Mechanical Measurements and Metrology

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Module - 2
Lecture - 15

Measurement of Transient Temperature and Resistance Thermometry

So this will be lecture number 15 on mechanical measurements. In the previous lecture we were looking at the errors in measurement of temperature under various circumstances. Now we are going to look at the measurement of temperatures which vary with time.

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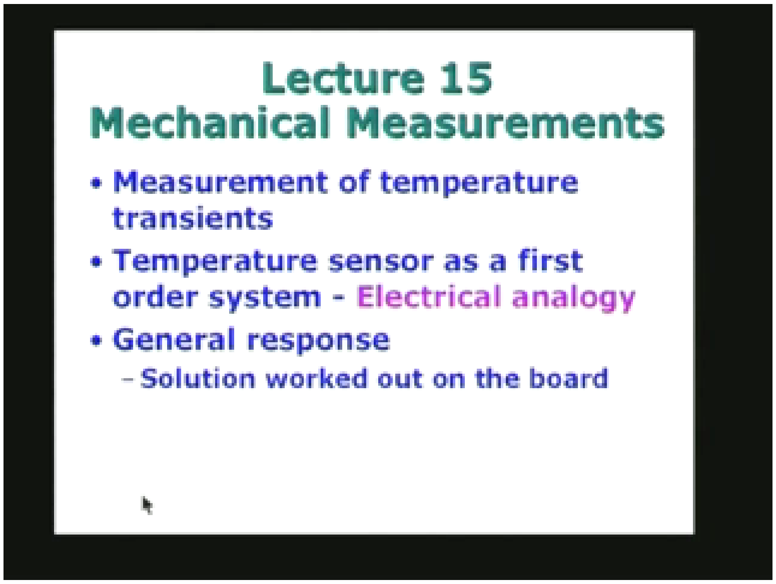

So, these are supposed to be so called temperature transients or measurements of temperature which varies with respect to time. What we will be doing in this lecture is to look at a temperature sensor and describe it mathematically in terms of a first order system behavior. And we will also draw an analogy with electrical circuit theory and we look at the similarities between the source systems. We will look at the governing equation, we will derive it and then workout what we call as a general response for any variation of the temperature. And after doing that, I am going to take three particular cases.

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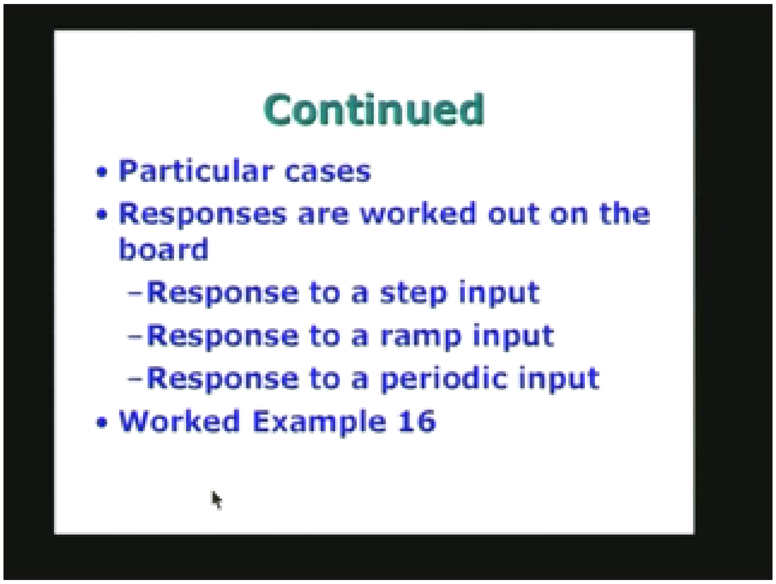

The first one will be response of the sensor to a step input or step change in temperature. It will become clearer when we look in to the details. Then response to a ramp input where the temperature is varying linearly with respect to time and response to a periodic input, a pure sinusoidal or pure co sinusoidal variation of the temperature with respect to time. And we will see what is the reason why we do that as we go along and we will also look at few representative cases and consider one example and work it out on the board.

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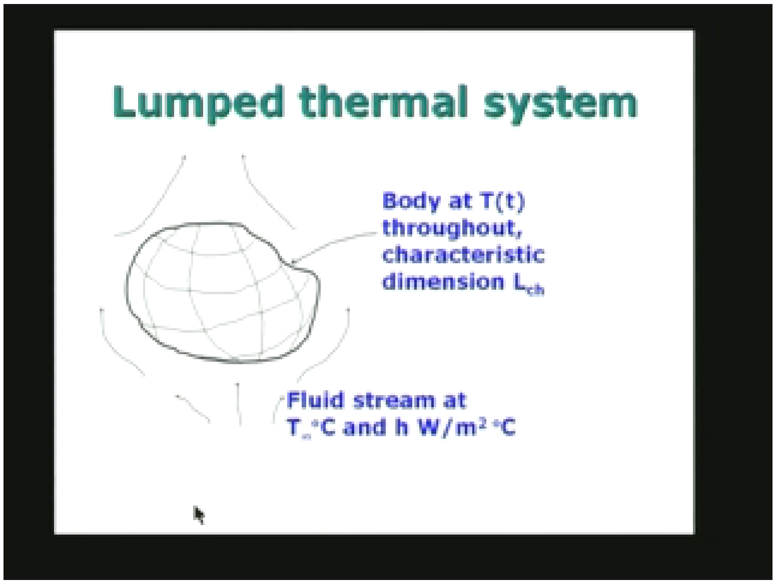

The basic model which we are going to use for describing a temperature sensor is in terms of a lumped thermal system which is schematically shown in the slide here. We have an object. Suppose the sensor is to be represented by this object which is shown here. Of course the shape and size and other things will come later on.

We will assume that at any time as t, the entire sensor which is represented by this regular shape body here is at the temperature T which is same throughout. It is a homogenous temperature field within the sensor. We will also characterize the sensor by a characteristic dimension we call it L subscript ch which is the characteristic dimension as far as the geometric variables describing the body is concerned. The characteristic dimension is what is going to come in the description of the thermal behavior of the system. We also visualize a fluid stream which is at a temperature equal to T infinity which may, of course be varying with respect to time and it subjects the sensor at its surface to a heat transfer coefficient h watts per square meter degree Celsius.

There are different ways of looking at it. For example if the temperature is the same T infinity is constant, and initially the body is at temperature difference from the temperature of the fluid, and let us say we turn on the heat transfer coefficient by turning on a fan, for example, or some method by which the flow takes place, then immediately the temperature of the system starts changing.

Another way of visualizing the problem is suppose the temperature T infinity is one value to start with and suddenly there is some mechanism by which I change it I want to find out what happens to the temperature indicated by sensor. It will be ideal if the sensor exactly follows what is happening to the temperature of the surrounding which it is supposed to be measuring.

Invariably there will be a difference between the temperature indicated at any particular time by the sensor as compared to temperature of the ambient, whose temperature I want to measure. Therefore we expect a difference between the indicated temperature or the temperature of the body sensor at a particular time t, this will be different from this temperature. So the idea is to look at what are the differences? How they depend on various parameters which govern the problem?

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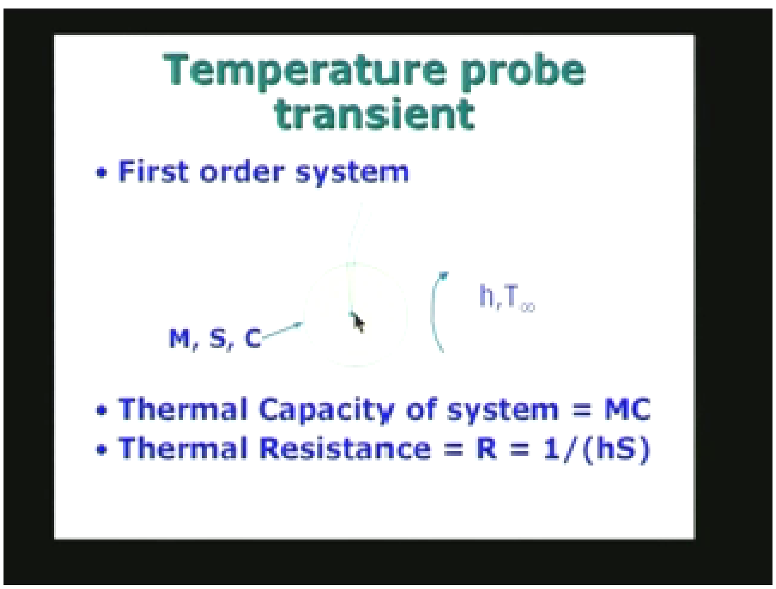

And first for doing that I have given a slightly different schematic here. I have a sensor idealized as a spherical object here. It may be visualized as a spherical shell and I have introduced a thermometer or a temperature sensor somewhere in the shell, and because we are assuming that the shell is entirely at a uniform temperature at any given time, there is no variation of temperature within the shell. The thermocouple position in the shell is of no significance.

We will also assume that the lead wires are very very thin, so that there is no heat loss due to the lead wires which are attached to it. Of course, if the lead wires are going to conduct significantly, then we will have to take into account, that is not very difficult, but it becomes a little more difficult to in terms of the description of the problem.

We will assume that the shell which is in a spherical shaped object here is characterized by a mass of m. It has got a surface area which is in contact with the ambient fluid which is flowing over the surface of capital S and it has got a specific heat capacity of C. The units will be mass is kg S is square meters and C will be joules per kilogram Kelvin. And the ambient fluid is going to subject the surface to a heat transfer coefficient h watts per square meter Kelvin or watts per meter square degree Celsius and the temperature T infinity is the ambient fluid temperature which may be either stationary that is constant or it may be varying with respect to time in a particular fashion or it may be even periodic. So we are interested in three different cases where the temperature is in fact constant and the initial temperature of the sensor is different, from that temperature it may be either higher or lower.

The second case where the temperature T infinity is actually varying with respect to time in a linear fashion which is normally what happens if you start heating for example, an oven which is in the off position you turn it on, initially the heat is input to the system and the temperature is likely to vary in a linear fashion.
Or you may set up an experiment where you want to specifically heat an object at a constant rate of temperature increase with respect to time. (Slide time 8:11). So then I would like to follow the temperature as it changes for that particular experimental purpose.

The third case is where the temperature is actually varying periodically with respect to time, cyclically in terms of a sinusoidal variation. And then I would like to find out how the system is going to respond to the changes or the variation in T infinity. Here, the system is actually the thermometer.

We will see in a moment, I will work it out on the board. The thermal capacity of the system is the product of mass and the specific heat which is important. The mass time which is specific heat that is M in to C. So C is joules per kilogram Kelvin and M is kilogram. Therefore it becomes joules per Kelvin, that is the unit of thermal capacity. And then you have the thermal resistance which is nothing but 1 over the heat transfer coefficient surface area product. And again you will notice that 1 over hS is units of resistance, this will be in Kelvin per watt which will be unit of that. So the first order system which I have not yet defined in terms of an equation which I will do in a little while from now is defined by the quantities which are shown in this figure and the mass specific heat product is going to be one of the important parameters,and the second parameter is going to be the resistance heat transfer from the surface which is given by 1 over heat transfer coefficient area product. So with this background, let me go to the board and look at the system from a thermal point of view.

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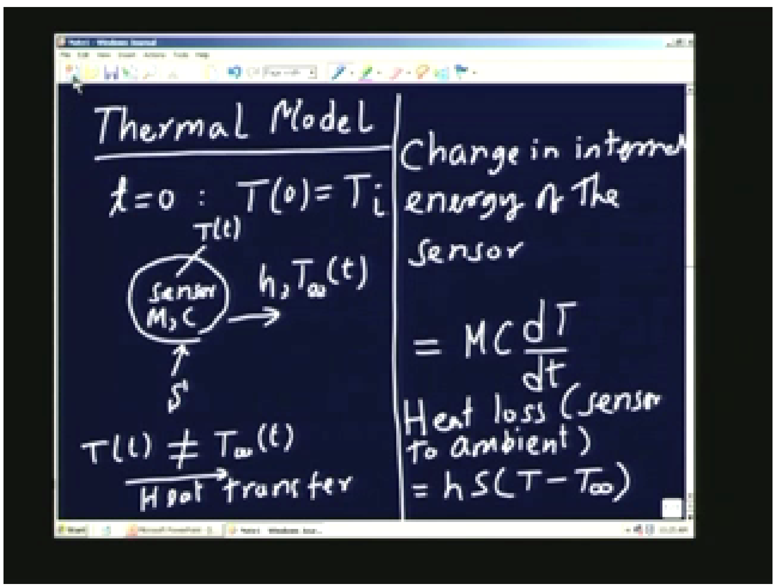

So we will consider the thermal model. So I will just indicate the sensor and we have already seen that mass and specific heat and the surface area is S and the heat transfer coefficient, and T infinity are given. To be more specific I will say that T infinity is possibly a functional temperature. Let us assume that initially at T equal to 0, T of the system will be indicated by T subscript i. So, let us look at the way the temperature of the sensor is going to vary. This is your T subscript C at T throughout. I am assuming that it is uniformly the same temperature throughout that object. Let us look at the rate of change of the temperature of the system. Suppose, if we say that T at any instant T is not equal to T infinity, at that particular instant of time there will be heat transfer in this direction. Of course depending on which temperature is higher, the heat transfer will depend on the direction of decreasing temperature.

So let us look at the way if we are going to model this. So the change in the internal energy of the sensor, if it is going to lose heat that means if the temperature at any instant of the sensor is greater than the temperature of the ambient when it loses heat the temperature will reduce. So the rate at which the internal energy is changing or decreasing is given by mass into specific heat. I am assuming that all these are constants and are not varying with respect to temperature. dT by dt, is the rate at which the internal energy of the system is changing. And the heat loss or heat loss from sensor to ambient whose temperature is what I am trying to follow is given by h into S (Slide time 13:13) into T minus T infinity. Both are of course functions of time.

I am not showing specifically here, and it is understood that both temperatures T and T infinity are essentially functions of time. So, if the heat loss is from the sensor that is heat is going in this direction, the temperature of the sensor is going to reduce, therefore dT by dt is negative. dT by dt is negative, therefore if I take negative of this MC dT by dt that will be equal to heat loss and that is how you write the equation.

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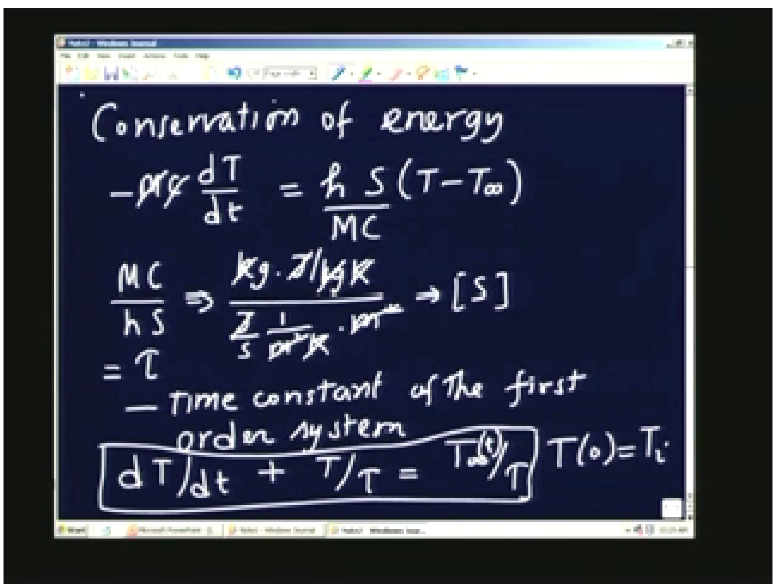

So what conservation of energy requires is that, the change in internal energy which is minus MC dT by dt must be equal to hS into T minus T infinity. This equation is valid whether T is greater than T infinity or T infinity is greater than T because automatically the sign of dT by dt will change to accommodate these changes. Therefore when I derive the equation, I assume that the temperature of the sensor is greater than the temperature the ambient. So that I derive the equation with that assumption but in a case where the temperature of the sensor is actually smaller or lower than the temperature of the ambient, both the left hand side and the right hand side will change their sign and therefore there is no change in the equation. So this equation can be rewritten in the following form.

So what I will do is, I will remove this MC from here, I will take it to the denominator. And look at this MC by hS ,is the mass specific heat product. Let us look at the unit of this quantity, this will be kilogram, this will be joules per kilogram Kelvin divided by h is watts per square meter Kelvin. So watts is nothing but joules per second into 1 over meter square Kelvin and this is meter square. So you see that this meter square will cancel off with this, Kelvin will cancel off with this, kg will cancel off with this kg and joules will be canceled. So this becomes unit is second. This MC by hS has the units of second, usually it is return as tau and we call this the time constant of the first order system.

Why is it first order system?
It is because the equation has a first order derivative in the equation, that is, it is governed by a first order equation. So with this notation MC by hS equal to tau, I can rewrite the equation very simply as dT by dt plus T by tau equal to T infinity divided by tau. This is the equation governing the problem. So the first derivative with temperature of the system or the thermometer with respect to time plus T by tau equal to T infinity by tau and we will reaffirm that T infinity could be a function of time.

Therefore, if you want to you can indicate here specifically there is a function of time. It could be different function. And also we know that the temperature of the sensor is initially T(0) is equal to T_i. So we have the equation governing the problem which is the first order differential equation, ordinary differential equation with the initial temperature given as T at t equal to 0 equal to T_i. Hence, the solution of this is essentially what we are going to look for. So before we proceed with the solution, let us look at some other details. And the details I am looking at essentially is the quantity tau which we have just now seen that is nothing but that MC divided by hS.

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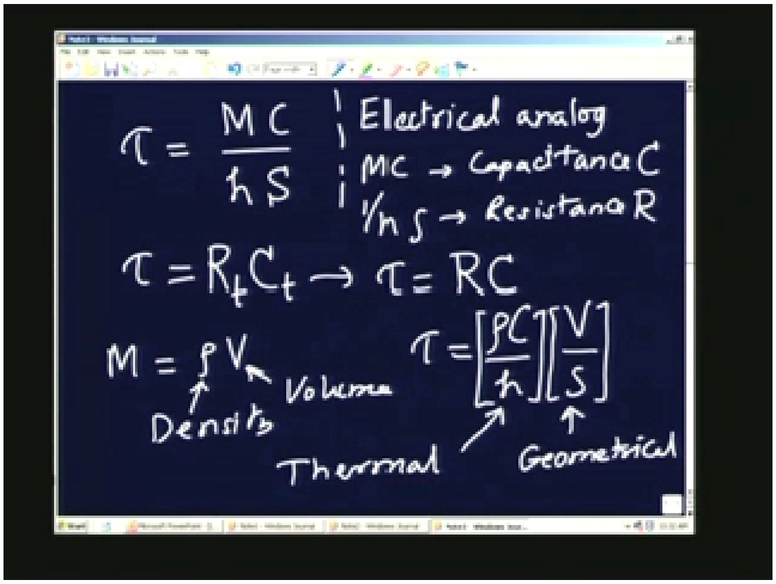

If I look at the electrical analogy or analog MC is like the capacitance, and 1 over hS is like a resistance. And you see that if I show it as capital C and that as R, the time constant for a electrical system is R times C. And exactly that is what we have here. If you want you can show it as 1 over h is the thermal resistance. So I can say it is R thermal and C, MC is actually the C thermal. tau is equal to RC. This is the analogy between the two system that is number one.

Number two, the time constant is dependent directly on the mass specific heat product. So the larger the mass, or larger the specific heat, the time constant is larger. And the larger the resistance that means that hS product the smaller the value of heat transfer coefficient surface area product the larger the time constant. And in fact I can rewrite the MC by hS slightly in a different fashion. M is also equal to rho times the volume of the sensor. So rho is the density and V is the volume. So if I do that, then I can write tau as M is rho into V. I will write rho C multiplied by V divided by h into divided by S. So I am dividing it into two groups, this first group is thermal in origin and this is geometrical which has to do something with the size and the shape of the sensor. So it is actually the volume to surface area ratio.

Now you can see that, if I make the volume to surface area ratio as small as possible or I reduce it, I will be able to get a smaller time constant. And we will see in few minutes from now, that a smaller time constant is better if you want to follow the changes as accurately as possible and as closely as possible. Therefore, the control I have over how a temperature sensor is going to respond, there are two factors which I can vary or I can choose or I can manipulate. The geometrical one is very easy to manipulate because all I have to do is to see that V by S is as small as possible. The following example I can take. Suppose I take a sensor in the form of a spherical shell. So that is let us say R and the thickness is delta. This is a spherical shell.

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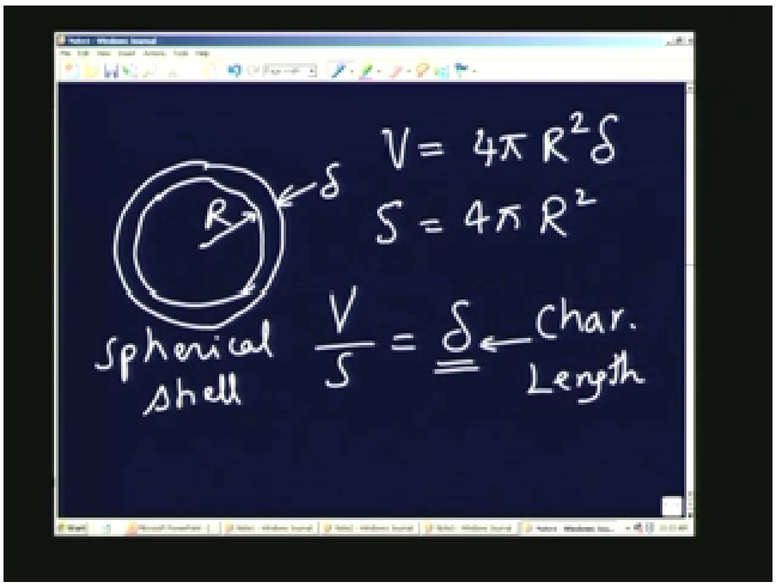

Therefore, we can see that volume is equal to 4 pi R squared delta. And this surface area is 4 pi R squared. So the volume to surface ratio, area ratio is nothing but equal to delta. In fact we will be referring to this as the characteristic length. So, if I have the sensor attached to a small spherical shell, what is the advantage I am going to have? I am going to have a very small characteristic length. And this characteristic length being small, it means that it is going to reduce the time constant of the system. Therefore, instead of taking a spherical solid object, if I take the object in the form of a spherical shell, I will be able to reduce the time constant because the characteristic length is going to reduce.

However, if you look at the heat transfer point of view, I can choose a very large value of R, so that it is a large enough object, so that the heat transfer can be improved by having a good heat transfer coefficient in this particular case. You can see that there are several things which are manipulated at the same time by this technique of taking a spherical shell object. In fact I can also take the object in the form of a thin film of very small thickness like this. It has got two surfaces; one on this side and one on the right side. That is, the total area is the two surface areas and the volume is this, surface area multiplied by the small thickness.

So, again the thickness, if it is delta, the characteristic dimension becomes exactly equal to delta by 2. So I will say L_{ch} equal to, because there are two surface areas and the volume is proportional to surface area multiplied by thickness, therefore I get a delta here. That means, time constant can be manipulated by simple geometric arrangement, where I am going to take the sensor in the form of a thin shell or in the form of a thin film and this thin film can be very thin. It could be a few micro meters in the thickness and it could be very large surface area may be a few millimeters in width and a few millimeters in height. So this is one part of the description of the system. I have not solved any problem, I have not solved any equation. But by just looking at the equation I am able to get some insight into the thermal behavior of the system.

With this background let us look at the governing equation which was derived on the board and I have written it in this slide just to recapitulate. What I have is the change in temperature with respect to time dT by dt is the rate at which it is changing is given by a difference between temperature of the ambient which may be varying with respect to time minus the temperature T which is the temperature of the sensor divided by the tau which is the time constant.

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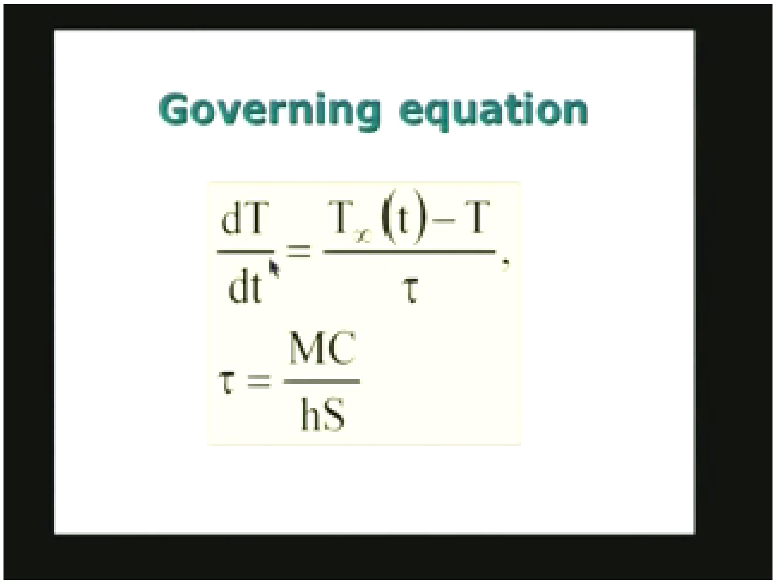

So dT by dt equal to T infinity minus T divided by tau. This is the temperature difference or it is like the voltage potential difference and divided by time constant. Time constant is something like a characteristic time for the particular problem which is given by MC by hS. And in the next slide I am showing the electrical analog of the same situation.

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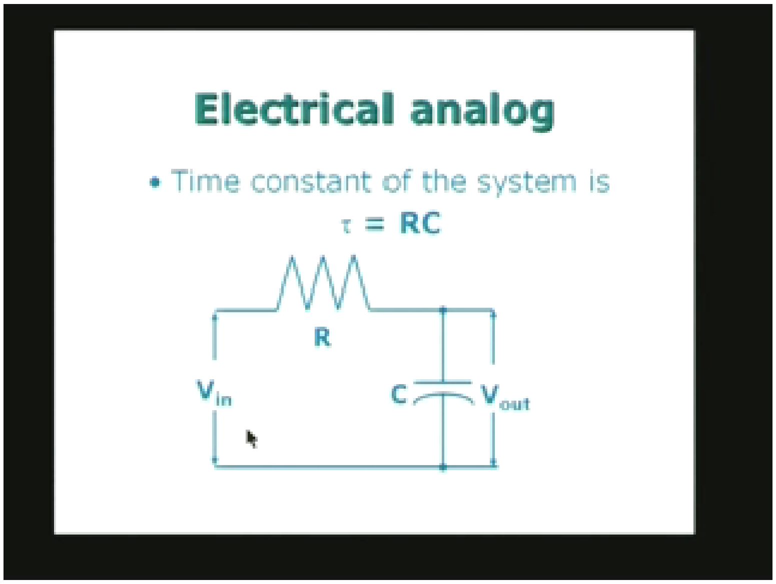

The time constant system is RC where R is the resistance and C is the capacitance and suppose I connect the resistance and capacitance like this, I input a voltage V_{in}. This is like the T infinity and the output voltage V_{out} is the response of the system. So RC is in series like this, and I am putting the input as I have shown here. This input may be either constant, or varying with respect to time, and then the output is going to appear across the across the object which is the system heat capacity.

Actually you can see that, the resistance which is in series with the capacitance is something which connects the output which is what we want from this system, the temperature of the object to the temperature variation on the outside. This is something like an interaction. The resistance allows an interaction between the ambient and the system. And the system itself is characterized by the capacitance C of the system. So the relationship between V_{out} and V_{in} is what we are going to study and let us look at the way we are going to do that by again going back to the board. And let me just try to work out the so called general solution to the problem which is very simple.

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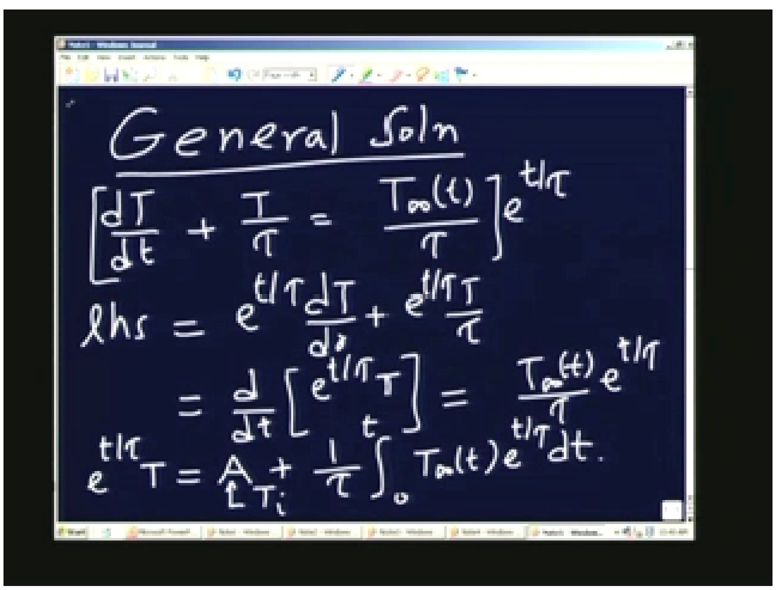

So, general solution to the first order equation, we know that it is dT by dt plus T by tau is equal to T infinity which is a function of time divided by tau. Suppose I multiply throughout by e to the power of T by tau which is an integrating factor. So if I multiply these two, the left hand side becomes e to the power of t by tau dT by dt plus e to the power of t by tau multiplied by T by tau is nothing but d by dt of e to the power of t by tau multiplied by T. You can verify that. So the right hand side is equal to, then T infinity by tau multiplied by e to the power of t by tau and all I have is to do is integrate it once. If I integrate, I will get e to the power of t by tau multiplied by T is equal to some constant of integration A, I will say , plus 1 over tau,because tau is a constant I will just take it outside, integral 0 to t, T infinity which is a given function of time, e to the power of t by tau dt. And in fact you can see that the integral shown here for t equal to 0, the upper limit and the lower limit will be the same, and this will become 1, e to the power of 0 by tau becomes 1. Therefore A is nothing but T_i, and therefore I can write the solution as T which is a function of t.

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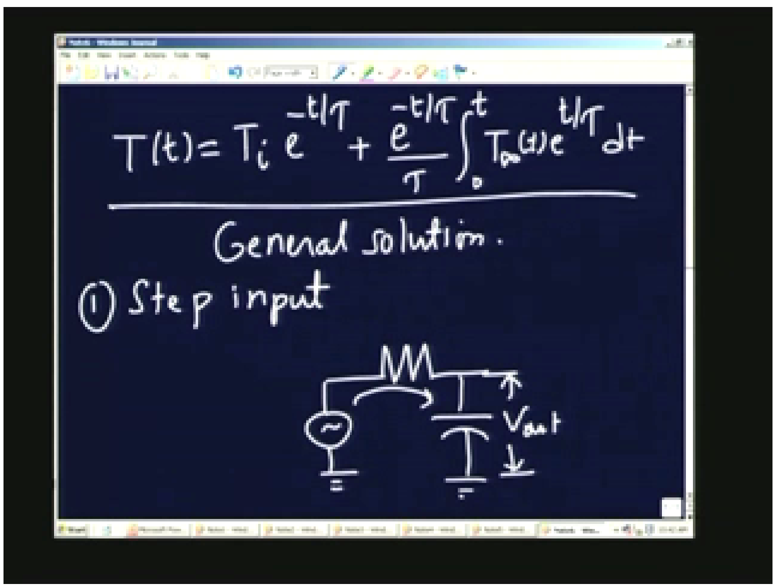

I will multiply throughout by e to the power of minus t by tau, so that it becomes T_i e to the power of minus t by tau plus e to the power of minus t by tau divided by tau integral 0 to t, T infinity dt. This is the general solution. And in fact, I can use this as the starting point to write down the solutions for all the three different cases we are going to consider. The three different cases, if you recall one will be a step input. I will explain this using the electrical analogy. You have resistance and then you have the capacitance and you are going to connect it and you are going to measure the output here.

So in this case, initially I do not have any input. I just give a voltage at the input by turning on a switch and let us say, you have a battery and then you connect it. So what initially will happen is, from the battery the current will flow through this circuit and this current flow will be taking place till the capacitance gets charged. Once it gets charged completely the current will stop. That is the transient. The transient is when the capacitance is getting charged. So what is the equivalent in terms of thermal system? In the thermal system, initially the temperature of the system is different from the temperature of the ambient and once we communicate the two at T equal to 0 the temperature of the system of the thermometer will start going towards the temperature of the ambient which is a constant value and therefore the temperature will approach it as T tends to infinity as time becomes large compared to the natural time much larger than time constant system. The two temperatures are going to be the same. So we can say that we have equilibration of the temperatures, equilibration of the system and the surroundings. So let us look at the way we are going to get the solution to the problem. All I am going to do is to use the general solution here.

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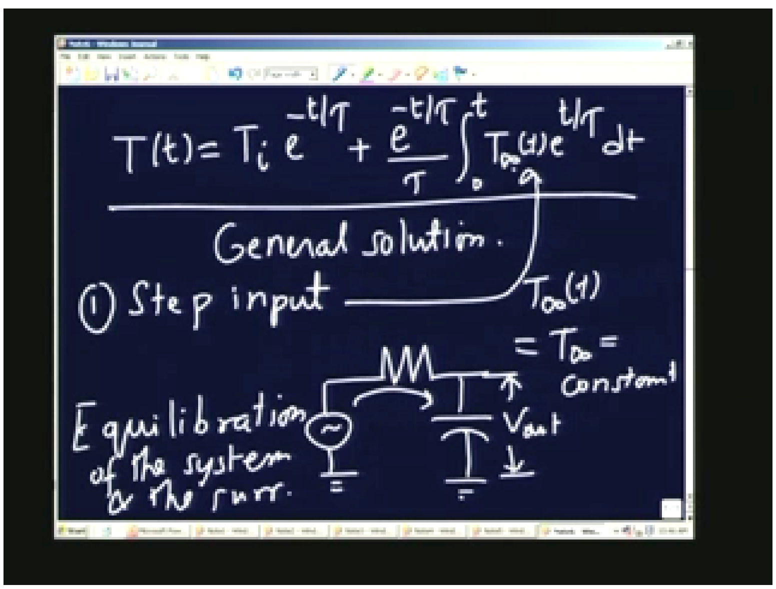

I am going to put T infinity equal to constant. So this will be replaced by constant value T infinity and all I have to do is to obtain this integral and then I will be able to get the solution to the problem. Let us look at that one.

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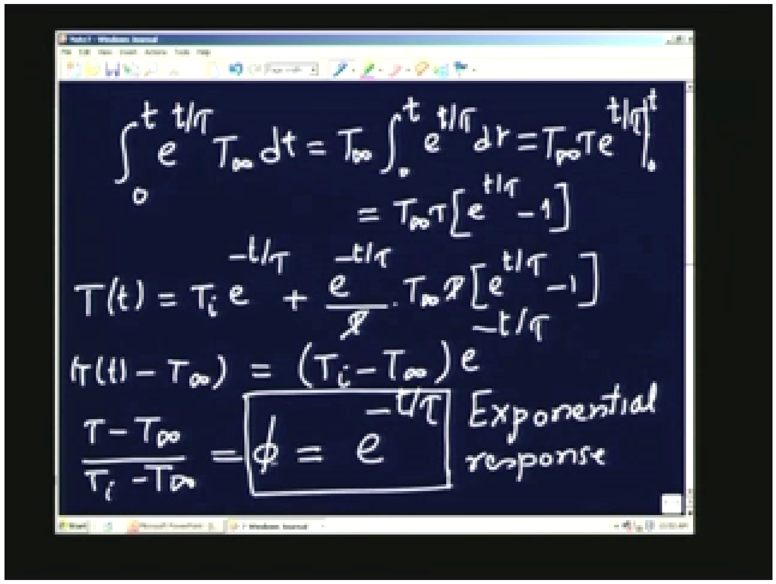

So the thing I require is 0 to t integral, e to the power of t by tau multiplied by T infinity dt.Because ,T infinity is now constant, I can take it outside and write it as 0 to t e to the power of t by tau dt. This is a very simple integral. So this becomes T infinity tau e to the power of t by tau. It is because I integrate it I get this one, if you differentiate this you will get this one. And therefore the temperature T as a function of time is given by, of course you have to take it with respect to only between 0 and t. So let us just complete that. This will become $T_{infinity}$ multiplied by tau, e to the power of t by tau is e to the power of t by tau minus 1. So if I put it back into the general solution of the problem, T as a function of time, I have T_i e to the power of minus t by tau plus e to the power of minus t by tau divided by tau multiplied by that integral. That integral is now $T_{infinity}$ tau into e to the power of t by tau minus 1. This tau will cancel off with this tau and therefore this e to the power of minus t by tau multiplied by e to the power of t by tau is going to give 1. So it is plus T infinity here. Therefore I will see that T of t minus $T_{infinity}$. This term will go to the left hand side. This T infinity is multiplied by this T infinity will go to the left hand side is equal to T_i minus 1 into T infinity is minus T infinity e to the power of minus t by tau or T minus T infinity divided by T_i minus $T_{infinity}$.

I will call it as ratio of two temperature differences equal to phi which is a non dimensional temperature ratio which of course is a function of time. This is equal to e to the power of minus T by tau is again a non-dimensional ratio of time in T seconds or T in whatever units to the time constant is the same unit. So you see that this equation represents the solution to the problem. There is an exponential response. The advantage of writing in this particular form is that I have only one universal curve which is going to give the response of the first order system to a step input. So let us look at the step input in the presentation. I have made a plot of whatever I showed in the solution there.

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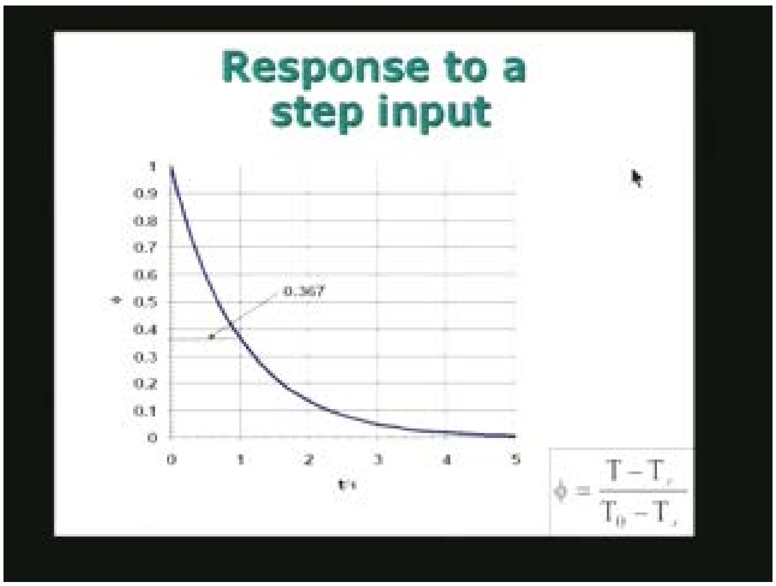

The ratio of the temperatures T minus $T_{infinity}$ divided by T_0 minus $T_{infinity}$ at T_i minus $T_{infinity}$, I have taken T_i there. It does not matter, T_0 here represents the initial temperature. $T_{infinity}$ is the temperature of the ambient which is what we want to measure and initially the temperature of the sensor is different from the ambient temperature. So it will vary according to this exponential and you see that the characteristic of the exponential variation is that, at the end of one time constant t by tau equal to 1 it is about .367 of the value here, and if I go to another time constant it will be point 367 of this value. And every time constant reduces by a factor of 1 over e. That point 367 is nothing but 1 over e. So you see that, in principle of course it takes infinite amount of time to approach the ambient temperature. But as you can see from this graph, around 5 time constants, the response is almost like it has come to equilibration. So let us look at the equation again.

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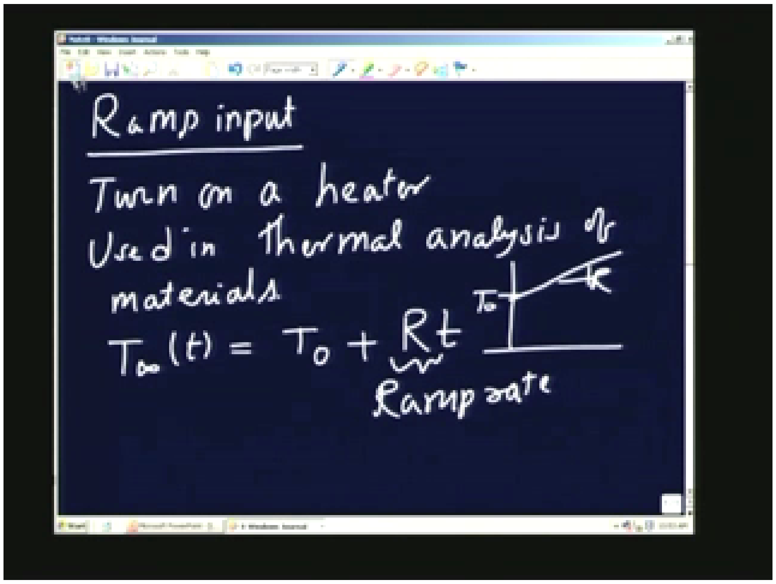

And what I am going to do is, I am going to look at a slightly different change of temperature of the ambient. So if I have a ramp input, a ramp input is very common. For example, when you turn on a heater or control it to vary with respect to time in a linear fashion, used in thermal analysis of materials. Instruments which study the thermal behavior of material usually use a programmed heating mode in which the linear or ramp input is a common feature. So, increase the temperature with respect to time in a linear fashion.

Suppose I take the temperature T infinity to vary, initially it is some T_0 plus R times t where R is the ramp rate. So if I were to plot this temperature, this is T_0 and it will go like this and the slope of this line is R linearly it is increasing with time. And now the temperature of the sensor may be either higher or lower than the T_0 to start with and we would like to know what is going to happen. So all I have to do is to go to this general solution which we worked out previously.

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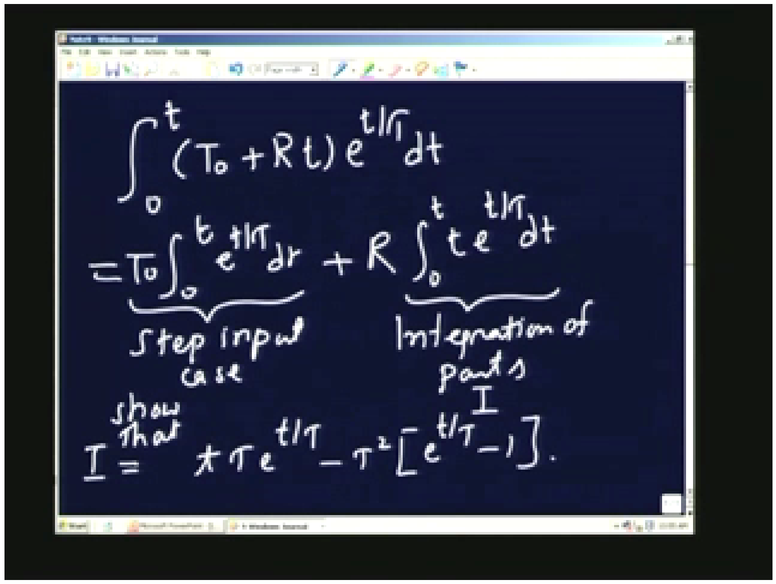

And all I need is an integral of this form 0 to t. Instead of T infinity I have T_0 plus Rt into e to the power of t by tau dt. This can be written as two integrals plus R times integral 0 to t, t e to the power of t by tau dt. This integral is similar to what we did in the case of a step input. Here we have an integral which involves the product of t and e to the power of t by tau, this can be done by integration by parts. And in fact I am going to leave it as an exercise for the student to work out and I will give only the final answer to the problem. So what I am going to do is, I am going to give the final answers to this. Step input case has already been done. Instead of T_0 we had T infinity there, this integral 0 to t, t e to the power of t by tau dt can be integrated by parts and it can be shown.

So this becomes, I will call it as I [show that] which is equal to, I will just give the final expression, t multiplied by tau, e to the power of t by tau minus tau squared into e to the power of t by tau minus 1, by integration of parts, which is very simple and straight forward. Hence, what I am going to do is to substitute it into the general solution and simplify and write it in this particular form. So the solution turns out to be, T of t is equal to T_i minus T_0 plus R into tau e to the power of minus t by tau is the first part and plus T_0 plus Rt minus Rtau is the second part.

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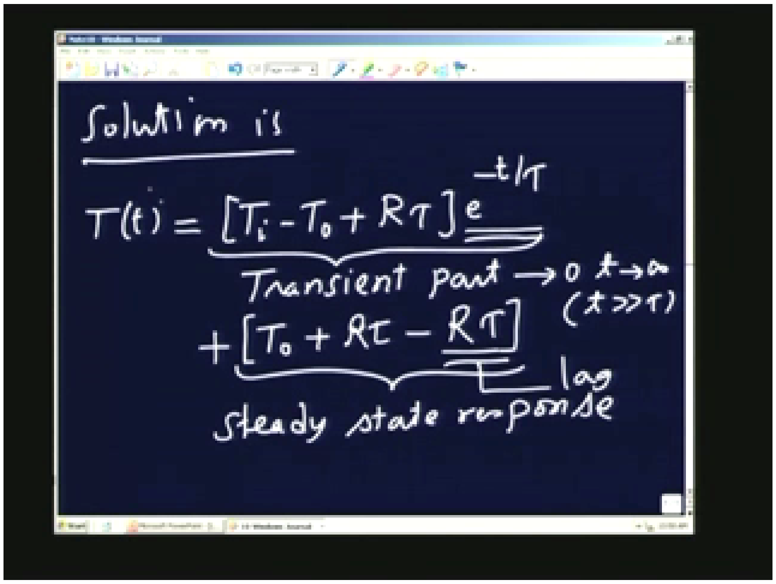

So if you look at the first term, there is an exponential term and as T becomes larger and larger this is going to become smaller and smaller, so we will call this as the transient part which tends to 0, as t tends to infinity which we will interpret as t greater than tau. t varies largely when compared to the time constant system. And this we will call as the steady state response wherein if you look at this you will see that when the transient part has already become 0, the surviving part, the part which remains is called the steady state response. It is nothing but this is T_0 plus Rt which is nothing but the input. That is the rate at which the ambient temperature is changing. Therefore there is a small or a lag with respect to the input which that means that the two are going to vary alike with a lag, the response of the sensor will lag behind the response of the input. The input is changing with respect to time according to a ramp input. This will be slightly lagging behind that. Therefore it will always be behind that. Actually we will see that if we were to look at the solution by plotting it as I am showing here.

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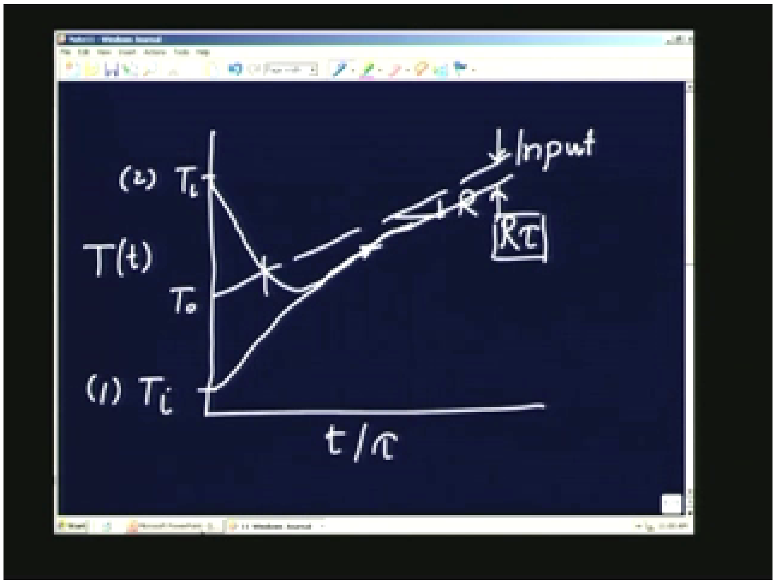

So I am going to plot it again t by tau on the time axis and then I am going to plot T here on the y axis. So this is your input, this value is T_0 and the slope of this line is R. Suppose the initial temperature of the sensor is here, it will first try to catch up with this. So it will vary like this, and go and become parallel to this. This will be equal to R into tau. If it is staring with a value higher, it will first decrease and will go below this and then go here. Either way, let us say case one, case two T_i is here. Initially what happens is that the sensor cools as it is at a higher temperature because the ramp is starting with a lower temperature. So it will cool and it has to cool below the temperature of the ambient and it will cross over and then it will start increasing again. And then ultimately, in this case also it will lag behind by a factor equal to R into tau. This is the general behavior we are going to see and I have made a plot for a particular case in this slide.

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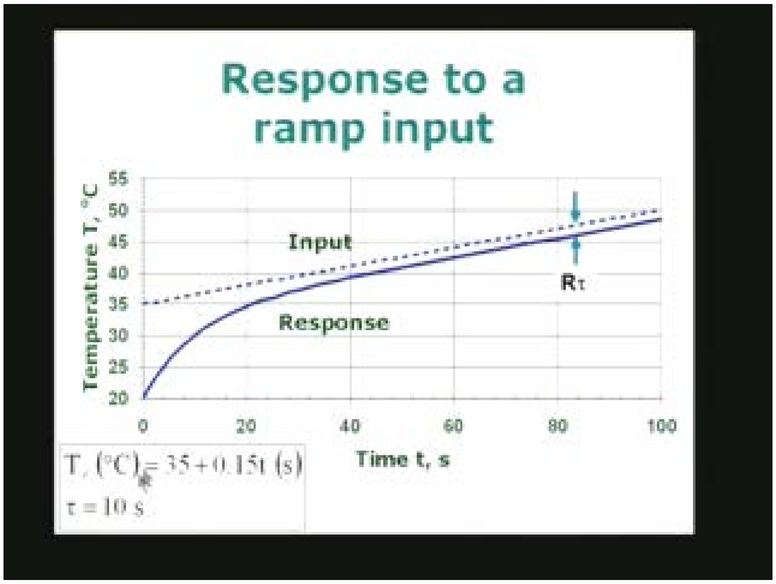

I have taken T infinity which is in degree Celsius. It is given by 35 plus point 15t, where t is in seconds. I have taken the time constant of the sensor as equal to 10 seconds, and I have shown the plot when the temperature of the sensor was 20 degree to start with. So, initially you see that the temperature is increasing and it is trying to catch up with the temperature of the ambient which itself is again increasing and therefore it will never catch up completely.

So if you remember the time constant is 10 seconds, after about 50 seconds if you look at here that is 5 time constant, the difference between these two graphs, these two are going to become parallel, they both are going to be linear but with a small difference R into tau. In this case R is equal to point 15 and tau is equal to 10. Therefore R into tau is equal to 10 multiplied by point 15. So 1 point 5 degree difference is there. So the difference is 1 point 5 degree between the temperature of the ambient which is changing according to a linear relationship and the temperature indicated by the sensor. So this is what is going to happen in the case of a first order system whose response we have plotted for a ramp input.

When the temperature of the ambient is changing linearly with respect to time there is always a lag whether it starts with a lower temperature as in this case or a higher temperature. If it were to start with a higher temperature to somewhere it will cross over. So what is this cross over? Because initially the temperature of the sensor is lower than the ambient temperature there will be heating problem. The temperature of the sensor will start increasing from t equal to 0.

However, if the temperature of the sensor is more than the temperature of the ambient to start with, it will have to first cool. The temperature has to cool down and it will actually cool down to a temperature lower than the temperature of the ambient in some time and then it will start heating again. That is what I indicated on the board. So let us look at the last case I am going to consider. That is the case of sinusoidal input or we will call it as a periodic input problem or periodic variation in temperature.

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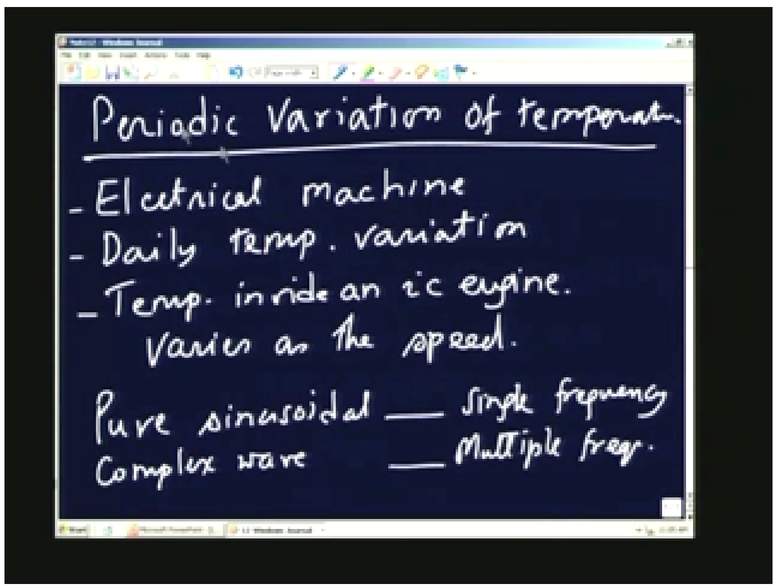

There are many instances where periodic changes in temperatures are to be measured. Example, you have an electrical machine or variation in daily temperature. For example, temperature inside an engine, inside an IC engine, inside the cylinder there is a gas and the temperature or the wall of the IC engine will also have a periodic temperature variation. This period of the periodicity of the wave will be determined by the speed of the engine. The temperature inside the engine varies with the speed. So in the case of the electric machine if it is 50 cycles we expect the temperature to vary somewhere in that vicinity of 50 cycles. In the case of an IC engine depending on the speed, the temperature will vary with respect to time.

And in fact what I will do is I will just take a simple case where I will assume, it could be a pure sinusoidal or pure periodic wave given by one single frequency or it could have a shape which is not necessarily sinusoidal. So if it is a pure wave, pure sinusoidal, it is characterized by a single frequency. If it is a more complex wave which has the shape it will have multiple frequencies. So there may be a frequency which we call as the fundamental, which is given by the periodicity of that wave and there will be harmonics which are at higher frequencies. So if you look at the solution to the problem, what I have to do is to obtain that integral which is given by integral 0 to t, T infinity as a function of time, e to the power of t by tau dt. This is what I have to obtain.

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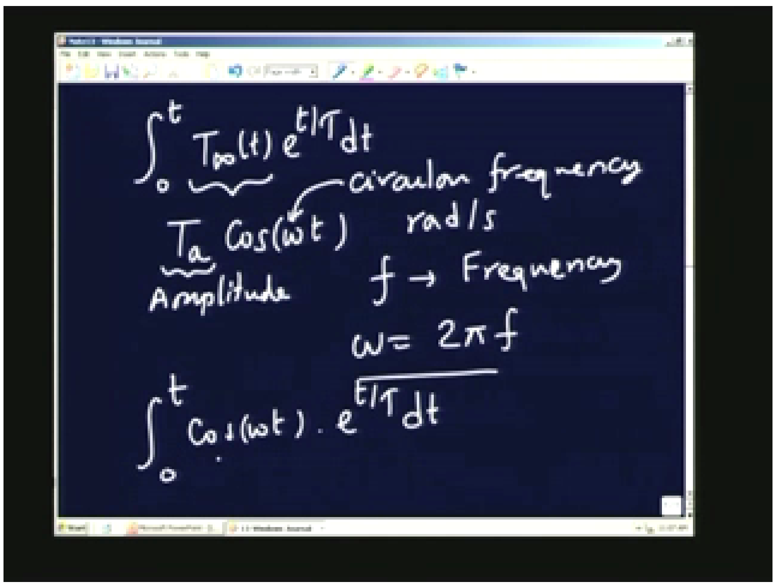

And if I assume that T infinity t is a pure sinusoidal wave or pure periodic wave with one frequency, I can take for example, amplitude T_a cos omega t. I can either take cos omega t or sin omega t it does not matter. I am just taking T_a cos omega t. And T_a is a constant, this is called the amplitude and this is the angular frequency, actually in radians per second. If the frequency is f, then omega will be nothing but 2 pi times f omega ,equal to 2 pi f. So let me just indicate how we are going to solve this problem and I will actually leave the details to the student to workout.

What I want is the integral given by cos omega t multiplied by e to the power of t by tau dt. Again I will integrate it by parts, so in the first integration what will happen? I will be integrating e to the power of t by tau keeping this as such and then I will have to differentiate this when we do the integration of parts. And the differentiation will give you a sinusoidal because cos becomes sine. Suppose I repeat again, afterwards I will get an integral which involves sine and then second time I integrate I will get back the cos omega t. That means if I integrate once by parts the integral will change over to from cosine to sine and then when I integrate the second time this sine will again become cosine and therefore the integral will repeat itself. And to conserve time I will not indicate the steps involved, I will just give the final answer to the problem. Those who are interested can work it out on their own.

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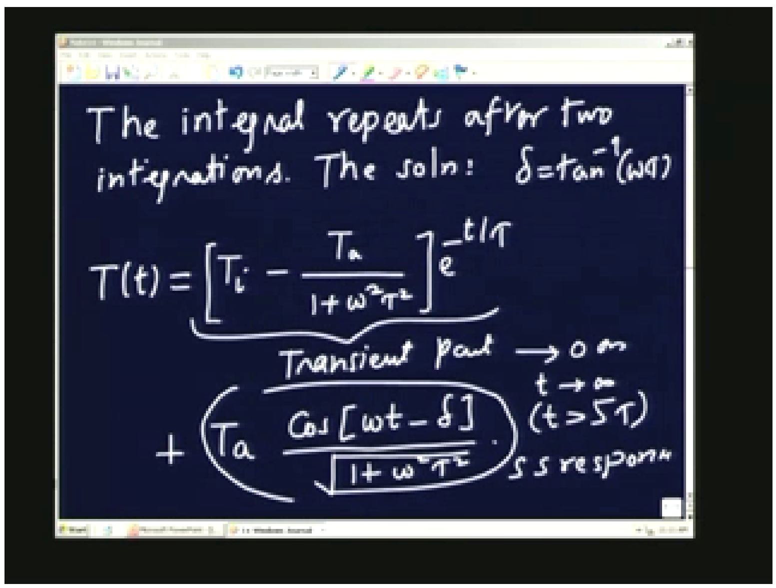

So the integral repeats after two integrations. This is the observation we are making. So mathematically we can look at it but we are not inclined to do that. So the solution can be written as,T_i minus T_a divided by 1 plus omega square tau square whole multiplied by e to the power of minus t by tau. We will call this as a transient part just like what we had in the case of a ramp input. The solution is made up of two parts; the transient part plus again we have what is called a steady state response part which is given by T_a multiplied by cosine omega T minus delta which is called the phase angle divided by square root of 1 plus omega square tau square. This solution can be worked out by the method I have suggested. So again the transient part tends to 0 as t tends to infinity which means t is something greater than above 5 tau let us say.

So what will happen is, as the time becomes larger and larger the response is given by the second part, this is called the steady state response. And what we notice from the steady state response is that it is also co- sinusoidal but it is omega t minus delta that means there is going to be a lag with respect to the input and in fact delta is given by tan inverse of omega tau. And the coefficient is T_a divided by square root of 1 plus omega square tau square and omega tau 1 plus omega. This is smaller than T_a by a factor greater than 1 therefore the amplitude is reduced.

Therefore we say that there is an amplitude reduction. So T_a divided by square root of 1 plus omega square tau square and delta is equal to tan inverse omega tau. And in fact I have worked out a simple example to show what is going to happen in the case of a response to a cosine input as we said. So I have taken a specific case T_a by T_0 equal to .25, omega equal to 1 radian per second, and tau equal to 1 second and you see that the temperature where it starts, this is the input which varies like this and the response of the system is following the other curve and you see that initially there is a transient and it dies down and then both of them become periodic and there is a time lag as shown here,and the amplitude is smaller than the other amplitude by a certain factor.

So what I will do is in the next lecture I will briefly touch upon this case again and indicate what happens when you have a general periodic input which may not be a pure sinusoidal and we will look at that case. And then what I am going to do is to look at other methods of measurement of temperature using different types of sensors. Thank you.