Mechanical Measurements and Metrology
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Module - 2
Lecture - 16
Resistance Thermometry

This will be lecture number 16, Mechanical Measurements. During the, toward the end of the last lecture, we were looking at the measurement of the transient temperature, and we discussed three different cases, and the last case, that of a sinusoidally varying temperature was not completely covered, and therefore, what I am going to do in the present lecture is to start with that particular case and look at the details of how the response of the system behaves with the sinusoidal input and then we will give an example, which I call as example 16.

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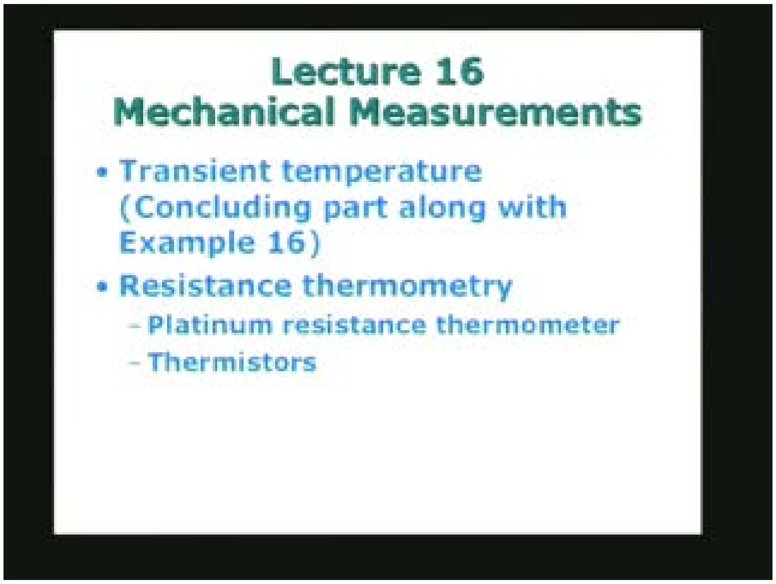

I will work it out on the board, and then I will look at the resistance thermometry. Two different aspects are there; one is platinum resistance thermometer which is very often used, and many times we also use what are called thermistors which are also the temperature sensors where the resistance of the element changes with respect to temperature with definite fashion.

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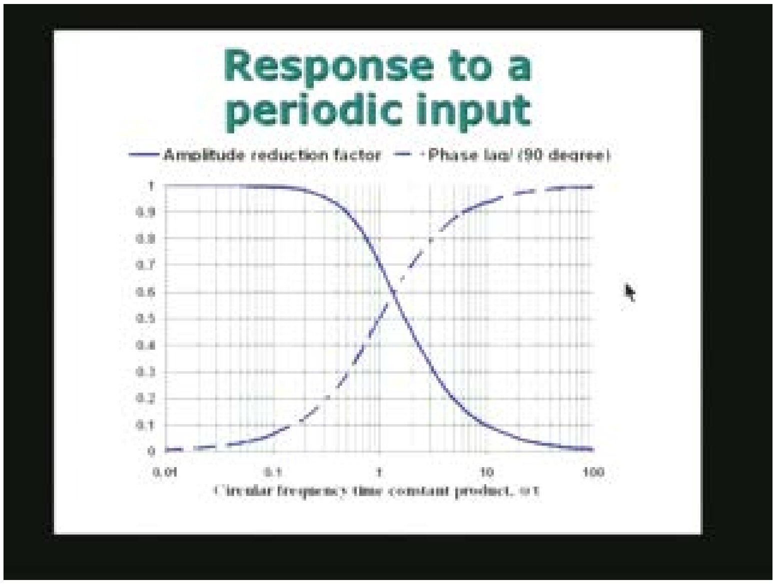

So to begin our discussion, let us look at the response of the first order system, the temperature sensor treated as a first order system. I am subjecting it to a periodic input in the form of a function, T_a cos omega t, where omega is the angular frequency, and T_a is the amplitude, and initially, the temperature of the system and the sensor are not the same. They may be different. And then starting from t is equal to 0, for t greater than 0, we are going to input a sinusoidal variation of temperature of the ambient fluid.

If you remember, the general solution was obtained, and then by putting the appropriate form of T infinity as the function of time on the inside, the integral, after integrating it twice, we get back the integral, and therefore, we will be able to find out the response in terms of the amplitude reduction factor and the phase lag. The output is again a sinusoidal. That means that there is no change in the shape of the variation with respect to time. Only the size is changed, that is, the amplitude is changed, and also, there is a phase lag that means that the response of the system, lags behind the response or the input which we are going to provide.

The important parameter which is going to play a role in this particular case is the product of omega, the angular frequency, and the time constant tau, omega tau product. Of course, omega tau product can go from anywhere, from 0 to infinity in principle. That means very small values to very large values. If omega symbol tends to 0 obviously it is like steady state. There is no variation with time, and when omega tau tends to infinity, that means that the input frequency is very high, or the time constant is very large. It is either this or that.

And in fact, the figure I have drawn here shows the logarithm of the omega tau product on the x axis, and on the y axis I have either the amplitude reduction factor which was given by 1 over square root of 1 plus omega squared tau squared, or the phase lag which was given by tan inverse of omega tau, and I have divided the phase lag by 90 degrees so that I can normalize the phase information, and put it, both the amplitude reduction factor and the phase lag become normalized and therefore the line between 0 and 1.What we notice is that when omega tau tends to 0 the phase lag tends to 0 and the amplitude factor tends to 1 and when the omega tau product becomes very large (in this case I have taken up to 100) you see that the amplitude reduction factor is almost 0. That means that there is no response.

The periodic input is not indicated. That means there is no response of the system to the input which is very high frequency. The tau, the time constant, is very large, therefore, it is not able to respond to that, so it will just show an average value and you will also see that the phase lag approaches 90 degrees as omega tau product becomes infinity, or very large. There is a crossover, and you can see that,in fact, if I plotted logarithm of on this axis, it will show a nice behavior. It will be almost like straight, and then it will become, it will drop down straight according to straight-line kind of relationship at this point onwards.

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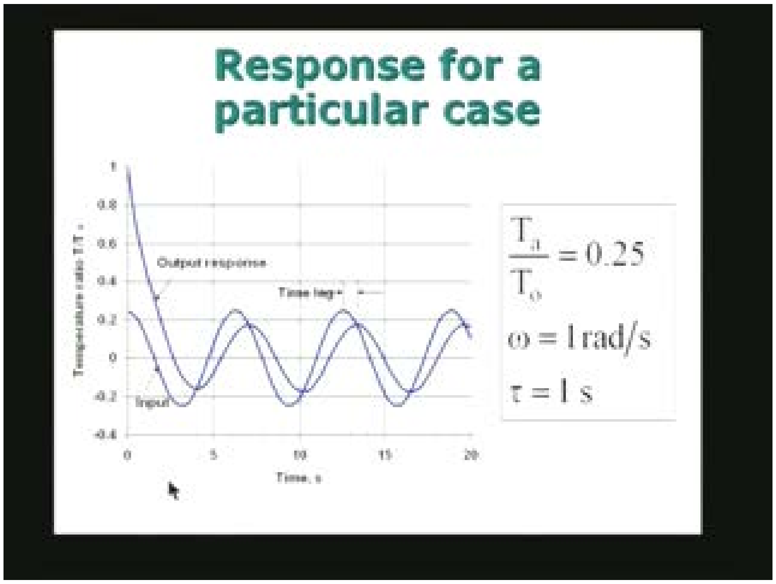

So the typical case I take for which I can plot the response in the real time temperature time graph so that we can appreciate what is going on. So if I have the temperature ratio T divided by T_0 plotted as time function of time, and the input is T_a by T_0 is equal to 0.25. This amplitude is 0.25 times the initial value, and the omega is 1 radian per second. I have just taken a particular case, and tau is equal to 1 second, so that omega tau product is equal to 1 in this particular case, and you will see that the input response of the input, the variation of the temperature, is according to this relationship. It starts with the value of 0.25 at t is equal to 0, and it cycles according to the formula T_a by T_0 is equal to 0.25 cos omega t, and the output, it starts initially from value of 1 because T by T_0 is equal to 1 at t is equal to 0, and then initially, there is a transient wherein the output response falls quite drastically, and then immediately it starts responding to the variation which is periodic, and therefore quickly the solution of the output response tends to become a sinusoidal as shown by this curve, and we notice that the amplitude of this curve is smaller than the amplitude of the input. And that is one of the consequences of the periodic input in the case of a system of first order system with the given tau.

You will also notice that the input is reaching a maximum and somewhat, sometime later, the output response of the system is catching up with it and it is giving a value which is of course lower, but the maximum of shifting with respect to time. Therefore, time lag is there. In fact, I can measure the time lag by taking the phase lag. Phase lag is nothing but we can interpret it as omega times some t lag, and therefore, omega t lag is equal to the phase lag, and therefore, phase lag divided by omega will give you time lag equivalent to time lag. So in this case you see that there is a time lag of something like between 1 second, or whatever equivalent of that, because omega is equal to 1, tau is equal to 1. There is a time lag like this, so the response for a particular case is shown here, and let us now look at what is going to happen in a general case.

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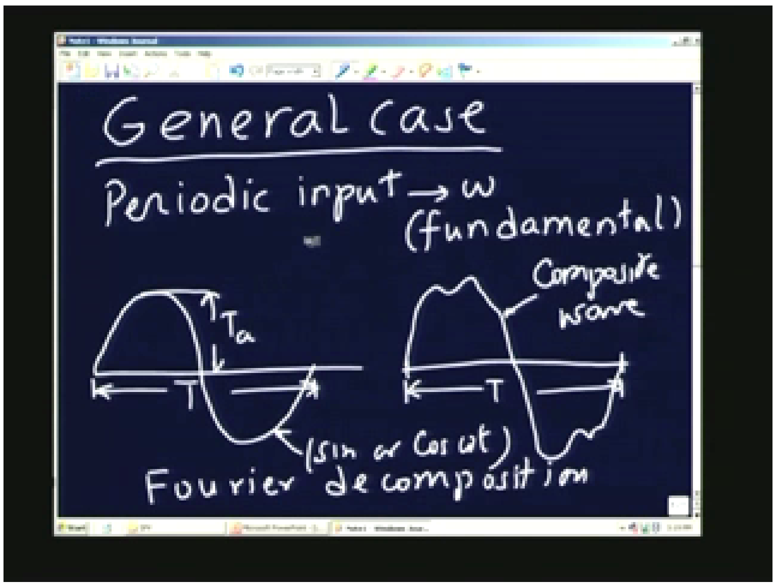

So, if I look at the general case, I am just going to make a very simple analysis without too much of mathematical details so that we understand what we are talking about. So if you remember what we did, we had a periodic input at one single frequency, omega. We can call this as the fundamental frequency. That means that I have a wave varying like this, and this is the period of the wave, this is the amplitude, and this is sine or cos omega t.

The difference between sine and cos is that they are shifted with respect to each other by 90 degrees. That’s it. Suppose, instead of this, let us say I have a slight, the same wave, with a something like this. This is again T; this has many more small wriggles here. That means this is a composite wave. That means that is going to be given as a sum of terms like this. We can, in fact, write what is called Fourier decomposition, and I will just briefly indicate this Fourier decomposition in the next page.

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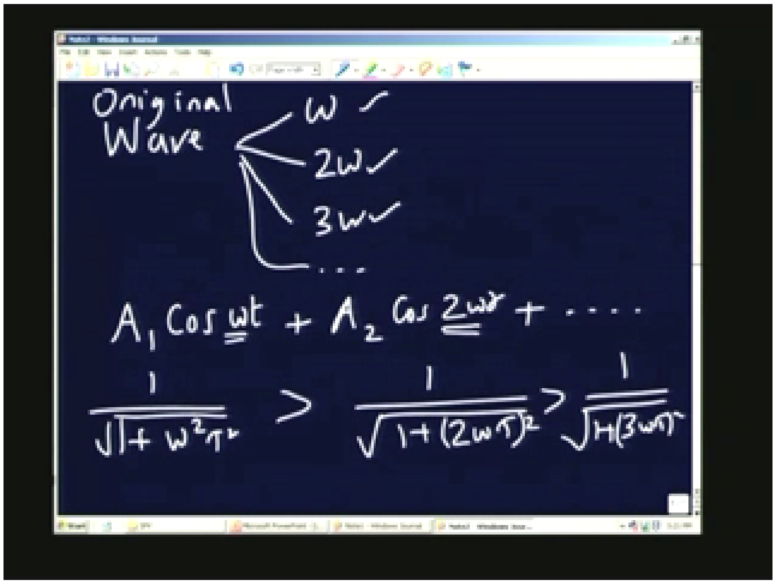

So what we are going to do is, the wave is split up into omega, 2 omega,3 omega, and so on, that means that the original wave is now consisting of a fraction of this, a certain fraction of this, etc. That means that I am going to have, we can say that I can write it as A_1 cos omega t plus A_2 cos 2 omega t plus, etc. So in other words, the original wave is now made up of components where A_1, A_2, etc, are to be determined by comparing the series with the original wave, and using the Fourier series method, we can do that. Let us not worry about it right now because it is not of importance to us. We want to just look at the, how the solution is going to be.

The first component is that omega, the second one is 2 omega, and if you remember, the amplitude reduction factor for this will be 1 plus omega squared tau squared under square root of, and this will be 1 over square root of 1 plus 2 omega tau whole squared. That means that this is greater than this, and this will be greater than the third term. It will be 1 over 1 plus 3 omega tau whole squared, and so on. That means that different components of the waves which are present in the original wave are going to be attenuated with respect to attenuated or the amplitude reduced by different amounts. This is smaller, this is less attenuated, this one is more attenuated and so on. And therefore, we will get a change in the shape of the wave, that is, number 1.

And number 2 also, this will be phase lag, will be tan inverse omega tau. This will be tan inverse 2 omega tau, and so on, that the phase lag is also changing. That means that there is a shift in the wave, or the shift in the response with respect to time, and also there is a change in the reduction, or change in the amplitude of each one of these components. Therefore, we are going to have some distortion in the shape of the wave. The output will be now a complex wave which is going to consist of several components, each component attenuated to a different extent, and each component having a different amount of phase lag with respect to the original wave which we have given.

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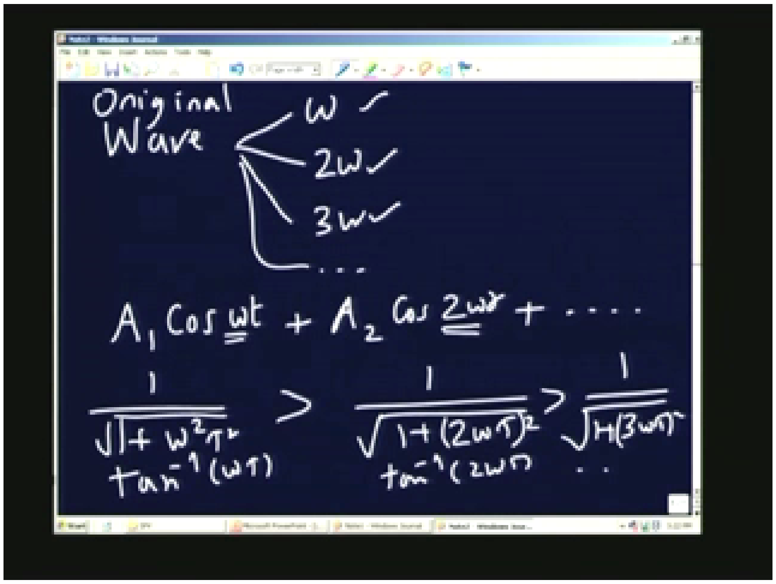

So the idea with which we have done this exercise is to just look at what happens to one component, and then if we understand that we will be able to understand what happens to the other components, and therefore, when we have a original wave which is a complicated shaped curve, not a pure sinusoidal, you can always look at the overall response by looking at the component responses and then adding them and getting the response of the system, because the system is essentially a linear system.

So we will say that this is a linear system and the response is made up of all these components in most applications. We may be interested only in the fundamental, more or less, and therefore, many times we need to worry only about the first term in the series, but there may be occasions when we want to look at the other terms also, and it can be done by this particular method I have indicated. So with this background let us look at a simple application of what we have done. This is the example number 16 which I am going to just indicate by looking at a simple case.

I have a small foil type sensor, is aluminum foil with a small thickness, and it is subjected to a heat transfer coefficient of 50 watts per meter squared degrees Celsius, and the size of the foil is given 0.25 meters multiplied by 0.03 meter multiplied by the thickness is 0.00005 meter. This is the size of the foil, and the foil is loosing heat by, with the heat transfer coefficient h is equal to 50 watts per meter squared Celsius, and the properties of aluminium, the value of C, especially heat capacity, is 903 Joules per kg Kelvin, or kg degrees Celsius, and the density of aluminum is 2700 kg per cubic meter.

If you remember what we have discussed earlier, the volume-to-surface ratio, assuming that it is losing heat from both the sides, that is the, this side as well as this side, it is nothing but delta by 2, where delta is the thickness of the foil, ok, of the foil thickness is the volume to surface area ratio. What I would like to calculate, there are three things which I am going to do:

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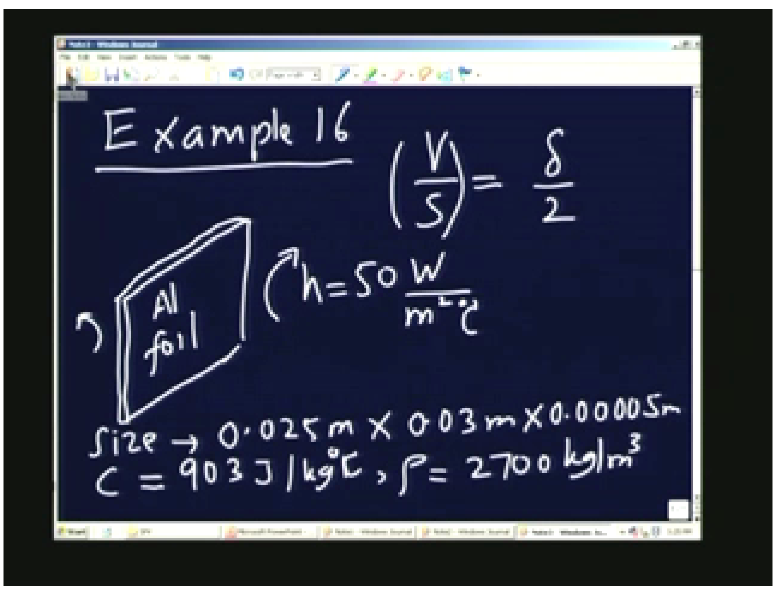

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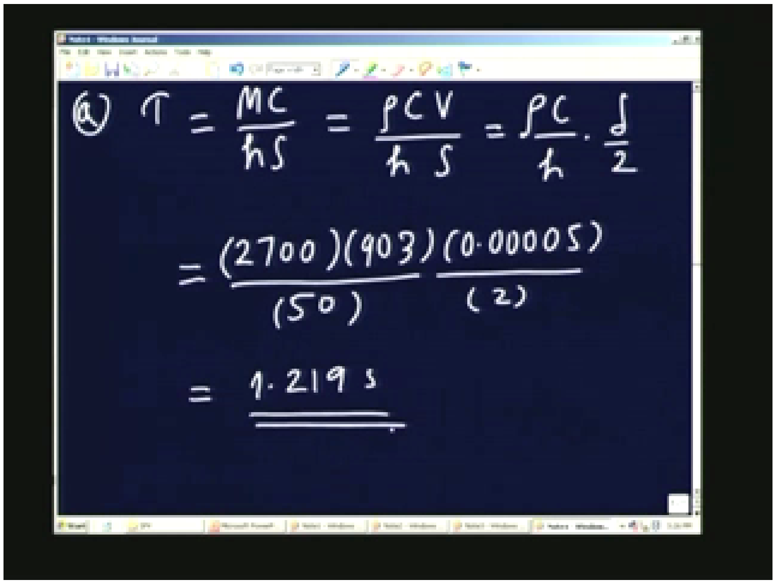

One is, I will calculate first the time constant, tau, which is easily calculated. By using the formula tau is equal to the mass specific heat product divided by the area heat transfer coefficient product, and this is written as rho CV divided by hS, and V by S, we have just now seen, is rho delta by 2, therefore, this becomes rho C by h into delta by 2. I can plug in all the values. rho is 2700, C is 903, h is 50, and delta is 0.00005 divided by 2. So the time constant comes to 1 0.219 seconds, so the time constant of this particular system is 1 0.219 seconds.

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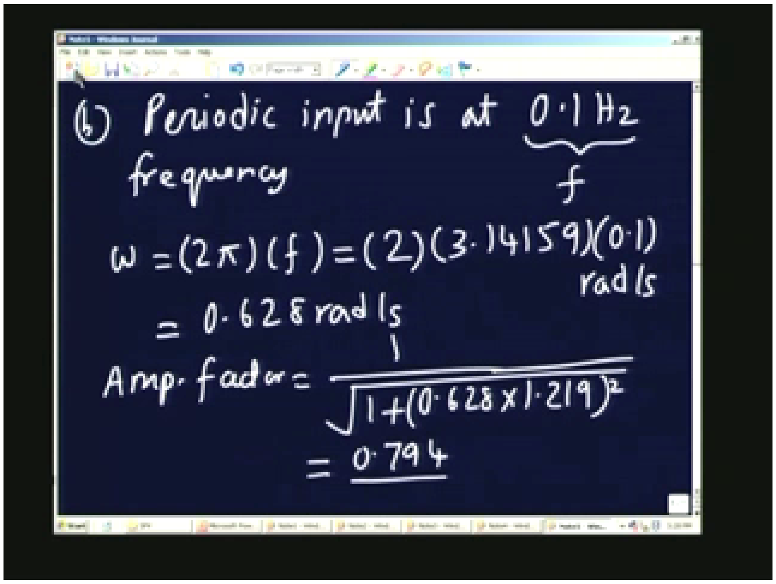

Now let's look at the typical case, where a periodic input is at point 1 hertz frequency. You would like to find out what is the amplitude reduction, and also you would like to find out what is the time lag, what is the phase lag, so it is very simple to do. So if you remember f, it is nothing but the frequency, omega will be given by 2 pi times f, that will be 2 into 3.14159 multiplied by 0.1. This will be radians per second, and this works out to 0.628 radians per second. So we can find out the omega tau product, and the amplitude factor is nothing but 1 over square root of 1 plus omega tau whole squared, 0.628 multiplied by 1.219 whole squared whole under the square root sign. This comes to 0.794. So, only 79 percent of the input amplitude is registered by the sensor.

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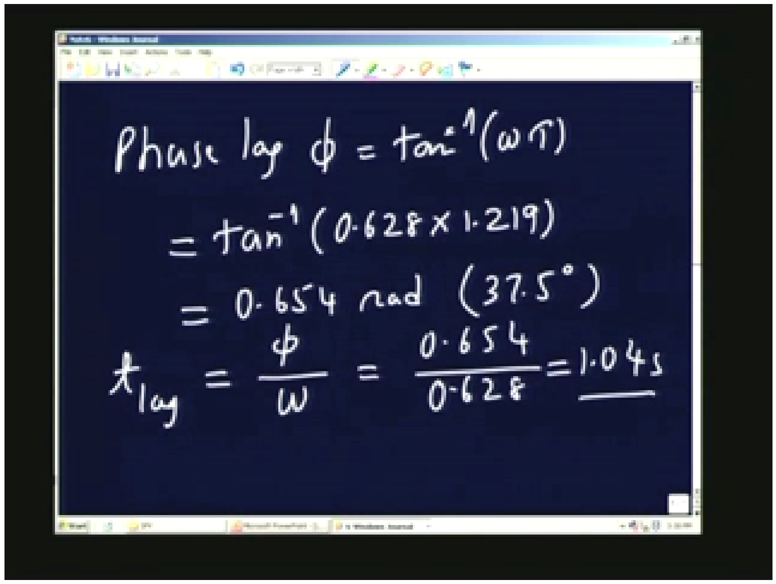

We can also look at the phase lag, pi equal to tan inverse omega tau, which is equal to tan inverse omega is 0.628 multiplied by 1.219 which will come out, turn out to be 0.654 radians, and in terms of angles, this will turn out to be 37.5 degrees. And as I indicated earlier, the time lag is nothing but phase lag divided by omega. So that will be 0.654, and you should remember that we have to use the phase lag in radians not in degrees for the calculation of the lag in time, this will be divided by 0.628, this will give about 1.04 seconds. That means that the maximum indicated by the sensor will be after 1 second, almost one second, a little more than 1 second after the maximum actually occurs in the sensor's temperature which is varying. There is one second lag between the two.

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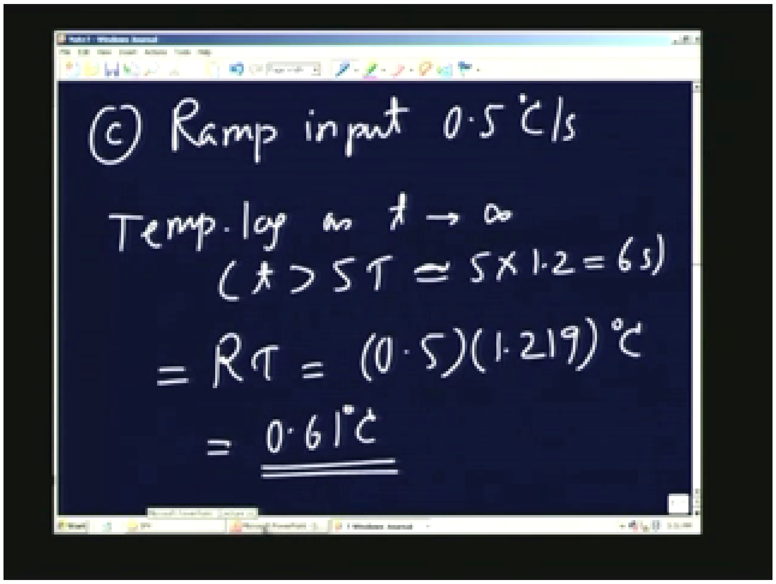

And the third case I am going to consider is, if there is a ramp input that means that the temperature T infinity is varying linearly with respect to time at point 5 degrees Celsius per second. What is the lag in the temperature? So we will say that it is the temperature lag as t tends to infinity. In this case, t must be greater than about 5 tau, let us say, and tau is almost 1.2 seconds. This will be equal to 5 into 1.2 roughly, and this about 6 seconds. For time greater than 6 seconds, you can expect the temperature to lag by an amount equal to R into tau. R is 0.5 multiplied by 1.219. This is so many degrees Celsius, and this comes to 0.61 degrees Celsius. That means that there is 0.61 degrees difference between the temperature indicated by the sensor and the actual temperature of the ambient, which is varying with respect to time, according to a ramp input. So this sort of completes our discussion on the transient temperature measurement.

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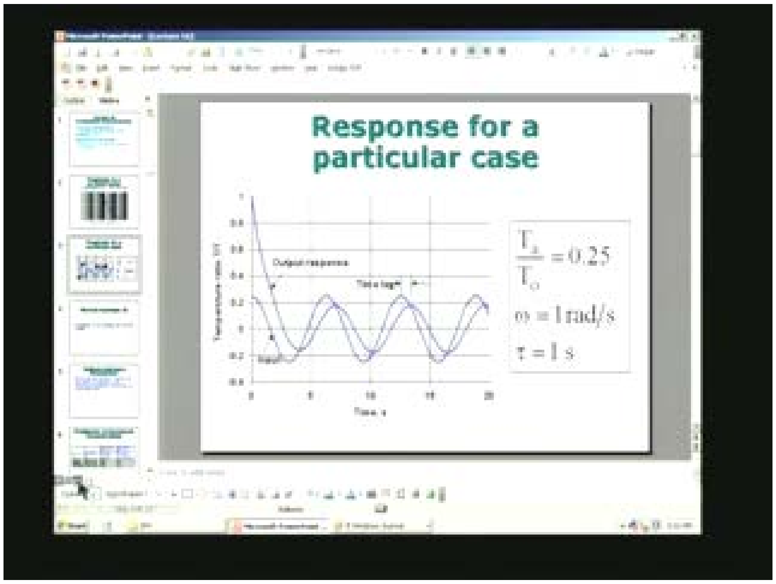

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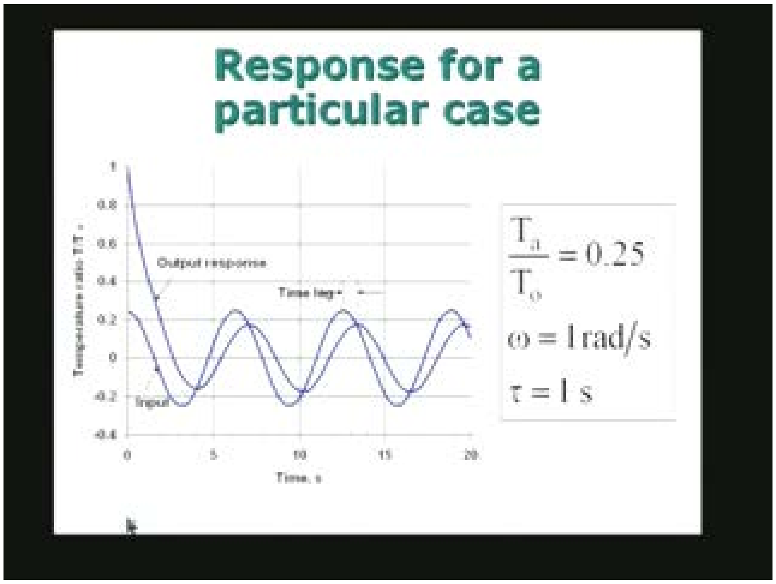

And then we are ready to go back to our main theme of measuring temperature.

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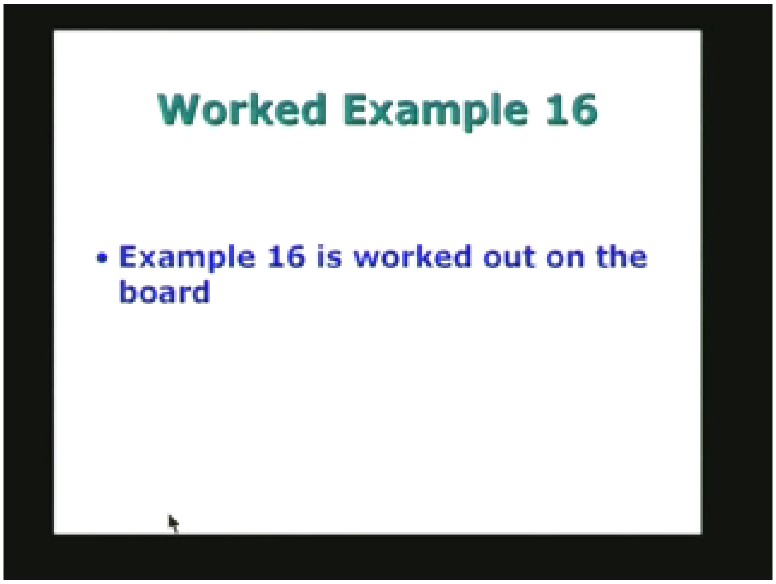

In this case, I am going to look at the resistance thermometer.

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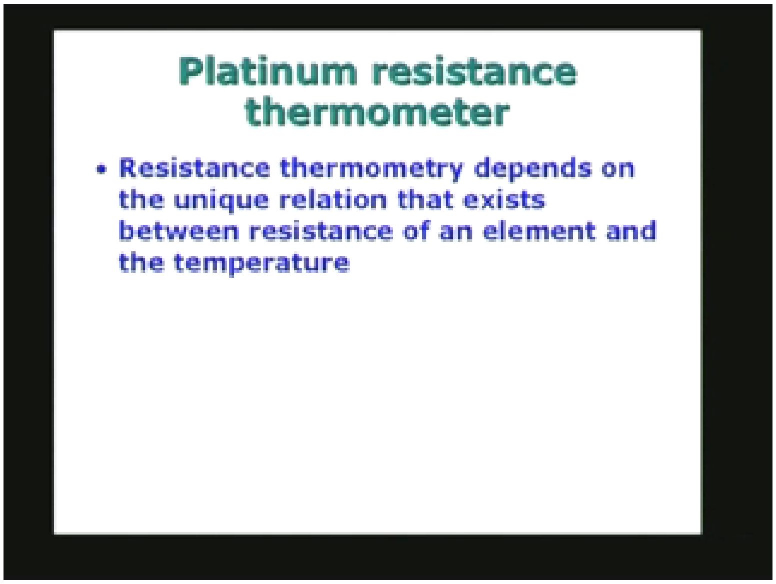

Two thermometers I am going to look at the resistance thermometry at resistance thermometer, platinum resistance thermometer in particular, and then I will also look at the case of thermistors, and we will see where these two things, these two types of sensors are useful. And we will also look at, in detail, how to use them and what are the characteristics of each, and so on. So the basic idea of resistance thermometry is, goes back to what we said several lectures ago, the idea is to see if there is a definite relationship between the temperature and the property of the resistance thermometer which is the resistance, electrical resistance. If there is a definite one to one relationship between temperature and the resistance, the resistance can be used as a thermometric property, and the resistance thermometer can be used as an instrument to measure the temperature. So the resistance thermometry depends on the unique relation that exists between resistance of an element and the temperature.

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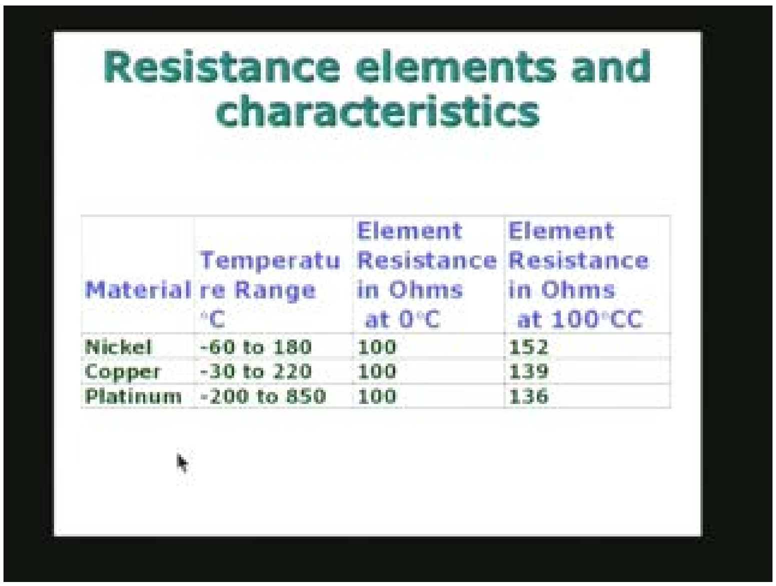

Now let’s look at some of the resistance elements which are used in practice. I have indicated three of them: nickel, then copper, and platinum. Of course, platinum is the most common, others are not that common, but they are also useful and used. So we have three different materials: nickel, the temperature range in which it can be used is minus 60 to 180 in degrees Celsius, and the characteristics given by specifying the resistance of the element at 0 degrees Celsius and 100 degrees Celsius. It is only 1C here, so the nickel can be used between minus 60 and 180. The value is 100 at 0 degrees Celsius, then the value at 100 degrees Celsius is 152. There must, therefore, almost 50% change in the resistance between 0 and 100 degrees.

Copper is useful between minus 30 and 220. It is 100 and 139. Of course, this is less sensitive than this one because this variation of nickel is more than in the case of copper. And in the case of platinum it is useful from minus 200 to plus 850 degrees Celsius, and the variation of the resistance is 100 to 136 for about 36% variation between 0 degrees and 100 degrees. So this is the steam, the ice point, and the steam point. So I am talking about the ice point, and the steam point, and the variation of the resistance is something like 100 to 136.

The main difference between resistance thermometers and thermistors, which you will see later, is that this ratio, this variation of resistance, is very small in the case of resistance thermometers, and the resistance increases with temperature. In the case of thermistors,it is going to decrease with respect to temperature, and also decrease by a larger fraction, and therefore, thermistors are more sensitive, but they have a smaller range compared to the resistance thermometers, the case being we are talking mostly about the platinum resistance thermometer.

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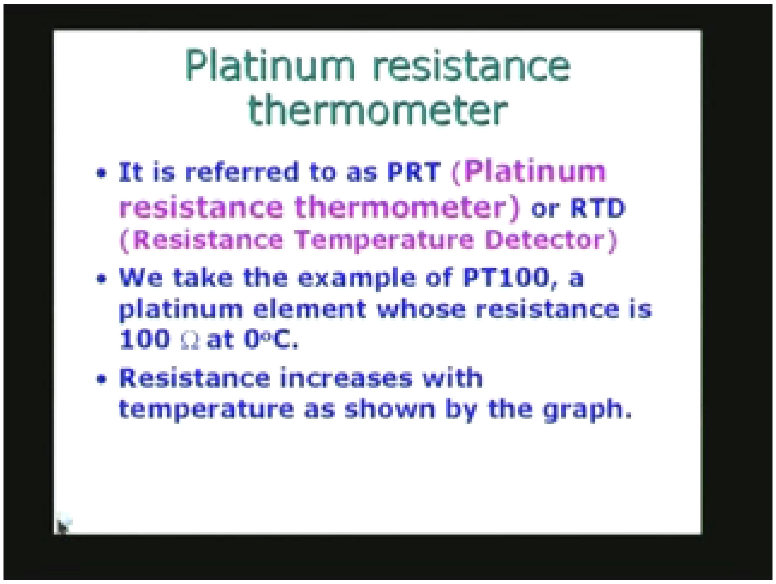

So the platinum resistance thermometer is also referred to as PRT. PRT is nothing but P is for platinum, R for resistance, T for temperature. Or it is also referred to as RTD. RTD is simply resistance temperature detector. RTD can refer to any resistance temperature detector, but PRT will refer to only a platinum resistance thermometer. So these terminologies are used very extensively in the literature, in books, and so on. So you should not be confused if you see PRT. You know what it is, RTD? RTD is a very general term used for resistance temperature detector. We shall take the example of PT100.

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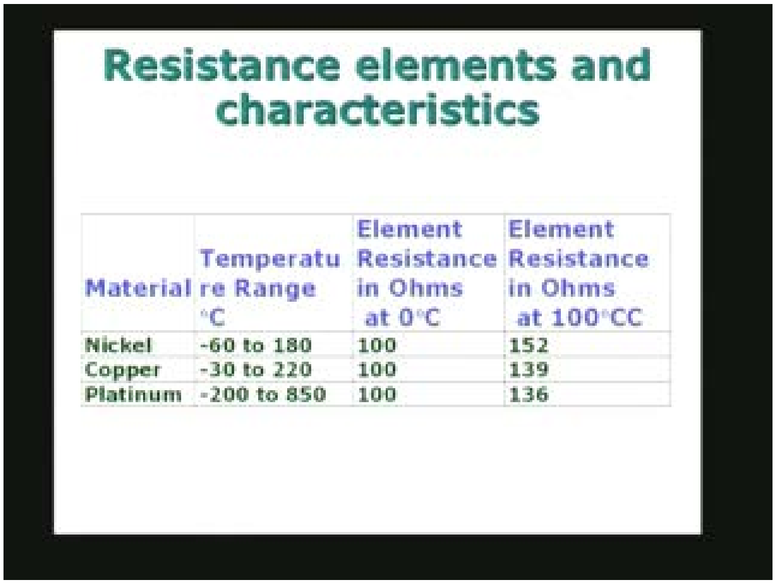

In fact, if you remember, if you look at the previous case, I took the element resistance at 0 degree Celsius, 100 Ohms, so this platinum having a resistance of equal to 100 Ohms at 0 degree Celsius is called the plati PT100. PT100 means platinum resistance thermometer with a resistance equal to 100 ohms at 0 degree Celsius. That's the ice point. This is the standard element which is used in practice, PT100. Of course, you can also have PT with a different value of resistance. That means the number of terms or the length of the wire and the cross-section of the wire has to be determined, taken suitably so that you can get different values for the resistance.

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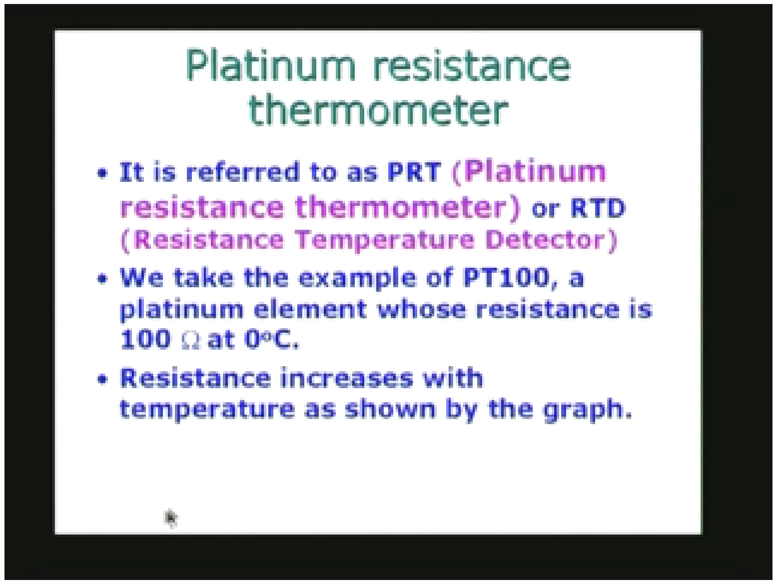

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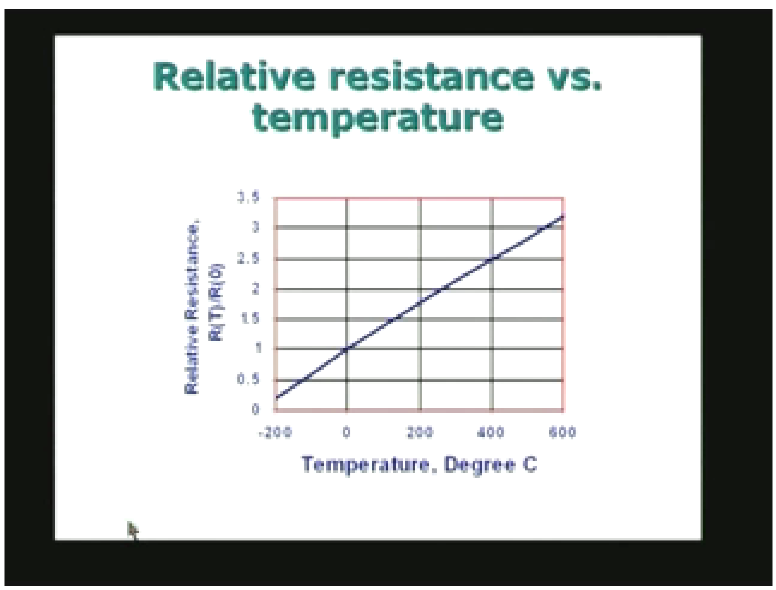

We also know that as we explained in the earlier case, resistance increases with temperature and this is shown graphically in the next graph. Thus, I have used a terminology here known as the relative resistance. Relative resistance is nothing but the resistance at temperature T divided by resistance at the 0 degree Celsius. So this is called the relative resistance, and you can see the relative resistance is equal to 1 at 0 degrees, because that is the reference anyway, and for temperatures below the room temperature or below the ice point, the ratio is smaller than 1. That means, the resistance is decreasing with respect to when you decrease the temperature, and it is greater than 1 for temperatures above the ice point. So it increases, and the way I have plotted, if you are not very careful, you will think it is almost like a straight line, but there is some straight nonlinearity. So the idea is to see how to characterize this nonlinearity, how to look, how to use the platinum resistance thermometer for measurement of temperature. This is what is going to be the discussion from now onwards.

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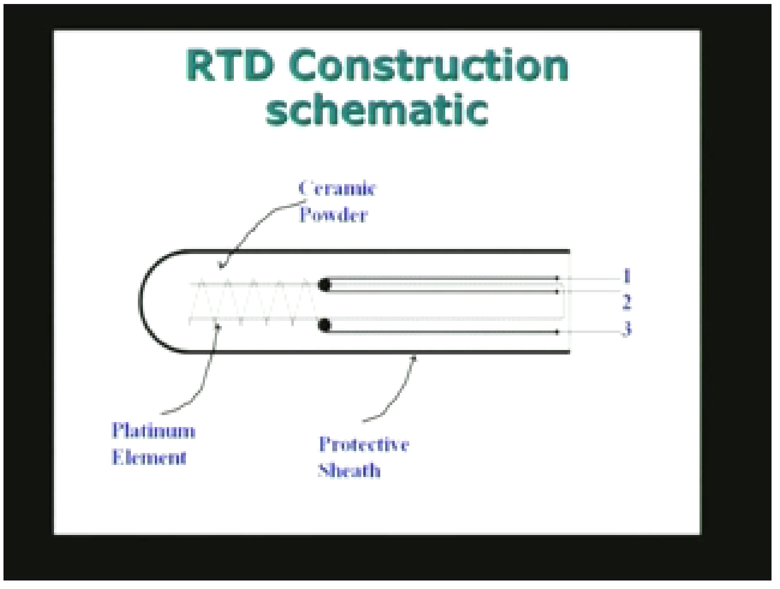

The construction details of an RTD are shown in the schematic way. In this particular, case we usually take it in the form of an element, resistance element, honed over some kind of mica or some material like that, and the platinum element is connected to the outside because we need to take the leads out of that to make the measurement of resistance. And in this case I have shown a three-wire arrangement, that means that the two terminals coming from this point, that is one and two and there is three.

Sometimes, we also have four terminals, or four wire RTD, in which case there will be one more wire coming out from this side, and I will explain later why we have the three-line, three-lead arrangement and the four-lead arrangement. That will become clear when we talk more about the method of measurement of the resistance itself. So the important requirement is the resistance thermometer should be, the resistance element must be allowed to freely expand, and it should not be confined, and therefore, it should not experience any stresses during the operation of the element, and therefore, it is allowed to freely expand along this direction. It is loosely put inside a sheath material which is the protective sheath. It may be of either metal or an alloy, and inside that you have a ceramic powder which is going to be loosely filled between the element and the sheath, and the platinum element is itself protected from the outside by the sheath itself, and the three, either the three leads or the four leads, are taken out as shown here. This is only a schematic. The entire dimension of this RTD construction is may be only a few centimeters in length and may be a few millimeters in diameter. Here I have shown in a very enlarged view just to make the things clear, and also I have shown very few turns. Actually there will be a large number of turns to achieve a 100 ohm resistance at 0 degree Celsius for a PT100. So let us look at the way we are going to characterize, the PRT.

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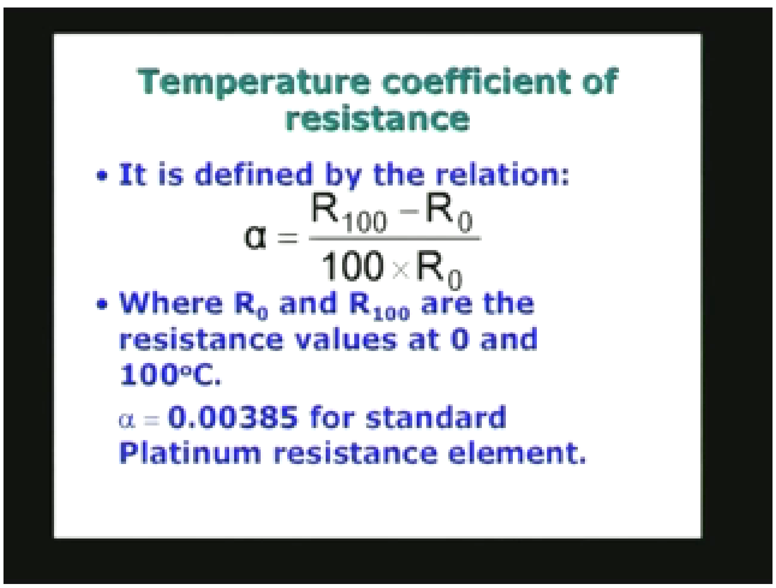

So we use some terms which are useful in describing the platinum resistance thermometer. One of them is the alpha value, it is also called the temperature coefficient of resistance. So it is defined by the following relation R of the platinum element at 100 degrees at the ice point, at the, I am sorry, at the steam point, minus R at the ice point, divided by, this is actually 100, it is nothing but 100 minus 0. I am not showing it as 100 minus 0, just 100. It is nothing but the temperature of the ice steam point minus the ice point temperature divided by R_0. R_0 is the resistance at 0 degree Celsius.

So this alpha is an important characteristic of platinum resistance thermometer. The, internationally, the purity of the therma platinum element, is specified, and the alpha value which should characterize that is also specified by giving a value equal to alpha equal to 0.00385 for standard platinum resistance element. This is very important, and I have indicated R_0 and R_{100} are the resistance values at 0 and 100 degrees Celsius at the ice point and the steam point.

If one were to actually use a platinum resistance thermometer, one would like to find out whether it is according to this alpha value is correct or not by actually measuring the R_{100} and R_0, and then calculating the alpha value in the laboratory and finding out whether it is agreeing within reasonable limits.

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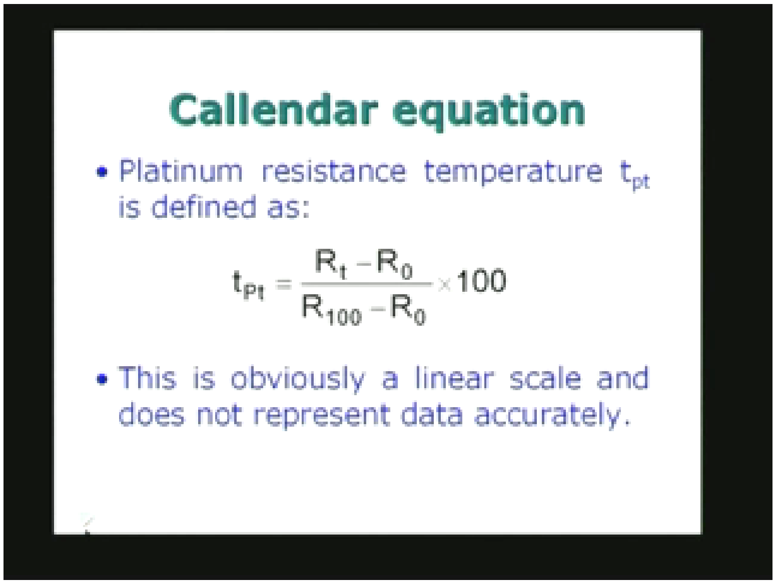

In fact, the manufacturer will do that and give you a guarantee that it is according to this particular value, so you can take it if it is a reputed manufacturer and what I will do is now is to slightly digress, and if you go back to the response relative resistance with temperature, if you look at this curve, it is almost straight line, but really not.

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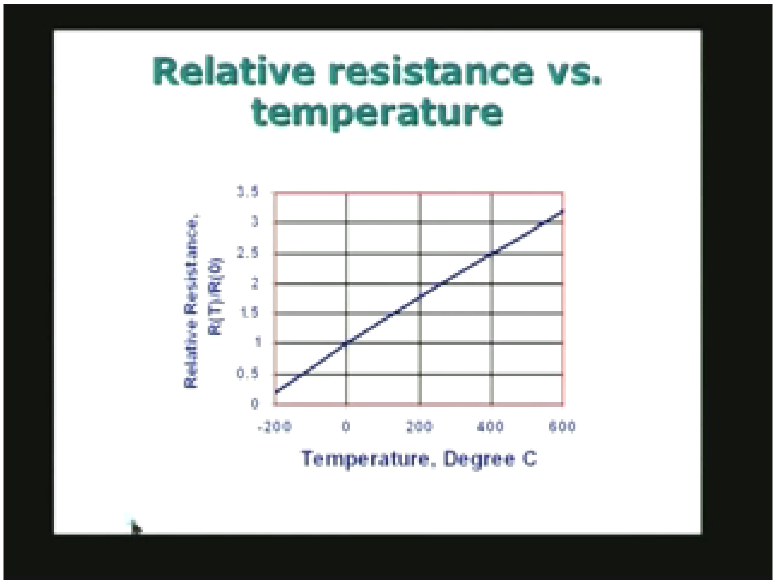

There is slight nonlinearity, so what we would like to do is to see how to take care of this nonlinearity, and this aspect I will cover on the board.

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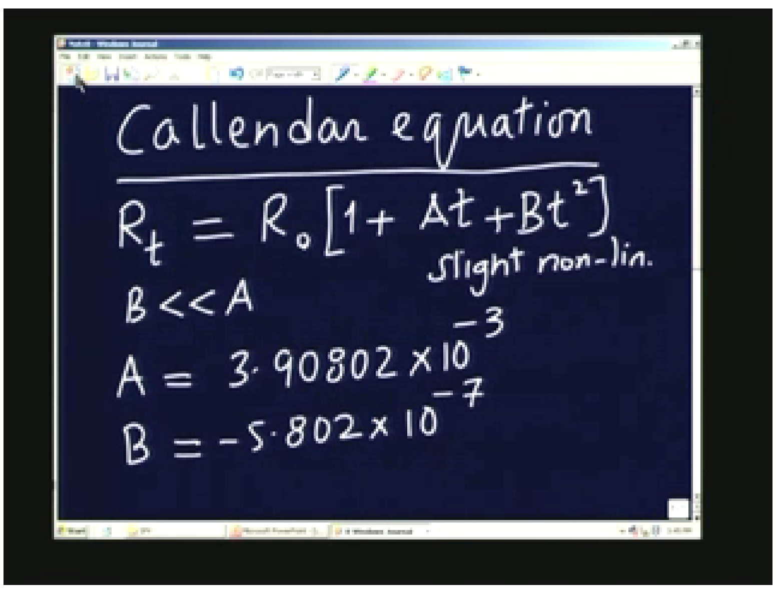

I will give a little discussion on that and I will indicate how we are going to take into account this nonlinearity by looking at what is called the Callendar equation. I will derive this equation in a very simple fashion. I will indicate the steps. This is the name of the person. Callendar was a Scientist who worked with platinum resistance thermometers in the early days of the development, and essentially, the Callendar equation is basically based on his recommendations and understanding of what was going on.

So I said that the resistance of the thermometer, I will say R_t, is the resistance with therma platinum resistance thermometer at any temperature t, can be written as R_0 into 1 plus A times t, plus a small nonlinear term. I will say it is equal to Bt squared, where B is very small compared to A. This is because slight nonlinearity, and in fact, for the internationally accepted value for A and B are given here. The value of A is equal to 3.90802 into 10 to the power of minus 3, and B value is minus 5.802 into 10 to the power of minus 7, and obviously B is very small compared to a, because you can see this 10 to the power of minus 3, 10 to the power of minus 7. That is the big difference.

So you will appreciate now that when we use the resistance thermometer over a wide range of temperatures the non linearity is going to become important because B is going to be multiplied by the square of the temperature, and the square of the temperature, if it is 100, for example, it becomes 10 to the power of 4. If it is more than 100 it becomes even greater than that, and therefore, I am multiplying this actually by a factor of 10 to the power of four to see that it is going to become more or less as big as this term. So at high temperatures, when you are measuring using the PRT, over a rest over an extended range of temperatures to be measured it is necessary to take into account the nonlinearity. But how are we going to take nonlinearity into account? That is what we are going to look at now.

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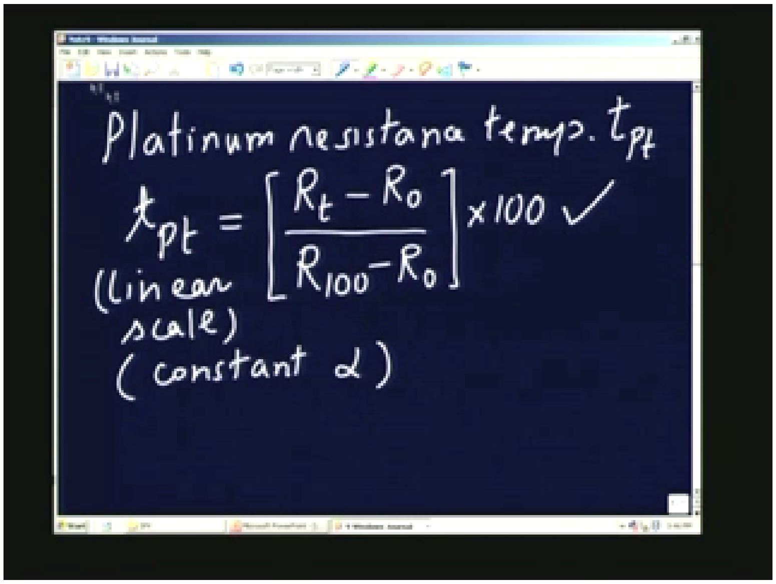

What we will do is we will define what is called a platinum resistance thermometer scale, platinum resistance temperature which we call as t subscript pt. This I will define as a linear scale. This is actually a linear scale. It doesn't take into account the nonlinearity, so I will define it simply as R_t minus R_0 divided by R_{100} minus R_0 multiplied by 100. This is assuming a constant alpha. You remember the alpha how it was defined. I am using that definition to write the temperature as t subscript pt. Now let us look at the nonlinear case.

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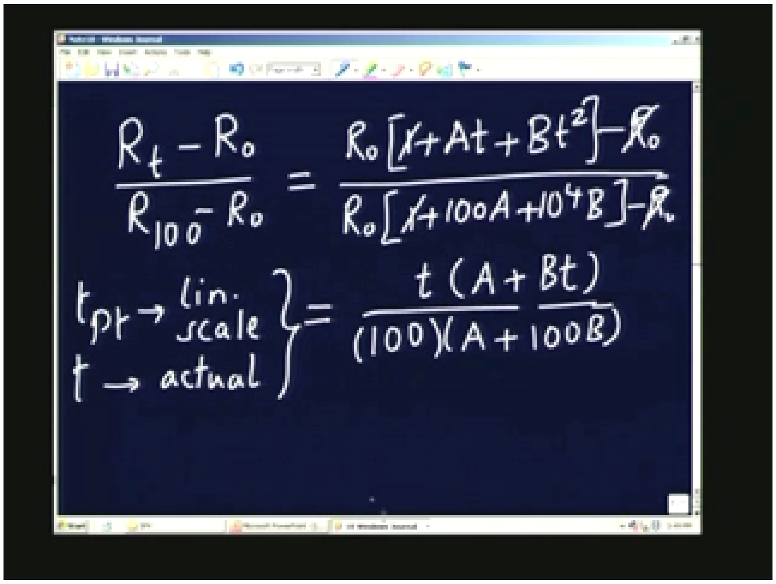

So if I write R_t minus R_0 divided by R_{100} minus R_0, you know that it is equal to R_0 into 1 plus At plus Bt squared. This t is not the platinum resistance temperature. It is the actual temperature I am talking about. So, when I say t_{pt}, this is the platinum resistance temperature. t is the actual temperature, this is the actual, this is the temperature according to the linear scale. This is the linear scale minus R_0 divided by R_{100}.

In fact, I can do the following. I can write the R_{100} as R_0 into 1 plus 100t plus 10 to the power of 4 t squared minus R_0, and this can be further written as, this R_0 will cancel with R_0, and similarly, this will cancel with this. This becomes R_0 into t. I can take it outside into A plus Bt divided by 100. I can take outside into t plus 100. I think I left out something here, 100A plus 10 to the power of 4B. So I take 100 outside. This becomes (A plus 100 B), (R_t minus R_0 divided by R_{100} minus R_0) is equal to R_0 will also cancel. t into (A plus Bt) divided by (100) into (A plus 100B). So why did I write like this? It is because, actually I want to find out t, and what I have written on the left hand side, if you go back to the previous, you can see that t_{pt} is nothing but R_t minus R_0 divided by R_{100} minus R_0 into 100, and what I can do now is to substitute that here for the left hand side R_t minus R_0 divided by R_{100} minus R_0 into 100 is t_{pt}.

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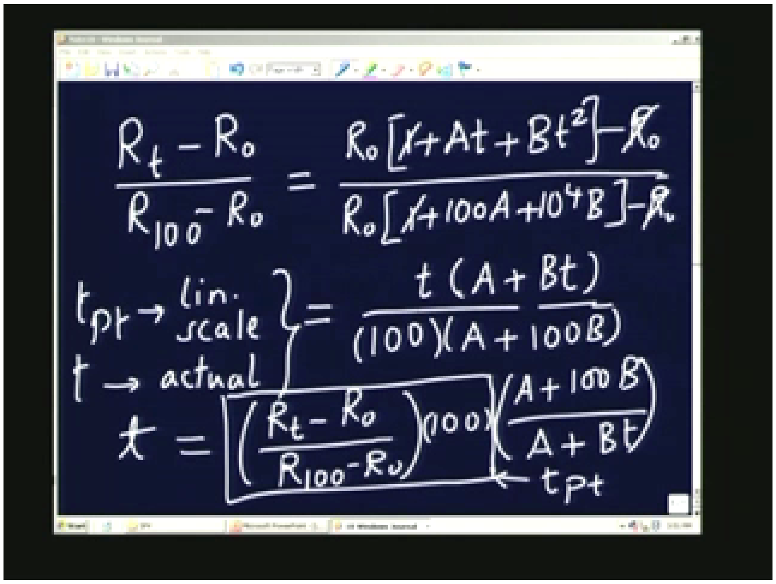

So I can write it here as t_{pt} divided by 100. That's what we are going to do, that one, and in fact, what I want to do is to write in the following form, I want to write for t so t will be this t here. I will cross-multiply and so on, this will become (R_t minus R_0 divided by R_{100} minus R_0) multiplied by (100) (A plus 100B divided by A plus Bt. This factor, first factor, is actually t_{pt}. This is your t_{pt}, so the t is equal to t_{pt} multiplied by some factor which I want to find out what is that factor in terms of the characteristics of the platinum resistance thermometer that is next in terms of A and B. So I can rewrite that as t_{pt}. That is the first factor, and multiplied by, if I go back and do a little manipulation, you have A plus 100B divided by A plus Bt.

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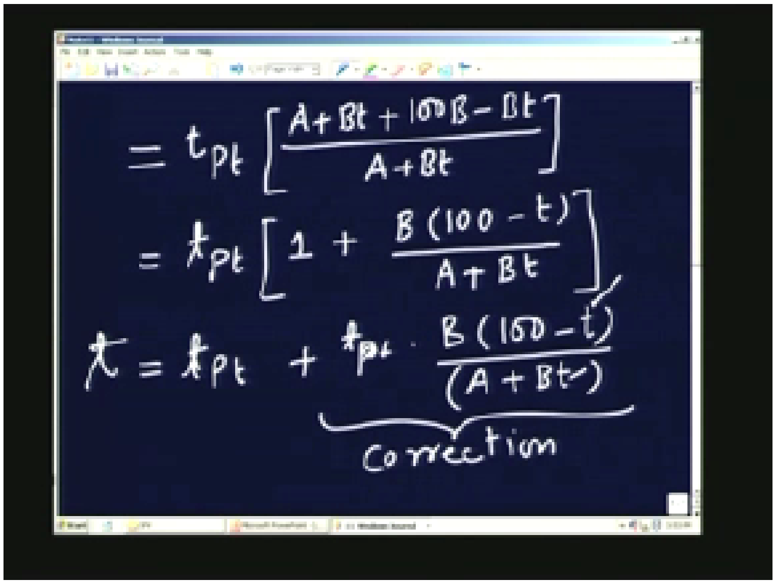

Therefore, I will add plus Bt to the numerator and also subtract minus Bt from the numerator so that I can write it in a slightly different form. So this becomes [A plus Bt plus 100B minus Bt divided by A plus Bt] and this gives you 1 plus B into(100 minus t) divided by(A plus Bt). So I can say this is t_{pt} plus t_{pt} multiplied by (100 minus t) divided by (A plus Bt). This will be your correction because the left hand side is nothing but the temperature.

Therefore, now if you look at the second term you will see that it contains, let me write it back, it contains the temperature again. There is a temperature again. So what we will do is we will assume that this temperature will be approximated by the platinum resistance thermometer scale which is indicated by t_{pt}, number 1. And number 2, this is (A plus Bt). We will assume that in the denominator this term is not as important as this term, therefore, we will neglect this term Bt. If you do that you will get a very simple expression for the resistance nonlinearity.

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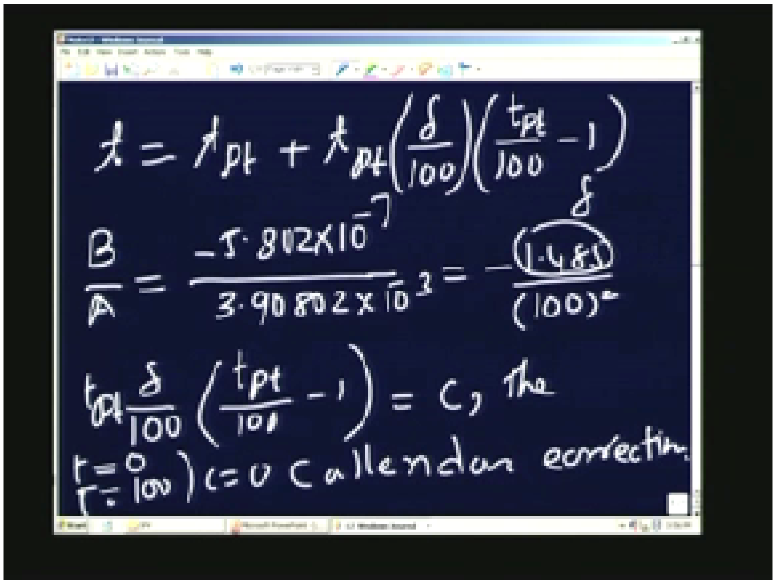

So we can write it as t_{pt} plus t_{pt} multiplied by the value of B by A, if you remember is minus 5.802 into 10 to the power of minus 7 divided by 3.90802 into 10 to the power of minus 3. This gives you a value of 1.485 into 10 to the power of 4 minus 4. I will write it as 100 squared. And why do I do that?

Let me go back to the previous one to see that if I remove A outside, if I take this A outside and this becomes B by A here, and A will come out, this B by A is what I get here, and this B by A is going to be this number for the resistance thermometer 1.485 divided by 100 squared. This I am going to write as delta. The symbol given for 1.485 is delta, therefore, delta divided by 100 squared is B by A, and if I now go back to that expression and write the correction, the correction will become simply delta by 100.

You can verify that, t by 100 minus 1, and I am going to approximate it by t_{pt} by 100 minus 1. Hence, this is the factor, delta by 100 multiplied by t_{pt} multiplied by t_{pt} by 100 minus 1 is called the C, the Callendar correction, and we will notice immediately that when the temperature is 0 this is multiplied by t_{pt}. Temperature is 0, this will become 0. Correction is 0, at temperature equal to 0. Again, if temperature equal to 100, you see that 100 divided by 100 minus 1, this becomes 0. Therefore, the correction is 0 at the, at t equal to 0, and t equal to 100, C equal to 0. Actually, what I have done is to make a plot of this, the so-called Callendar correction.

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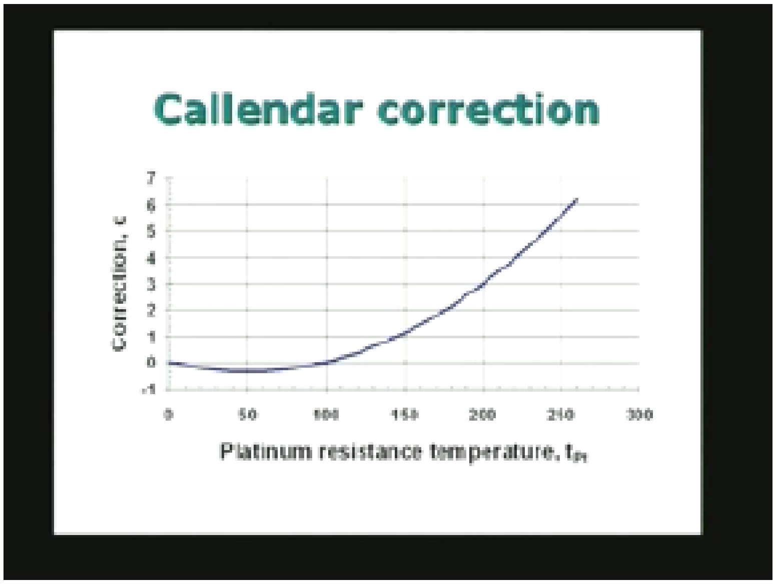

You see that the Callendar correction is 0 at the ice point, at t equal to 0, it's also 0 here, and it is also 0 at 100 degrees. That means, at these two points, t is equal to 0 and t is equal to 100 degrees, both the resistance thermometer, the platinum resistance scale, and the actual temperature coincide, and for all other temperatures, there is a correction, and the correction is somewhat small between 0 and 100, the maximum being about minus point 4 or something like that.

And as you go to higher and higher temperatures (I have shown up to about 250 here) the temperature correction could be as big as 6 degrees. What is the advantage of this method of calculation? So what I will be doing is I will be calculating t_{pt} using the linear relationship, and then calculate the corresponding value of the correction using the linear temperature which has been calculated and putting into the equation for C, and then I will be able to calculate the temperature without much difficulty. So the actual temperature is estimated by using the correction based on the linear platinum resistance thermometer scale. That is the advantage of the Callendar equation.

So the Callendar equation is very useful, because without much of an arithmetic or algebraic effort I will be able to calculate the actual temperature from the linear t_{pt} scale. Let's look at the details of how we are going to make the measurement resistance. This is the important thing which we have to discuss because we have the temperature detector and it is going to be changing resistance. Resistance is going to change with the temperature at which it is going to be, to which it is going to be exposed, and I am indicating the RTD here.

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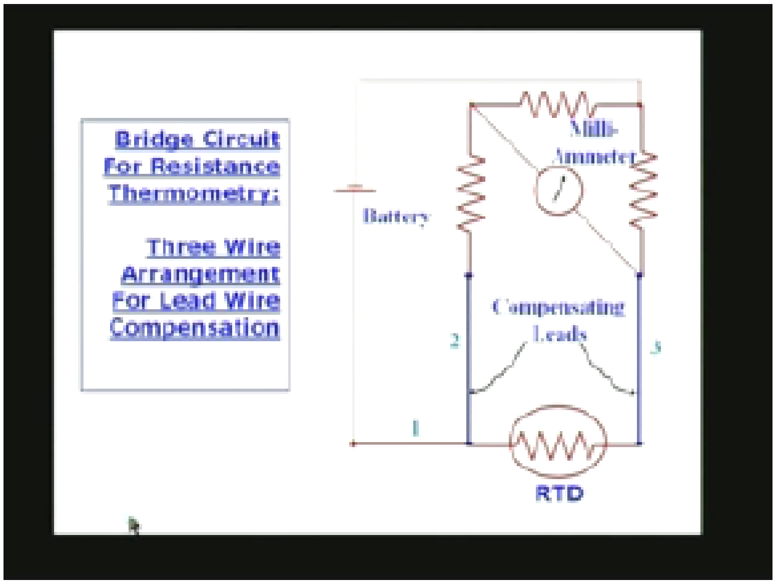

In this circuit, the RTD is in the one arm of the bridge, and there are two resistances in one of the arms, and the other arm I have got a resistance here which can be varied, and the RTD, there are two ways of doing it. I will just indicate what I have shown here, a three lead RTD. That means, there are three leads: one lead here; lead number 2; lead number 3. What is the advantage of using a three lead or three leads for the correct connection? This point is connected to the battery, and this opposite corner is connected to the battery, and therefore, there are two arms of the bridge, one arm consisting of this resistance and the RTD plus this lead wire, and the other arm consists of this resistance, this variable resistance, and the lead wire.

Therefore, the lead wire is now going to be on both the arms, and therefore, the lead wire is going to compensate for each other these two; 2 and 3, they have certain resistance of their own, and these resistances are going to be in the two arms of the bridge, and therefore, they are going to cancel out, and the lead number 1 is going to help us in connecting it to the external world, external battery, which is shown here. So the 2 and 3 are referred to as the compensating leads because they are going to be in the two sides, two arms of the bridge of the measurement.

So if I vary the resistance here, so that there is a balance, because the RTD resistance has changed, the change in the resistance indicated by this one is going to be, if for example, I choose these two resistances equal in magnitude, these two are equal in magnitude, at some particular point, let us say 0 degrees Celsius, that means, when RTD is at 0 degrees, if I choose such that these two are equal, then I will have to choose this equal to 100 if PRT, if the RTD is the PRT 100, PT100, I am going to have 100 ohms here.

Now when the temperature changes, this resistance increases, I have to increase this resistance so that the balance is restored because it is the resistance changes this resistance arm, this arm, the ratio will be different from this arm, and to compensate for that I have to change this by an amount equal to the change in the resistance RTD, and therefore, I have to restore the balance, and I can measure the resistance and immediately calculate the t_{pt} by using the linear relationship, and use the correction by Callendar to calculate the other one, the second arrangement, which is even better than the first arrangement, is where we have a four wire arrangement for lead wire compensation, and there are four lead wires I have shown here, one, two, three, and four.

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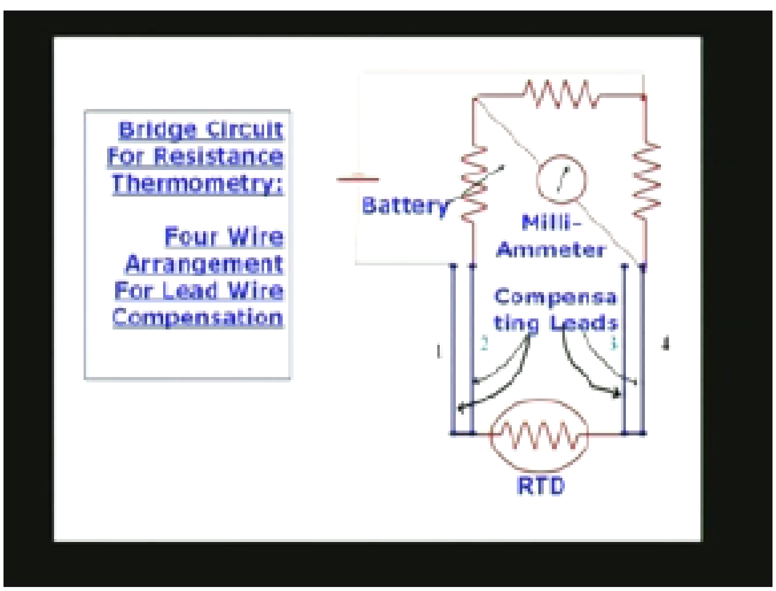

These are the compensating leads, four leads, are there. RTD is connected, the rest of the connections are the same, the battery, and the variable resistance which is in this arm, the two other resistances, and so on, exactly the same. Now let me explain briefly the reason for four leads, and let us discuss it on the board.

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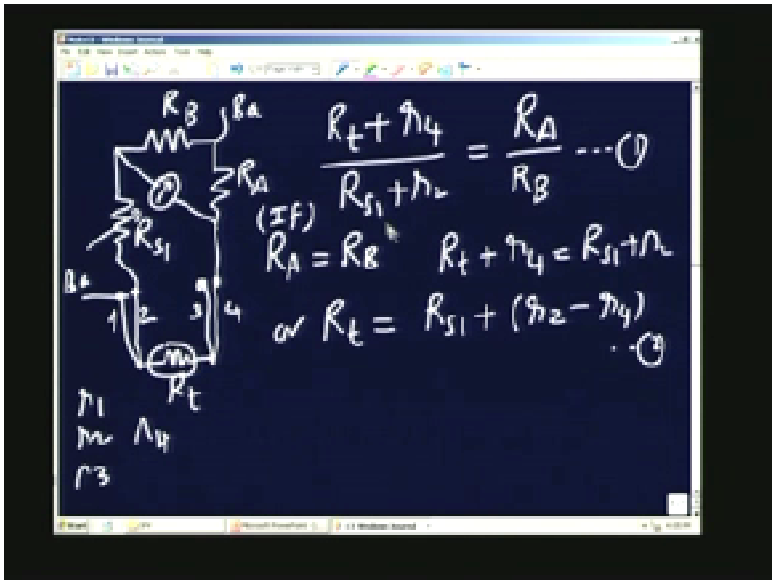

I am going to take a look at the arrangement. So in the first case, this is R_B. Let me just call that as R_A and R_B, and the milliammeter is here to indicate the balance. I am just drawing a simple sketch corresponding to what we had, is the resistance which is going to be varied, R_S, we will call it, and here we have the RTD with the four lead wires, the PRT. Let me indicate them like this. So I am going to connect this to this.

We will label them as 1, 2, 3, and 4. This I am going to connect here. It is connected to four, and this is connected to two. Three is floating, not connected in the first case, and this is connected to the battery, and this corner is also connected to the battery as we have seen earlier. This goes to the battery. I have already indicated, this corner gets to the battery. So let us assume that the resistance of the lead wires, r_1, r_2, r_3, and r_4. So let us call this as R_t, the resistance of the temperature detector, and this will be, we will call it as R_{S1} the resistance of the, in the first case. Now I have obtained the balance by adjusting R_S suitably.

Therefore, you see that if R_B and R_A are constant, so we know that R_t, that is this R_t plus r_4, this resistance divided by R_{S1} plus r_2. So, I have taken this resistance,this resistance, the ratio of these two, but it is same as R_A by R_B.

R_A by R_B equal to this resistance divided by this resistance. We will call this as equation number 1. If we choose R_A equal to R_B, if, which can always be done, I will have R_t plus r_4 equal to R_{S1} plus r_2, or R_t equal to R_{S1} plus r_2 minus r_4. We can call this equation number 2.

Now let us look at the second arrangement, where what I am going to do is, I will have the same, these two resistances, no change, and here I am going to change the terminals. This is R_t. I will allow the r_1 to float. I am going to connect r_2 here, and on the other side this will be R_{S2} in the second arrangement, and this will be connected to r_4, and the battery will be connected through r_3. This is a battery; this is also a battery.

What I have done is, I have simply changed 2 and 4 and the connections now, r_3 which was floating in the first case, is going to be used for the connection to the battery, and by an arrangement similar, the argument similar to what we did in the previous case, we can see that R_t in this case is R_{S2} plus r_4 minus r_2.

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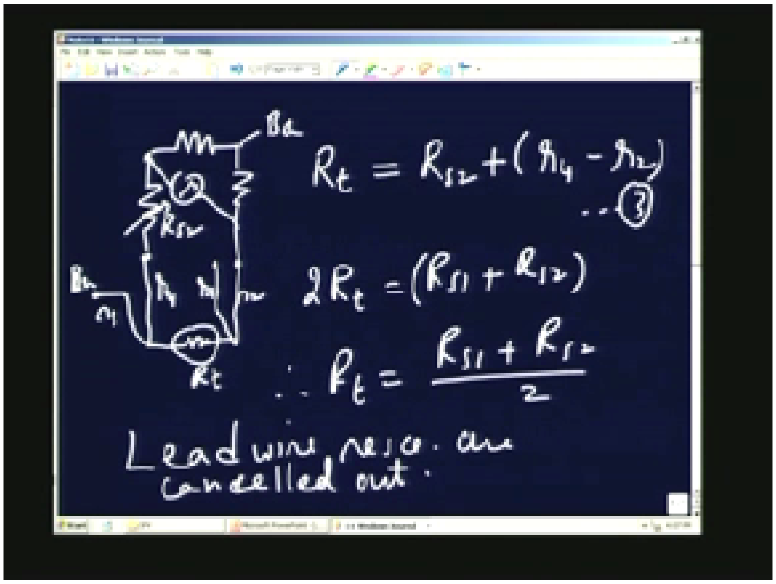

So if you go back to the previous one, r_2 minus r_4 plus R_{S1} equal to R_t. So the second case is equation 3. So, if I take the sum of these two, R_t equal to R_{S1} plus R_{S2}, these two r_4 minus r_2 here, and r_2 minus r_4 are going to cancel, and therefore, R_t equal to R_{S1} plus R_{S2} divided by 2 means that the lead wire resistance is compensated or cancelled out which means that the resistance of the lead wires is going to be compensated. We will continue in the next lecture. Thank you.