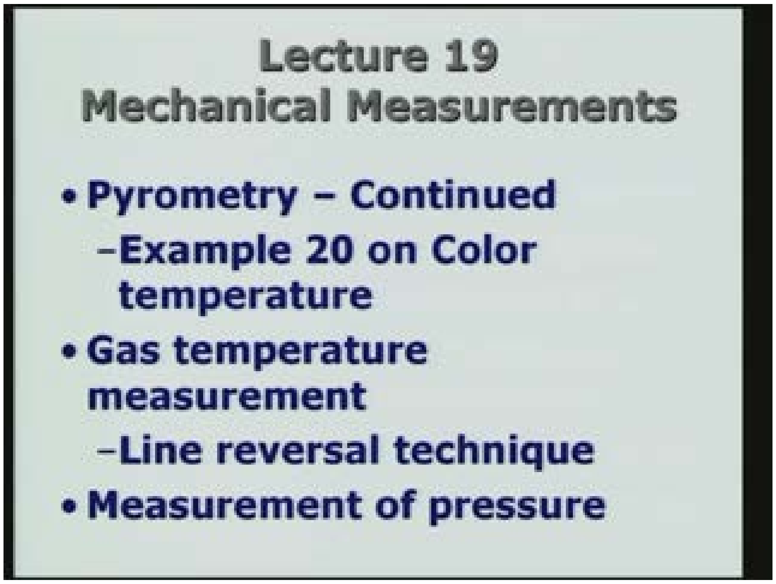
Mechanical Measurements and Metrology
Prof. S. P. Venkateshan
Department of Mechanical Engineering
Indian Institute of Technology, Madras
Module - 2
Lecture - 19
Pyrometry (continued), Gas Temperature Measurement and Measurement of Pressure

This will be lecture number 19, ongoing series of Mechanical Measurements. Towards the end of the last lecture we were looking at the color pyrometer, two-color pyrometer, which is one way of measuring the temperature of a hot object which is luminous, that means, it is able to give out radiation which can be used for the measurement of the temperature. Essentially, it is used for high temperature measurement. So we will close the discussion with an example which will determine actually the color temperature, given the input data.

(Refer Slide Time: 1:33)

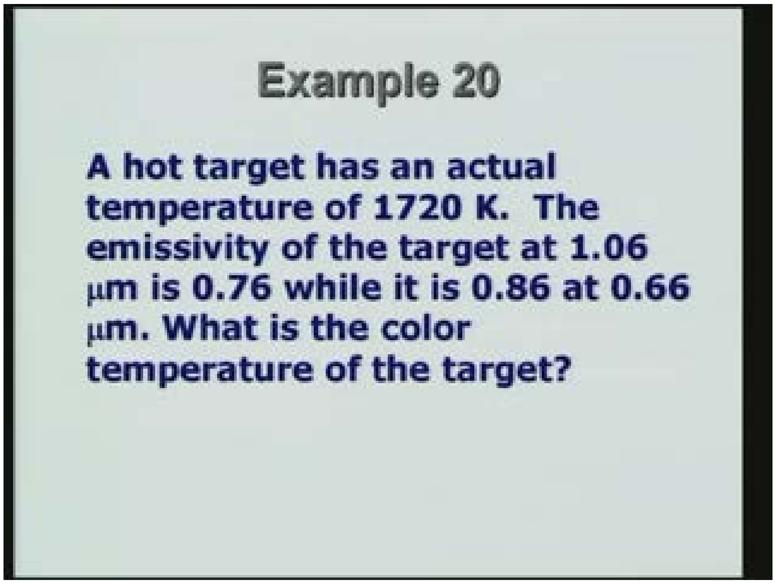


Lecture 19
Mechanical Measurements

- Pyrometry – Continued
  - Example 20 on Color temperature
- Gas temperature measurement
  - Line reversal technique
- Measurement of pressure

Subsequent to that, I am going to look at gas temperature measurement, the temperature of the gas which either absorbs or emits radiation. It can be the temperature, can be measured using essentially the ideas of Pyrometry, and it will deal with what is called line reversal technique which I will describe that in some brief detail, not too much, just enough to understand. Then we will switch over to the next topic which is the measurement of pressure. So we will look at the example.

(Refer Slide Time: 2:15)

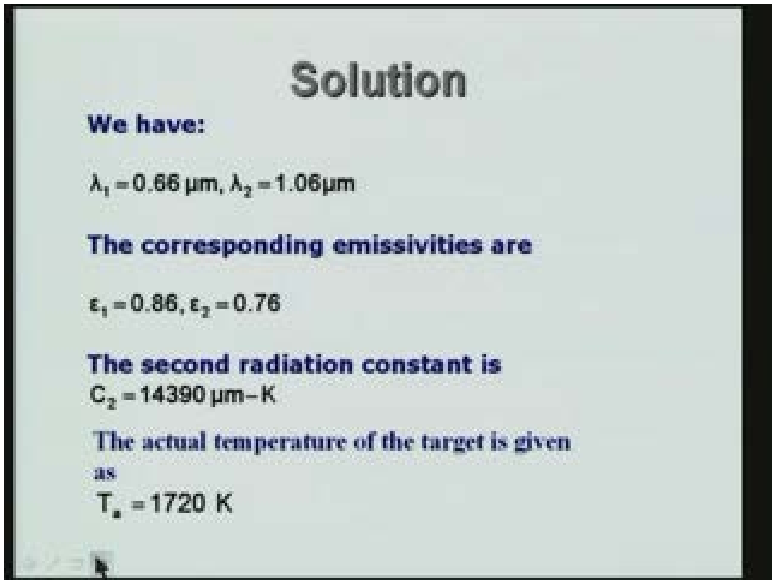


Example 20

A hot target has an actual temperature of 1720 K. The emissivity of the target at 1.06 μm is 0.76 while it is 0.86 at 0.66 μm. What is the color temperature of the target?

A hot target, its temperature is measured by a different, an independent technique, is given as 1720 Kelvin. The emissivity of the target is known at two different wavelengths. First one is at 1.06 micrometers and its value is 0.76, and at 0.86, it is, 0.66 micrometers. So two different wavelengths are given, and two different emissivities are also given. So we would like to find out what is the temperature, color temperature, of the target. The solution is worked out here. We have the following data.

(Refer Slide Time: 02:55)



Solution

We have:

$\lambda_1 = 0.66\ \mu m,\ \lambda_2 = 1.06 \mu m$

The corresponding emissivities are

$\varepsilon_1 = 0.86,\ \varepsilon_2 = 0.76$

The second radiation constant is

$C_2 = 14390\ \mu m{-}K$

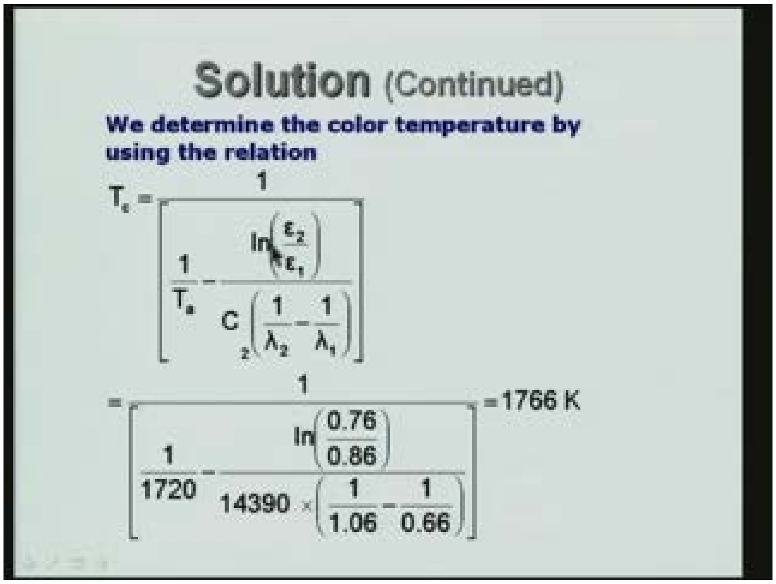
The actual temperature of the target is given as

$T_a = 1720\ K$

I am taking the first wavelength as 0.66 micrometer. The second one at 1.06 micrometers. These are the two wavelengths specified in the problem. The corresponding emissivities are also known and given by epsilon$_1$ equal to 0.86 at 0.66 micrometers, and epsilon$_2$ corresponds to lambda $_2$ of 0.76. The second radiation constant, we already know, is required for the calculation. We have written it down as 14,390 micrometers Kelvin, and consistently, what we are doing in solving problems is to use micrometer for the wavelength, and the radiation constants, C_1, C_2, etc, will also consist of micrometers as the unit of wavelength in that definition. So the actual temperature of the target is also given, is given by T_a equal to 1720 Kelvin. Thus, the problem is very simple.

All I have to do is to use the definition of the color temperature, and in the previous lecture we have actually derived this equation, and I am just writing it down again. T_C, the color temperature, is given by 1 over this entire thing, 1 over T_a, where T_a is actual temperature minus logarithm of the ratio of the emissivities, epsilon$_2$ to epsilon$_1$, In fact, you can identify epsilon$_2$ consistently. Epsilon$_2$, lambda$_2$, epsilon$_1$, lambda$_1$, correspondingly, have to be identified which you want to consider as lambda$_2$, which is lambda$_1$, this is essentially immaterial as long as the epsilon, lambda, the two quantities, are related to each other properly.

(Refer Slide Time: 04:39)



Solution (Continued)

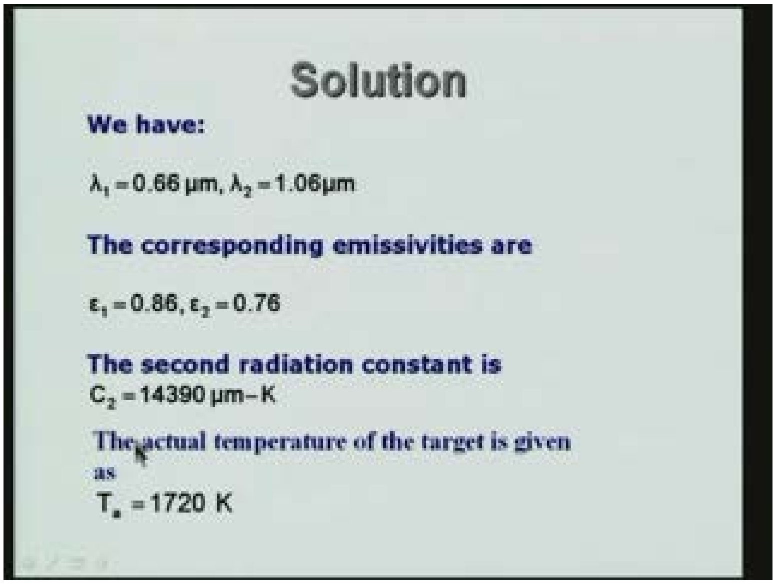
We determine the color temperature by using the relation

$$T_c = \frac{1}{\left[\frac{1}{T_a} - \frac{\ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right)}{C_2\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)}\right]}$$

$$= \frac{1}{\left[\frac{1}{1720} - \frac{\ln\left(\frac{0.76}{0.86}\right)}{14390 \times \left(\frac{1}{1.06} - \frac{1}{0.66}\right)}\right]} = 1766\ \text{K}$$

So ln of epsilon$_2$ by epsilon$_1$ divided by C$_2$, the second radiation constant multiplied by 1 over lambda $_2$ minus 1 over lambda$_1$. So all I have to do is to substitute the values 1 over 1720. It is 1 divided by the entire thing, minus 14,390 is C$_2$ logarithm of 0.76 by 0.86. It is epsilon$_2$ by epsilon$_1$ and lambda$_2$ is 1 over 1.06 minus 1 over lambda$_1$ is 0.66 gives you the value of 1766 Kelvin.

(Refer Slide Time: 05:14)



Solution

We have:

$\lambda_1 = 0.66\ \mu m$, $\lambda_2 = 1.06 \mu m$

The corresponding emissivities are

$\varepsilon_1 = 0.86$, $\varepsilon_2 = 0.76$

The second radiation constant is

$C_2 = 14390\ \mu m - K$

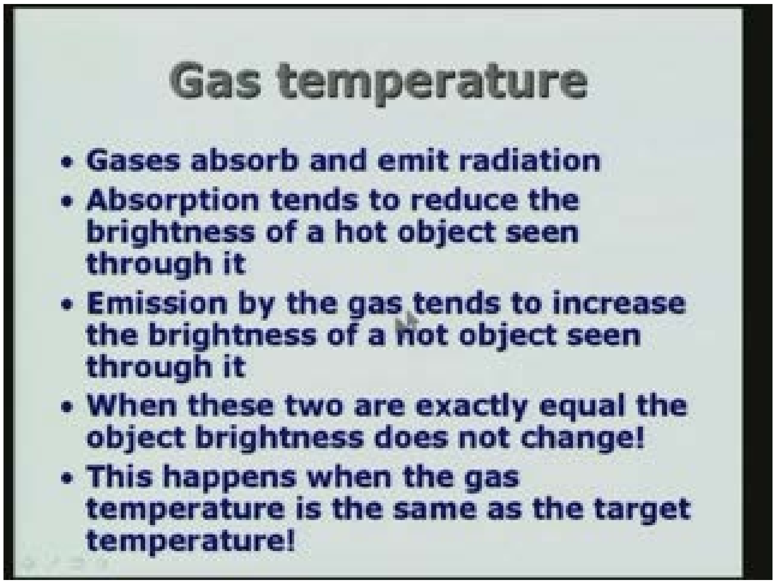
The actual temperature of the target is given as

$T_a = 1720\ K$

Therefore, if you go back, so you see that the actual temperature is 1720 Kelvin, and the color temperature is actually higher. It is actually 1766 Kelvin here. That is because the variation of the emissivity with wavelength is a particular variation which is given here. In this case, you can see that the lower wavelength, the emissivity is higher, and the higher wavelength, the emissivity is lower. If this pattern is, for example, if both of them increase together or decrease together, you will see that the value of the color temperature will change, and you will also notice that if epsilon$_2$ equal to epsilon$_1$ this whole thing will become logarithm of 1 equal to 0, and the color temperature become exactly equal to the actual temperature. That means that if the ratios of emissivities of the two wavelengths are the same, is equal to 1, that is, the two emissivities are the same, then the current temperature and the actual temperature are one and the same.

However, if epsilon$_2$ and epsilon$_1$ are different then you see that the temperature, T_C, is actually, it is not equal to the actual temperature, it may be either greater or lower. In this particular case it so happens it is going to be higher than the actual temperature. Let us now look at the measurement of gas temperature.

(Refer Slide Time: 06:29)



Gas temperature

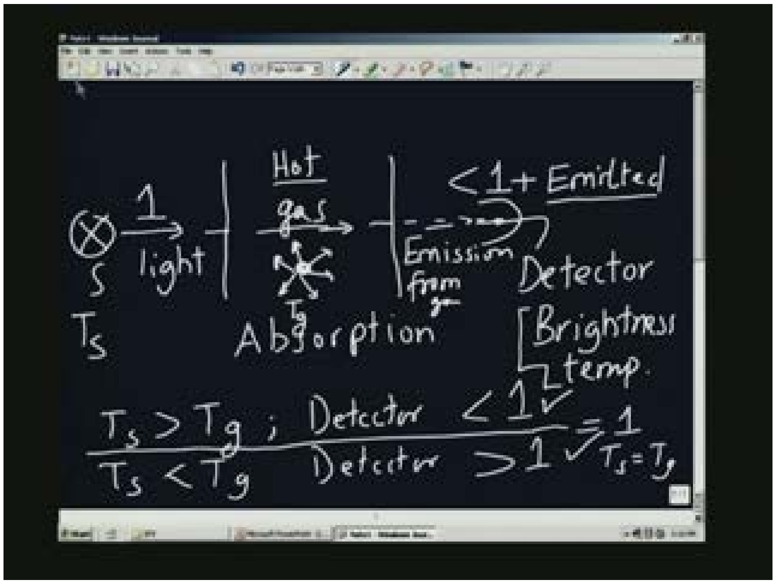
- Gases absorb and emit radiation
- Absorption tends to reduce the brightness of a hot object seen through it
- Emission by the gas tends to increase the brightness of a hot object seen through it
- When these two are exactly equal the object brightness does not change!
- This happens when the gas temperature is the same as the target temperature!

This is slightly more involved. I am going to cut down all the details and just give a brief description of what is expected. The main thing with gases is that they absorb and emit radiations. Of course, they don't absorb and emit radiations in all parts of the spectrum of electromagnetic radiation. They selectively absorb and emit. Gas is absorbed in what are called bands, and if it is an atomic gas, gas containing atoms, it may absorb over lines, that means at a particular specified wavelength.

Therefore, instead of going into the details about absorption and emission of radiation by gases we will just look at what are the highlights, what are the important things from the point of view of gas temperature measurement. The first thing to notice is gases at absorb and emit radiation, absorption becomes dominant if the temperature is low, it will only absorb the radiation which is incident on the gas, and if the temperature of the gas is also high enough it will emit radiation, and the absorption and emission process are something like the opposite of each other.

If the external radiation is falling on the gas it is absorbed by the gas, whereas, because of the temperature of the gas itself it is emitted, or if you want to call it re-emitted. So the absorption and emission process is some kind of a competing process which is taking place all the time. So in a sense, absorption tends to reduce the brightness of a hot object seen through it. So let me just explain what I mean by this.

(Refer Slide Time: 8:15)



Suppose I have a gas confined. This is a hot gas and I have a source of light which I will call it S, and the emitted photons, or emitted light, are going to pass through the gas. And suppose I have a detector here, I will show it, and the radiation is going to pass through the gas, and it is going to be falling on the detector, and because of the gas which is present the radiation leaving the source, if I say 1 unit is going to leave from here the detector will detect less than 1 unit. The difference of the change from 1 to less than 1 is because of absorption. So absorption reduces the amount of light which is passing through and falling on the detector.

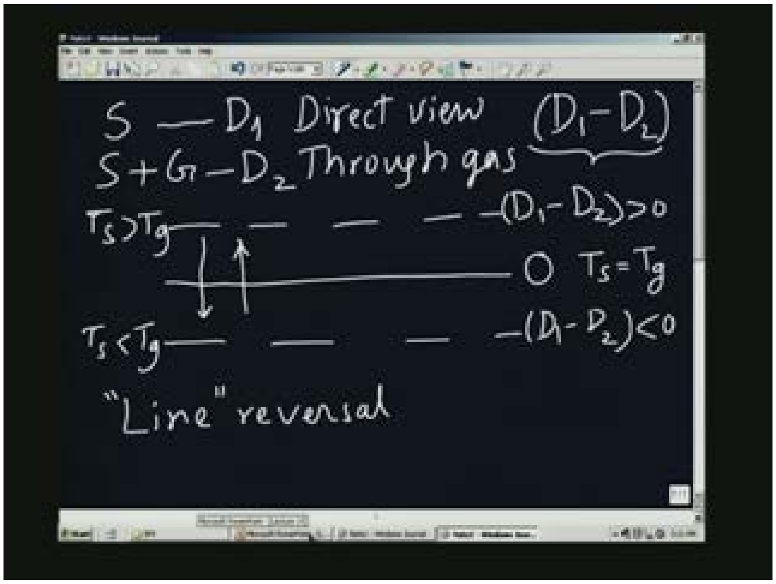
Now imagine that the gas is hot, so the gas molecules, suppose you take a molecule here, it's also emitting in all directions because of the temperature, T_g. The gas is at temperature T_g, and it is emitting radiation, and therefore, some of the radiation which is going in this direction from the gas molecules within will also be here. So we will say emission from the gas, so, plus that, less than 1, because the light which is leaving the source here is reduced because of the passage with gas, but plus emitted. So what we expect is if the gas is at a temperature lower than the source temperature, so we will say this is the temperature of the source.

If T_s is greater than T_g, the detector is less than 1 because of the amount of emitted radiation, because of the gas emission is not compensating for the reduction in the absorption, so the detector will still detect something less than the amount which is leaving the source which I call as 1. If T_s is less than T_g, the detector will actually give you something greater than 1 because it is over-compensated. There is more radiation from the hot gas which is contributing to the signal at the detector, and therefore, the amount of reduction of the source which is going to be also detected by the detector, that the compensation is over-compensated. That means the detector is going to give you a net which is greater than the amount which is leaving the source, S.

So just remember, or recapitulate what we did earlier when we talked about the brightness temperature. If you remember the definition of the brightness temperature we talked about the temperature of an object being equal to the brightness temperature which is in comparison with a black body. So we compare the actual radiation coming from the target with that of a hypothetical black body which is going to give you, in other words, which is going to look or appear to be equally bright. That means that the temperature of the object, the brightness of the object, is the temperature of the black body.

Now you can relate that here. So if I look at the brightness of the target here, if the target which is viewed through the hot gas, if it gives you a value of signal less than 1, it means that the source temperature must be greater than the gas temperature. And if it is greater than 1, the source temperature must be less than the gas temperature, and therefore, if you see that if it is equal to 1, the temperature, T_s, and the T_g must be equal. So we will say T_s equal to T_g. That means there is a full compensation. Whatever is absorbed from the radiation coming from the source absorbed by the hot gas is exactly equally compensated by the amount of emission from the gas.

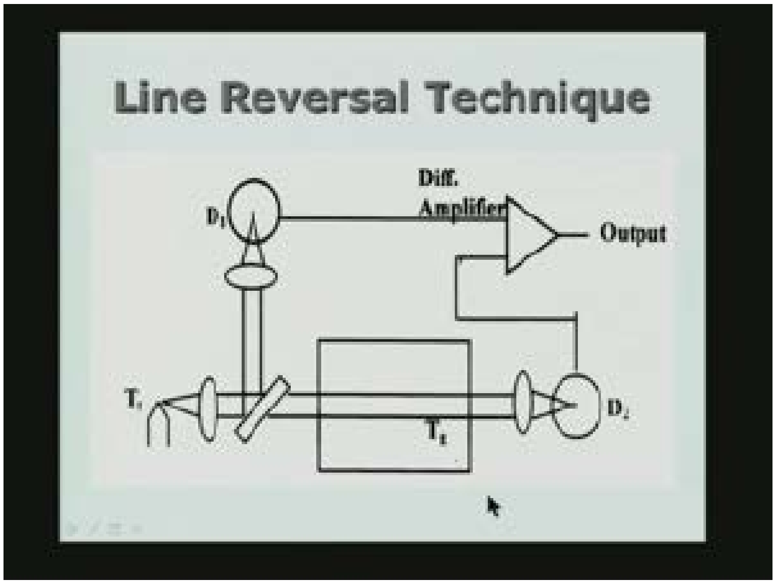
(Refer Slide Time: 13:59)



So if you were to look at the signal, you remember, we have the source, we have the source plus gas and now suppose I have a detector D_1 for this, and detector D_2 for this. That means that I am able to look at the source directly, and I am also able to look at the source through the gas. So this is direct view. I will explain how this can be done in a simple way. This is through gas. So if I look at the D_1 minus D_2, the difference between the two signals which is direct viewing of the source, and the second one through the gas, this D_1 minus D_2 is the difference between the two. The D_1 minus D_2 will be 0, if T_s equal to T_g.D_1 minus D_2 equal to 0, if T_s is equal to T_g, if T_s is greater than T_g, we know that the direct view, D_1 will be greater than D_2. So this will be positive. Hence, T_1 minus T_2 greater than 0, if T_s is greater than T_g, and I can show another line here, T_s is less than T_g, then D_1 minus D_2 is less than 0, so we can do the following. We can start looking at the direct view, and through the gas, and look at the difference in the two signals if you have two detectors, and then systematically vary the temperature of the source, T_s, so that you move from either this direction, this to this side, or to this side.

So in one case I have T_s greater than T_g to start with, and I start reducing it at one particular value equal to T_g, it will give you 0. It will go through the 0. So we say that this is called line reversal. So let us go back to the slides and look at how we are going to do that. So I am just explaining whatever I said on the board. Emission of the gas tends to increase the brightness of an object seen through it, and when these two are exactly equal the object's brightness does not change. This is the same as the two, D_1 minus D_2 equal to 0. This happens when the gas temperature is the same as the target temperature. The way of doing this measurement is shown schematically here.

(Refer Slide Time: 17:16)



Line Reversal Technique

I have the hot target; in this case I call it T subscript t. It is actually T_s in the, what I explained on the board. I collect the light coming from the target and I make two beams out of it by having a plate tilted at 45 degrees, one beam is going to be reflected off in this direction, the other beam is going to pass through directly straight. This one beam is made to pass the gas contained in a vessel.

Obviously, there must be some windows which are transparent to radiation at this side and this side, and the other radiation which is going through the reflected radiation is corrected directly by the detector, D_1, so that at the same time I am able to see the, detect the target directly, like what I explained earlier. So, direct view, and the other one is through the gas, and what I have to do is to vary the temperature of the target by using some arrangement, like you have a filament whose current is varied systematically.

What you will notice is that if you connect the output from D_1, the detector 1 and detector 2 to a differential amplifier, the output will be 0 under the condition of line reversal, insipient line reversal. So, if the output is positive then we know that the target is at a temperature higher than the gas temperature; if the output is negative we know that the target is at lower temperature than the gas temperature, and when there is a balance between the two, the line reversal, insipient line reversal takes place. So the output will swing from the positive side to the negative side and just when that happens you have equality with the temperature. Let me also explain a few more things.

If you take gases which are, whose temperature we want to measure, they may not themselves emit or absorb radiation at the wavelength we are interested. For example, suppose we have a gas which does not radiate in the visible part of the spectrum, you can introduce some small amount of trace material, like for example, sodium. So, if you inject a little bit of sodium chloride solution into the flame the flame will start radiating with yellow light, and therefore, the gas can have a small bit of sodium which will do the emission, and the assumption is that the sodium molecules, or sodium atoms, are going to be in thermal equilibrium with the gas, and therefore, they have the same temperature.

Therefore, what we do is look at the radiation coming from sodium atoms, and we know that it comes out in the yellow part of the spectrum. So we can use a monochromatic light from the target. We have a monochromator which will allow only light of the correct wavelength, for example, in this case, sodium emission wavelength, so that you are looking at that part of the spectrum alone, and the D_1 will give you the direct view of the target, and the D_2 will give you the signal corresponding to the view through the gas, and when the two detectors are going to give you an output of equal to 0, then we know that the temperatures are the same, and of course, we have to calibrate the target by having the temperature measured independently for the target. So, whatever target temperature we will measure is actually the gas temperature.

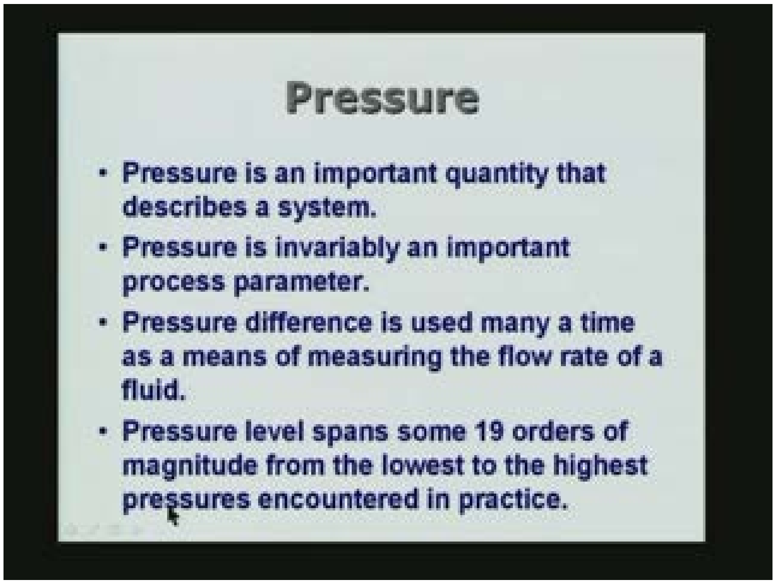
Therefore, you see here that the temperature of the hot gas is measured actually by measuring the temperature of a surface or a filament, and this is just what Pyrometry is all about. Pyrometry, if you remember, is the measurement of the temperature of a surface, and we have, by a clever arrangement of arranging an absorbing gas to be in the path of the beam we are able to now measure the temperature of the gas by using the temperature of the surface which is measured by using the pyrometer. So with this we will complete our discussion on temperature measurement. Of course, this discussion has been very brief. I have not talked about the nature of the gas radiation, how it originates, and so on. It is a little more than what we can cover in this particular course. All I wanted to do was to just give a brief idea what the gas temperature measurement by Pyrometry means. We will now look at the measurement of pressure.

(Refer Slide Time: 22:00)

Measurement of Pressure

In fact, the measurement of pressure is a very important business in the laboratory because pressure, just like temperature which we took up as the first quantity of interest to us, is a quantity that defines or describes many systems,especially in thermal engineering, and therefore, pressure is an important quantity that needs to be measured.

(Refer Slide Time: 22:07)



Pressure

- Pressure is an important quantity that describes a system.
- Pressure is invariably an important process parameter.
- Pressure difference is used many a time as a means of measuring the flow rate of a fluid.
- Pressure level spans some 19 orders of magnitude from the lowest to the highest pressures encountered in practice.

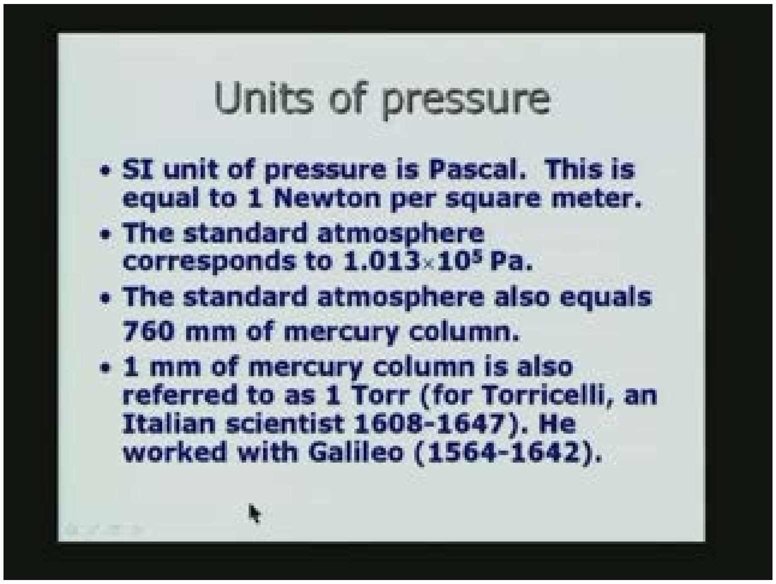
And also, in many processes it’s also an important parameter of interest because the parameters which affect either the speed or the rate at which a particular process is completed is dependent under the various parameters. The pressure and temperature happen to be the most important concept.

In other words, measurement of pressure is invariably involved in many process conditions or processes, and therefore, measurement of pressure, as an independent variable, is an important activity. Secondly, in many situations if you want to measure, for example, the flow of a moving fluid, the flow itself is not directly measured, but it is inferred from some kind of a pressure difference which is created by either changing the area of cross-section of the flow tube or introducing some kind of obstruction in the flow, you can convert the flow information to that of a delta p or a change in the pressure.

So the differential pressure measurement is an alternate way of measuring the flow rate of a fluid. If we look at the various situations where pressures are measured, we have very low pressures in vacuum systems. Actually, when the pressure is very low, we call it as high vacuum. Very high vacuum, ultra high vacuum, refer to very very small pressures.

We also meet with very high pressures in many industrial applications, manufacturing applications, and so on. And in geo-physical applications within the mantle of the earth the pressures are severely high, and therefore, very high pressures are also met with in practice. So the pressure level spans some nineteen orders of magnitude. It will become clear from the table which I am going to show in the next slide. Before we show the slide containing the information on the pressure measuring devices and so on, let us look at the units of pressure.

(Refer Slide Time: 24:46)



Units of pressure

- **SI unit of pressure is Pascal. This is equal to 1 Newton per square meter.**
- **The standard atmosphere corresponds to 1.013×10^5 Pa.**
- **The standard atmosphere also equals 760 mm of mercury column.**
- **1 mm of mercury column is also referred to as 1 Torr (for Torricelli, an Italian scientist 1608-1647). He worked with Galileo (1564-1642).**

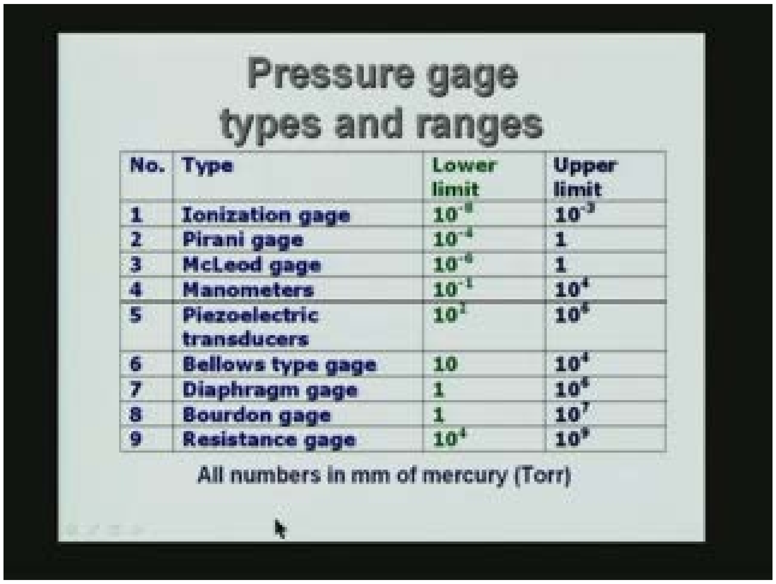
It is therefore the SI unit of pressure is Pascal, and this is equal to 1 Newton per square meter. That is the definition. Newton happens to be the unit of force, square meter is the unit of area, 1 Newton acting on 1 square meter of area gives you a pressure of 1 Pascal. In fact, 1 Pascal is a very small pressure. The standard atmosphere, if you look at it, corresponds to 1.013 into 10 to the power of 5 Pascals. It’s a very large number in terms of the Pascals, but it is just about one standard atmosphere and you see that the Pascal in relation to the standard atmosphere is a very small quantity. Indeed, this is about 1.10 to the power of 5 times the Pascal.

The standard atmosphere is also sometimes measured in terms of the hydrostatic pressure developed by column mercury, and the standard atmosphere actually is defined such that it is exactly equal to 760 mm of mercury column. That means, if you have a column of mercury which is 760 mm tall, at the base of that column the pressure will be exactly equal to 1 standard atmosphere.

Sometimes the unit of bar is also used. One bar is roughly equal to 1 atmosphere, but slightly less than 1 atmosphere. So 1 mm of mercury column is also referred to as 1 Torr. Torr is named after Torricelli, an Italian scientist who lived in the 16th century, from 1608 to 1647. He worked with Galileo. He was, in fact, an assistant to Galileo who lived from 1564 to 1642. In fact, Galileo and Torricelli are very important from our standpoint because Galileo was the one who measured and quantified the measurement of the temperature for the first time. He suggested the use of, what is called a baroscope which actually converted the information regarding pressure, the temperature, to the measurement of the pressure.

Torricelli actually designed the first barometer. And therefore, the unit of pressure is appropriately named after him. One Torr is actually 1 mm of mercury, and in fact, it can be referred given in terms of the other units also which we will do as and when we come across them. The pressure gage types and ranges are given in this table.

(Refer Slide Time: 27:26)



Pressure gage types and ranges

No.	Type	Lower limit	Upper limit
1	Ionization gage	10^{-8}	10^{-3}
2	Pirani gage	10^{-4}	1
3	McLeod gage	10^{-6}	1
4	Manometers	10^{-1}	10^{4}
5	Piezoelectric transducers	10^{2}	10^{6}
6	Bellows type gage	10	10^{4}
7	Diaphragm gage	1	10^{6}
8	Bourdon gage	1	10^{7}
9	Resistance gage	10^{4}	10^{9}

All numbers in mm of mercury (Torr)

This gives an idea as to the range of measurements, which we do in practice. We start from the lowest pressures and go towards the highest pressures as we go down the table. So, ionization gage is actually used for very low pressures. Incidentally, all the numbers in this table are given in terms of mm of mercury, and I am talking about 10 to the power of minus 8 mm of mercury is a very very small number. 10 to the power of minus 8 mm of mercury is a very small number. This is the smallest pressure that is normally measured in practice.

For example, if you go out, go into the space outside the atmosphere of the planets, you will reach pressures as low as this, what is given here. And also in many instruments which are used for chemical analysis, and so on, these, such pressures are met with. The ionization gage can be used from 10 to the power of minus 8 mm of mercury at the lower end, to about 10 to the power of minus 3 mm of mercury at the upper end, and you will see that 10 to the power of minus 3 is in the vacuum. Ten to the power of minus 8, we will call it as ultra-high vacuum. The second one is the Pirani gage.

We will just look at the names now. Later, we will describe them in more detail, which can be used between 10 to the power of minus 4 mm of mercury, 1 mm of mercury still in the vacuum side. McLeod gage is also an instrument which is used for the measurement of vacuum, 10 to the power of minus 6 to 1 so 10 to the power of minus 6 mm of mercury to 1 mm of mercury at the upper limit. And then we have the familiar manometers which are used for measuring from a lower limit of 10 to the power of minus 1 to about 10 to the power of 4 at the upper limit. And in fact, the McLeod gage is also a manometer in some sense, but the way it is used is slightly different from an ordinary manometer, and therefore, we will be describing that later on in some more detail. Then beyond the number 4 here, we are now getting into larger and larger pressures. So we have different types of transducers.

Just like we had transducers for measurement of temperature we have a transducer for measurement of pressure, and basically, we have the piezoelectric transducers which is nothing but based on piezoelectricity. Application of pressure on the material produces an electrical output, and the electrical output is directly related to the pressure, and therefore, a piezoelectric transducer converts directly the pressure information to an electrical signal.

We can also use what are called bellows type gages. A bellows type gage is essentially a spring. The bellows acts as a spring, and the bellows, inside the bellows, you have the pressure which needs to be measured, and the bellows will either expand or contract, depending on the pressure difference between the inside and the outside. And what we are going to do in the case of a bellows type gage is to convert the pressure information to a displacement.

The pressure gives rise to a displacement of the spring under equilibrium; the spring force is exactly equal to the pressure times the area of cross-section.

Therefore, the force balance gives you the, converts the pressure information to a displacement, and displacement is actually measured by a displacement meter diaphragm gage. In fact, the bellows and diaphragm gages are basically similar because both of them work on the principle that the pressure is converted into displacement. The way it is done is slightly different. In the case of a diaphragm gage we have a diaphragm which is a circular membrane, with pressure difference acts across the two sides, and the membrane is going to undergo a change in shape, and this change in shape is in terms of a displacement, and the displacement can be measured either directly as a displacement or it can be converted into an electrical signal by making it part of a capacitance. So the membrane is 1 electrode of capacitance, and you have an anvil which is going to be a fixed electrode. By-the-by, the application of the pressure the membrane is undergoing is going to undergo a change in shape, and because of this the spacing between the two electrodes is going to change, and because of that the capacitance will change.

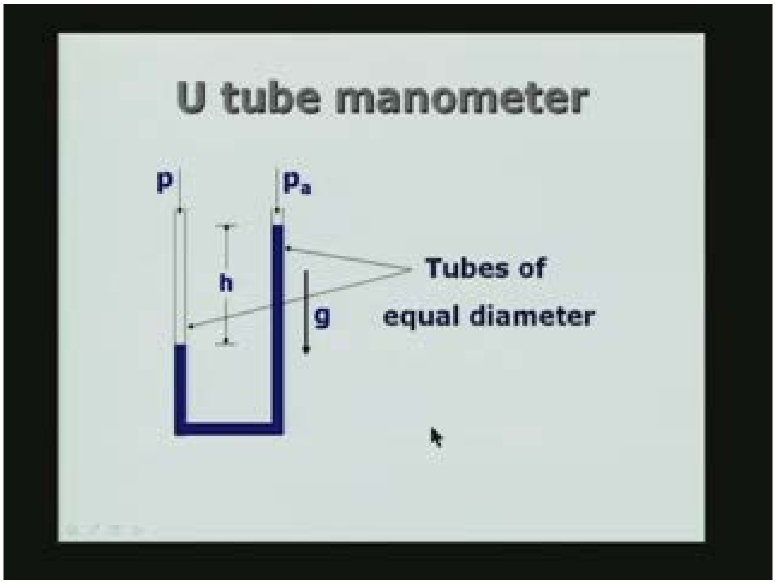
So a capacitance gage which measures the pressure, essentially a diaphragm gage, where the spacing between two electrodes is going to change. Then we have the Bourdon gage which is very popular because it is very inexpensive, and it covers a very large range from a lower limit of 1 mm of mercury to about 10 to the power of 7 mm of mercury. We are now already talking about very high pressures 10 to the power of 7 mm.

If you remember 760 mm,1atmosphere. So, if I take 10 to the power of 3 as roughly 1 atmosphere, this is 10 to the power of 4 atmospheres. So already we are talking about very large pressures. In fact, we can see here the bellows type can go up to 10 to the power of 4 which is about 10 atmospheres. This is about 100 atmospheres. This is 10 to the power of 7. Another 1000 atmospheres and you see that resistance gage can go from 10 to the power of 4 mm of mercury to 10 to the power of 9 mm of mercury. This is about the highest pressure which is measured in the laboratory.

The resistance gage works as follows. If you have a material and it is subjected to a pressure, it undergoes displacement of change in the shape, and because of this the electrical resistance of the material will change.

Therefore, the change in the resistance is measured just like, we had, if you remember, in the case of temperature measurement, we had a resistance temperature detector where the temperature information was converted into change in resistance because of the expansion of the material, and this resistance change was inferred as being related to the temperature. Similarly here, the resistance will change and it is in relation to the pressure which is acting on it, so this gives a broad overview of the kind of things we are going to be met with.

(Refer Slide Time: 34:12)



U tube manometer

And let us look at, in some detail, the individual pressure of measuring devices and the starting point, of course, is the u-tube manometer which is very familiar to us and most of us do use it in the laboratory, and it simply consists of two tubes as shown here. Two tubes of equal diameter connected by a tube here. What is important is the diameter of this tube and the diameter of this tube. The vertical legs of the u-tube must have the same diameter. So, u-tube manometer, the two vertical legs have the same diameter. The one which is bridging the two need not have the same diameter. It could be of any diameter.

Let’s look at the principle of operation of this. We have two tubes of equal diameter. Gravity is acting in the lower direction like here, and when I apply a pressure difference, p is the pressure to be measured,p_a is the pressure which is the atmospheric pressure which is acting on the other limb. So we have a liquid which is filling this tube and this liquid is called as the ceiling liquid. It's also called the manometer liquid. So this liquid has a certain density, and under the action of gravity, it has got some weight.

Let us look at what happens when the pressure p is greater than the atmospheric pressure. The pressure is acting on this side of the u-tube. This side of the u-tube the pressure acting is p_a. There is a difference in pressure, and this difference in pressure is balanced by a certain column of liquid of the manometer liquid. The manometer liquid can be mercury. In the case of mercury u-tube manometer, it could be a medium like water in the water manometer. Sometimes we use other liquids. Some organic liquids are also used.

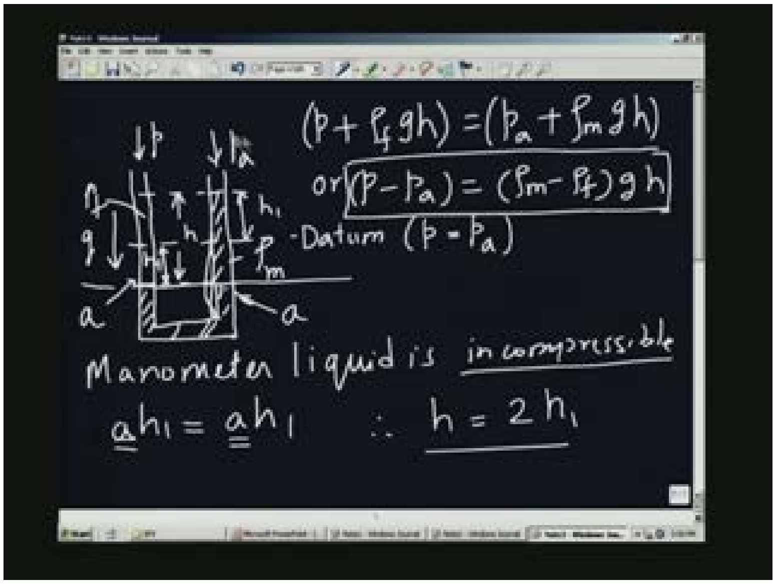
For example, kerosene can be used as the liquid. The reason why we use different liquids is to have different pressures, different pressure ranges, because, if you take mercury it has got about 13 point 6 density in grams per cc, water has got 1 gram per cc, and other liquids will have different densities. Kerosene is lower than 1 gram per cc, so if you want to convert the difference in pressure to the height of a column of a liquid, if you choose the proper liquid the column height which you are going to measure can be a convenient number. If the pressure difference is large I can use a mercury manometer because mercury has a high density, and therefore, the mg, the mass times the gravitation of force is going to be large, and therefore, large pressure can be measured of the order of atmosphere.

Anyway, you would appreciate that 760 mm of mercury is 1 atmosphere. Therefore, you can have probably a manometer which is a column, which is about 1 meter, or 1 point 5 meters tall, or even 2 meters tall. So you can see that the amount of pressure I can measure using the mercury manometer is limited to the size of the u-tube. If the density is very small, you can either measure large … Large pressure measurements require a large u-tube, but you can measure small pressure differences using a small size or a normal size u-tube manometer using a lower density liquid. So the density of liquid is an important parameter, and depending on the range of pressure which we are interested in, we may be interested in a few mm of water or a few centimeters of water. You may be interested in a few mm of mercury or a few centimeters of mercury.

Therefore, depending on the range, you can choose the appropriate liquid. The diameters of the two vertical tubes here, this one and this one, have to be equal. They need not be; they can have any value in principle. Normally in practice, we use diameters of the order of 3 to 4, 3 to 5 mm of inside diameter usually made of glass, so that you can see the manometer liquid inside, so that you can, and also, we have a scale which is attached to the column here, so that we can measure this meniscus of this level, as well as the other level, and by taking the difference will be able to measure the head, h.

The pressure is simply pressure, p minus p_a. p minus p_a is given by the hydrostatic pressure due to the column of liquid of this density which I will take it as rho_m. rho_m is the density of the manometric liquid multiplied by g, multiplied by h, is the pressure acting at this place. And in principle, the pressure acting at this place is due to the pressure, p, plus the density of the fluid in which the pressure is measured, multiplied by the same h into g. So the difference in pressure, p, minus this p_a is given by the difference in density multiplied by gh. So let us just look at that, the formula, how we are going to. I will make a simple sketch here of the same u-tube manometer.

(Refer Slide Time: 45:01)



$$(p + \rho_f g h) = (p_a + \rho_m g h)$$

$$\text{or } \boxed{(p - p_a) = (\rho_m - \rho_f) g h}$$

Manometer liquid is incompressible

$$\underline{a} h_1 = \underline{\underline{a}} h_1 \quad \therefore \underline{h = 2 h_1}$$

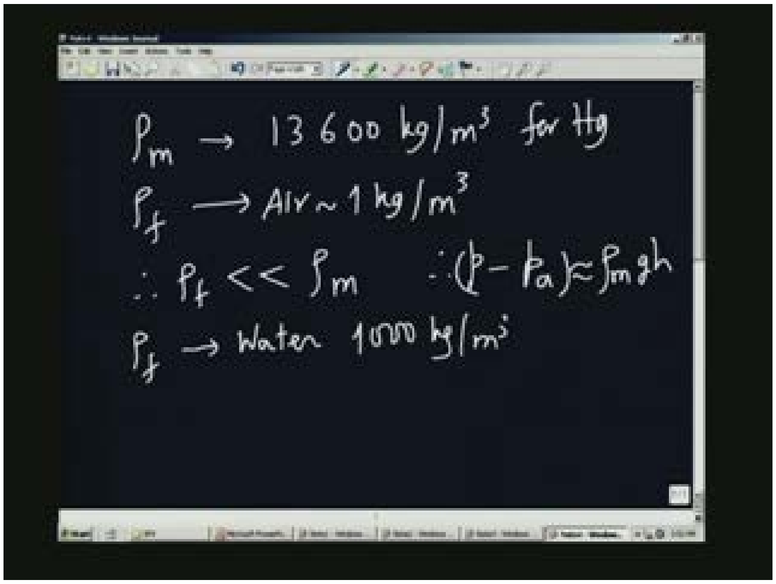
So we have , this is, we will call it rho_m, and the pressure p_a is acting here, p is acting here. I am assuming p is greater than p_a, therefore, it is depressed on this side, and the g is in this direction. Of course, the liquid, the manometer will not work where there is no gravity, and fortunately, we have gravity all over the world, and wherever you go you can use this instrument, but it has to be vertical orientation, that means that you have to make sure that the u-tube is actually vertically oriented. The two limbs of the tube must be exactly vertical.

Now, I will draw an imaginary line here, and a line here, and I will say this is the h. To start with the p, if the pressure p_a and p,are equal you would have had the, this is the datum p equal to p_a. If the two sides are subject to the same pressure, you will have the same level on the two sides, and that will give you the data p equal to p_a. When the pressure p is greater than p_a, the liquid will depress on the left side and will go up on the right side. So if the area of cross-section is a, and this is also a, if we assume that the manometer liquid is incompressible–incompressible means the volume of the liquid does not change– so suppose it goes up by a certain amount, h_1, it also has to go down by a distance equal to h_1 because the volume of the liquid which has gone up should be equal to the volume which has gone down. These two must be equal because the manometer liquid is incompressible. So a into h_1 is equal to a h_1 because area is the same. These two are the same area. Therefore, it goes up and goes down equally.

Therefore, we can say that h is nothing but two times h_1. This is the consequence of the fact that the liquid is incompressible, and the area of cross-section of the two sides of the two limbs of the u-tube are exactly the same, and we can also see now that p minus p_a. So I draw a line here. This is our line now. On this side you have p plus a certain height of the column of the fluid whose density, whose pressure, I am measuring. So this will be, let us rewrite it as slightly differently. So, we have p plus rho_f into gh here, and on the other side I have p_a plus rho_m, and these two are equal because , if you look at the fluid or liquid inside this portion, the force acting must be exactly the same here and here, otherwise this will move about. So equilibrium requires that p plus rho_f times gh equal to p_a plus rho_mgh, or p minus p_a equal to rho_m minus rho_f into gh.

This is the manometer equation, or the basic manometer. So you see that in the use of the manometer, the density of the manometer liquid and the density of the fluid whose pressure I want to measure, both are going to come into the picture. In practice, if you are using the measurement, if you are using the mercury manometer for the measurement of pressure in a medium like air, the density of air is like 1 kilogram per cubic meter compared to about 13,000 kilograms of meter cube of mercury, therefore, in the case of mercury manometer, rho$_f$ is very small compared to rho$_m$. Let me write it down in the next.

(Refer Slide Time: 46:21)



$\rho_m \rightarrow 13600 \text{ kg/m}^3$ for Hg

$\rho_f \longrightarrow$ Air $\sim 1 \text{ kg/m}^3$

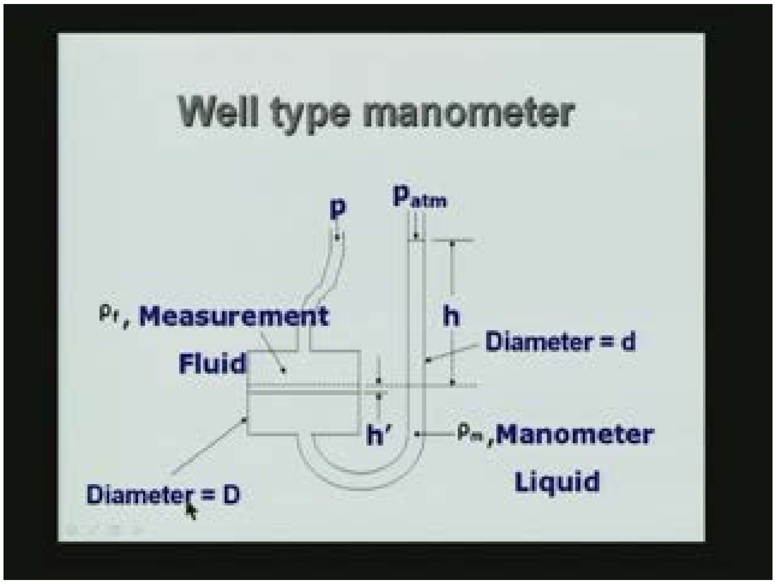
$\therefore \rho_f << \rho_m \quad \therefore (p - p_a) \approx \rho_m gh$

$\rho_f \rightarrow$ Water 1000 kg/m^3

So rho$_m$ is about 13,600 kg per meter cube for mercury. rho$_f$, if I have air, it's about 1 kilogram per meter cube. Slightly more, but it is alright to do that, therefore, rho$_f$ is very small compared to rho$_m$, and therefore, p minus p$_a$, I can approximate it as rho$_m$ gh. Suppose I am measuring in water, rho$_f$. For water, it's about 1000 kg per meter cube. I cannot neglect, so I will have to take into account the pressure due to the column of water, and that is going to have a density of 1000. Therefore, 13,600, the difference is only a factor of ten, a little more than that, whereas here, the factor is almost 13,000.

So in the case of measurement of pressure in air, if you are using a manometer with mercury as the manometer liquid, you can neglect the density of air in relation to the density of mercury, and therefore, this simple formula can be used: p minus p$_a$ equal to rho$_m$gh. But in all other cases where the density difference rho$_m$ minus rho$_f$ if rho$_f$ is not negligible compared to rho$_m$ then we have to take into account the rho$_m$ rho$_f$, both in the calculation. It is very important to remember, and we have now discussed the case of a u-tube manometer where the two limbs of the u-tube have the same diameter, but many times we use a well type manometer which I will show you, a well tube, well type manometer which is going to have a large reservoir as shown here the left side I have a large reservoir, right side I have a smaller diameter vertical limb. So we have a large diameter large reservoir here, and a small diameter tube as the manometer, and let us see what is going to happen.

(Refer Slide Time: 47:20)



Well type manometer

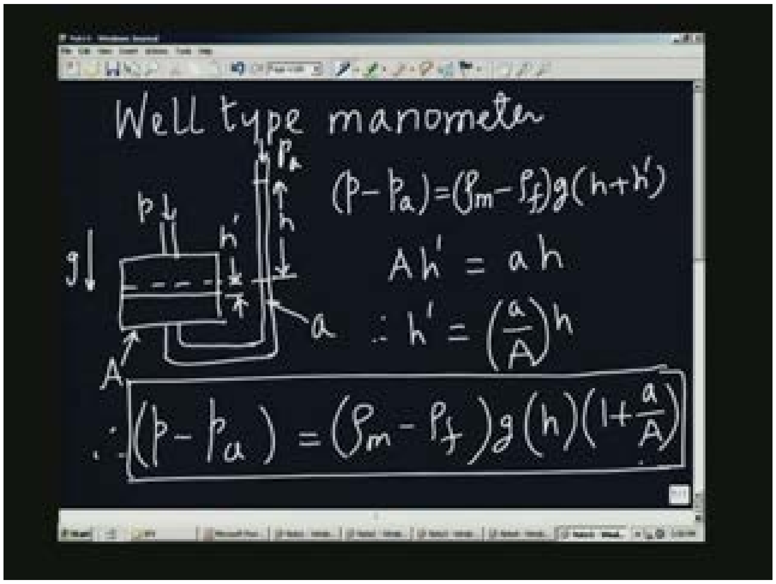
I am going to connect the reservoir to the pressure to be measured, not the other way. The pressure to be measured is always connected to the reservoir side, or the well. In the case of well type manometer the reservoir or well has got a diameter equal to D which is larger than the diameter of the tube which is d. For example,d could be a few millimeters and D could be a few centimeters,so a factor of 10. And the measurement fluid is going to communicate to the reservoir here, and the atmosphere is going to communicate to the top of the manometer liquid in the limb which is here. And again, the gravity direction is in the direction of vertical – vertical downwards.

So what is going to happen in this case is slightly different. The amount of depression, if you remember, in the u-tube column, the depression on the left side was exactly the same as the elevation on the right side. The amount of liquid level going down on the left limb was exactly the same as on the other side. Therefore, h_1 plus h_1 is equal to h. That's what we said, two h_1 is equal to h. But here, the amount of the depression in the reservoir is going to be a small quantity, h prime. h prime is the depression in the well side, and h is the increase in the height on the tube side.

Again I am going to invoke the incompressibility argument that the manometer liquid is incompressible, therefore, you can see that the volume change, volume this change in the level inside the well is going to be a change in volume which is given by the area of cross-section of the well multiplied by the h prime, this will be exactly equal to the height, h, multiplied by the area of cross-section of the tube.

So now, I can find out what is the true manometric height. That will be nothing but h plus h prime. It is not just h. h is of course measured. h prime can be calculated and used as a correction for correcting the manometer head. So h plus h prime multiplied by again rho$_m$ minus rho$_f$, as we indicated earlier, so let me write it on the board, and we will take the well type manometer.

(Refer Slide Time: 53:18)



Well type manometer

$$(p - p_a) = (\rho_m - \rho_f) g (h + h')$$

$$A h' = a h$$

$$\therefore h' = \left(\frac{a}{A}\right) h$$

$$\therefore \boxed{(p - p_a) = (\rho_m - \rho_f) g (h) \left(1 + \frac{a}{A}\right)}$$

I will schematically draw a sketch with pressure is exerted here. This is the original datum. This is communicated. This is the original level. This is the level after the pressure has been, so h is this portion, this I will call as h prime. h prime is the depression in the well; h is the amount by which it is gone up on the right side, and the gravity direction is like this. We can see that p minus p$_a$, this p$_a$ is equal to rho$_m$ minus rho$_f$ into g into h plus h prime.

So in the u-tube manometer h prime would have been equal to h, and therefore, you would have actually, h we call it as h_1, h prime would have been called as h_1. This h would be corresponding to h_1 in the previous case, and become 2 h_1 equal to h.

Suppose I take the area of cross-section, area A here, and area of cross-section a here, you see that A into h prime must be equal to a into h because of volume increase on this side, the volume decrease on this side. This is again incompressibility assumption, therefore, h prime is equal to area ratio a by A times h, and therefore, I can substitute there p minus p_a equal to rho_m minus rho_f into g into h plus h prime. So h, I can take it outside multiplied by 1 plus a by A. This we will use in the case of a well tube, well type manometer with a large reservoir on the left side to which the pressure to be measured is communicated, and on the right side I am going to have a vertical tube of small diameter, and the ratio of area a to A is going to come into the picture.

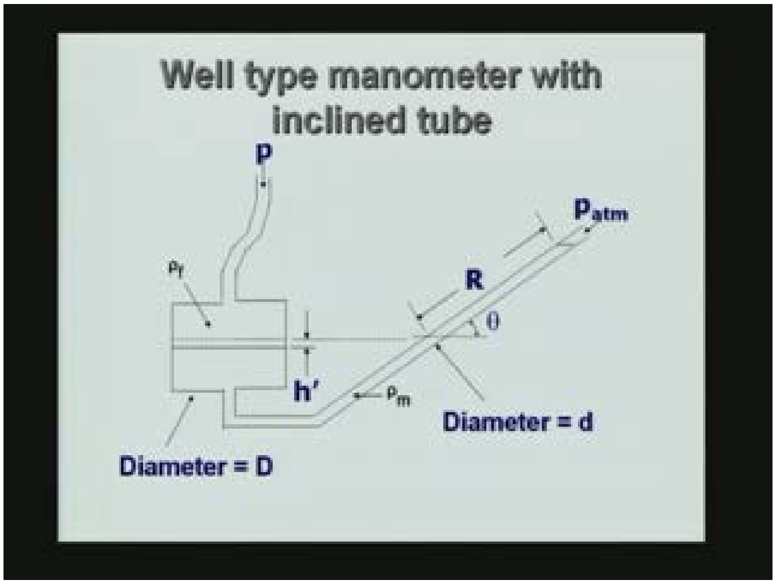
If a is few mm and A is few centimeters, and if you remember, a is proportional, the area is proportional to square of the diameter, if the diameter is, let us say, 3 mm here, and it is 3 centimeters here, it will be 3 divided by 30 whole squared. This will be a small factor, and sometimes we can neglect this; a by A when we are measuring a large pressure. This, a by A may be neglected, and therefore, you can use only h, or what is done in practice is to expand the scale along this direction. Instead of using a 1 centimeter equal to 1 centimeter, I will expand it slightly, or I will change the scale on that such that 1 plus a by A into h is what is indicated here. This 1 plus a by A is a factor is going to be taken into account in constructing the scale on the right hand side.

That means, if I have the datum fixed properly, that means, when the pressure p and p_a are the same, the datum is taken as the level of the liquid on the two sides, the well and the tube, and there will be a mark on the u-tube, the vertical tube, to indicate the data. So the liquid filled inside the manometer when the two pressures are the same must be exactly at that particular level, and I will use a scale on the right side which is corrected for the factor 1 plus a by A.

So if I measure 1 centimeter there, it is actually 1 into 1 plus a by A centimeters. So this factor is already included, so that the scale which is going to be next, placed next to the vertical limb, will have that correction factor built into it so that you can directly use. So the height measured using the scale which is attached here will give you, not h, it will give you h into 1 plus a by A directly. Therefore, you can say that this is the corrected height. We will call it the corrected height. The height is h; h into 1 plus a by A, is a corrected height, and the scale can be adjusted such that the corrected height can be directly measured.

Now, we will be go back to the case here. So to recapitulate, what we are doing is, instead of measuring the height using an ordinary scale, I am going to use a scale which is going to have the factor 1 plus a by A already built into it. Let's look at the slightly changed type of well tube manometer.

(Refer Slide Time: 56:35)



Well type manometer with inclined tube

Here, purposefully instead of the vertical limb I have got the inclined limb. The tube is inclined at a small angle. So what is the advantage of this? If, for a given height of liquid, (I am talking about a manometer height h) h will be the vertical component of this travel. So, the vertical component will be, if you, what you are going to measure in this case is R, but R is not the actual manometer head. The manometer head is r sine theta. So if I change a small value of theta, the scale which I am going to attach to the tube is going to measure a large distance for a small change in the pressure for a h. Therefore, the inclined tube is going to give a mechanical advantage, and when you are measuring small changes in pressure I can convert the pressured signal to a large manometer reading which will correspond to a small manometer head.

So, in the next lecture, I will take a problem which will concern itself with the measurement of pressure using an inclined tube manometer, or a well type manometer in general, and we will look at the sources of error. You look at how to analyze the problem of this particular type. Thank you.