

**Mechanical Measurements and Metrology**  
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**Module -2**  
**Lecture - 20**  
**Pressure Measurement**

So this will be lecture number 20, on Mechanical Measurements. The topic we are discussing is pressure measurement. We have already covered the basic aspects of manometry that is using either a u tube or a well type manometer for making the measurement of pressure. So what we will do in this lecture to start with we will take a simple example of a well type manometer with an inclined tube and see how we can calculate the pressure given the various input data for the problem. And then we will also look at how to calculate the error in the measured value of pressure by a simple analysis which is based on the error propagation formula which has already been covered in an earlier lecture. After doing that we will look at different types of pressure measuring instruments.

To start with we will look at the bourdon gage and then we will also look at what is called a dead weight tester which is used basically for calibrating pressure gages or comparing different pressure gages in order to ascertain whether the gage is properly calibrated or not. And then we will continue our discussion on various types of pressure gages which are based on some advanced sensors.

And specifically for example we will look at the gages which depend on the displacement of the bending of a diaphragm and the diaphragm movement itself is sensed by a transducer which is going to be a respond to the displacement. So, displacement transducers of different types are used and we will see how they work and also how they how they can be analyzed from the point of view of understanding the working principle of this.

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### Example 21

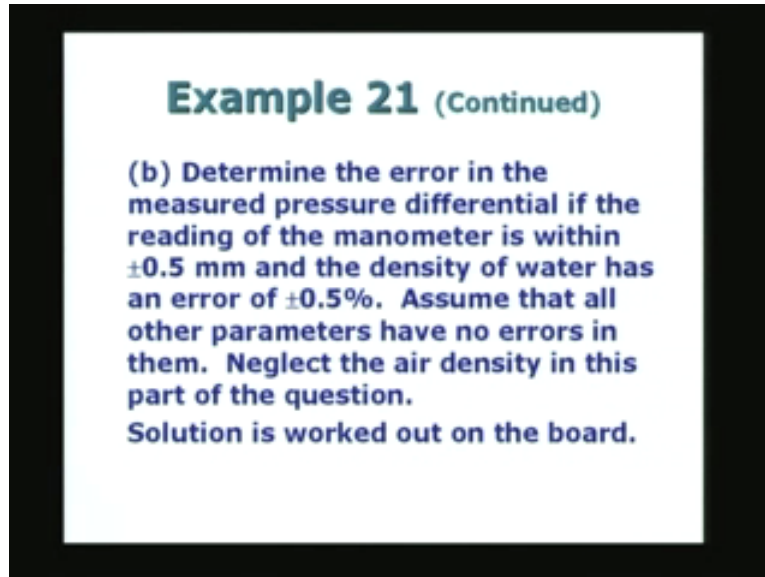
(a) In an inclined tube manometer the manometer fluid is water at  $20^{\circ}\text{C}$  while the fluid whose pressure is to be measured is air. The angle of the inclined tube is  $20^{\circ}$ . The well is a cylinder of diameter 0.05 m while the tube has a diameter of 0.001 m. The manometer reading is given to be 150 mm. Determine the pressure differential in mm water and Pa. What is the error in per cent if the density of air is neglected?

To start with, let me just move on to the example 21, which basically is a problem based on what we have covered in the last lecture. So we are given an inclined tube manometer and the fluid is water, at 20 degree Celsius that is the manometer fluid and the pressure to be measured is that of air and the angle of the inclination of the tube is 20 degree. If you remember theta the angle the well is a cylinder of the diameter 5 centimeters or .05 meters or the tube itself is of 1 millimeter diameter. The manometer reading is given in terms of the column length along the inclined tube which is measured to be 150 millimeters and we are asked to determine the differential pressure or the pressure of the air with respect to atmosphere in millimeters of water and also Pascals.

Also we would like to find out what the error is in percent if the density of air is neglected because if you remember in the last lecture we talked about the fact that the density of the manometric fluid and the density of the fluid in which the pressure is being measured, both are going to play a role. And in many applications we could in fact neglect the density of the fluid in which the pressure is measured. Specifically, in the case of mercury and air for example, the mercury density is so large compared to that of air that this neglect may be justified. But if it is water and air as we have taken up in this particular example it may be a cause or a considerable amount of error.

The second part of the question is to determine the error in the measured value of the pressure.

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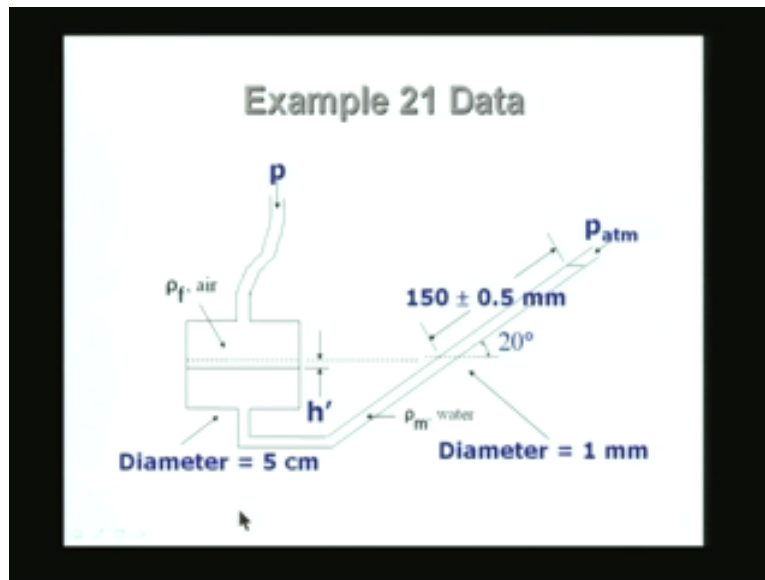
**Example 21 (Continued)**

(b) Determine the error in the measured pressure differential if the reading of the manometer is within  $\pm 0.5$  mm and the density of water has an error of  $\pm 0.5\%$ . Assume that all other parameters have no errors in them. Neglect the air density in this part of the question.

Solution is worked out on the board.

If the manometer reading itself is subjected to some error of plus or minus 0.5 millimeters while taking the reading we make that kind of an error. And also the density of water also has an error of point 0.5% because it is uncertain because of various reasons especially the temperature measurement or the temperature is given at 20 degree if the value of temperature has an error the density will have a corresponding error and so on. For the second part of the problem we will assume that all other parameters are subject to no error and will also neglect the air density in this part of question just to highlight some of the differences we find. So what I will do is I will work out the solution on the board. In fact I will show you exactly what the data is. So I have made a sketch for the example.

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We have the well on the left hand side and the pressure to be measured is spread into the well as you see here and the density of the air is given as  $\rho_{f, \text{air}}$  and the inclined tube is at an angle of 20 degree as indicated here and the distance between the datum and the reading which is taken in this particular case when the pressure is applied as  $P$  is given as ,150 plus or minus 0.5 millimeters. So the angle is 20 degree and you see the advantage of this measurement is that even though we are measuring a distance of 150 millimeters for a change in the level of liquid in the tube the actual height of the manometer is only this, vertical distance between datum and meniscus level here.

In fact if you remember or recall in the last lecture we said that the inclined tube is like an inclined plane and it is a simple machine and it is going to give a mechanical advantage. In this case what it does going to give a larger difference in the level of the meniscus which is measured along the length of the tube using this I am going to calculate a smaller height which is what we have to be measured to start with. So we are amplifying the length by the inclined tube and that is the mechanical advantage we can think in terms of.

So with this basis let us just make a calculation. We are given some of the things, the density of the manometry liquid, this is water, at 20 degree so we have to go to a hand book which gives the density of water, and I have taken the value as 997 point 6 kg per cubic meter. The air whose pressure is being

measured the density has to be calculated. For this we will assume because the pressure differential between the atmosphere and what is measured is related to 150 millimeters and 150 millimeters is very small compared to one atmosphere therefore we can assume that the air is essentially at one atmospheric pressure as far as the density calculations are concerned. Therefore we will say that the pressure of air for calculating density is essentially one standard atmosphere. This is also equal to the 1.013 into 10 to the power of 5 Pascals.

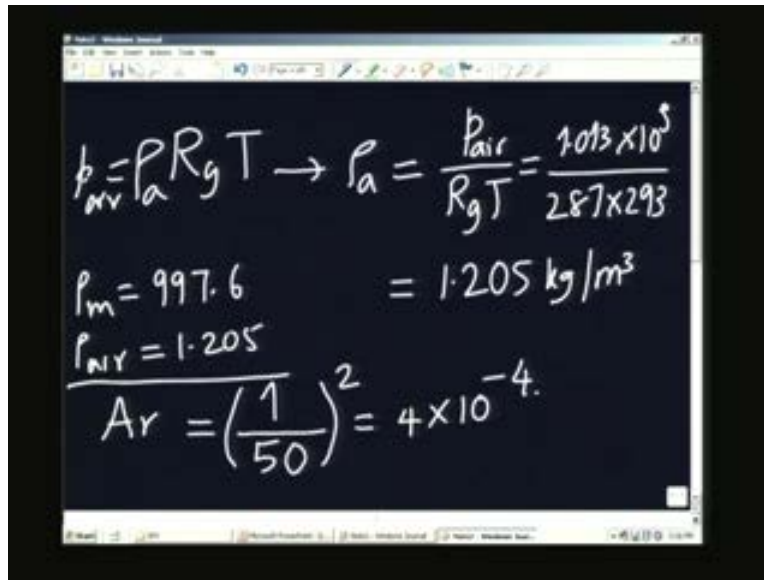
We also know the gas constant for the air assuming it is an ideal gas so we will say  $R_g$ , a gas constant is 287 joules per kg Kelvin and the temperature is the same as this temperature because the air and water are at the same temperature and this is also the temperature of the air equal to 20 degree Celsius. We have to add 273 to make it in Kelvin so this becomes 293 Kelvin.

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$P_m = 997.6 \text{ kg/m}^3$   
 (Water at  $20^\circ\text{C}$ )  $\rightarrow T_{\text{air}} = 20 + 273 = 293 \text{ K}$   
 Air  $\rightarrow p_{\text{air}}$  for calculating density = 1 atm  
 $\downarrow$   
 $R_g = 287 \frac{\text{J}}{\text{kg-K}}$   $= 1.013 \times 10^5 \text{ Pa}$

So, by using the equation of state we can calculate  $\rho R_g$  times  $T$  is equal to  $p_{\text{air}}$ , from this I can calculate  $\rho$  of air or the fluid in which the measurement is being made equal to  $p$  by  $R_g$  times  $T$  is 1.013 into 10 to the power of 5 by 287 that is  $R_g$  into 293 the density comes out to 1.205 kg per cubic meter.

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$$p = p_a R_g T \rightarrow \rho_a = \frac{p_{air}}{R_g T} = \frac{1013 \times 10^3}{287 \times 293}$$
$$\rho_m = 997.6 \quad \rho_a = 1.205 \text{ kg/m}^3$$
$$\frac{\rho_m}{\rho_a} = 1.205$$
$$A_r = \left(\frac{1}{50}\right)^2 = 4 \times 10^{-4}$$

So you see that we have the water density or  $\rho_m$  is equal to 997.6 and the air density is 1.205 both in kilograms per cubic meter. So even in this case you can see that the difference between the densities, this density is very much higher than this, and it will be only one part in 1000 roughly or a little more than one part in 1000. So in essence, it may be alright to neglect this but of course we will take into account then we will see what is going to be error in that.

The second part of the calculation requires two things, one is the area ratio, if you remember, the area ratio becomes because we assume that the fluid the manometric fluid becomes the incompressible therefore, to determine how much is the change in the height of the liquid within the well itself we have to use the area ratio, this is how it comes into the picture. This will be given by 1 millimeter diameter for the tube and the diameter of the well is 55 centimeters so this will be proportional to the square of this and this comes to 4 into 10 to the power of minus 4. There is no dimension between the ratios of two areas. Let us look at the appropriate equation in this case.

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$$\begin{aligned}
 (p - p_{atm}) &= (p_m - p_a) g R (\sin \theta + A_r) \\
 \text{(gage pressure)} \quad &\quad \sin 20^\circ = 0.342 \\
 &= (997.6 - 1.205)(9.81)(0.15) \\
 &\quad (0.342 + 4 \times 10^{-4}) \\
 \text{Error } &\boxed{\pm 0.6 \text{ Pa}} \quad \text{quite negligible} \\
 &= 502.06 \text{ Pa.} \quad \text{(Neglecting } p_a \text{ } 502.66 \text{ Pa)}
 \end{aligned}$$

So the pressure difference if you remember will be  $p$  minus  $p$  atmosphere, what we are measuring is the difference between the pressure of the air with respect to atmosphere, this is also referred to as the gage pressure.

I am just trying to introduce some of the terminology using this particular example. I am measuring the gage pressure. If I am interested only in the absolute pressure  $p$  will be called the absolute pressure the gage pressure plus the atmospheric pressure will give you that. However, I am interested in the gage pressure in this particular case. This is given by  $\rho_m$  minus  $\rho_a$  or  $\rho$  fluid in general, multiplied by the gravitational constant  $g$  multiplied by the reading  $R$  multiplied by there are two factors inside this, one is because the of height which is measured which is actually  $R$  times  $\sin \theta$  so I will put  $\sin \theta$  here plus the depression of the level within the well which is calculated based on the incompressibility assumption,  $\sin \theta$  will not there, therefore this will be simply the area ratio  $A_r$ . And let me just look at this formula again. If  $\theta$  is equal to 90 degrees it corresponds to the vertical tube case 1 plus  $A_r$  and that is what we had in the case which we treated earlier.

Therefore if  $\theta$  is equal to 90 degrees the inclined tube manometer becomes the vertical tube manometer and this will become  $\sin 90$  is equal to 1 plus  $A_r$  instead of that I have got  $\sin \theta$  plus  $A_r$ . So all I have to do is to

plug in the values  $\rho_m$  is determined 997.6 minus  $\rho_a$  is 1.205, g I am assuming 9.81 and R is 150 millimeters so I will convert it into point 15 meters. In this formula everything should be in SI units otherwise you will not get Pascals that is very important. This is multiplied by sin of 20 degrees and sin of 20 degrees happens to be point 342. So you can make a note here that this is point 342 which is the same as sin of 20 degrees plus area ratio is 4 into 10 to the power of minus 4. Actually you will notice one thing because the well has a very large diameter compared to tube the area ratio is very small and actually this is negligible even compared to this, this is four units in the fourth decimal place therefore we can say that this is quite negligible.

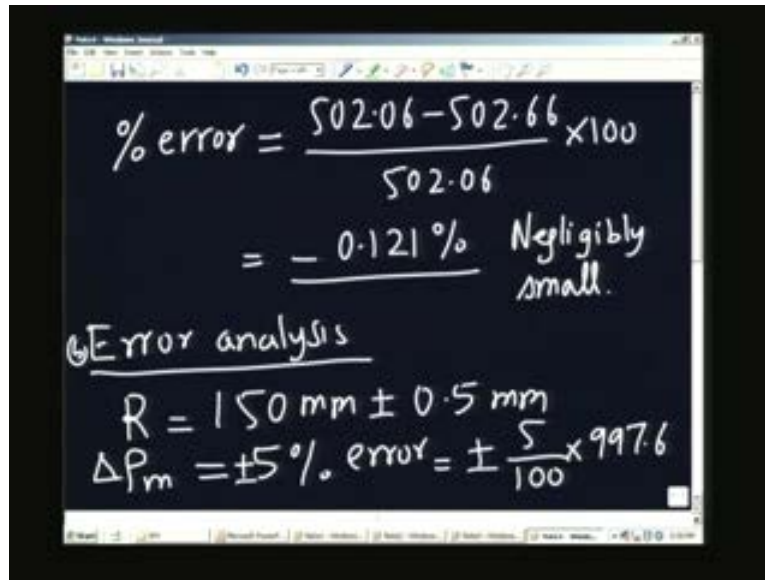
So the idea of having taken an example is to indicate or to get a feel for the apparatus or the instrument and understand what are the significant things and what are the insignificant things in that particular example, find out what kind of approximation is valid. In this case you can see that the area ratio being very small in fact we can ignore it and most of the times it is actually done that way. So in other words I can simply say that I can remove this  $A_r$  from this formula inside this can be ignored or cut this factor so that you get  $g r \sin \theta$  and  $\rho_m$  minus  $\rho_a$  and indeed if you ignore  $\rho_a$  also it simply becomes  $\rho_m g r \sin \theta$  and this value comes out to 502 point 06 Pascals.

This is first the part of the question which says please find out what is the measured pressure from this calculation. Now in this calculation we have taken into an account  $\rho_a$ , suppose I neglect  $\rho_a$ , I will just give the value in the bracket, neglecting  $\rho_a$  you will get a value of 502 point 662 so you get, 502 point 06 if you do not ignore the value of density of air and if you neglect it you get 502 point 66.

Actually by neglecting the density of air you are actually over estimating the pressure by a small amount. In this case it will be point 06 Pascal. I will write it here, error in neglecting is plus point 06 Pascals. Of course you can represent it also as a percentage value. So the percent error when you ignore the density of the fluid is the difference between the two in which the first one is 502 point 06 minus 502 point 66 by 502 point 06 into 100% and this comes to minus point 121%. So, for this you get a value of minus point 121% which is quite negligible, negligibly small.



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The image shows a digital blackboard with handwritten mathematical calculations. The first part calculates the percentage error using the formula  $\% \text{ error} = \frac{502.06 - 502.66}{502.06} \times 100$ , resulting in  $-0.121\%$ , which is noted as "Negligibly small." The second part is titled "Error analysis" and shows  $R = 150 \text{ mm} \pm 0.5 \text{ mm}$  and  $\Delta P_m = \pm 5\% \text{ error} = \pm \frac{5}{100} \times 997.6$ .

$$\% \text{ error} = \frac{502.06 - 502.66}{502.06} \times 100$$
$$= -0.121\% \quad \text{Negligibly small.}$$

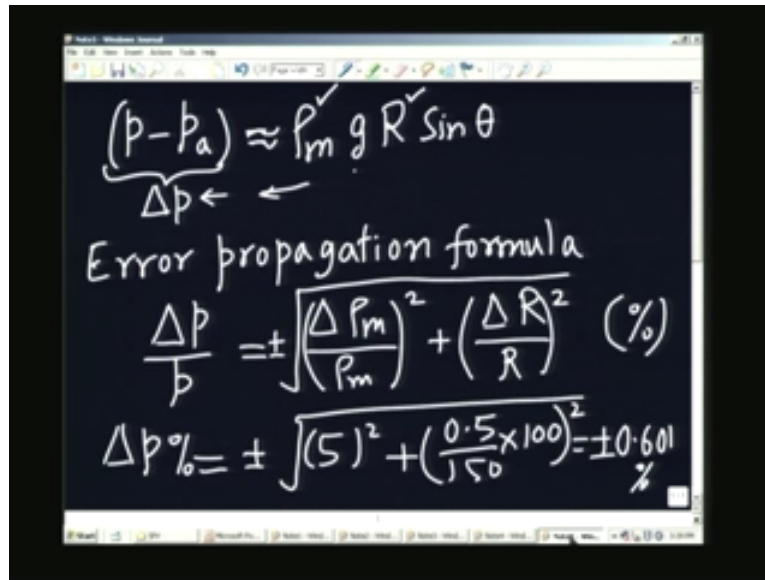
Error analysis

$$R = 150 \text{ mm} \pm 0.5 \text{ mm}$$
$$\Delta P_m = \pm 5\% \text{ error} = \pm \frac{5}{100} \times 997.6$$

The second part of the question concerns the analysis of errors. So, the next one is error analysis which will actually complete the solution of the problem. This is actually part b. So what are the errors due to, we know that the value of  $R$  which is measured, So we have  $R$  is equal to 150 millimeters plus or minus point 5 millimeters, this is one of the sources of the error. We also know that  $\rho_m$  has a 5% error so I will write it as a plus or minus 5% error. So I will say  $\Delta \rho_m$  5% that means plus or minus 5 by 100 into 997 point 6.

Actually the error which is specified here is quite artificial just to indicate the way you do the error analysis. In actual practice the error may be much smaller than this. It is just to get an idea of how sensitive it is and just deliberately take a large error. In practice the error is going to be much smaller than this. So the point is, we go back to the formula which gives  $\Delta p$  or  $p$  minus  $p_a$ .

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The image shows a digital blackboard with handwritten mathematical notes. At the top, the equation  $(p - p_a) \approx \rho_m g R \sin \theta$  is written, with a bracket under  $(p - p_a)$  labeled  $\Delta p$ . Below this, the text "Error propagation formula" is written. The formula for the relative error is given as  $\frac{\Delta p}{p} = \pm \sqrt{\left(\frac{\Delta \rho_m}{\rho_m}\right)^2 + \left(\frac{\Delta R}{R}\right)^2} (\%)$ . Finally, a numerical calculation is shown:  $\Delta p \% = \pm \sqrt{(5)^2 + \left(\frac{0.5 \times 100}{150}\right)^2} = \pm 0.601 \%$ .

I am going to make the approximation that the density of air itself is neglected. So I will say approximately  $\rho_m g R$  and in fact I can also ignore the area ratio there. If you want you can take into account plus  $A_r$  this is approximately this one. And I will just call this as delta p as a symbol representing p minus p or the gage pressure is delta p.

So if you remember the error propagation formula which was covered in a earlier lecture, what does it say? It says that we have certain error in this measurement and these two errors are going to propagate to this so the propagation is from the right hand side to the left hand side. And we are also given that the other quantities are all known without any error. That means g and sin theta has no error and therefore we will ignore it. But in practice sin theta or theta itself may be subject to some error and in which case we have to take into an account also the error in the value of theta and hence sin theta.

So if you write the error propagation formula delta p which can be obtained by simply logarithmic differentiation because this is in the form of a product of quantities, so this becomes delta  $\rho_m$  by  $\rho_m$  whole squared plus delta R by R whole squared plus or minus of under square root, because in this particular case delta p is a function of product of gm and R, g and sin theta are not going to coming into the picture because they do not have any error

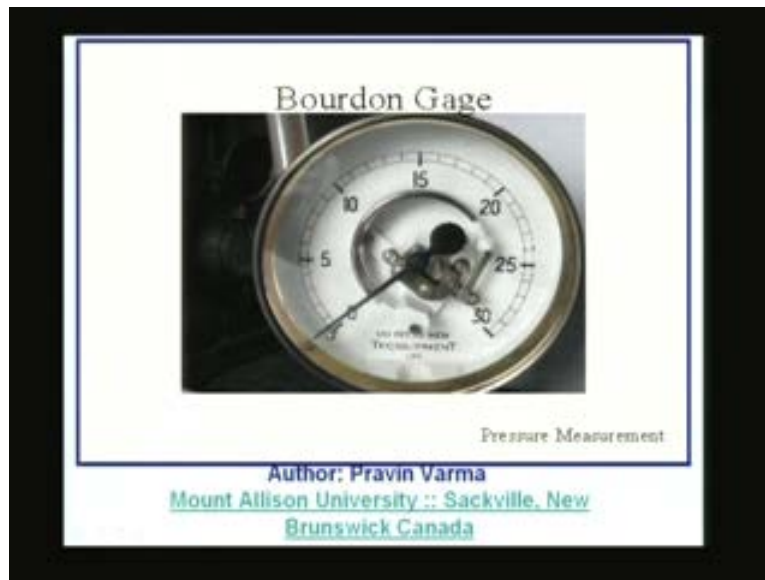
assigned to them.

Therefore there only two sources of error; error in  $\rho_m$  error in R. And if I substitute these errors in terms of percentages this whole thing can be written in terms of percentages,  $\Delta p$  by  $p$  percent plus or minus this percent whole squared plus this percent whole squared that is another way of writing it. So we mentioned that it can be all in terms of percentages. Both the left hand side as well as the right hand side will be in percentages, this is 5%. 5 point 0 whole squared plus  $\Delta R$  by  $R$  is point 5 millimeters by 150 into 100 whole squared because the value was not given in percentage. I have calculated in percentage by taking .5 which is the expected error divided by 150 is the actual measured quantity multiplied by 100 whole squared gives you the value. So the  $\Delta p$  I will say is this error in the percentage is plus or minus point 601.

I think I have made a mistake earlier, let me just go back, this is point 5%, let me just correct it, not 5% actually this point 05 by 100 into 997 point 6. So the point I was making at the time was with respect to point 05%. I should have said that point 05% itself may be too big to expect in this particular case. Therefore the pressure is subject to an error of point 06% because of the two uncertainties in the density and uncertainty in the measured value R.

You just you go back and calculate or compare this error, what happened when you neglected the value of the density of air. Previously, I had a point 12% if you go back you will see, so the percentage error between the two cases was point 12 or point 6 about 5 4 this error is not negligible. Thus, in the previous case we said that the error due to the neglect of the density of the air itself is very small it can be neglected but not this, this is about five times that error. So in this problem what comes out through the numbers is that some errors may be neglected, some errors may not be neglected depending on the circumstance. So we will go back and continue with our discussion on the other types of pressure gages. So the first type we are going to look at is called Bourdon Gage.

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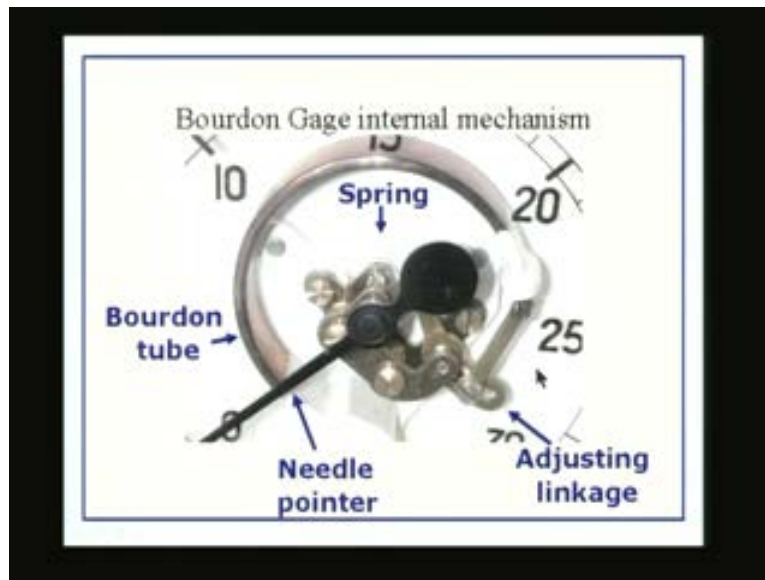


Let me just briefly explain the mechanism of the way it is going to function. I have taken a picture which available on the net and I have presented it here. It is from the net in which the Author Pravin Varma has put this particular picture and few more interesting pictures were available there. If one would like to look at them one can go to website and look at them.

This is typically the appearance of a Bourdon Gage. The Bourdon Gage essentially consists of, if you see here you are able to recognize that there is a tube in the form of C, this is the tube and this is connected to some linkage here and eventually it is connected to a needle or the pointer which is going to move over the scale.

Let me just go to next figure which is some what clear. What it consists is the Bourdon tube which is some kind of a flattened tube, you can see that here, it is flattened and the cross section is somewhat elliptical and the inside is communicated to the inside of the tube which is sealed.

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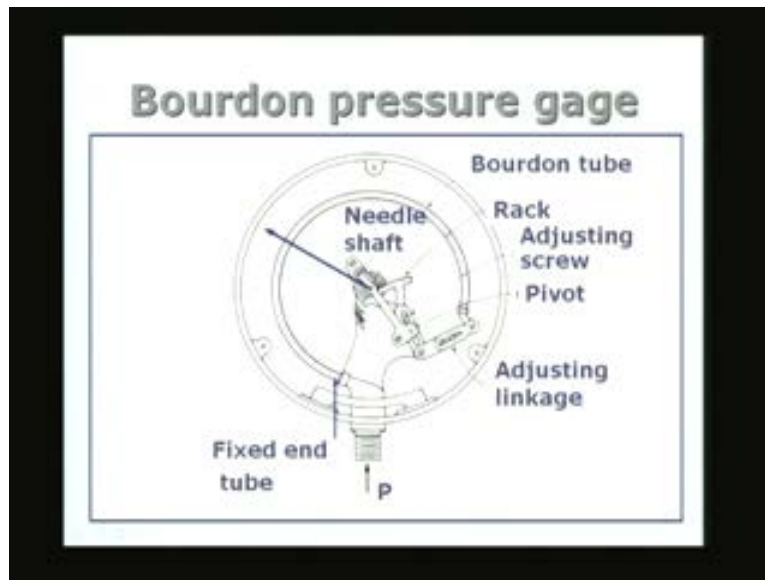


Through the sealed tube we communicate the pressure to be measured. And of course at the outside pressure is the atmospheric pressure. Or in some cases the outside pressure may be a different pressure.

We can use this for measuring the differential measure or the gage pressure. By now you will realize what the gage pressure means. Essentially it consists of this Bourdon tube which is fixed at one end and then it is connected to a linkage mechanism. It is not very clear here but in the next sketch I am going to make it clearer. It is got some arrangement by which it is going to move a rack against a pinion and the pinion is connected to the shaft of this needle pointer and there is a spring. So the spring is going to resist the movement of the Bourdon Tube.

If you have a pressure difference between the inside of the Bourdon tube and the outside the tube tends to open up, when it tends to open up it tends to move the pointer and the spring force is going to oppose this. So under equilibrium the needle will take such a position that the forces due to the torque on the needle pointer and the force exerted by the Bourdon tube is exactly compensated by this spring. The spring is a restraining element and the pointer is going to take a position which will correspond to a certain angle along this scale and the angle angular position can be calibrated with a particular  $\Delta p$ , a very simple instrument.

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And as we have already made it clear in the previous lecture it can be used for various ranges starting from even below atmosphere to above atmosphere to very large pressures, very small pressures the Bourdon gage covers a very large range of pressure measurement. So let us look at the internal construction. The internal construction is very clear from this particular sketch.

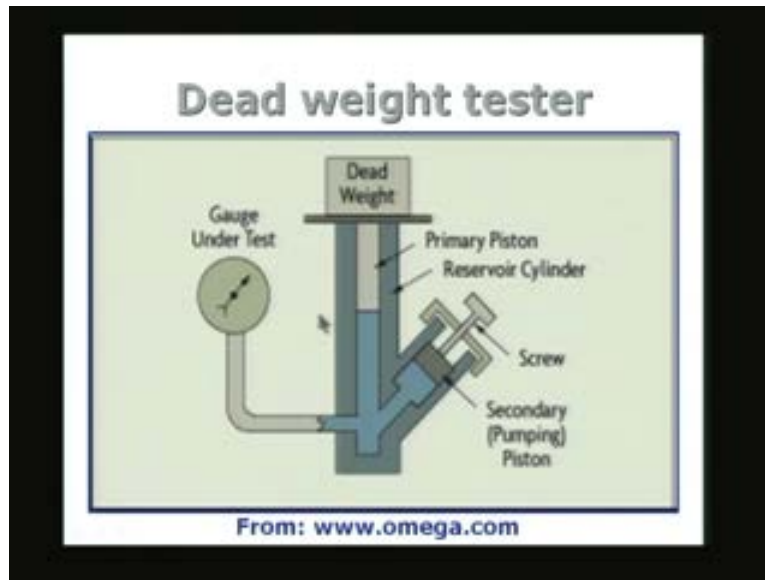
You have the fixed end of the tube, this end of the tube is fixed and it is in the form of a C as I said, and it is connected to an adjusting linkage this adjustment is for adjusting the 0 position. And then it is connected to the rack arrangement and the pinion is actually connected to a needle shaft and you have the adjusting screw here and pivot here and it is very clear. So the pressure is communicated through this tube to the inside and outside is the atmospheric pressure. Or if you want to measure the differential pressure it is  $p_1$  inside and  $p_2$  outside. So the difference in the pressure is what is going to be measured and the mechanism of working has already been explained.

With this background let us look at, for example, if I were to look at the calibration of a pressure gage of the type which is indicated here then what is calibration?

Calibration in this case is, the pointer is going to occupy an angular position and each angular position will correspond to a certain pressure difference between inside and outside. and I would like to make a scale on this

circumference of this circle or the indicating position of the pointer and at the corresponding position of the pointer a pressure reading must be indicated so that is what we call it as a calibration exercise. So the calibration can be done by using what is called a dead weight tester.

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It is a very simple instrument. So what we have in the dead weight tester is, we have a fluid in this case oil which is in this cylinder and the oil can be pressurized by using a screw as is shown here, this plunger is going to go in and out and increase or decrease the pressure as the case may be and there is a dead weight and there is a piston on which the dead weight is placed, this whole thing is vertically oriented. And if you can realize the pressure of the oil here, if the dead weight is just floating, that is if the oil pressure is such that the pressure exerted by the pressure weight on the piston which is simply given by the weight which is placed on the piston, the dead weight here divided by area of cross section of the piston is the pressure.

Therefore, calibration is done by having a known weight and the known area of cross section of the piston such that the pressure can be calculated directly as the weight divided by the area of the piston as simple as that and this pressure is acting on the oil and there is a side tube through which it is communicated to the gage under test and the gage will experience the same hydro static pressure, the fluid pressure, the oil pressure is communicated

what ever is experienced by the liquid here which is the oil usually is communicated to the gage under test.

Now what I have to do is to use different weights and adjust the position of the screw such that the dead weight is going to just float which means it is going to move down freely, it should not be restrained. In that case the dead weight divided by the area of cross section the piston gives you the pressure which can be expressed in terms of the weight in Newtons divided by area in square meters that will be in Pascals and directly the position of the needle here will correspond to the same pressure, a very simple arrangement, a very accurate arrangement this is in fact considered as a standard for pressure calibration. A typical dead weight tester is available in the market, this is manufactured in Australia, you have the gage under tester which is indicated here, the weights are added here and then the screw is actually this handle and this is usually put on the table top and then you can see that it will also have an arrangement by which you can find out whether it is leveled and so on, leveling screws are there and so on.

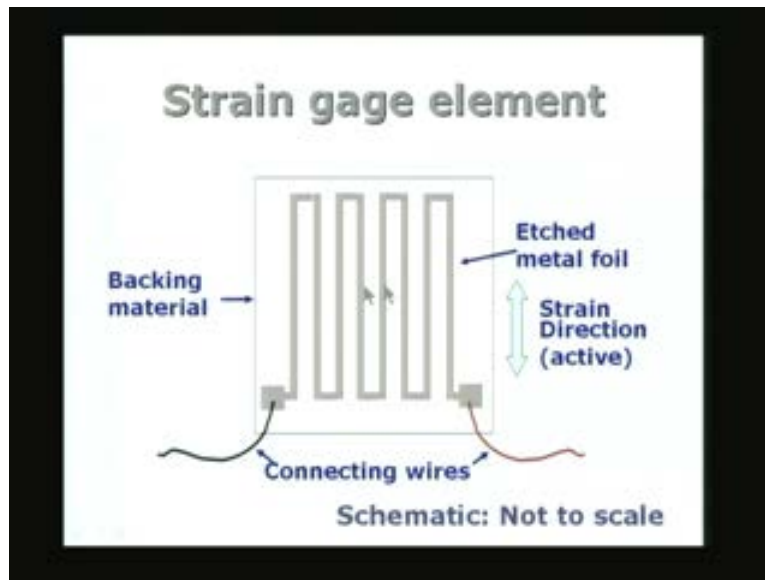
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This is a dead weight tester which is commercially available and it is used for calibration purpose. Now, let us look at the other ways of measuring pressure.



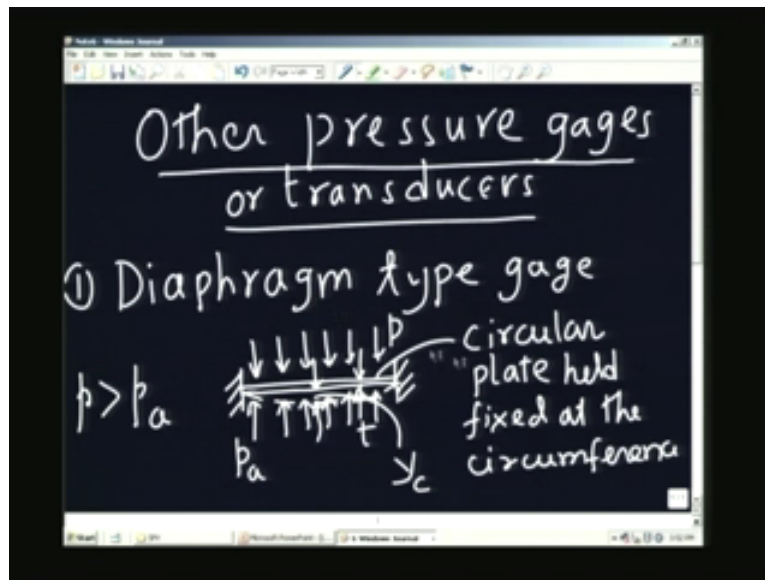
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I am going to go to the board and indicate some of the methods. Other methods of measuring, I will simply say other pressure gages or transducers. Let me just look at the principle of operation of different types of pressure gages one by one.

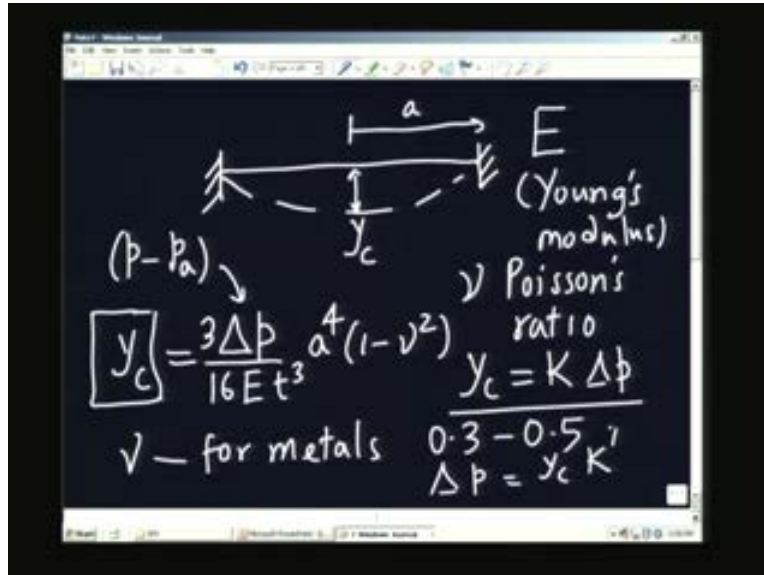
As I indicated earlier we could have a diaphragm type gage and what is the diaphragm type gage? Suppose I have a diaphragm which is held fixed at the periphery, this is actually a circular plate of small thickness so we will say that thickness  $t$  and if I apply a uniform pressure  $p$  on this side, and the uniform pressure of  $p_a$  on the other side then I have a circular plate or a diaphragm held fixed at the circumference at the periphery, it is held fixed.

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And now I am applying the uniform pressure on one side and the atmospheric pressure of course which is uniform on the other side. So if the  $p$  is greater than  $p_a$  then the diaphragm is going to undergo a displacement like this. It is going to undergo a change in shape, it is going to open out like this so if you look here, there is a displacement  $y$  at the center or any other place I am just taking the center. To make the figure more clear let me just indicate it here, so I have the diaphragm which is fixed and it is going to undergo a change in shape like this.

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I have a displacement which is measured at the center I will call it  $y_c$  and let us take this as radius  $a$  and the material has got some mechanical properties which are going to be important.

Let me look at the mechanical properties the Young's Modulus from Strength of Materials, you must be already knowing this quantity, Young's Modulus is going to play an important role. The Poisson's ratio  $\nu$  is a Poisson's ratio is also going to play a role in determining the displacement of the diaphragm and the formula which describes this is given by  $y_c$  is equal to  $\Delta p$  where  $\Delta p$  is  $p$  minus  $p_a$  as we already have written down earlier,  $p_a$  is the atmospheric pressure,  $p$  is the pressure which is being measured,  $p$  minus  $p_a$  is  $\Delta p$  multiplied by three times  $\Delta p$  divided by sixteen times Young's Modulus times cube of the thickness  $t$  cube multiplied by fourth power of radius  $a$  to the power of four multiplied by 1 minus the square of the Poisson's ratio. Poisson's ratio for metals, if you are using a metal diaphragm is a stainless steel or some other metal it may be any where between 0.3 and 0.5. And Young's Modulus is known,  $a$  is the radius of the diaphragm and  $\Delta p$ .

Therefore you see that the displacement at the center is proportional directly  $\Delta p$  and all the other things are constants, material constants or geometrical constants. Therefore you can see that  $y_c$  is equal to some  $k$  times  $\Delta p$ . Or you can also write it as  $\Delta p$  equal to  $y_c$  into  $k$  dash where  $k$  dash is nothing but 1 by  $k$  and  $k$  dash is nothing but the gage constant.

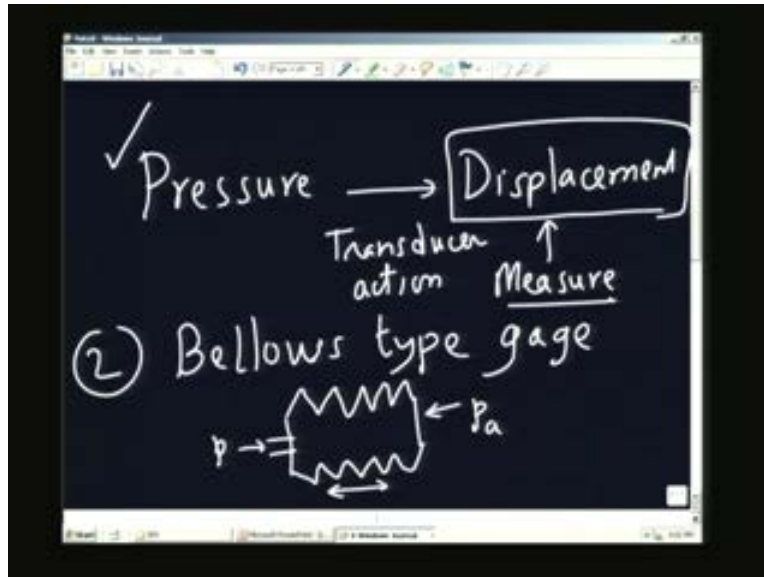
Therefore if you have a diaphragm as indicated here, and if you have a method of measuring the displacement at the center, of course you can measure the displacement in some other location also.

At the center it will be maximum and therefore presumably it will be easier to measure the maximum there at a some other location away from the maximum. So if you are able to measure the displacement you can relate it to the pressure by a simple linear relationship given by  $\Delta p = \frac{y}{k}$  where  $k$  is a constant which once for all determined by  $\frac{3}{16} E t^3 a^{-1} (1 - \nu^2)$ .

Thus, if the diameter or this radius of the gage is known, thickness of the material is known and the material itself is known its property is Young's Modulus in the Poisson's ratio then we directly have a formula which can give you the value of  $K$ . Again I can use again a calibration procedure like what we discussed with respect to Bourdon gage to calibrate the instrument instead of looking at all these properties and then they are subject to there own errors and so on. We do not know exactly what these values are so we can actually calibrate the instrument instead of going through a formula like that. But the formula helps us in identifying the fact that there is a direct linear relationship between the displacement and the pressure.

Therefore in this particular transducer, we can say that pressure is converted to a displacement. And in fact this is called transducer action. Of course we have come across similar things earlier.

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We talked about thermometers; the change in resistance is related to temperature so we measure the resistance change when we went for the temperature change. Here we are finding out what the pressure is by measuring the displacement of a diaphragm element.

The second one corresponds to what is called a Bellows type gage. In fact we briefly referred to it earlier. In the Bellows type gage we have actually a Bellows which is very familiar to us, it is a tube with accordion type shape. So one side you have the pressure, outside you have the atmospheric pressure. So if you introduce the pressure inside, this is going to compress. Of course it is like a spring and it has resistance to this motion so there will be some equilibrium when the spring force is exactly equal to the force due to the  $p$  minus  $p_a$ .

Actually you can look at the diaphragm gage also the same way. The diaphragm is nothing but a spring and the displacement is such that when the diaphragm has displaced itself from its original position to the changed position the internal is exerting a force exactly opposite to the force which is giving rise to the displacement that is where it came to equilibrium. Therefore the element is actually spring element and what we have is simply pressure being measured by the spring constant of the system.

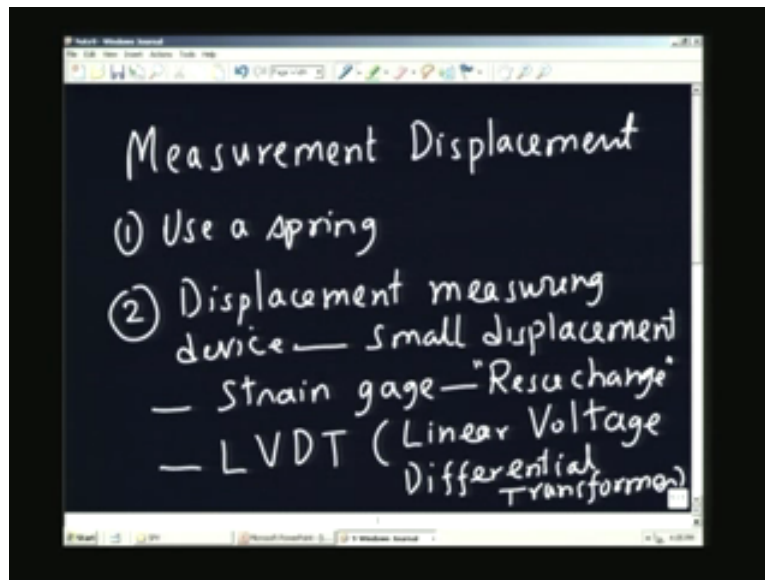
Actually if you remember we used a spring balance for measuring weight

actually it is the same principle the force due to the weight balanced by the spring force which is in the opposite direction. Therefore the equilibrium position corresponds to the weight being equal to the spring force and the spring force is proportional to the displacement of the spring multiplied by spring constant and is exactly the same, so it is Hook's law in action. And if you look at what we did just now in the diaphragm the diaphragm displacement is also according to Hook's law. The material is undergoing a displacement by Hook's law kind of analysis, elastic deformation, Young's Modulus has come into the picture only because of that.

In other words to summarize the two or three different methods of measuring the pressure I have given now, they are all based on the same principle. The displacement, the spring and then a force due to pressure which is balanced by the force of the spring and that is what it is all about. So the Bellow type gage also will undergo a displacement and again I can say that the pressure is going to be related to a displacement and therefore you see that we have changed the original problem measuring the pressure to that of measuring the displacement, so this is what we have measured, measure displacement of the change in the position of some datum, you want to measure that change and then relate it to the pressure. So the point is the original problem of measuring the pressure has been now replaced by the measurement of displacement.

What are the different ways of measuring the displacement is what we have to now look at.

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So if we are now thinking in terms of converting the pressure information to displacement information. Measuring the displacement information by using a suitable instrument or suitable transducer and then working backwards and trying to find out what the pressure is. So there are several ways of doing it. Actually we can use a spring, one is to use a spring itself.

Second one, we can measure the displacement by using a displacement measuring device. If the displacement is small we need a very sensitive method of measurement and usually the displacements are in fact, small displacements. We will be able to see why the displacements are small because, in the case of diaphragm the displacement must not be such that it is going to give rise to plastic deformation it should be operating all the time within the elastic limit.

Elasticity is what gives rise to a spring because once you remove the force it goes back to the original position so elasticity means, the displacement vanishes when the force is removed. When the force is brought back it is going to displace. So the relation between the displacement and force are one to one. Remove the displacement or remove the force there is no displacement. So generally small displacements are required. Very small displacements cannot be measured by using a normal scale. Therefore we have to devise some way of measuring the small displacements.

So what are the displacement measuring devices?

One is, we can use a strain gage, we can also use what is called LVDT this is a Linear Voltage Differential Transformer. LVDT stands for the first letters of these words here Linear Voltage Differential Transformer. This strain gage in this case the resistance change is measured. Electrical resistance of an element undergoes a change when it is subjected to a displacement and therefore what I am going to do is I am going to have one more transduction or one more transducer action pressure, the pressure gives rise to a displacement and displacement is now going to apply on an element which we call as a strain gage and this strain gage element is going to undergo the same displacement and because of this displacement the strain gage is going to change its resistance, this change in resistance I am going to measure using an electrical circuit. Therefore we have a chain of transducer action here. We started with pressure, displacement, displacement to resistance change, resistance change to some change in electrical voltage or current. So there is a chain of activity.

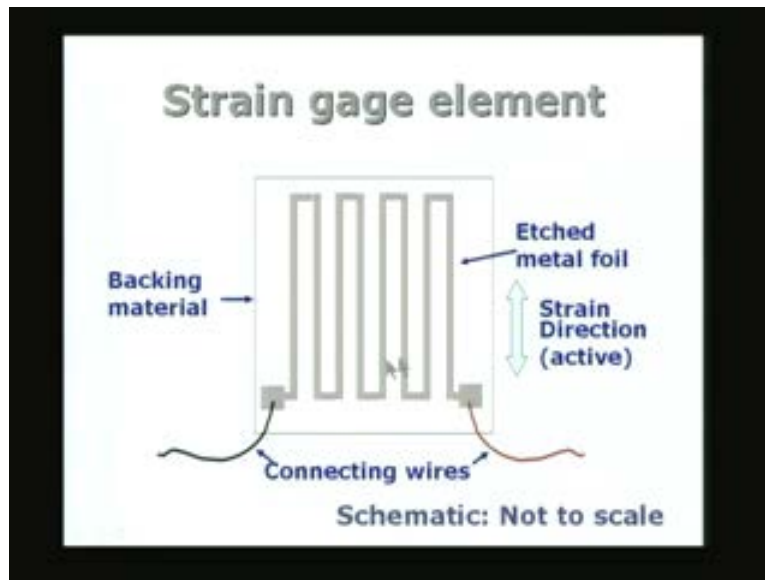
In the case of LVDT the displacement is going to be communicated to this differential transformer and it is going to give an output which is expectedly or designed to be linearly dependent on the displacement. So we will be very happy to have a linear relationship because it is easy to model. And therefore LVDTs can be used for measuring very small displacement, a strain gage can be used for measuring very small displacement, strain gages are also called as load cells, they are used actually for measuring stress in terms of the strain, stress gives rise to strain and strain gives rise to a change in the resistance and change in resistance gives rise to change in voltage.

So you measure the stress, in this case of course pressure is the stress we are talking about. In the case of application where strain gage is used for measuring the displacement of the strain in order to determine the stress there is one area of activity where strain gage is used, in elasticity the study of stresses in structural members and so on.

But in this particular example we are interested in measuring a pressure using the strain gage as the displacement transducers, it measures the displacement. Or we can use the LVDT it is a linear voltage differential transformer which can be used for measuring the displacement. So let us look at some of the things. I will come back to some of these in the next lecture.



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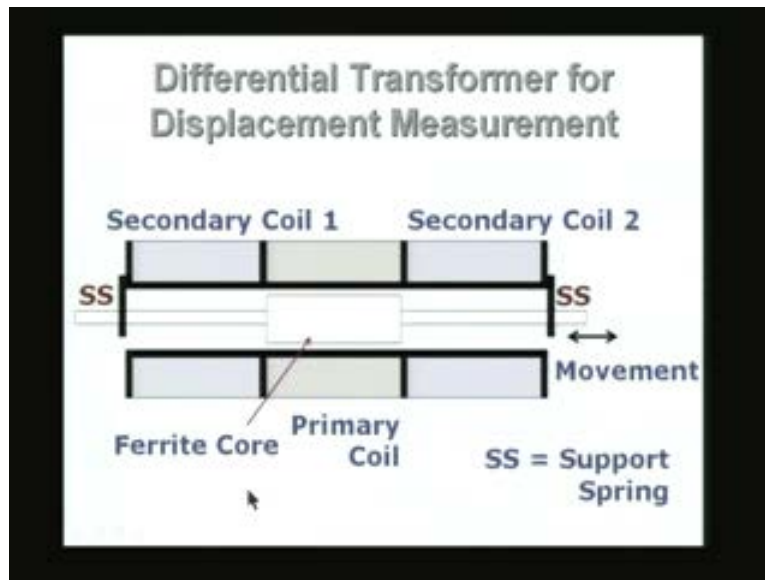


A strain gage element I said just a little while ago that we can measure the displacement using a strain gage element.

What is this strain gage element?

It consists of an electrical conductor or it could be also a semi conductor. Now-a-days semi conductors are also used because they are much more sensitive than metal elements and it is arranged in this form like what is shown here. in this case you have a long length of wire twisted around like this, and you see that this is usually edged on the backing material which may be a plastic for example, and the resistance between these two ends is the resistance of this whole element that depends on the strain, and in this case the strain direction or the active direction for this gage is this particular direction, if this is subjected to a force in this direction shown here then the resistance will change and we will be able to get a change in resistance which can be measured in terms of change in voltage, we will see that the next lecture of course. The differential transformer for displacement measurement consists essentially of a primary coil in the center two secondary coils one on the left side and the other one on this side and there is a core which is a ferrite core which is restrained or controlled by a spring force.

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There is a spring here support spring and in our case what I am going to do is I am going to get the displacement from my pressure transducer I am going to communicate to this core and this core will either move to the either left or right and it will undergo a certain displacement. Suppose I give an AC alternating current to the primary coil, the two secondary coils are going to differ to generate alternating current voltages. And if the ferrite core is moved to the left there will be coupling between the center coil and the secondary coil here so that this will generate more voltage and this will generate less voltage.

Suppose this moves to the right hand side the secondary coil two will give a high voltage than the secondary coil one. And if I connect these secondary coils back to back, in other words if I measure the difference in voltage generated by the secondary coil one and secondary coil two that will be proportional to the movement of the displacement. That is why it is called the linear voltage differential transformer, linear voltage because if it moves to the right or to the left the voltage which is developed between the secondary coil one and secondary coil two in the differential mode that is the differential transformer is proportional to the displacement so a linear relationship exists between the voltages.

Of course a secondary coil one generated more voltage it will be in one direction, it will change the direction when this generates more so the

variation of the voltage will depend on whether the secondary coil two is in more intimate, it is actually magnetic coupling, the magnetic coupling between coil one and secondary coil two determined by the position of the coil.

So I think we will stop there and in next lecture what I am going to do is I am going to take up each one of these in some more detail, work out one or two examples to show and demonstrate how these particular instruments work and then move on to a very important topic that is measurement of unsteady pressure or time varying pressure which is also very important from view point of applications. Thank you.