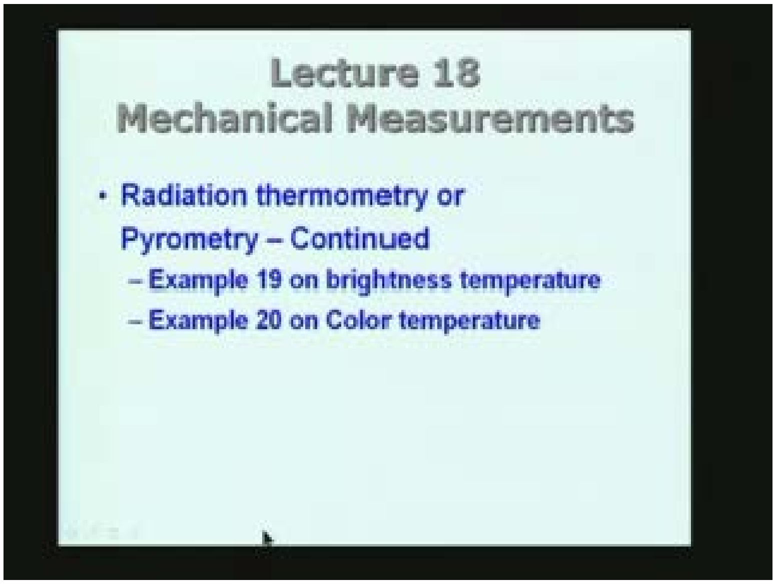
**Mechanical Measurements and Metrology**
**Prof. S. P. Venkateshan**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Madras**
**Module - 2**
**Lecture - 18**
**Pyrometry (continued)**

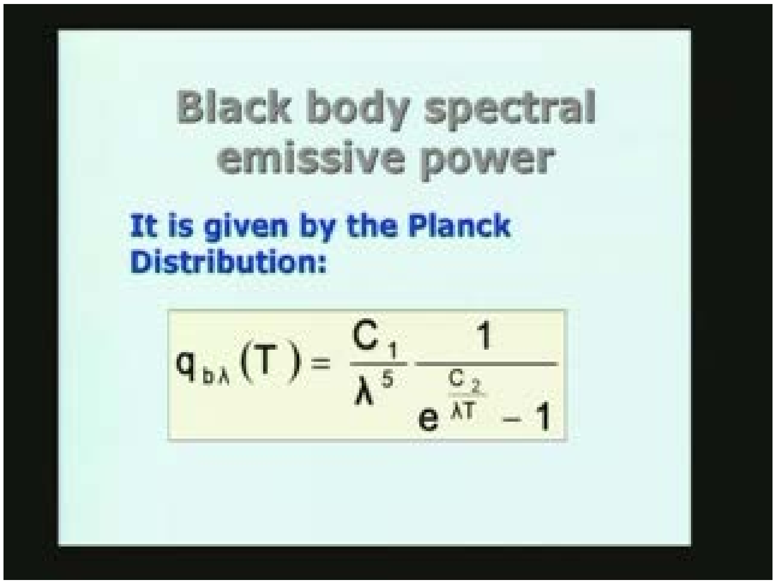
So this will be lecture number 18 on the ongoing series of Mechanical Measurements. Towards the last lecture, end of the last lecture, you were looking at Pyrometry. We briefly considered the nature of the black body radiation, and we just had enough time to discuss or mention about two concept: the brightness temperature; and the color temperature.

(Refer Slide Time: 1:17)



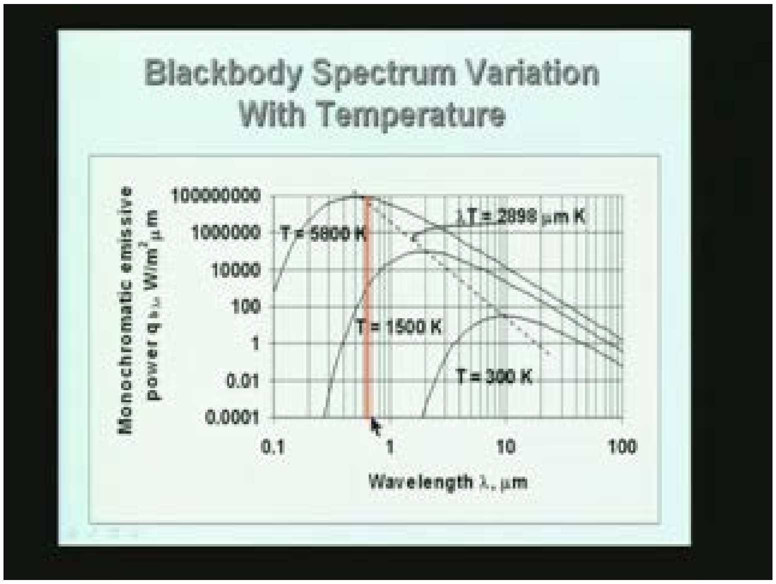

So what we will do in this lecture is to continue from where we left off, introduce these terminologies like the brightness and color temperatures, and indicate how this can be used for measuring high temperatures, specifically, surface temperature of bodies at high temperatures. When I say high temperatures, we are talking about the temperatures greater than about the silver point or the gold point. These are already familiar to us from our discussion on the international temperature scale of 90. To just recapitulate, we go over again some of the basic concepts.

(Refer Slide Time: 02:18)




The first one is the concept of the black body whose spectral emissive power depends on the wavelength lambda and the temperature T, and as I indicated earlier, the black body emissive power in watts per square meter micrometer is a unique function of wavelength as well as the temperature. So, if I fix the wavelength, for example, if I fix the wavelength, then it becomes a unique function only of temperature. So that is the basic idea of using this equation as a basis for thermometry in what is called a pyrometer. So let us look at some of the details.

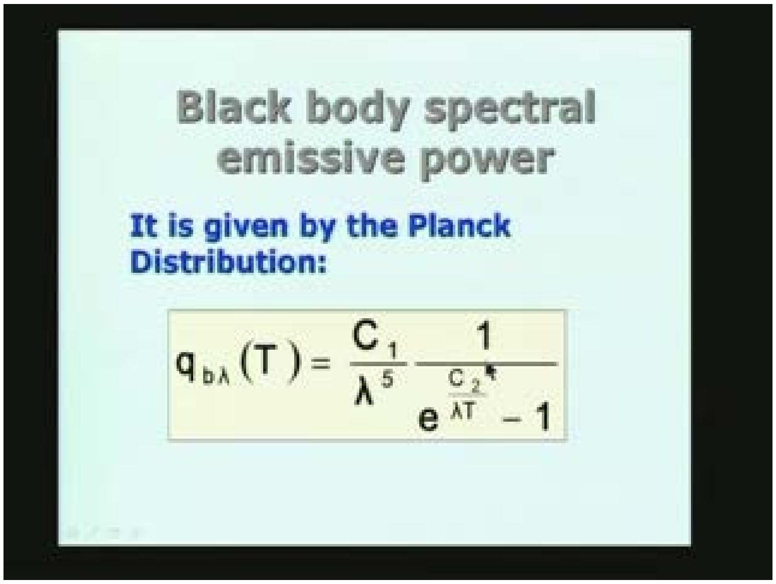
(Refer Slide Time: 02:58)




We have already done this earlier. I have taken an example of, I have plotted the monochromatic emissive power as a function of temperature, and just to recapitulate what I said a while ago, who will fix the wavelength at some particular value so you see that the value is given by this vertical line, the red line, which is shown here. The vertical line, the color of the line is red because this is in the red region of the electromagnetic spectrum. It is in the visible part of the spectrum. You can see this 0.1, 0.2, 0.3, 0.4, 0.5, 0.6. Between 0.6 and 0.7 micrometer, the value which is shown is actually used in parameter, and if you go up on this line you see that there is a unique temperature variation of the monochromatic emissive power as a function of temperature.

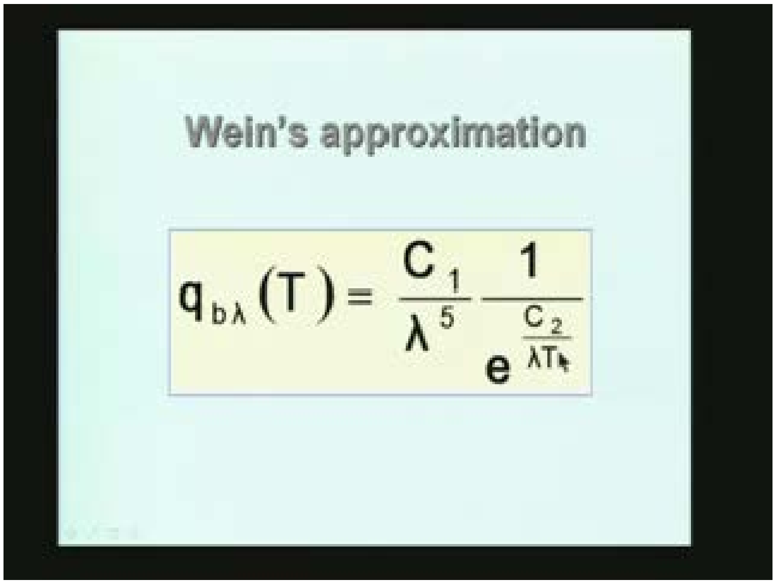
As you increase the temperature the intensity of the value on the y axis is going up, and it is going up by orders of magnitude. And here I have taken examples, first one being at 1500, the second one is 5800, and these have been chosen physically because the 5800 is roughly the upper limit up to which I am going to use the pyrometer. So, the point to notice is that there is unique relationship between the y axis here, which is $q_{b\lambda}$, and the temperature, once I fix the wavelength at a particular value .

(Refer Slide Time: 04:46)




What we normally do in practice, is, because in the Planck distribution function, if you go back and look at it, in the denominator I have e to the power of $C_2$ by lambda T, where $C_2$ is the second radiation constant which has a value of about 10,000 and odd, so this is a large number, and you see that if lambda times T product, typically, for example, lambda equal to 0.5 or 0.6 micrometer, even if you take the temperature of the order of 5000 or 8000 which we talked about as the solar temperature, you see that this lambda T product will be 0.6 times 6000, about 3600, $C_2$ is about 14,400, and therefore, 14,400 divided by this will be 0.6 into 6, 3600. It is a fact of e to the power of 4 or 5, and this factor is much larger than the factor 1 here. Therefore, sometimes what we do is especially for the sake of considering a pyrometer measurement of temperature we will ignore this factor, minus 1, and that is called the Wein's approximation to the black body function, and that's what I have given here.

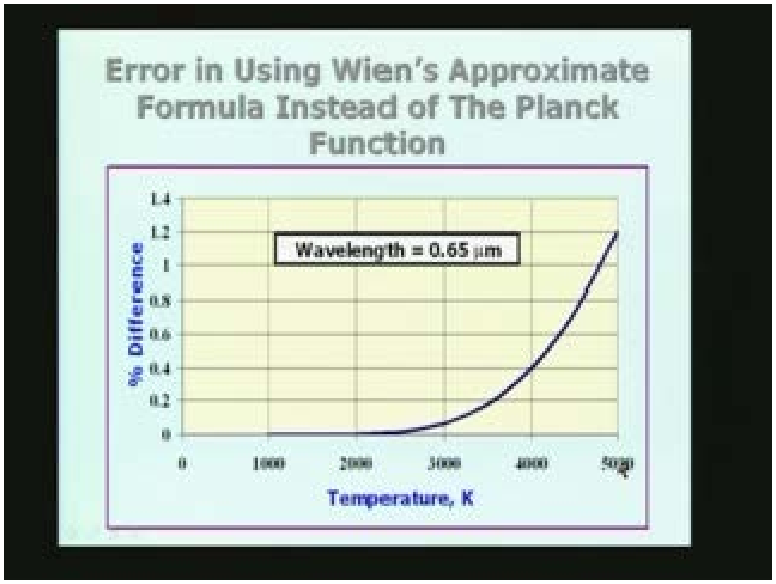
(Refer Slide Time: 05:52)




So the Wein's approximation says that we can ignore the factor minus 1 in the denominator. The difference between the Planck function and this function is only that I have neglected the minus 1 in the denominator. So $q_{b\lambda}(T)$, is equal to $C_1$ by lambda to the power of 5, 1 over e to the power of $C_2$ by lambdaT. And, if I fix the value of lambda this is the fact, this is the fixed value, $C_1$ is a constant, $C_2$ is a constant, lambda is fixed. To see that $q_{b\lambda}(T)$, is proportional, is equal to some constant divided by e to the power of some constant divided by T, so there is a definite relationship between $q_{b\lambda}(T)$, and the temperature.

Let's look at the error which is committed by using the Wein's approximation instead of the Planck function. You notice that I am going to neglect that minus 1 factor in the denominator as compared to e to the power of $C_2$ by lambda T.

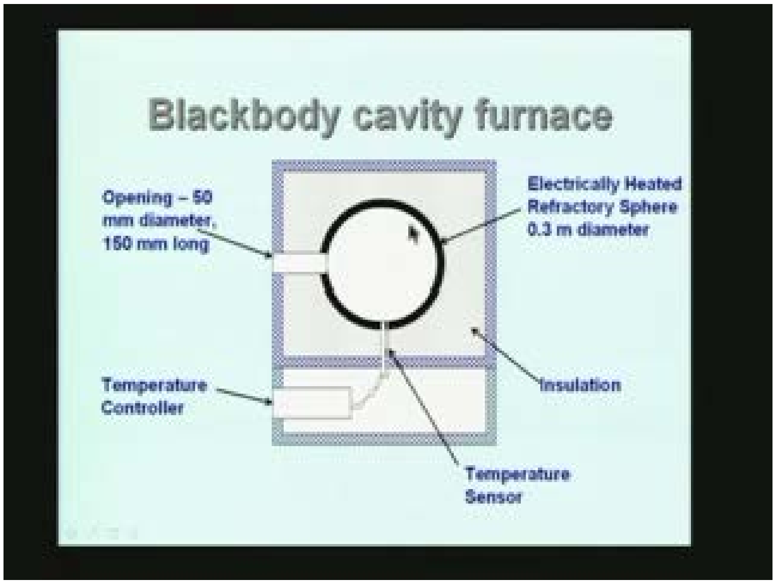
(Refer Slide Time: 06:55)




And for the kind of values here, I have taken from 0 to 5000 Kelvin, which is the normal range we are going to be interested in practice, to see that the maximum error in using the Wein's approximation as compared to the Planck function is only about the 1 percent, and this is, so this is small enough to be of no concern to us. Therefore, we will assume that for the purpose of Pyrometry, I can use the Wein's approximation, and therefore, that simplifies the calculations a little bit even though there is nothing wrong in using the full Planck function. And here I have, you should notice that I have used 0.65 micrometer as the fixed value for the wavelength, so the thermometer which uses the black body function fixes the wavelength at a particular value, and makes the black body function a function of temperature alone. And that is the basis for thermometry.

So let us look at the source of black body radiation, because in a laboratory, if you want to calibrate a pyrometer, I must have a source of radiation which is a black body. The normal way of making or obtaining black body radiation is to use what is called a cavity furnace.

(Refer Slide Time: 08:09)

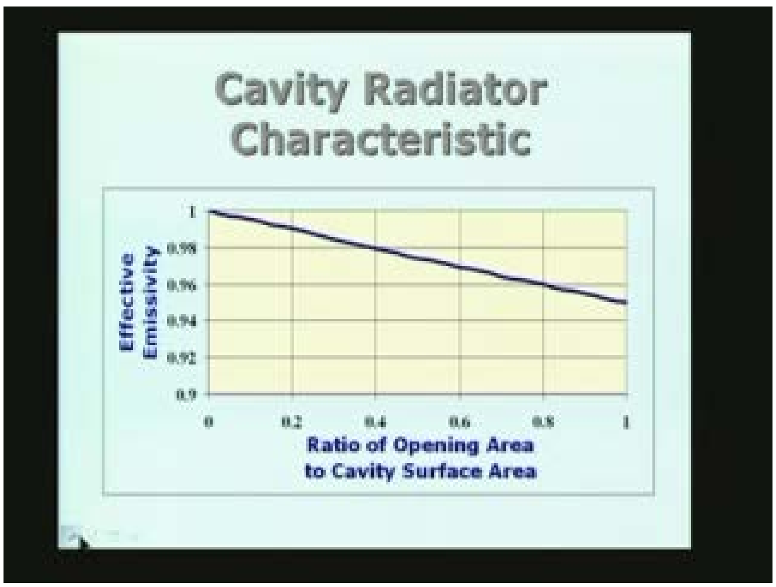



So the cavity furnace consists of, you see here this black thing is electrically heated, a refractory sphere, something like 0.3 meter diameter, and this sphere is placed in a box with insulation here, so that the heat loss from the sides of the spherical cavity is reduced. We also have a temperature control controller which will adjust the heat input to the electrical heaters such that the temperature is fixed, or kept fixed, at a particular value we want, and the temperature is sensed by a sensor which is attached to the wall of the electrically heated sphere. And there is a narrow opening up to 50 mm diameter, as I have shown here, 50 mm diameter opening with a 150 mm diameter. Some kind of a tube is attached to it, and the radiation which is leaving from this sphere through this opening to the outside, approximates black body radiation very closely, and sometimes what is done is, behind the heated sphere I can also have a phase change taking place in a material of given melting point.

For example, I can have the cavity. Behind the cavity I have a receptacle or an enclosure in which I am going to have a metal which is going to be allowed to undergo a phase change. That means, it’s going to melt. You start from a low temperature approach, the temperature of melting. So the material starts melting. During the process in which it is melting, when the material is present both in the form of solid and liquid, you will have a constant temperature. That constant temperature is nothing but the liquidous temperature of the substance, and the cavity will be exactly at that temperature, and therefore, the black body will give you black body radiation corresponding to the melting temperature of the solid which is undergoing a phase change.

In fact, we can choose different materials with different melting points, so that the black body can operate at constant temperatures during the time it is undergoing this melting process, it will be at constant temperature, and therefore, I can have different temperatures, achieved. For example, gold point, you can have gold which is melting, or you can have the silver point, zinc point, lead point, and so on, depending on the material you have chosen.

(Refer Slide Time: 10:43)

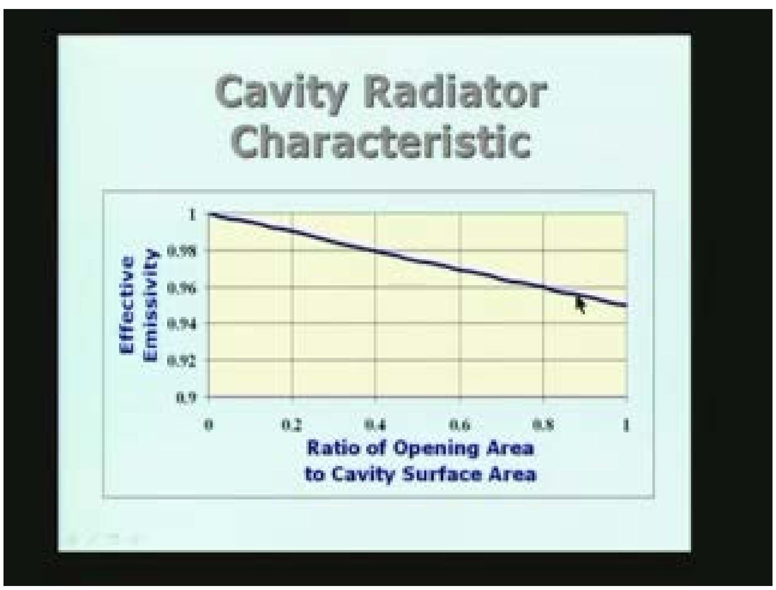



The radiator cast characteristic of the cavity is shown here, and in fact, if you remember what I said I have a large spherical cavity here. It is a large spherical cavity of .3 meter diameter with a small hole on the side. So the opening has an area of pi d square by 4 pi into 50 square mm, 50 mm square divided by 4. That is the area, whereas the surface area of this sphere is pi d square and .3 meter diameter. You can see that this is a very large area compared to the area of the opening. What I have done in the next slide is to show what happens to the effective emissivity of the radiation coming from the opening.

Emissivity is something which tells me how close is the radiation to that of a black body radiation. If emissivity is very small, that means that the radiation coming out from that opening is not at all like a black body radiation. If it is very small it is not like black body. If it is very close to 1 it is like black body radiation. Therefore, what we want to have is a black body radiation coming out of the hole because that is the basis for thermometry. Therefore, what I should do is I should have (1) a ratio opening area to cavity surface area as small as possible, and (2) I must choose the surface to have essentially a very large emissivity.

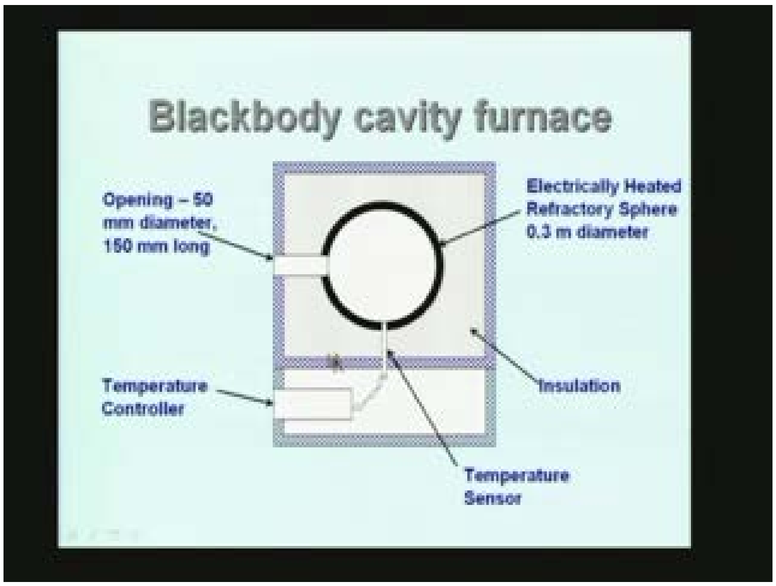
To start with, different surfaces have different emissivities. That means that if you have two surfaces, one black surface and one surface which is not black, if you heat them to the same temperature, the black surface will give you a heat, will emit radiation, according to the Planck distribution function full amount, whereas, if the surface is not black it has got, let us say, that it emits a fraction of that emitted by a black body at the same temperature. That fraction we call as the emissivity.

(Refer Slide Time: 12:45)



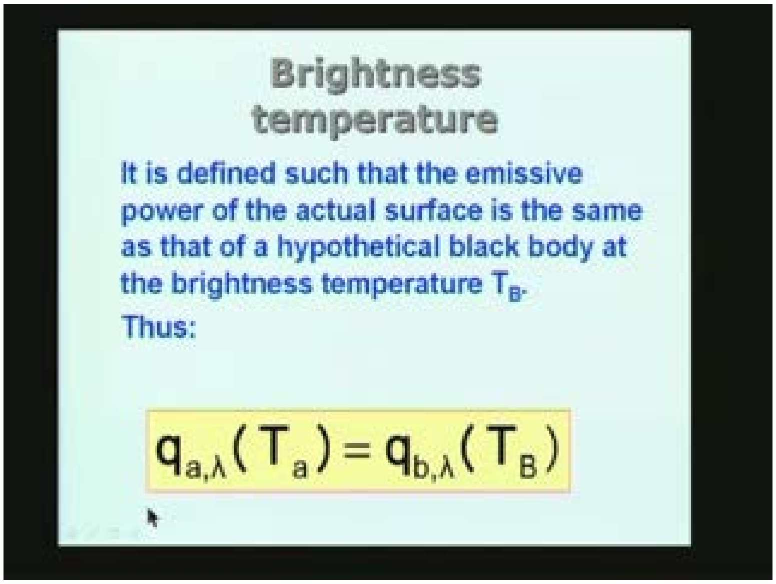

Therefore, remembering emissivity as the property of the surface, if I have a cavity whose surface already has a high enough emissivity, in this case I have taken a cavity surface which has got a emissivity power of 0.95, and if I make the opening very small compared to that, for example, I can have an opening 1 over less than 10 percent, less than 5% of the area of the surface of the sphere, then you can see that approaching the emissivity equal to 1, that means it is approaching a black body radiation. Therefore, the requirement of a furnace of the black body cavity furnace is that you have a very large area for this surface of that cavity. Let it have as high an emissivity as possible to start with, and then make this opening as small as possible.

(Refer Slide Time: 13:31)




Of course, if you make the opening very small the amount of radiation coming out will be small, but that is part of the price we pay for getting, or approximating, the black body, the characteristic, very close. So what comes out of this hole if you have a very narrow hole compared to the size of the cavity is black body radiation. So the black body radiation we use basically as a calibration.

(Refer Slide Time: 13:55)




Now let us look at the, we go back to, we showed earlier, if you fix the wavelength at a particular value we know that the brightness, the amount of radiation coming from the surface is a function of temperature given by the Wein's displacement, Wein's law now, instead of the Planck law, because we are approximating. So the brightness temperature is actually defined such that the emissive power of the actual surface is the same as that of a hypothetical black body at the brightness temperature, $T_B$.

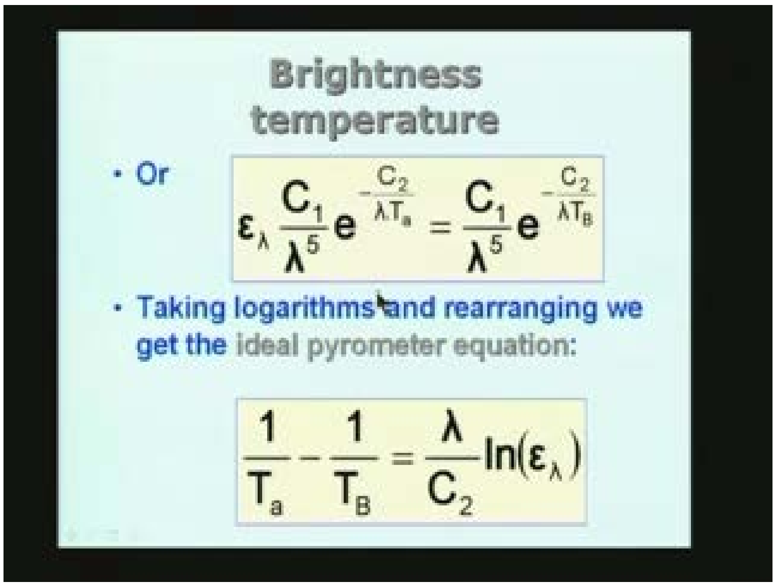
Why do we do like this?

Suppose the surface in question is not a black surface. We know that it will emit less than a black body at the same temperature, so I want to know, compare it with a black body which is going to emit the same amount. That means, it will be at a different temperature, obviously lower than the actual temperature of the body, such that, it is going to give the same amount of heat per unit area, per unit frequency. So if you fix the lambda constant, lambda, if you fix the lambda value by some suitable arrangement, the amount of energy emitted per unit time, per unit area, per unit wavelength interval, by the black body is given by $q_{b,\lambda}(T)$,which is now at the temperature, $T_B$, which I call as the brightness temperature which should be exactly equal to the $q_{a,\lambda}(T)$, a stands for the actual surface at the temperature. The actual temperature $T_a$ is the temperature.

In fact, what is my intention?

My intention is to know what is this $T_a$ in reference to this $T_B$. So if I can get a relationship between $T_a$ and $T_B$, I will be able to use this method. By measuring $T_B$ I will be able to find out what is the actual temperature. So let us look at the calculation and how we are going to do that. So the brightness temperature simply says that if you go back to the previous slide, $q_{a,\lambda}(T_a)$, is equal to $q_{b,\lambda}(T_B)$.

(Refer Slide Time: 16:16)




And let us assume that the actual surface as an epsilon, or emissivity equal to epsilon lambda. This is a fraction less than 1 multiplied by, according to Wein's law, $C_1$ by lambda to the power of 5 e to the power of minus $C_2$ by lambda $T_A$. This is $q_{a,\lambda}(T)$. What is given here is $q_{a,\lambda}(T)$. This must be equal to $q_{b,\lambda}$ at $T_B$, so $q_{b,\lambda}$ at $T_B$ black body epsilon is 1. So there is nothing here. That is factor 1 $C_1$ by lambda to the power of 5 e to the power of minus $C_2$ by lambda into $T_B$.

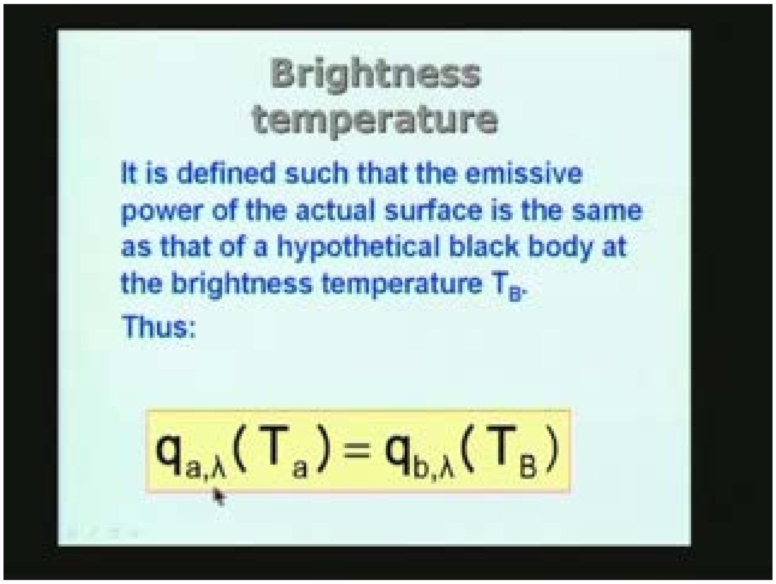
Now notice that $C_1$ by lambda to the power of 5 can be canceled off from the two sides. Therefore, I have epsilon lambda e to the power of minus $C_2$ divided by lambda $T_A$ is equal to e to the power of minus $C_2$ by lambda $T_B$. I can take logarithms on both the sides and rearrange to get the equation which is given here. This is called the ideal pyrometer equation which says that reciprocal of the actual temperature minus reciprocal of the brightness temperature is equal to lambda by $C_2$. This is coming only from here, lambda by $C_2$ logarithm of epsilon lambda.

So you notice that lambda is fixed. The brightness temperature is obtained by measurement which we will see how it is done in a little while from now. The actual temperature is what I want. If I know the epsilon lambda, that is, if I know the emissivity is the surface whose temperature I am trying to measure, then I will be able to obtain the actual temperature knowing the brightness temperature, knowing the value of lambda at which measurement has been made, knowing the emissivity of the surface. This is basically what we do in Pyrometry.

The pyrometer consists of an instrument which is going to help us in measuring $T_B$, and if you know epsilon lambda by some means, by some other measurement, then we will be able to use this pyrometer equation to calculate the temperature. It is called the ideal pyrometer equation, because I am using logarithm, or epsilon lambda here.

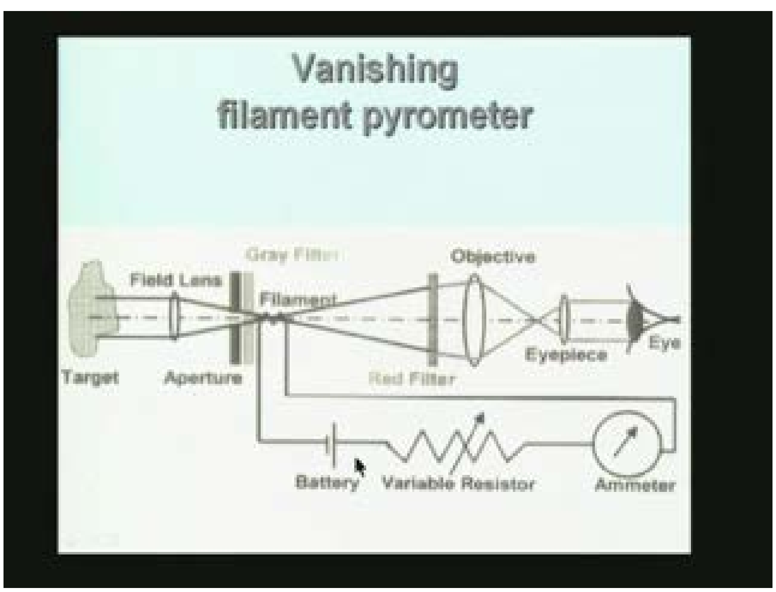
Later, we will see that we will have to modify that epsilon lambda as an effective emissivity which will take into account some losses which we will talk about a little later on. So the idea is to measure the brightness temperature. The instrument I am going to use for this purpose for measuring the brightness temperature is actually a comparator which compares the brightness of two objects. The object whose temperature I want to measure, I want to compare it with a black body whose temperature I can measure, or I know. So I compare these two and I bring a match between these two. If you remember what we talked about earlier, this is the match.

(Refer Slide Time: 18:58)




We are talking the amount of heat coming out per unit area per unit time must be the same for the two at a given wavelength, so this match is what we are doing. This match is called, if the two are in match, we say that they are equally bright. That is why the brightness temperature. So what I do is, I will use an instrument which is shown schematically here. Let me just describe all the parts of this instrument.

(Refer Slide Time: 19:19)



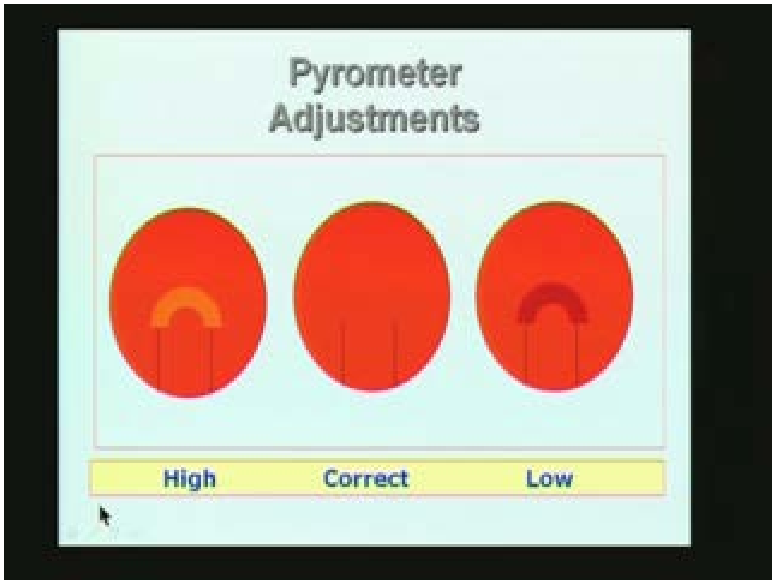
We have the target whose temperature I want to measure. I have a field lens which will gather light coming from this target, and then it will be focused on to the plane of a filament which is shown here, and the filament is actually run by a battery, a variable resistor, an ammeter and so on.

So we will come back to this in a little while from now. The radiation coming from the target is made to pass through an aperture so that straight light is not entering the instrument. It is an optical instrument. Then it passes through a gray filter which will, if necessary, reduce the brightness by a known factor. Then it falls on the filament, and now, as far as the objective lens is concerned, it receives light which is coming, both from the target which has passed through the gray filter, and then it also gets, receives radiation from the filament which is run by this battery.

By variable resistor, by varying the resistance here, I can vary the temperature of the filament, and therefore, the amount of radiation which comes from the filament. So I can, in other words, change the brightness of the filament by changing the variable resistor position. When I change the variable resistor position the current through the filament is changed, and the current, in fact, becomes a measure of the brightness of the filament, or it becomes a measure of the temperature of the filament. So it's usually a tungsten filament which is run by a battery, a variable resistor, and an ammeter. So before it is gathered it is looked at by the eye of the person who is making the measurement. It also passes through both, the light gathered from the target, as well as from the filament passes through a red filter. This red filter, along with the eye of the person who is looking at it, it has got a certain response to the light which is falling on the retina, so the two together will help us in finding out, or selecting a certain wavelength of operation for this instrument.

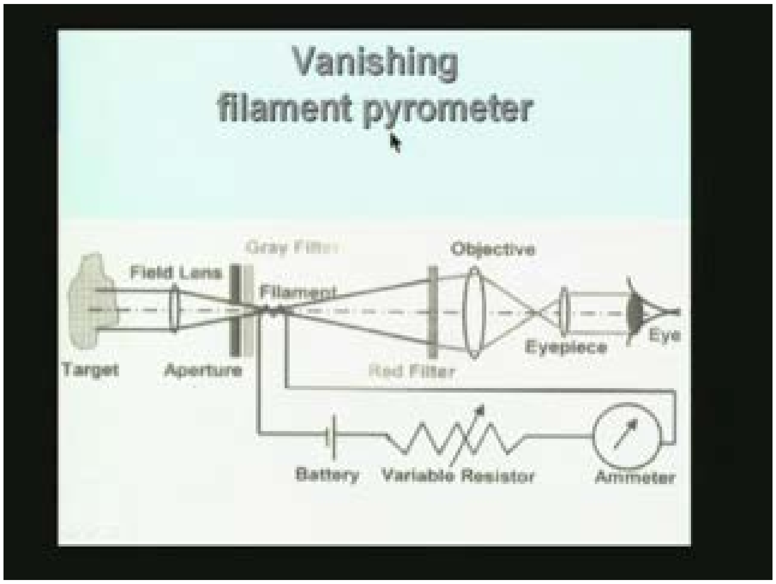
I am going to look simultaneously at two things: the object whose temperature I want to measure; and I am also going to, or the operator is going to, look at also the filament whose brightness I can vary. So the operator will have to adjust the resistance such that the two, the target and the filament, appear equally bright, and which can be verified by making the proper adjustment.

(Refer Slide Time: 22:16)




For example, if the temperature of the filament is very high, you can see here the filament is appearing bright in a background, red background, which is, which appears to be dull. If you go here, if the temperature of the filament is too small, then you will see that it appears dark, looks like a shadow in the background, which is brighter, and in fact, if the adjustment has been made properly you will see that this is the correct adjustment. Then the brightness, the filament, and the background are equally bright. There is no contrast between them. So what happens is that the filament and the background have merged together. The filament has become invisible, and therefore, it has vanished. Therefore, we say it is a vanishing filament pyrometer.

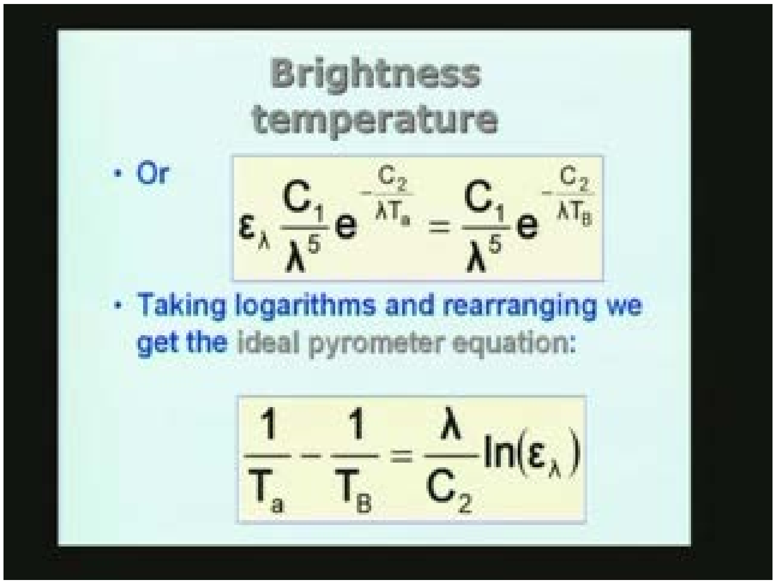
(Refer Slide Time: 23:08)




Therefore, in vanishing filament pyrometer we make the adjustment of the variable resistor so that the ammeter current is varied, so that you will see that the target and the filament appear equally bright. So with this basic understanding of this instrument let us look at some of the features. Suppose the filament, assume that the filament can work at a maximum temperature of, let us say, 3000 Kelvin. Because the filament cannot appear at any temperature you want, it has got a limitation. If it goes beyond some particular value, the filament will melt and you will have to replace the filament. So the filament has a maximum temperature up to which it can go, and therefore, theoretically the maximum brightness temperature I can measure using this filament is the maximum temperature of the filament itself.

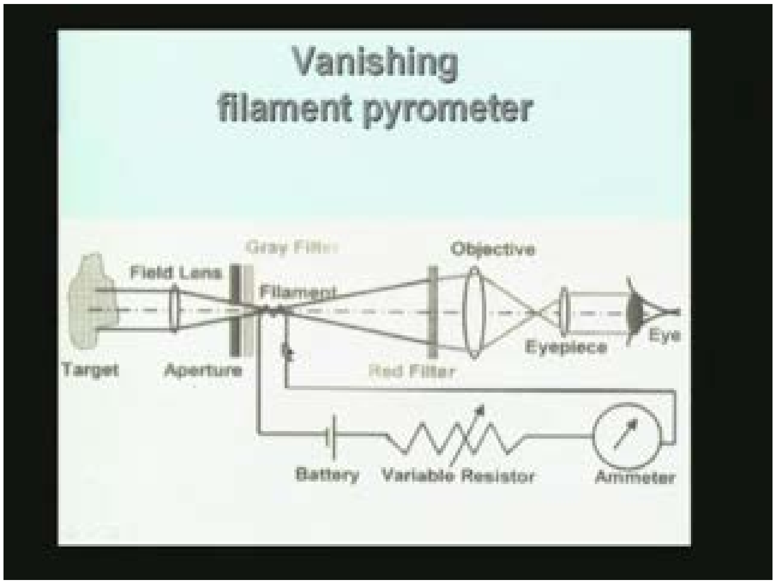
Therefore, if the target is at a temperature higher than the filament temperature what I am going to do is, I am going to use a gray filter which will reduce the intensity by a factor which can be calibrated or known factor. In other words, I can reduce the amount of radiation which is coming from the target by using this gray filter, and reduce it by a fraction, known fraction, for example, half or one-fourth or one-eighth, and so on, so that the intensity of light which is coming from the target is equal to or less than the maximum, the intensity of the filament at the maximum of the temperature. In other words, I am going to choose the range of the instrument by using the gray filter of desired transmittivity.

(Refer Slide Time: 25:00)




If you go back now to the ideal pyrometer equation we had lambda by $C_2$, ln epsilon lambda equal to 1 over $T_A$ minus 1 over $T_B$. Now the effective emissivity will be nothing but the epsilon lambda multiplied by, because we are going to put a gray filter it is going to reduce the intensity, it is like multiplying this by a factor equal to half or one-fourth or one-eighth, and so on. So I will say that this epsilon lambda will be nothing but the actual emissivity of the surface multiplied by the fraction which is allowed to get into the instrument, and in fact, we can also account for other losses like, for example, like in the case of vanishing filament pyrometer.

(Refer Slide Time: 25:43)



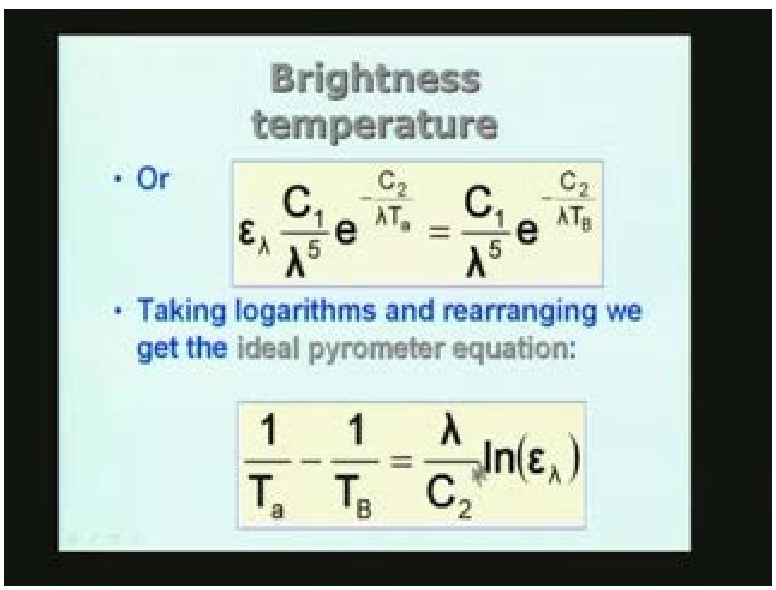

There are so many surfaces; there is a lens here, the front surface, the back surface, the gray filter, the front and back surfaces, and so on. At this surface some amount of reflection will take place. Therefore, it is like actually, like a loss, as far as the amount of radiation which is coming into the instrument is concerned. Therefore, we can club them, club together all these factor into what is called a transmittivity or a transmittance, and we can say that the, in this equation, epsilon lambda is actually the emissivity of the target multiplied by the transmittivity of the optical train. So if we do that, this epsilon lambda will be called the effective emissivity instead of the actual emissivity of the target.

Second point is that suppose I do the following; suppose I keep the filament at a constant temperature. Suppose I keep the filament at constant temperature, I adjust the factor in this gray filter, if I have a variable filter with various filtering coefficient, or various fractions which I can allow, then I can adjust the gray filter alone without adjusting the ammeter at all. Just keep the ammeter at a particular value which you choose so that it is at a given temperature.

For example, it may be maintained at the gold point temperature. Then by simply adjusting the gray filter, I will be able to bring a match between the target and the filament. So in this arrangement what is going to happen is that the filament is going to operate all the time at the same temperature, and therefore, theoretically it's life will be increased (1) from the point of view of the operation of the filament itself. It's very good because you are not subjecting into cycling in temperature, and (2) I am going to have a fixed point, fixed reference temperature, the cold point temperature here as the reference point with which I am going to calibrate the temperature of the target.

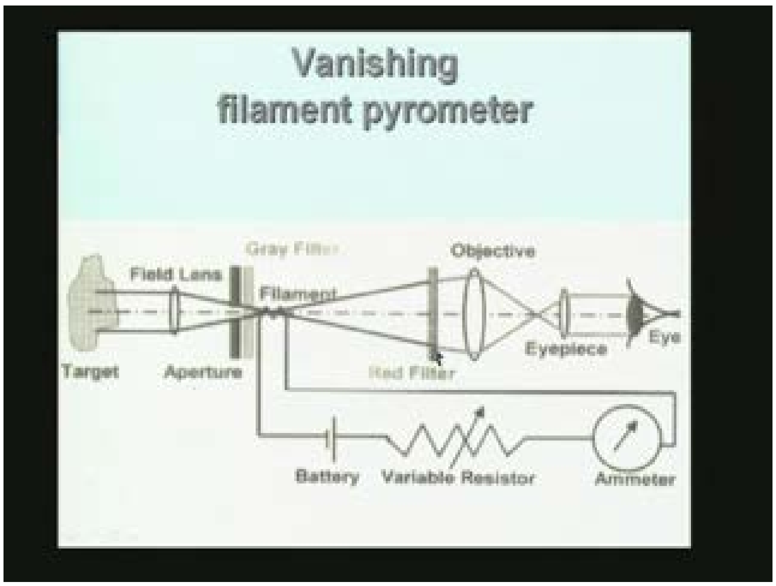
Therefore, just by varying the factor, epsilon lambda, or the effective emissivity of the target by changing the fraction which is allowed by the gray filter to pass through, I will be able to do the thermometry by using a single temperature of the filament which can be chosen as the gold point or the silver point, or whatever you would like to have which is usually at a temperature which is much lower than what is low enough for the filament to last for a long time. That is one way of doing it. That means that we are bringing back the idea of having a single fixed temperature as a part of the thermometry, or the ITS 90. That is the advantage of this. So there are two ways of operating the instrument; one is, of course, to have filament run at different temperatures, and in fact, the temperature of the filament it becomes the brightness temperature, and in the pyrometer equation this will keep on changing.

(Refer Slide Time: 28:44)



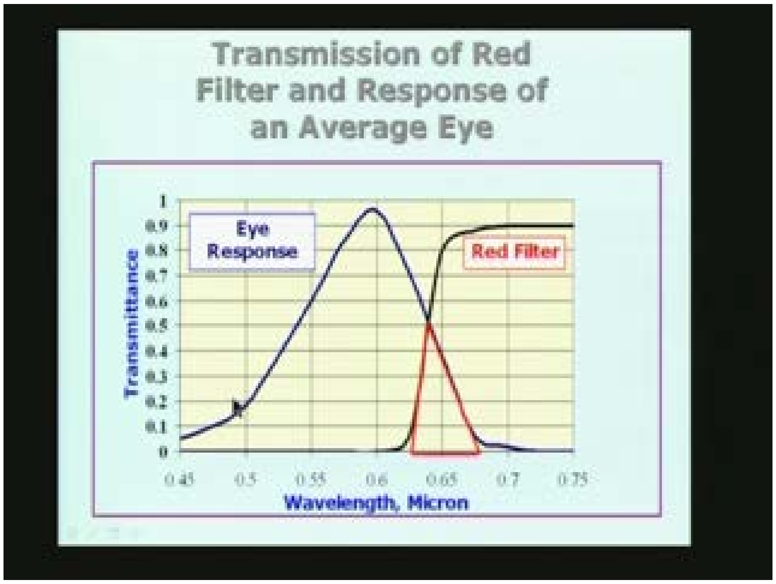

Then we use this to measure the temperature of the actual surface. And in the second arrangement I am going to keep this constant. I am going to vary this part. In other words, the pyrometer can be used in two ways. One is keeping the right hand side constant and working with the variation in the, this factor here, or you can keep this constant and work with the change in the right hand side. These are the two ways of using the pyrometer. And the second way is in fact more satisfactory from the point of view, from the theoretical point of view, also because we are going to have a single fixed temperature as the reference for thermometry.

(Refer Slide Time: 29:26)




Now, to complete our discussion on the vanishing filament pyrometer, I said that we are going to choose the particular wavelength by the combination of the red filter and the eye, and that is explained in the graph here.

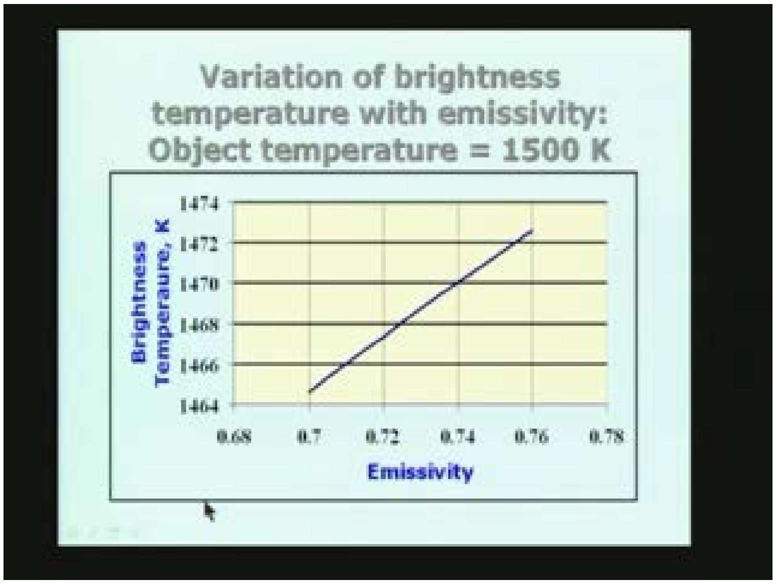
(Refer Slide Time: 29:42)




The eye response of a typical person is like this. The transmittance versus wavelength for the red filter, or in the case of eye response you just replace this transmittance by the response of the eye for different wavelengths, and you see that our vision has got this kind of a response, and the red filter will cut off everything above a certain wavelength, so below that it will transmit nothing, and above that it will transmit almost like 90 percent here.

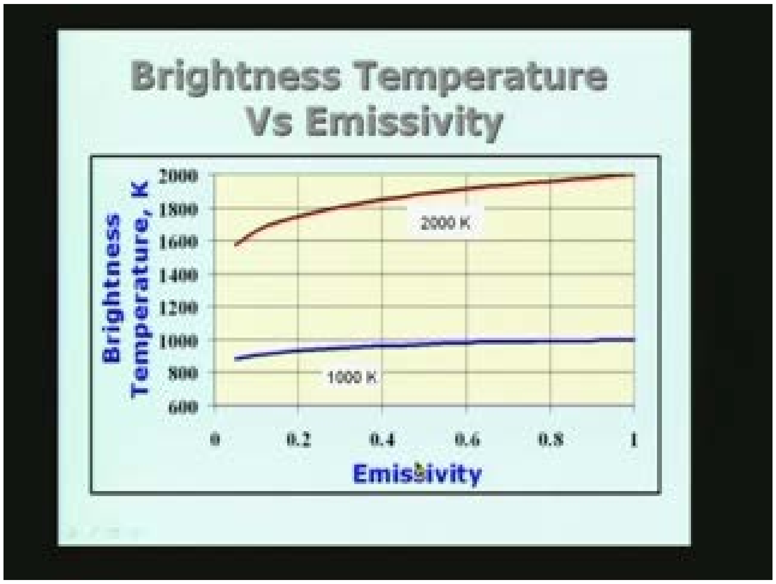
It starts cut off here. Below this it is not going to allow anything. Therefore, if the, if you look at the product of these two functions here, and then in fact this is something proportional to this triangle I have shown here, the radiation which is allowed to enter the eye is basically the, this portion which is roughly at 0.65 microns micrometers centered, and a small window around it. So, this is how you are going to fix the lambda in the case of the vanishing filament pyrometer.

(Refer Slide Time: 30:48)




Let us look at a few of the characteristics which can be derived from the ideal parametric equation. For example, if I vary the emissivity, what happens to brightness temperature? Suppose I take an object at 1500 Kelvin and I just find out what is the effect of the emissivity. You can see that between 0.7 and 0.76 I have varied, more or less there is a linear variation, and you can see that between and 0.7 and 0.76, it varies between 0.1465 and about 0.1473 also. So 65 and 73, about 8 or 9 degrees variation will be there. So it is not all that sensitive within this small, in other words, I can allow some amount of error in the emissivity without compromising the accuracy of the measurement. This is easily derived from the parametric equation.

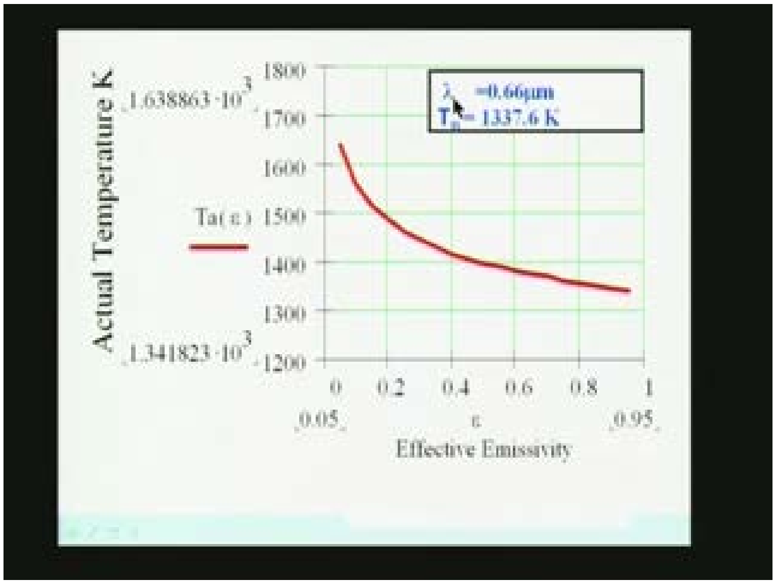
(Refer Slide Time: 31:40)




Suppose I take the brightness temperature as a function of emissivity I want to see. Suppose the actual temperature is 1000 Kelvin, 2000 Kelvin. You can see that if the emissivity is very low, like 0.1, the temperature, the brightness temperature of the object at 2000 Kelvin is only 1600. You will notice here that the brightness temperature is always less than, or equal to, the actual temperature. When epsilon becomes 1, the brightness temperature and the actual temperature become exactly equal to each other, because 1 over $T_A$ minus 1 over $T_B$ equal to whatever on the left hand side.

If you put epsilon lambda equal to 1 logarithm of 1 equal to 0, so the right hand side becomes 0, and therefore, the brightness temperature and the actual temperature are the same. This is, after all, a definition. The black body should have a brightness temperature equal to its own actual temperature. Any other surface will have a temperature brightness, temperature lower than that of a black body. That's what is clear from this. I have taken two examples, one at 1000 Kelvin, the other at 2000 Kelvin.

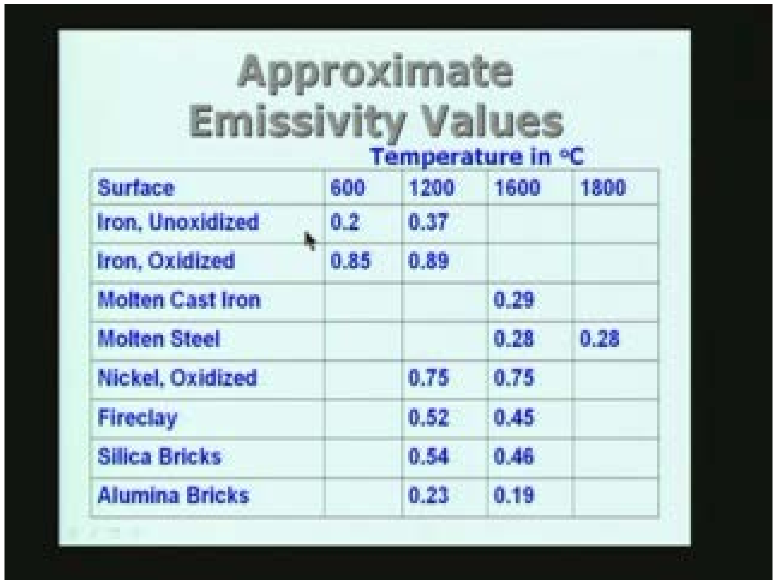
(Refer Slide Time: 32:44)




Suppose I do the following experiment. It's the brightest temperature, I am keeping constant equal to the gold point 1337.6 Kelvin, which you will remember from our ITS 90, this is the temperature of gold, melting gold, and I am taking lambda equal to 0.66 micrometer as in the case of a vanishing filament pyrometer. Then if I keep this constant with different emissivities you can see a surface with emissivity of 0.2 requires to be almost at 1500 Kelvin to be in match with gold point temperature.

However, if you go almost up to 1, very close to 1, you see that the temperature will match exactly. The actual temperatures in the gold point temperature are going to be more or less the same. Therefore, this will give you some kind of idea of what is going to happen to the difference between the actual temperature and the brightness temperature. In practice we also require some idea about the emissivity value of various surfaces and there is no best way of doing it. Most of the time what one has to do is to, if you are going to use a pyrometer regularly in your particular application it is better to do a set of experiments such that the emissivity is known for a particular surface which you are going to use in your application. Make a calibration of the emissivity of the surface so that you can use that. That is the best method. However, just as a guide, I had given some values which are taken from the references.

(Refer Slide Time: 34:11)



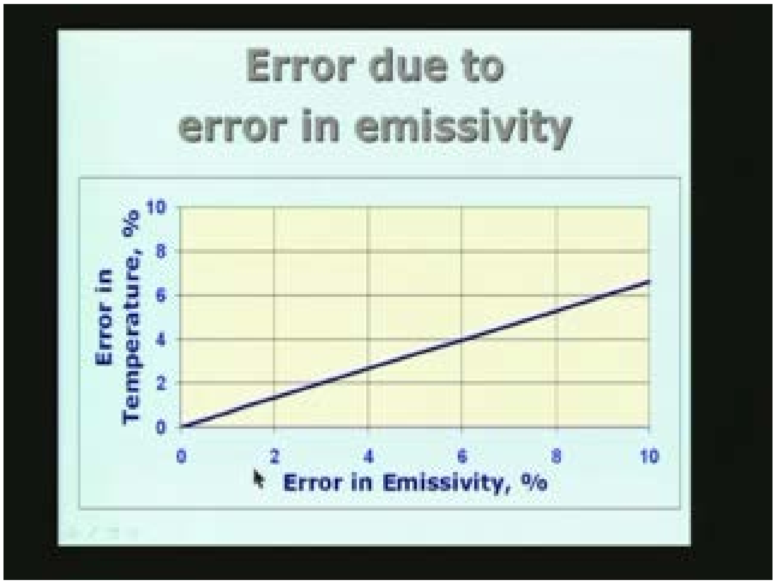
## Approximate Emissivity Values

Temperature in °C

| Surface | 600 | 1200 | 1600 | 1800 |
|---|---|---|---|---|
| Iron, Unoxidized | 0.2 | 0.37 | | |
| Iron, Oxidized | 0.85 | 0.89 | | |
| Molten Cast Iron | | | 0.29 | |
| Molten Steel | | | 0.28 | 0.28 |
| Nickel, Oxidized | | 0.75 | 0.75 | |
| Fireclay | | 0.52 | 0.45 | |
| Silica Bricks | | 0.54 | 0.46 | |
| Alumina Bricks | | 0.23 | 0.19 | |

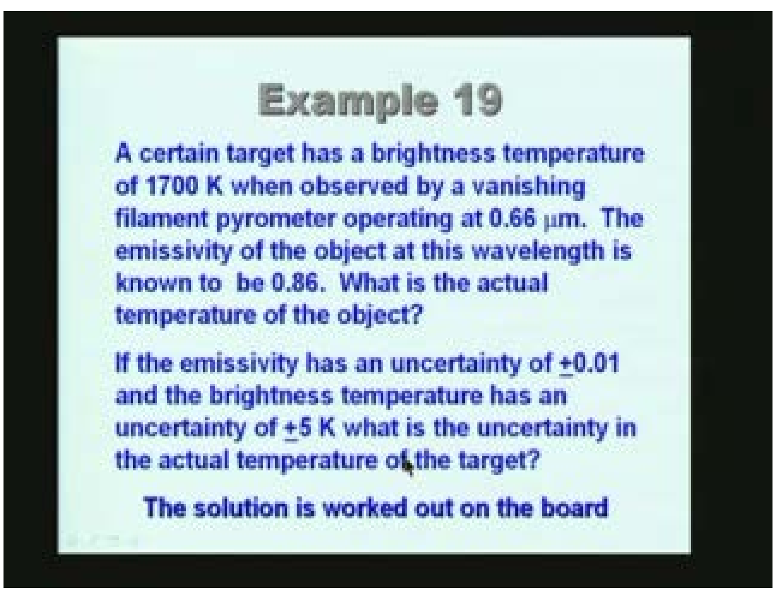
The surface temperature is given here 600, 1200, etc. These are the approximate emissive values which are given. For example, iron can vary between 0.2 and 0.37, between 600 and 1200 if it is unoxidized. If it is oxidized, you can see that it changes quite drastically to 0.85 and 0.89, and if you take molten cast iron 0.29, and molten steel 0.28, and nickel 0.75. Fireclay, this is very common. In furnace practice we use fireclay as the surface material for the furnace 0.52 and 0.45; silica bricks, again 0.54 and 0.46. These are all high temperature insulating materials. Alumina bricks, for example, 0.23 and 0.19. So, if I am going to measure the temperature of the surface of the brick using a pyrometer I can use the approximate value of 0.23 for the emissivity and make the measurement.

(Refer Slide Time: 35:12)



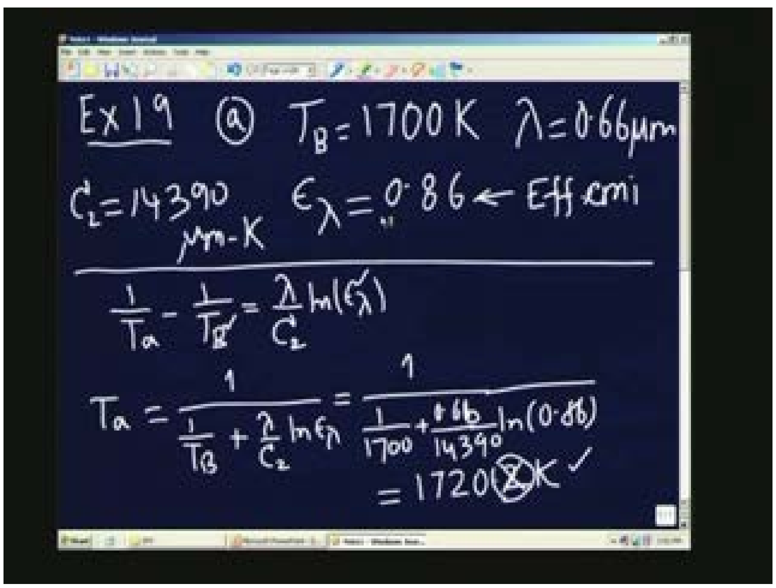

Now the other question is, (we have actually answered this question a little bit earlier also) if the error in emissivity is given, let us say I have taken between 0 and 10%, the error in temperature is something little smaller, so the percentage error in the temperature is slightly smaller than the percentage error in the emissivity as you can see from here. All these are derivable using the ideal parametric pyrometric equation.

(Refer Slide Time: 35:37)



So let me take an example and work out the number so that it becomes clear as to what we are talking about. So I have a certain target which has a brightness temperature of 1700 Kelvin when observed by a vanishing filament pyrometer of the type I just now described. It operates at a wavelength of 0.66 micrometers. So the emissivity of the object at this wavelength is known to be 0.86. It is given to us, so we want to find out what is the actual temperature of the object. This is basically an application of the ideal parametric equation. The second part, however, requires a little bit tough work. If the emissivity has an uncertainty of plus or minus 0.01, and the brightness temperature has an uncertainty of plus or minus 5 Kelvin, what is the uncertainty in the actual temperature of the target. The second part requires the use of the error propagation formula which we have discussed earlier. So we will work out this temporary solution on the board by taking the numbers given in that example.

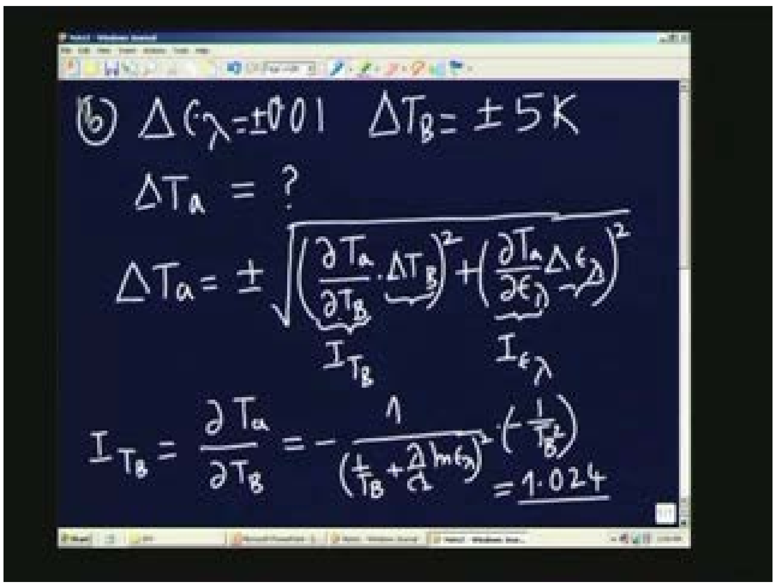
(Refer Slide Time: 36:41)




We are using this example 19. The first part, I am given the brightness temperature which is 1700 Kelvin. Lambda is specified as 0.66 micrometer. The emissivity of the target, is also specified as 0.86, and we know that the second radiation constant, $C_2$, is 14,390 micrometer Kelvin. So these are the data given. So all I have to use is to make use of the ideal pyrometer equation 1 over $T_A$ minus 1 over $T_B$ equal to lambda by $C_2$ logarithm of epsilon lambda.

We are making the assumption that this is the effective emissivity, because it is not specified. We can simply assume it to be the effective emissivity. So I have to, I have given this entire constant, this is given, this quantity is given. All the other things are known. All I have to do is to obtain the value of $T_A$. I can rewrite this equation $T_A$ equal to 1 over ,1 over $T_B$ plus lambda by $C_2$ logarithm of epsilon lambda, and I can substitute the values 1 over 1700 plus 0.66 divided by 14390 logarithm of 0.86. And if you work it out it comes to 1720 0.2 Kelvin. This 0.2 I can ignore. It is not significant. So I can simply say 1720 Kelvin. It is the actual temperature of the surface whose brightness temperature is given as 1700 Kelvin, and the effective emissivity of the target is 0.86.

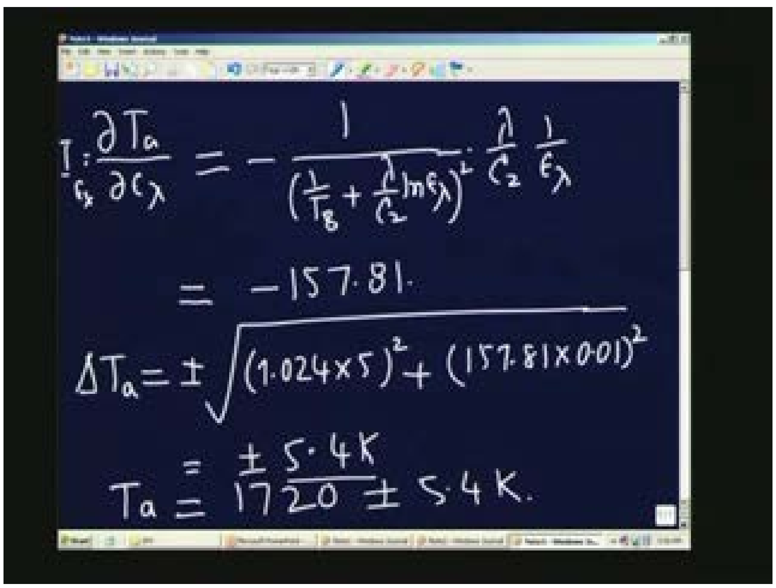
(Refer Slide Time: 42:06)




The second part requires the calculation of the error, so the part (b), I am given delta epsilon lambda equal to 0.01, and we are also given that delta $T_B$, actually you can say this plus or minus, again this is plus or minus 5 Kelvin. I would like to find out what is delta $T_a$. That is the problem which is given to us. So what I am going to do is, I am going to use the following formula so you will say that delta $T_a$ is equal to plus or minus square root of partial derivative of the expression for $T_a$ with respect to $T_B$ multiplied by delta $T_B$ whole squared, plus partial of $T_a$, with respect to epsilon lambda, multiplied by delta epsilon lambda whole squared.

Therefore, all I have to do is to calculate, this is known, this quantity is given, plus or minus 5 Kelvin. This is given as plus or minus 0.01. I have to obtain the two partial derivatives and these are also called the influence coefficient. So I can say that this *doh* $T_a$ by *doh* $T_B$ is nothing but the influence coefficient $I_{TB}$, and this is the $I_{\text{epsilon lambda}}$. So I have to take the expression which we used in the first part, take the partial derivative, and therefore, I can obtain the following. This will come out to be minus 1 over 1 plus 1 over $T_B$ plus lambda by $C_2$ log epsilon lambda whole squared multiplied by minus 1 over $T_B$ squared. So we substitute the values which are given in the problem and you can verify this gives you a value equal to 1.024. So similarly, we work out the other influence coefficient. This is given by epsilon lambda.

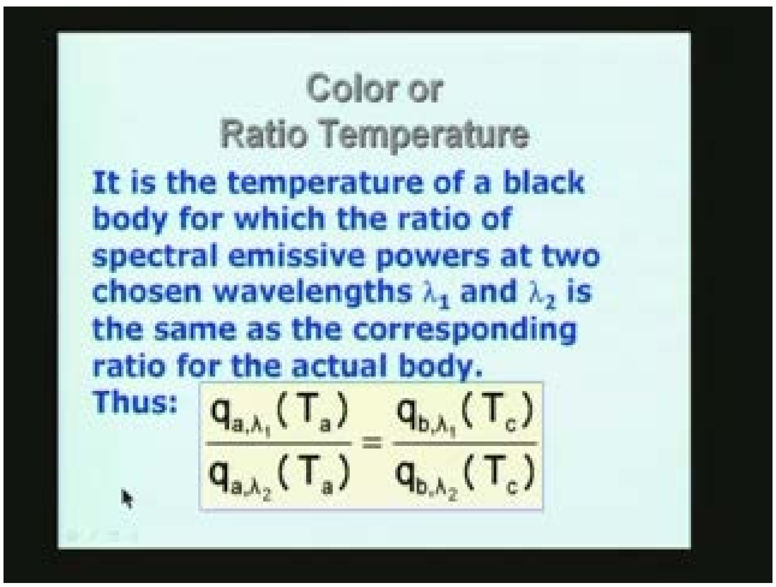
(Refer Slide Time: 42:12)




It is $I_{\text{epsilon lambda}}$. This will be minus, again 1 over $T_B$ plus lambda by $C_2$ whole squared multiplied by lambda by $C_2$ into 1 over epsilon lambda. All I have to do is to substitute all the values and you will get a value of minus 157.81. So the two influence coefficients are available to us. Now so I can calculate the error in the actual temperature as plus or minus square root of 1.024 into 5 whole squared, plus when you are using the error propagation formula, this minus will not come into the picture because it will be all squares of terms.

Therefore, this will be 157.81 into 0.01, because that is the error given whole squared, and this comes out to be plus or minus 5.4 Kelvin. So we will say that the temperature estimated of the target is 1720 plus or minus 5.4 Kelvin. So there is an error bar of plus or minus 5 which is quite acceptable when you are measuring temperature as large as 1700 Kelvin. It is simply not possible to have a better error specification for this because this is the nature of the quantity we are measuring. We are measuring a large temperature and a temperature of the order of plus or minus 5 is quite acceptable in practical applications. So this is a typical problem based on the vanishing filament pyrometer and the concept of the brightness temperature.

(Refer Slide Time: 44:27)




Now we look at a second concept which is called the color, or the ratio temperature. In the first case, if you remember, we compared the intensity or the brightness of two objects, and by comparing them we said that they share some common temperature which we call as the brightness temperature. Now I can also do the following. Suppose I choose two wavelengths, $\text{lambda}_1$ and $\text{lambda}_2$.

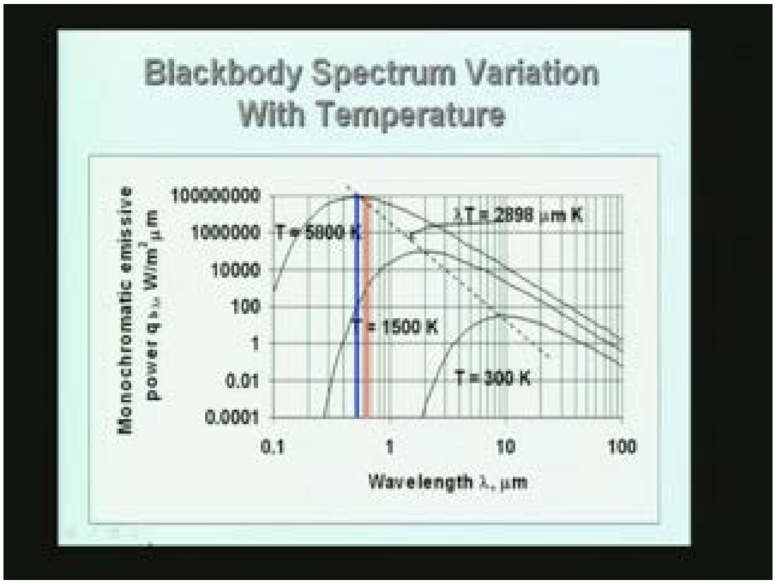
Suppose I have the ratio of the brightness at these two lambdas for the object whose temperature I want to determine. If I compare this ratio with that of a hypothetical black body which also has got the same ratio, then I can say that the temperature indicated by the ratio of the black body intensities, or black body brightness values at these two wavelengths, is going to give you a temperature which is shared by the two. This we call as the ratio temperature because it is the ratio of brightness of the two values. Or the temperature is called the color temperature. Color because we are going to use two different wavelengths.

And if you are using the visible part of the spectrum, $lambda_1$,$lambda_2$ will correspond to different colors of the visible spectrum, so the definition would be the actual temperature, actual surface, is the temperature $T_a$, and the ratio is $q_{a,\lambda 1}(T_a)$ divided by $q_{a,\lambda 2}(T_a)$. And please remember that at $lambda_1$ and $lambda_2$ the object may have different emissivities. It may have an emissivity of epsilon $lambda_1$ at $lambda_1$ and epsilon $lambda_2$ at $lambda_2$, whereas in the case of the black body epsilon is the same whether it's a $lambda_1$ or $lambda_2$, and therefore, the right hand side is simply the Wein's approximation written for the numerator and the denominator without any epsilon coming in the picture, whereas in the left hand side I have got, the epsilon will be coming in the picture.

Therefore, the ratio of the emissivities also is going to come here depending on whether the ratio of the emissivities at $lambda_1$ and $lambda_2$ are greater than 1 or less than 1, you may have $T_C$ which is less than $T_B$,$T_a$ or greater than $T_A$, whereas in the case of the brightness temperature, the brightness temperatures are always smaller than the actual temperature here. Because of the nature of the definition of the color temperature it may be either greater, or less than, or equal to the actual temperature.

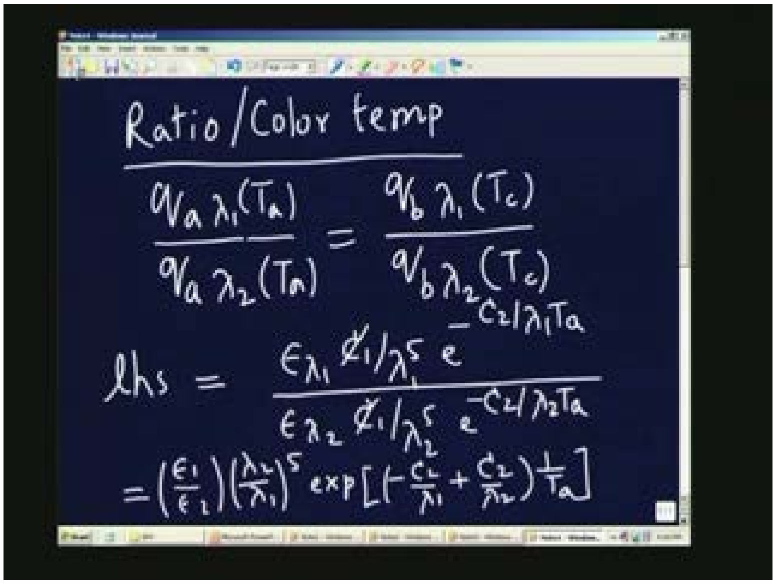
Suppose we choose two wavelengths which are very close to each other. $Lambda_1$ and $lambda_2$ are very close to each other, and suppose that the emissivities of the object or the target at these two temperatures are the same, are very close to each other, then you see that the ratio temperature, the $T_C$, will be the same as the actual temperature. In fact, that is the reason why we use it as the ratio, or the color temperature, because you would like to make the ratio temperature equal to the actual temperature by choosing two wavelengths close to each other so that the emissivity is not strongly varying between these two values, so that you get an equality between the actual and the color temperature. So that is the entire basis to it.

(Refer Slide Time: 47:43)




I have shown this using the same figure which I used earlier for the brightness temperature. I have chosen two colors, one red here, another blue. The blue line corresponds to the blue part of the visible spectrum. Red color corresponds to, so you take the ratios of these two quantities, red here and blue here, similarly, red here and blue here. So the ratio temperature will be characteristic of the black body here or the black body here, and so on. That is the basis for the color temperature. Let us look at some of the details. We will go back, go to the board and make a few derivations, we will make, so that we will understand what is going on.

(Refer Slide Time: 51:33)



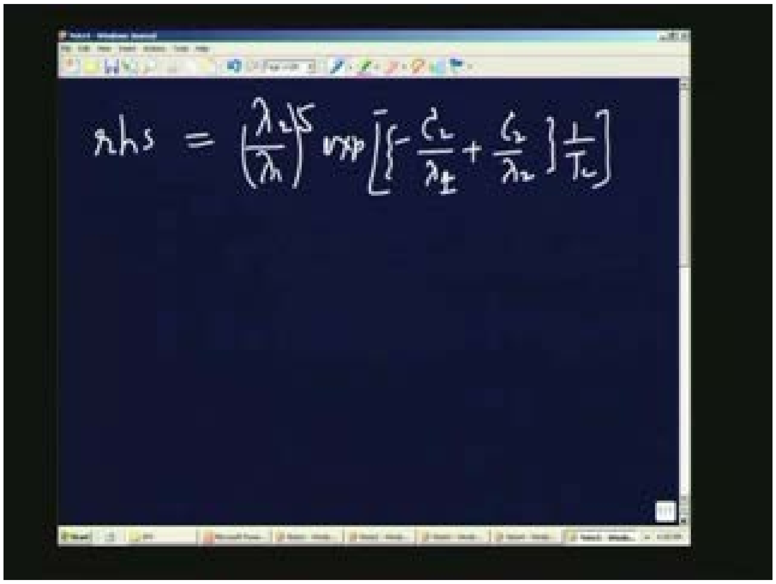

So basically, the ratio or color temperature is based on the following: $q_{a,\lambda 1}$ by $q_{a,\lambda 2}$ is equal to, we are just rewriting what we wrote on the slide there. So I am equating the ratios, the actual at temperature $T_a$, and the black body at a color temperature equal to $T_C$. So the left hand side I can write it. So I will say left hand side is the same as the actual, is nothing but epsilon $lambda_1$. I am using the Wein's approximation multiplied by $C_1$ by $lambda_1$ to the power of 5, e to the power of minus $C_2$ divided by $lambda_1$ into $T_a$ divided by epsilon $lambda_2$, again $C_1$ by $lambda_2$ to the power of 5, e to the power of minus $C_2$ divided by $lambda_2$ $T_a$. So the $C_1$ can be dispensed with.

Now this will become, actually ratio epsilon instead of $lambda_1$, I will just say $epsilon_1$ by $epsilon_2$, so that it is easier to write it; is multiplied by $lambda_2$ by $lambda_1$ to the power of 5, this e to the power of, I can write it as minus $C_2$ by $lambda_1$ minus plus $C_2$ by $lambda_2$ multiplied by 1 over $T_a$. So the left hand side is simply given by this point.

What about the right hand side?
The right hand side is given by a same quantity. If I replace $T_a$ by $T_C$, and I will put epsilon $_1$ by epsilon $_2$ equal to 1, that's all what I have to do. So that means that this fact will be there. This factor will become 1 and this will become 1 over $T_C$. So let me write it in the next. So the right hand side is nothing but epsilon $_1$ by epsilon $_2$ is now equal to 1. So $lambda_2$ by $lambda_1$

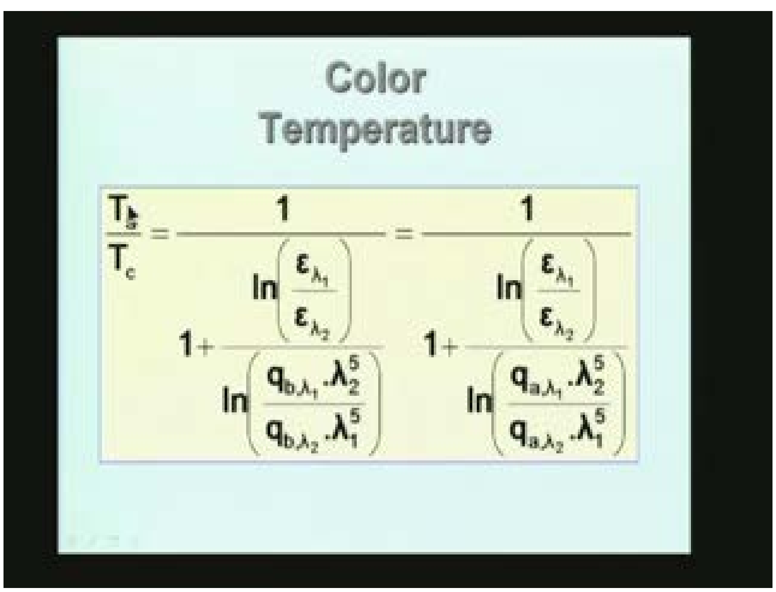
to the power of 5 exponential of minus $C_2$ by $lambda_1$ plus $C_2$ by $lambda_2$ into 1 over $T_C$.

(Refer Slide Time: 52:34)



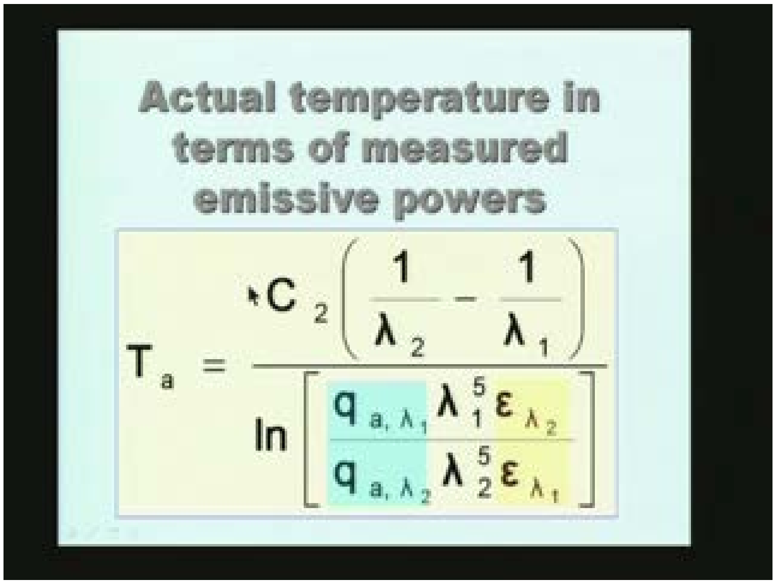
So if I equate the left hand side to the right hand side, and manipulate the two sides, I will be able to get several expressions which will be useful in practice.

(Refer Slide Time: 52:39)



So the slide shows the ratio of the actual temperature to the color temperature, and it can be shown by following the argument which I gave on the board: 1 over 1 plus logarithm of the ratio of the emissivity epsilon $\text{lambda}_1$ and epsilon $\text{lambda}_2$ divided by logarithm of the black body function at $\text{lambda}_1$ multiplied by $\text{lambda}_2$ to the power of 5 divided by black body function at $\text{lambda}_2$ multiplied by $\text{lambda}_1$ to the power of 5. And if you remember the definition of the color temperature, this ratio is the same as this ratio. Therefore, I can see, replace this by as the actual temperature of the body. This is the ratio at the color temperature. This is the ratio at the actual temperature. These are the same.

(Refer Slide Time: 53:35)

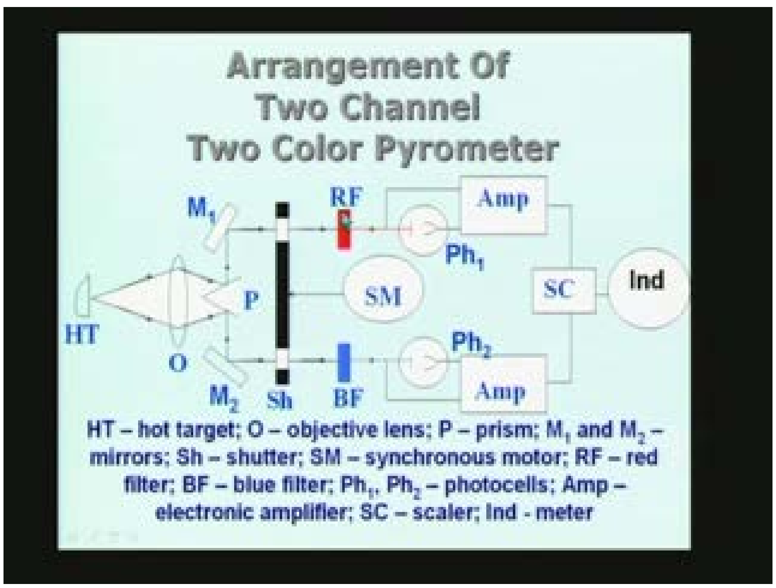



Another way of recasting these equations is to write the equation of the following form because this is what is going to be useful for Pyrometry, or the two-color Pyrometry because we are using two different wavelengths, $T_a$ equal to $C_2$ divided by 1 over $\text{lambda}_2$ minus 1 over $\text{lambda}_1$. This is the numerator divided by logarithm of $q_{a,\lambda 1}$ divided by $q_{a,\lambda 2}$. These are actually measured. The brightness of the object at the wavelength equal to $\text{lambda}_1$ at wavelength equal to $\text{lambda}_2$, this ratio $\text{lambda}_1$ by $\text{lambda}_2$ whole to the power of 5. These are the two wavelengths we already know, and epsilon $\text{lambda}_2$ and epsilon $\text{lambda}_1$ are also supposed to be known.

Hence, in practice, when we are using a pyrometer, or a two-color pyrometer, I am actually measuring this quantity. This $q_{a,\lambda 1}$ by $q_{a,\lambda 2}$ is actually measured, and the rest of the things are known. Epsilon lambda$_2$ and epsilon lambda$_1$ are known.Lambda$_1$ lambda$_2$ are specified wavelengths at which we are going to make the measurement. So everything is known. This is known, that the actual temperature can be calculated. This is basically what the Pyrometer does.

We are not, as such, interested in the color temperature. What we are interested in is the actual temperature which can be obtained by the measurement. The measurement we are going to make is the ratio of the brightnesses at two different wavelengths for the same target. So one way of doing it is to use a two-color pyrometer, and it consists of the hot target from which I am going to take the radiation that is coming out.

(Refer Slide Time: 55:10)




I am going to collect the radiation using the objective lens, and it falls on a prism, and one part is reflected upwards, and the other part is reflected downwards. There are two mirrors which are going to basically, to give me two beams from the same source. I am getting two beams from the same source and one of them is going to pass through a red filter. This red filter is different from the red filter we had earlier. This is going to allow a very narrow band around the red value equal to 0.65 plus or minus little bit.

Similarly, a blue filter, and there is in front of that, there is a wheel with two holes, or many holes if you want, it is run by a synchronized motor. This is rotating, so it will allow the radiation through this path to pass through this for some time, and after some time it will allow here. Therefore, the photodiode here, or photodetector here, is going to respond to the radiation coming out of the hot target at lambda$_1$. This photodetector is going to respond to the radiation coming at lambda$_2$. The blue radiation is lambda$_3$, red is lambda$_1$, then it is going to amplify the signal and then take care of any manipulations because we have the relation here. I want the ratio of these two, and the ratio of these two, and so on. These manipulations can be done by what is called a scalar, and then it is indicated by meter. The indication by the meter is the actual temperature.

So what the two-color pyrometer does is, to take two beams of radiation coming from the same source and one passes through the red filter and the other passes through the blue filter, takes the ratio of these two brightnesses, and then gives you the temperature directly. So what we will do in the next lecture is to consider example number 20, which we are not able to do now. We will work it out, and then once we have done, we will try to look at a new topic because thermometry, more or less, we are going to complete here. I may touch up on a little bit, on the measurement of the gas temperature, just to give a flavor, but we will not go deeply into it because it requires advanced knowledge. So what I will do is I will just touch up on temperature measurement of gases and then take a look at measurement of other quantities. Thank you.