinder

$$\sigma = \frac{pr}{t}, \quad \varepsilon = \frac{\sigma}{E} = \frac{pr}{tE}$$

Hoop strain in a thin cylinder is thus proportional to pressure.

The gage factor is thus given by \rightarrow $GF = \frac{r}{tE}$

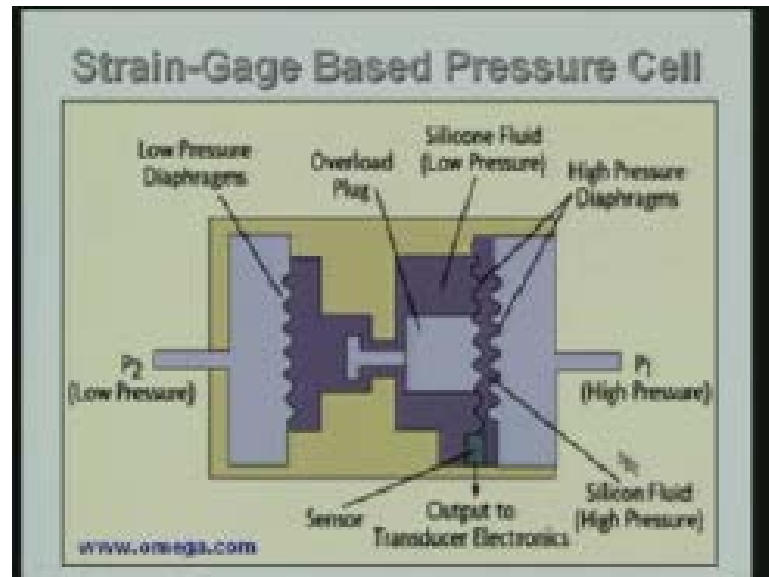
The hoop stress in a thin cylinder from Strength of Materials you will have learnt this, the hoop stress σ is equal to p into r by t where p is the internal pressure or actually the difference between the inside and outside or the gage pressure and r is the radius of the tube and t is the thickness. This is for a thin cylinder, when you have a thin cylinder the stress is essentially uniform across the thickness of the shell and you get a very simple expression σ is equal to pr by t which can be very easily derived but that is already known to you by Strength of Materials where you would have learnt some of these things. And the corresponding strain which we will call as the Hoop strain is simply given by Hoop stress divided by the Young's Modulus and the assumption we make is that the strain is so small that the material is within the elastic range.

Therefore the stress σ will never be very large compared, in other words, it will be small compared to the yield's stress of the material so much smaller. And therefore I can use ε equal σ by E where E is the Young's Modulus for material and substituting for σ the expression pr by t I will get pr by tE and if I know the radius the thickness and the Young's Modulus there is a direct relationship between the strain and pressure.

We will see later the strain is related to the resistance of the strain gage and therefore I will be measuring the strain and from that I will be working

backwards to find out what is the corresponding pressure which gives rise to it. In fact I can use this expression $\epsilon = \frac{pr}{tE}$ to define what is called a gage factor that means ϵ equal to some constant times pressure is linear relationship between the Hoop strain and the pressure which will be $\epsilon = \frac{p}{E} \times \frac{r}{t}$ so I can say that $\frac{r}{tE}$ is the gage factor I call it as GF, the gage factor is simply given by radius of the tube divided by the thickness of the cylinder divided by the Young's Modulus. So this is basically the way we are going to use the instrument to measure the pressure. So another example I am going to take before we look at the strain gage and its theoretical background and so on.

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We are looking at a strain gage based pressure cell, again I have taken it from the website of omega.com who is the manufacturer of these kinds of instruments and this is the schematic of how a pressure cell is made and you can see that, this is essentially a cylindrical casing within which you have certain elements, one you have a low pressure diaphragm on the left hand side this is a low pressure diaphragm the low pressure is fed in through the opening here, we have the high pressure P_1 and there is a high pressure diaphragm and there are in fact two of them and in between there is silicon oil or silicon fluid which is subjected to the high pressure, that means when I apply the high pressure here it is communicated to the silicon fluid in this particular region.

When I have the low pressure on this side the low pressure is going to be communicated to this region and therefore what happens is that there is a pressure difference between this side and this side and therefore when you apply a pressure here it is going to undergo a change in the strain and therefore it is going to compress the liquid here and in fact you can see that the gage itself is attached to the diaphragm here, and the output from the gage goes to the transducer electronic which is outside.

Let us look at some of the features; there is a overload plug, this plug is actually going to sit in this hole which is connecting the low pressure side and the high pressure side and if the pressure you apply is very high more than the rated pressure or which the pressure cell is designed this is going to go and close this opening and it will not work after that, it is going to protect the low pressure side from being damaged when we have high pressure applied here so this is just a safety feature.

Now in other words you see here that the pressure difference P_1 minus P_2 is communicated to a diaphragm and the diaphragm is going to undergo a strain and that strain is what I am going to measure by using the sensor, the sensor is nothing but a strain gage mounted on the diaphragm and then what I am going to measure is the deflection and this deflection is related to the pressure difference P_1 minus P_2 . And in fact, I can simply have a calibration done by having a known pressure applied here and a known pressure applied here I can choose the Δp and I can measure the Δp using a dead weight tester for example or some other arrangement so that I can have a calibration of the output of the transducer with respect to the Δp and that is one way of using the pressure cell.

We have discussed in other words two types of arrangements; one where I have a tube a short tube very thin wall material and the inside I am going to apply the pressure to be measured and the tube is going to undergo a strain because of the stress developed in the circumferential direction in fact we can also do in the longitudinal direction there is no reason why you should not do. Actually circumferential strain is slightly more than in the longitudinal direction that is the reason we use that and the strain is measured using the strain gage.

Now let us look at the theoretical background regarding the strain gage and for this I am going to go to the board and look at some of the principles. So what I will do is I will look at the principles of the strain gage and then look

at different properties determining the performance of strain gage or the strain measuring instrument then what are the different materials can be used, what are the gage factors which we are going to get or have in this case and how these measurements is made by using suitable electronic circuitry, this is the basic idea of this lecture.

So let us assume that we have a cylindrical object, for example it could be a wire. So I will say is the length is L and the area of cross section is A so I have a small piece of material in the form of a cylinder which has got a length L and a cross sectional area equal to A this is a solid cylinder. Now let us assume that it is subjected to some changes or some strain because of forces acting on it. So suppose I stretch the cylinder I elongate the cylinder by applying a tensile force so the length is going to become in other words I am going to change it to L plus dL this is the elongation. So elongation is dL due to the action of a stress or a force in the axial direction.

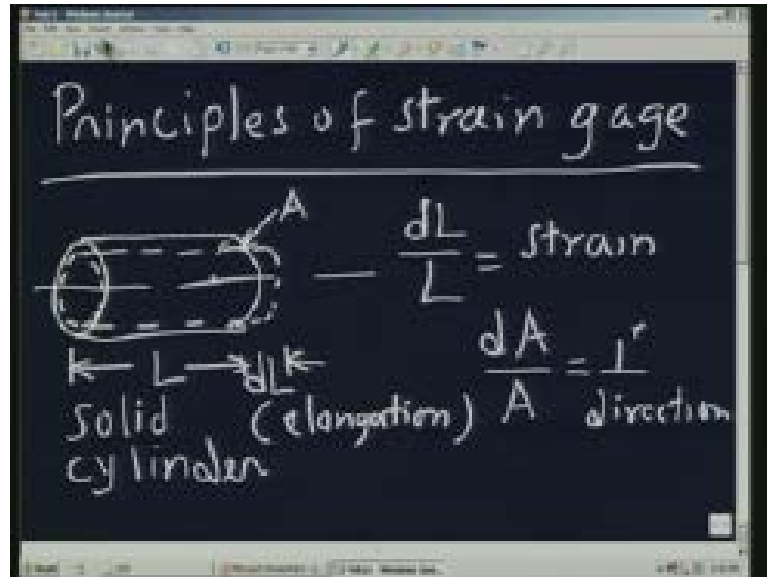
Therefore, what will happen when you have a elongation in this direction is that in the lateral direction it is going to undergo a change in the dimension, actually it is going to reduce in size, normally most materials are going to reduce in size, there are exceptional cases where the size actually increases in the other direction we are not interested in that, what we are interested in finding out is what happens in the case of a material which is going to undergo a change as shown here. So the dL by L is the strain in the length direction and dA by A is the change in the area per unit area in the normal direction. So this is dA by A is nothing but due to strain in the perpendicular direction, so perpendicular to the direction of the longitudinal strain there will be change in area this is in the perpendicular direction.

A very simple theory which we can look at is the case of an incompressible material. This is not necessarily correct. Suppose we do that let us assume that the material is incompressible. It means that the volume which is equal to the product of L and A is constant, the volume remains constant. That means when you have a deflection or change in L , A will automatically change such that the total volume is going to remain the same, so we can do the following:

I will call this as constant equal to some V which is equal to V_0 so let us say dV_0 by V_0 is equal to 0, and by logarithmic definition which is familiar to this is nothing but dL by L plus dA by A and therefore this is equal to 0. That means dA by A is equal to minus dL by L , that means positive strain in

the longitudinal direction leads to a negative strain or a negative change in the area of cross section.

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The cylinder becomes longer or any cause cross section in fact it is not necessary to have a cylinder you can have a prism, you can have square bar and the same thing is going to happen. The cross section is going to become smaller and the length has become larger. And conversely, suppose I subject this material to a compression the length is going to reduce the area is going to increase. And I am looking at very small changes. Let us not grow go close to the yield stress and so on. We are talking about very small changes in the length and the area of cross section, we are not talking about large changes.

Now let us look at the case of normal materials. The one I talked about is an incompressible case and most materials are not necessarily incompressible. So if I have a strain in the longitudinal direction so the lateral strain divided by the longitudinal strain, I am looking at two different strains and this ratio we call as the Poisson's Ratio. This particular value of ν is anywhere between 0 and 0.5 for most materials. There are exceptions also as I said earlier, there are materials for which may not be in this range but we are not interested in such materials in most cases.

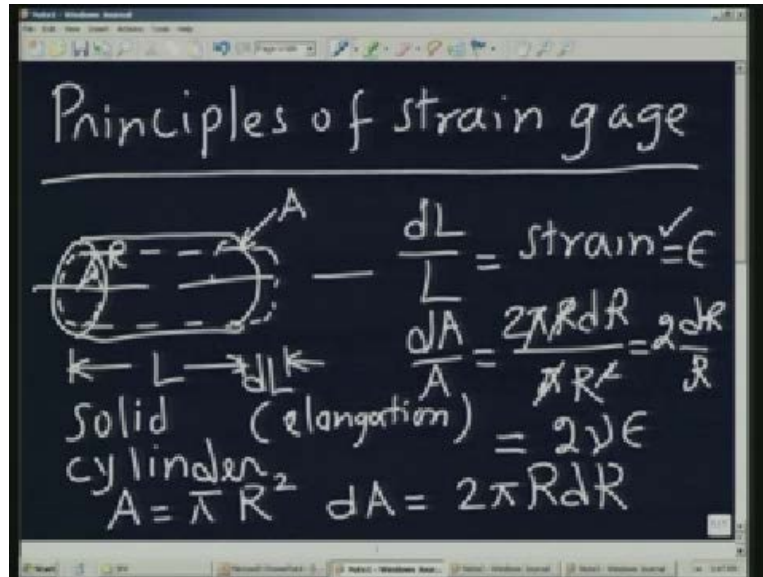
So if I have a lateral strain divided by longitudinal strain ν we can see what is going to happen in the case of what we had earlier. So we talked about a strain in the longitudinal direction, now we have talked about a change in area, now let us look at the strain in the longitudinal direction is nothing but dL by L and in this case if the original radius is R the radius is going to become smaller. Now I can write the value of A is the area of cross section which is πR^2 and therefore dA is equal to $2\pi R dR$.

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The image shows a digital blackboard with handwritten text and equations. The text reads: "Incompressible material", "Volume = LA = constant = V₀", " $\frac{dV_0}{V_0} = 0 = \frac{dL}{L} + \frac{dA}{A}$ ", and " $\therefore \boxed{\frac{dA}{A} = -\frac{dL}{L}}$ ".

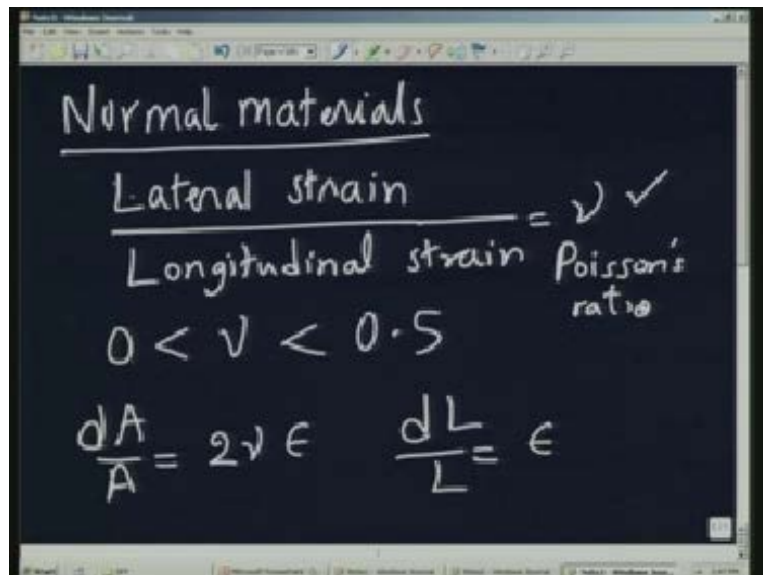
I am again assuming very small changes and therefore I can even use the differentiation to indicate the changes. So dA is equal to $2\pi R dR$ therefore what I want is dA by A , so I can write dA by A by dividing where dA is $2\pi R dR$ by πR^2 so π will cancel, one R will cancel, it is $2dR$ by R it is a very interesting and important thing to remember. The lateral strain is dR by R and the change in the area, the aerial strain or the area strain change in the area is simply twice the lateral strain. and we know that dR by R is equal to Poisson's Ratio times dL by L and therefore I can write this as two times ν times ϵ I will say L so ϵL is the longitudinal strain I will say this is ϵ or I will avoid using the subscript it is not necessary so I will just say ϵ . So if ϵ is the strain in the longitudinal direction given by dL by L then the change in the area or the strain in the area is given by 2ν times ϵ . With this background we can write the following; so with the Poisson's Ratio introduced here we have dA by A is equal to 2ν times ϵ where dL by L is equal to ϵ .

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Now let us look at the material of a strain gage. So I want to now link up whatever we talked in general about any material which is undergoing a strain, the longitudinal strain and lateral strain.

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I am going to now look at what is the consequence of strain gage. The strain gage is a material which has got a resistivity equal to ρ , let me just introduce the resistivity. We are talking about electrical resistance of the

material, the material is usually a conductor with a specific resistance or resistivity equal to ρ which is in units of ohm meter. So, the length of conductor is L and the area of cross section is A . So for the resistance R if I want to indicate that is electrical resistance because R was also used for the radius of the cylinder I will just call it electrical resistance R_e given by ρ into L by A this is a familiar expression which we know from our elementary electricity.

Now what is the change in the resistance?

I can again use logarithmic differentiation. So if you use logarithmic differentiation you get dR_e by R_e , what do I have to make logarithmic differentiation?

I take the logarithm both sides it becomes $\ln R_e$ is equal to $\ln \rho$ plus \ln of L minus \ln of A . And now I differentiate that term by term, this gives out 1 by $R_e dR_e$ that is what I have in the left hand side 1 by $\rho d\rho$ plus dL by L minus dA by A this is what we have. So dR_e by R_e is the ratio of the resistance change to the original resistance. So we are treating ratios in all these equations and the ratios are non dimensional that means they do not have any unit and therefore the left hand side is a change in the resistance with respect to original resistance equal to change in resistivity which may be due to either pressure or temperature because pressure and temperature also have an effect on the resistivity of the material and we will come to that in a few minutes from now plus the change in length with respect to its original length dL by L is the ratio minus the change in area divided by the original area.

We have three terms contributing to change in the resistance. What I am going to do using the strain gage as a transducer for the measurement of strain is to note the change in the resistance with respect to the original resistance. So it is just like strain, original length divided by length is mechanically termed as a strain, in the case of resistance it is the change in resistance divided by the original resistance. So in the right hand side the changes in these quantities lead to a change in the resistance, this is the basic theoretical framework on which strain gage is going to work.

Now I will combine this equation with what we have seen earlier. If you go back to the previous one (Refer Slide Time 33:01) I have got dA by A is equal to 2ν times of ϵ and dL by L is equal to ϵ so all I have to do is put them together, actually I should have put a minus sign here you will appreciate that. We will see here that dL by L is

positive and dA will be negative we saw that here minus dL by L so that has to be introduced and therefore dA by A is equal to minus 2ν epsilon, dL by L is epsilon so all I have to do is to introduce these two into that equation and you will see that dR_e by R_e $drho$ by rho plus dL by L is epsilon plus 2ν epsilon. Or I can write as $drho$ by rho plus epsilon into $1 + 2\nu$. If ν is equal to 0.05, this term will become 2ν is 2 into 1 by 2 is 1 and 1 plus 1 is 2, 2 into epsilon is 2 epsilon.

This is actually the incompressible material. For an incompressible material dR_e by R_e becomes $drho$ by rho plus 2epsilon. so you can see that this 2 is the factor which means the change in the resistance divided by resistance equal to 2epsilon and therefore you can see that epsilon is equal to 1 by 2 of dR_e by R_e . So this factor 2 is also referred to as gage factor and for an incompressible material the gage factor happens to be equal to two.

Of course I have not look at the $drho$ by rho yet, $drho$ by rho which is this quantity can be due to changes in the pressure or temperature. That is why the dummy gage, dummy gage is going to bring this extra term $drho$ by rho is going to be there because the temperature effect is also there and the pressure effect is also there.

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$$\begin{aligned}\frac{dR_e}{R_e} &= \frac{d\rho}{\rho} + \epsilon + 2\nu\epsilon \\ &= \frac{d\rho}{\rho} + \underbrace{\epsilon(1+2\nu)}_{\text{Gage factor}} \quad (\text{Ignore})\end{aligned}$$

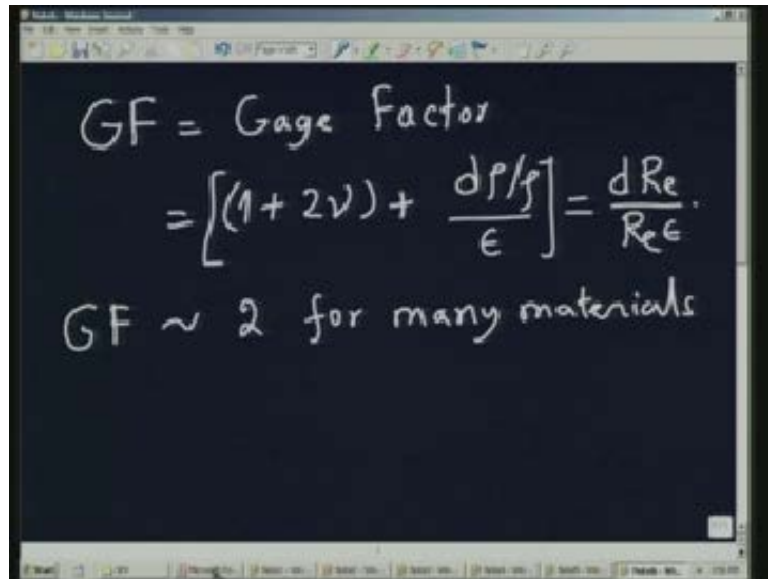
If $\nu = 0.5$,
(Incompressible material)

$$\therefore \epsilon = \frac{1}{2} \left[\frac{dR_e}{R_e} \right]$$

Of course this may be very small in normal practice and therefore it may be safe to ignore this. Sometimes we may ignore in which case dR_e by R_e

becomes equal to epsilon into 1 plus 2nu. In fact we can define gage factor in more complete terms this called the gage factor given by 1 plus 2nu plus drho by rho divided by epsilon, drho by rho divided by epsilon which is nothing but $\frac{dR_e}{R_e} \epsilon$ this is the ratio we call as the gage factor and for materials which are having a value of nu is equal to 0.5 Poisson's ratio equal to 0.5 will become 2 plus this small quantity which we normally would be ignoring it will become equal to 2 so the gage factor is roughly equal to 2 for many materials. We will see what are the materials used as strain gage elements. In other words, we use materials which have a gage factor roughly equal to 2 in most of the applications.

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The image shows a chalkboard with handwritten equations. The first line defines GF as the Gage Factor. The second line shows the formula: $GF = \left[(1 + 2\nu) + \frac{d\rho/\rho}{\epsilon} \right] = \frac{dR_e}{R_e \epsilon}$. The third line states: $GF \sim 2$ for many materials.

So let us go back to the presentation where I have some information on the Poisson's ratio of typical values for many materials. Steel is a very popular material for diaphragm. I am talking about the strain gage and the element which undergo strain which is also there, the diaphragm for example. The Poisson's ratio typically for steels is 0.3, concrete for example is 0.2, gold 0.42 you see that it is closure to 0.5 glasses may have between 0.2 and 0.25 and some elastomers have equal to 0.5. There are some materials which even have some negative Poisson's ratio and so on, let us not worry about those things as far as concerned here.

Now let us look at some of the materials which are used in practice. They are given in the form of a small table, I have taken it from same omega.com

website where this particular table is available. So some typical materials for example platinum 100% platinum the sensitivity or gage factor is 6.1 which is not equal to 2 is much larger, platinum iridium 95% platinum, 5% iridium some of these materials you have already seen are also candidates used in thermo couple manufacturing and are also used in other sensors. So that is the common feature between some of these materials.

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Poisson's ratio	
Poisson's ratio: typical values	
Steel	0.30
Concrete	0.20
Gold	0.42
Glass	0.20-0.25
Elastomers	→ 0.5

Platinum RTD resistance thermometers are made of 100% Platinum and they have a sensitivity of 6.1, platinum iridium 95% platinum iridium 5% 5.1, Platinum tungsten 92% Platinum and 8% tungsten 4 and then you see here I have shown some with a yellow background these are very common materials used in practice. The constantan is also given the name advance but some people call it copel, these are actually the names given by the manufacturers and they consist essentially of Nickel 45% and copper 50% alloy and the gage factor you see is very close to 2 which is 2.1. Nichrome nickel 80% Chromium 20% is called Nichrome V and 2.1 again. There is something called Karma, I think it is not coming right in this thing this karma it is nickel 74%, Chromium 20%, Aluminum 3%, iron 3% is again 2 there is something called Armour D Iron, Chromium Aluminum again you see it is close to 2. And then we have Monel, Nickel and Copper in 67% and 33% we have 1.9. So you see that most of these materials which I have indicated here these 1 2 3 4 5 these are normally used in practice for making the strain gage materials.

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Material	Sensitivity (GF)
Platinum (Pt 100%)	6.1
Platinum-Iridium (Pt 95%, Ir 5%)	5.1
Platinum-Tungsten (Pt 92%, W 8%)	4.0
Isoelastic (Fe 55.5%, Ni 36%, Cr 8%, Mn 0.5%) *	3.6
Constantan / Advance / Copel (Ni 45%, Cu 55%) *	2.1
Nichrome V (Ni 80%, Cr 20%) *	2.1
Karma (Ni 74%, Cr 20%, Al 3%, Fe 3%) *	2.0
Armour D (Fe 70%, Cr 20%, Al 10%) *	2.0
Monel (Ni 67%, Cu 33%) *	1.9
Manganin (Cu 84%, Mn 12%, Ni 4%) *	0.47
Nickel (Ni 100%)	-12.1

* Isoelastic, Constantan, Advance, Copel, Nichrome V, Karma, Armour D, Monel, and Manganin are all trade names owned by the respective owners.

Materials included are just to indicate a kind of sensitivity you will expect from these. With this background, let us further go back to a discussion on the use of strain gage and for this purpose I am going to give some of the expressions without proof which are useful and then I will take up a simple example and work it out, that is what is possible in the amount of time we have in this particular lecture. So let us do that.

So if you have a diaphragm I think, I have referred to it in the last lecture itself. You have a diaphragm which is fixed at the periphery and it has got a radius of a and thickness equal to t , this diaphragm is subjected to a uniform load that means the pressure is acting on two sides, one side is high pressure and the other side is low pressure so the diaphragm is going to undergo a change in shape, so we expect something like this to happen. And at the center you are going to have the maximum deflection, and at the periphery because it is held, fixed or clamped there is no displacement it is clamped here.

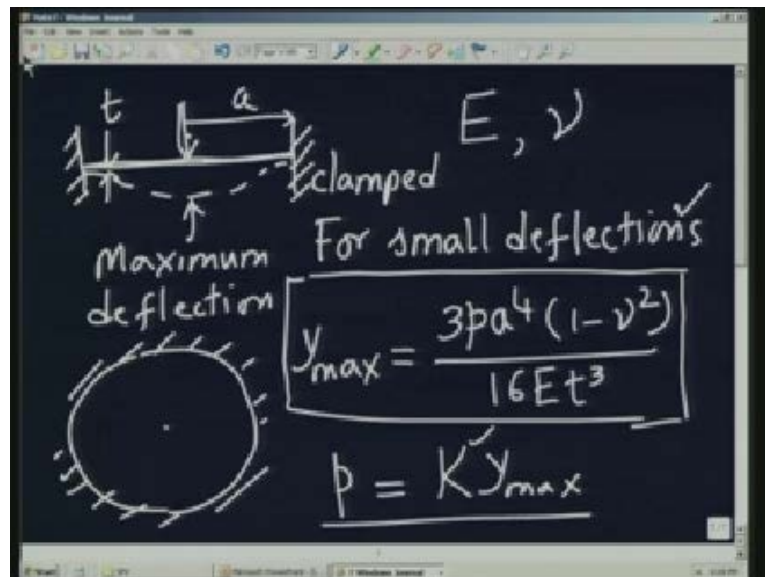
Actually if you look at it, it will be a circle which is fixed at periphery and I am talking about the deflection at the very middle of that circular diaphragm of thickness t , radius a and let us assume that E is the Young's Modulus and ν is the Poisson's ratio for the material of the diaphragm. Again, for small deflections, what do we mean by small deflection? It should be small compared to a characteristic dimension of the

diaphragm itself. For example, the change in or the movement which I have shown here must be smaller than the thickness. It will be in fact much smaller than the thickness and much much smaller than the radius of the diaphragm itself. Therefore for small deflections we have a linear relationship between the pressure and the deflection. So y_{\max} is given by $3pa$ to the power of 4 into $1 - \nu$ square by $16Et$ cube, this is a very important relationship.

Actually if you rewrite it then it is equal to some K times y_{\max} . So what I am going to do in my practical application? I am going to measure the y_{\max} and if I know the gage factor K which is nothing but you can see here p is equal to K times y_{\max} . K is nothing but $16Et$ cube by a to the power of 4 into $1 - \nu$ square. So you see that the gage factor is given in terms of geometric and physical parameters.

Here a and t geometrical parameters the radius and the thickness and the Poisson's ratio and the Young's Modulus are the properties of the material. Physical properties of the material and the geometric parameters are going to be combined together to determine the value of K and this is a linear relationship under the assumption of for small deflections that is going to hold good. Therefore I can use the expression as long as the deflection is very small, so let us remember that.

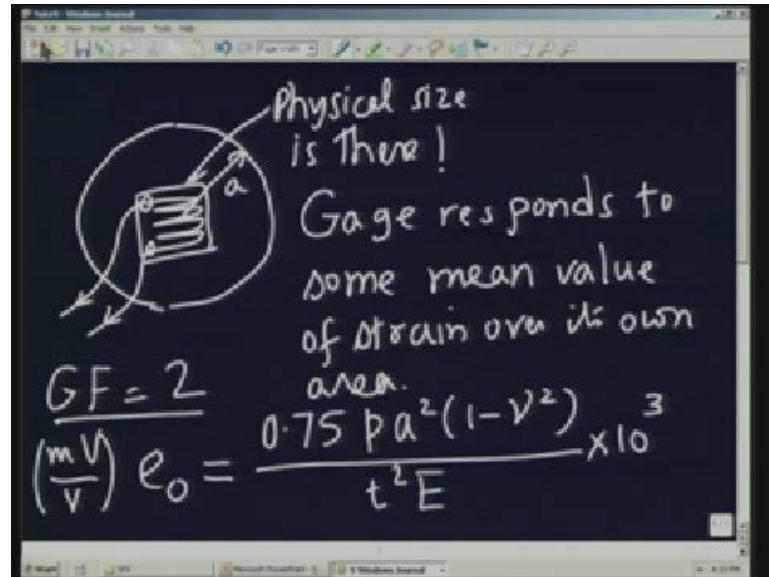
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The second formula I am going to give is something which is going to be useful that is the natural frequency of the diaphragm. The natural frequency of the diaphragm will play a role in measurement of transient pressures when the pressure is varying with respect to time. So the natural frequency is given by f_n , again for small deflections is $0.469 \frac{t}{a} \sqrt{\frac{E}{\rho(1-\nu^2)}}$ where ρ is the density this will give you value in hertz all others are in SI units, the lengths are in meters, Young's Modulus is in Pascal's ν is the ratio therefore it will have no unit, density will be in kilograms per cubic meters, thickness in meters and the radius also in meters so if you substitute you will get the value of the natural frequency in the hertz.

Now let us look at what we are going to do in the case of a gage. If you look at the gage or look at the diaphragm, I will write the diaphragm like this and gage if you remember it looked like this just schematic, from here I have taken the output leads so that is a gage attached to the diaphragm in the middle, and this is the radius equal to a , thickness t and the gage has got a certain physical size it is not a small thing it is quite a big thing. So it is going to occupy a certain region on the diaphragm which is not a point but a certain area. Therefore we can expect that the gage is actually going to respond to the average value of the shear or the average value of the strain over the area of the gage. So we can say that the gage responds to some mean value, it is not the maximum deflection it is the mean value of the strain over its own area, so a useful formula has been worked out for the value, the gage response assuming a gage factor 2 you remember the gage factor equal to 2 if you take which is true for those four materials I discussed earlier for those materials the gage factor is 2 and if I do that the average value of the gage output in terms of millivolts per volts, this e_0 is the gage output in millivolts per volt, I will explain it little later, this is given by $0.75 \frac{p_a}{E} \frac{1}{1-\nu^2} \frac{t}{a^3}$ so many millivolts per volt.

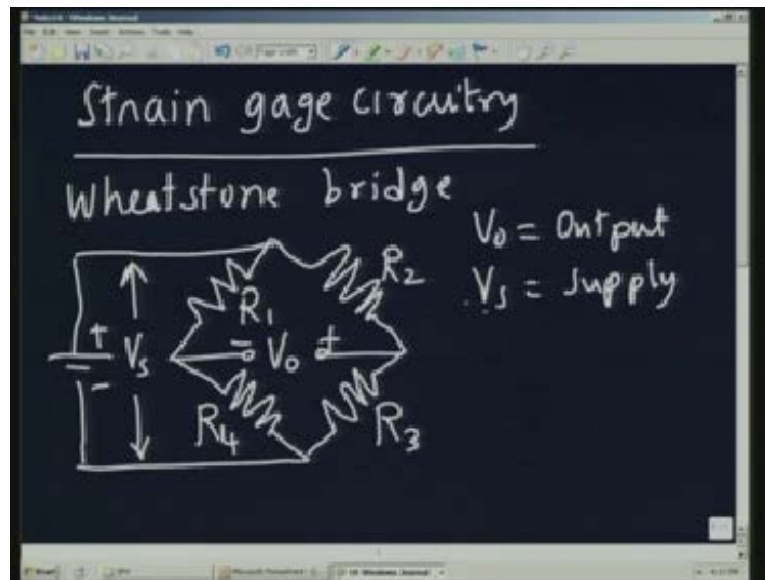
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So the gage responds according to this formula because this is obtained by integrating the effect felt by the gage over its own area when it is placed on the surface of an element which is undergoing strain or deflection. So this is the background for the physical aspects of the problem. Now, before we take up the example I will look at the way we are going to use the strain gage in a circuit. So I will be able to just discuss few elementary things and then of course we will continue in the next lecture.

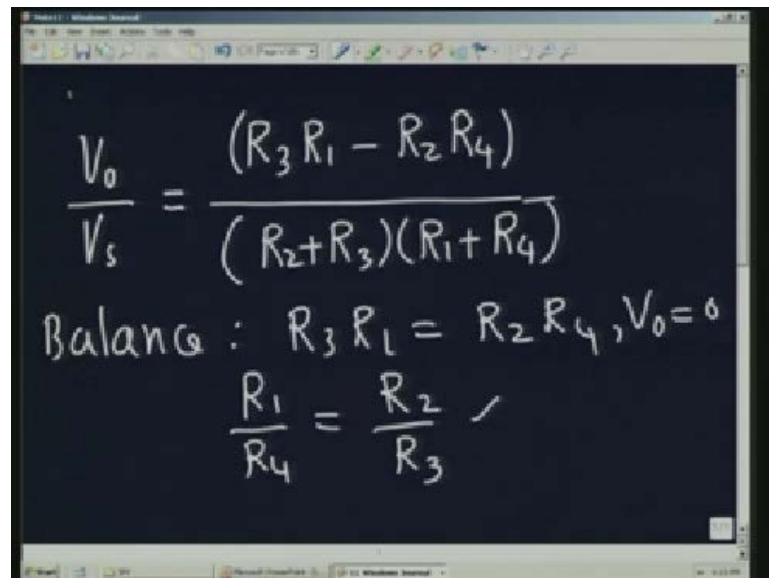
Basically what we are going to do is use a Wheatstone bridge which is very familiar to us. Let me just give a brief idea about how it is going to be used. We have four resistances we have already seen some of these things in case of temperature measurement also. We will label them as R_1, R_2, R_3 , and R_4 and I am going to inject some voltage from outside positive negative and I am going to measure the voltage here this is positive this is negative. We will call this voltage as V_s , the supply voltage and V_0 as the output so V_0 is equal to output V_s is the supply.

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I am talking only about DC volts and that is why I have shown a battery here. Or it could be a supply of DC voltage. In fact we can show the simple analysis of this circuit that the relationship between V_0 and V_s is given by the following formula. I am not deriving it, it is very easy to do that so we will say that V_0 by V_s the ratio of output voltage to the supply voltage is given by $R_3 R_1$ minus $R_2 R_4$ by R_2 plus R_3 into R_1 plus R_4 . This is a very important equation, it gives us an idea about the ratio of an output voltage which is in the numerator to the supply voltage given by $R_3 R_1$ minus $R_2 R_4$ and you can immediately see that when there is a balance the condition is $R_3 R_1$ is equal to $R_2 R_4$ and V_0 is equal to 0, this can also be written as $R_3 R_1$ by R_4 is equal to R_2 by R_3 so R_1 by R_4 is equal to R_2 by R_3 is the condition.

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Handwritten equations on a blackboard background:

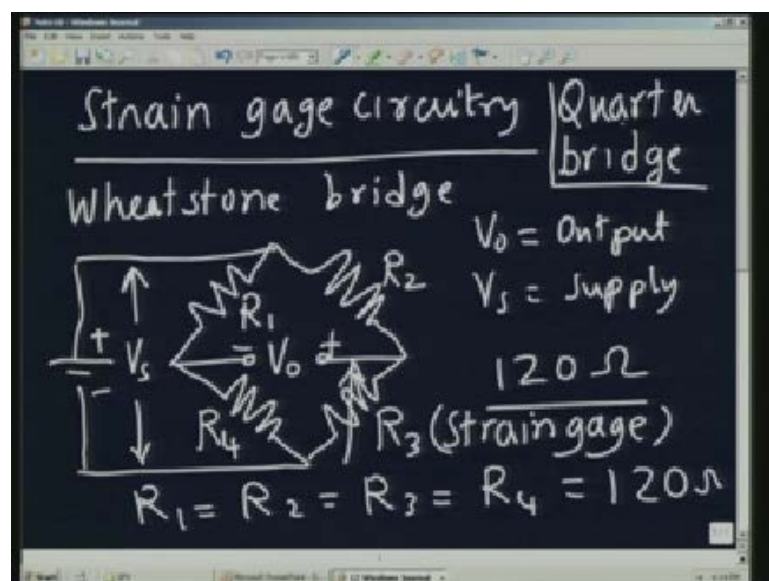
$$\frac{V_o}{V_s} = \frac{(R_3 R_1 - R_2 R_4)}{(R_2 + R_3)(R_1 + R_4)}$$

Balance: $R_3 R_1 = R_2 R_4, V_o = 0$

$$\frac{R_1}{R_4} = \frac{R_2}{R_3} \checkmark$$

Suppose we have the following; we have a gage with 120 ohms value for the resistor suppose I choose R_1 is equal to R_2 is equal to R_3 is equal to R_4 is equal to 120ohms it will be in balance to start with. And what I will do is I will identify this R_3 as the variable resistance which is the strain gage. So one of the resistance has been replaced by the strain gage and this is called quarter bridge arrangement, so this is called the quarter bridge arrangement.

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It is a quarter bridge because 1 by 4 of the bridge consists of variable resistance. So what we will do is to continue from here in the next lecture, look at the way this is going to respond to the changes in the pressure and therefore changes in the strain and then we will look at how to interpret the information in terms of the pressure which is being measured. Thank you.