Mechanical Measurements and Metrology
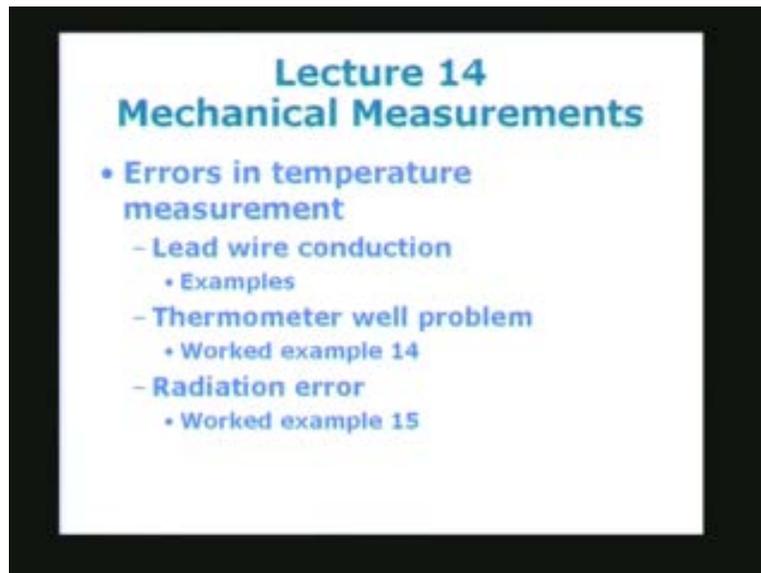**Prof. S. P. Venkateshan**
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Indian Institute of Technology, Madras
Module - 2
Lecture - 14
Errors in Temperature Measurement

So this will be lecture number 14 on our ongoing series on Mechanical Measurements. Towards the end of the last lecture we were in fact looking at the nature of errors

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in temperature measurement and we just gave a brief description of the kind of lead wire model one uses and what I am going to do now is to just recapitulate a few of the ideas from that lecture and then carry on with two examples, one, the thermometer well problem which will be followed by a worked out example and then I will also look at the calculation of error due to radiation. So these two examples will give a flavor of the kind of problems one faces when making measurement of temperature.

In fact in the last lecture we have talked about different situations or various situations where temperature is measured. The temperature may be within a solid somewhere inside, it may be on the surface of a solid, and it may be a

temperature of a flowing fluid, the fluid flowing through a duct or a pipe or a tube. And in each one of these cases there are several paths for heat transfer and in fact we identified the general nature of the interactive heat transfer paths or the heat loss paths and then we discussed that these can be modeled through a proper thermal model and then we will be able to estimate the systematic error due to heat transfer through various mechanisms. In the next slide I am showing, just to recapitulate the lead wire model, we indicated in the last lecture, we have two wires corresponding to the positive and the negative thermocouple wires and the two wires are encapsulated in the sheath material which is both electrically insulating and that is actually what we want, but it is also going to give some kind of insulation for heat transfer also.

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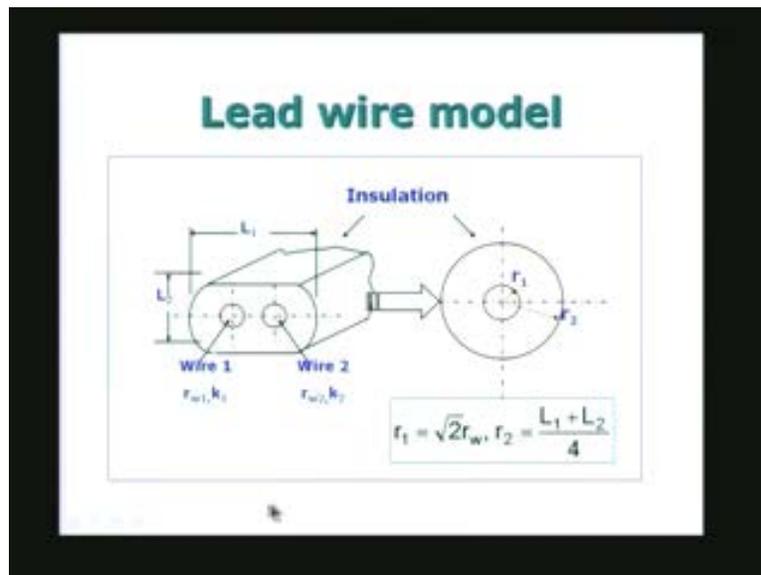
And the major dimensions were explained in the last lecture. And what we did towards the end of the lecture was to show that the two wire actual thermocouple with the sheath surrounding it is replaced by a single wire model consisting of a single wire of radius r_1 surrounded by a sheath of outer radius equal to r_2. We explain how to do that now and in fact we also explain that the mechanism of heat transfer will be the following.

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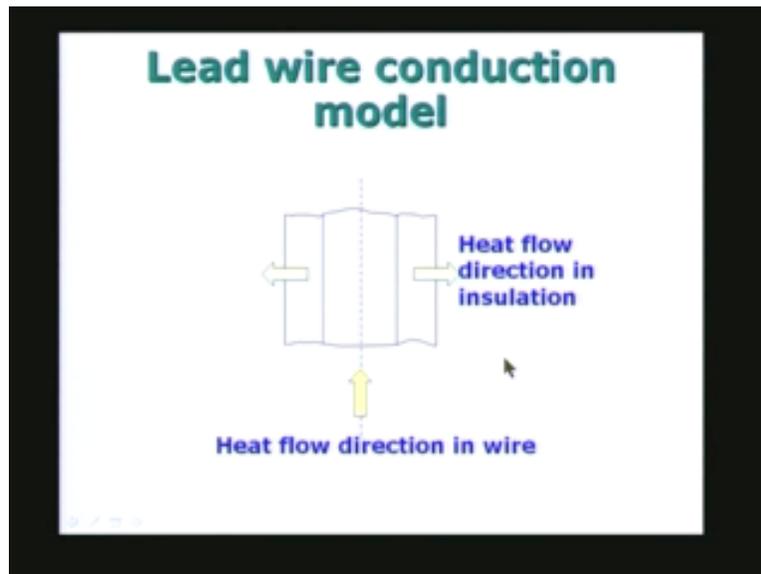
The wire is a very highly thermally conducting material because it is metallic and therefore you have heat transfer in the axial direction in the wire. So that is shown by this arrow and in the insulation which is surrounding the wire, in the single wire model we have a single wire surrounded by certain thickness of insulation and we have heat flow predominantly in the radial direction in the insulating layer. That means we are going to model the lead wire conduction as one dimensional conduction along the wire in the axial direction and the heat flow in the insulating layer in a direction normal to the access of the wire in the radial direction. So the idea is to look at a typical application where such a situation exists and find out how to make calculation of the heat loss due to conduction and then see how it is going to affect the temperature which we are going to measure.
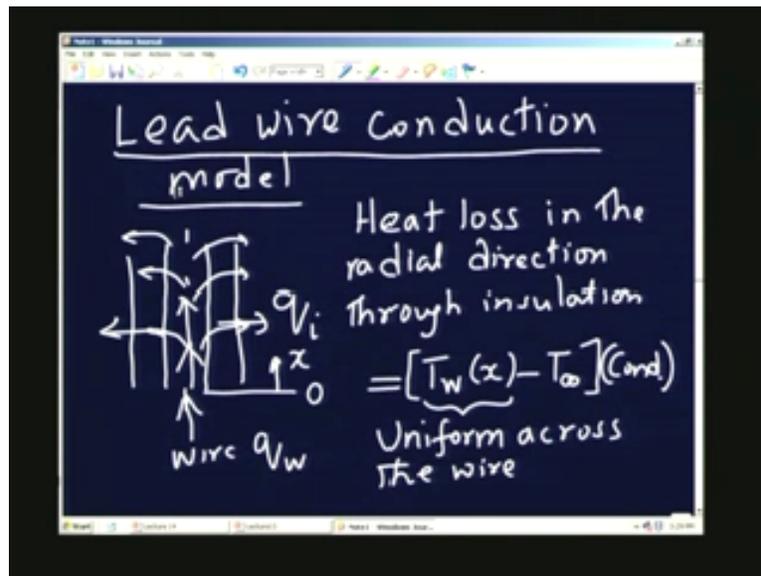
So in the lead wire conduction model what I will try to do is to basically give an appropriate formulae with some physical justification and not too much of mathematical derivation because at this time the students may not have had a course in heat transfer which we come to a little later. So here we will be using physical arguments instead of using mathematical arguments. So the wire is in this form and I said that the wire heat transfer is in the axial direction and surrounding that is our insulating layer and the heat transfer is in the radial direction in the insulation. That is as well as in the wire and in the insulation. So we expect temperature variation to be in the axial direction predominantly and this gives rise to heat transfer like what is shown by the arrows. So now the wire will conduct the heat axially and some of the heat will leave through the insulation like this all along the wire. Something like what is shown here like a tree.

The point to notice is that, the heat transfer in the radial direction keeps on decreasing as also the heat transfer in the axial direction. Some amount of heat comes in here and little bit is going to be lost here and the rest of the heat is going to flow in the axial direction, some amount of heat is going to leave. Therefore it is a continuous process. We can in fact show that the heat loss in the radial direction through the insulation, is due to a temperature difference between the wire temperature.

We are going to model this as follows: that is T_w which is a function of x, if I measure x from some datum here and then I am measuring x in this direction along the length of the wire, the heat transfer is T_w at x minus the ambient temperature which I will write as T infinity that is T_w which is the value at any particular value of x along the wire. In fact I will treat this as uniform across the wire that means if you take any cross section of the wire the temperature is uniform across that cross section given by T_w which of course is a function of x. So the heat loss in the radial direction though the insulation will be proportional to the temperature difference $T_w(x)$ minus T infinity multiplied by some conductance.

So the idea is to find out how to evaluate this conductance in the radial direction. So what I will do is just give a formula instead of working out mathematically. So I will work it out in terms of a resistance for heat transfer. In fact we can note that if I go here the conductance can also be looked upon as the inverse of a resistance because ohms law says that potential difference divided by resistance will give you the current. In this case the heat transfer is like a current, the temperature difference is like a potential difference and instead of conductance I can have a resistance in the denominator.

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So if I obtain a resistance to heat transfer and in fact we can show that the resistance heat transfer consists of two components, I will say it is R the

resistance of heat transfer, is one due to convection resistance at the surface of the insulation which is one component plus conductive resistance in the insulation.

In most books on heat transfer expressions are worked out for this and we will just take the expression without proof and this will be given by 1 over the area of heat transfer which will be 2 pi r_2 multiplied by h the heat transfer coefficient at the surface plus conductivity of resistance given by 1 over 2 pi k_i which is the resistance of the thermal conductivity of the insulating material multiplied by the logarithm of the ratio of radii r_2 by r_1. So the heat transfer in the radial direction is represented by a resistance of heat transfer given by the sum of two parts the convective which is also sometimes called as the film resistance which is given by the first expression as shown here, plus the resistance due to the finiteness of the thermal conductivity of the resonating material which is given by the second expressions here.

And if you remember, when we discussed the single wire model for the thermocouple, we had actually obtained relationship for r_1 and r_2 in terms of the dimensions of the two conductors and the insulating layer and so on. So, this is already available to us. And therefore I can use this expression, some of these two resistances for resistance to heat transfer in the radial direction. Now let me just make a simple sketch indicating what we are going to expect.

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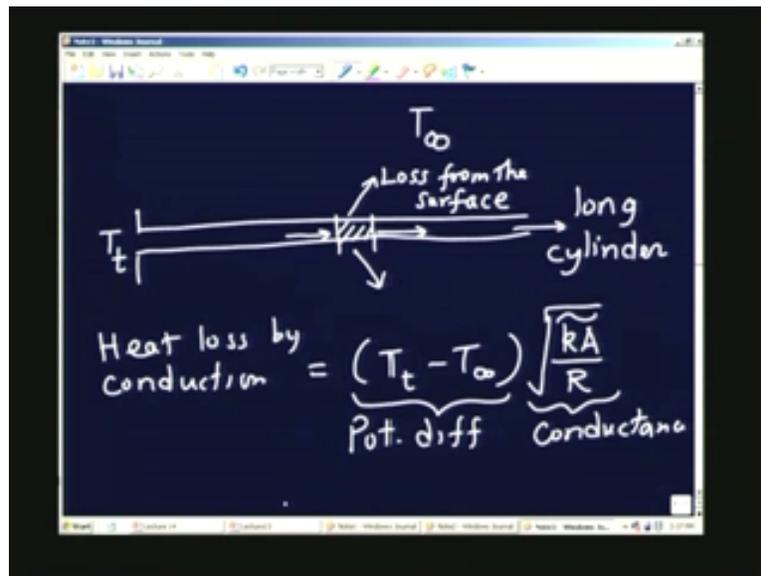
Suppose we have a wire which is in the form of a long cylinder, we remember that the wire is usually long because the thermocouple leads are taken far away and connected to a voltmeter to measure the temperature. Therefore this is relatively long when compared to the radius of the cylinder. The radius of the cylinder could be at the most a fraction of a millimeter and the length of this could be even a few meters or at least a meter. Therefore it is the very long cylinder for our purpose.

So now let us look at only the conductor. I will remove the resistance, I will remove the outer insulating layer and I am just looking at what is going to happen to the wire. At this place, it is connected to the measuring junction that is there. So I will just say it is the temperature of the measuring junction T subscript t. And here there is the ambient at a temperature equal to T infinity. And if I take a small portion of material of the wire, some amount of heat will come from the left side by conduction, some amount will leave to the axial direction and some amount will leave from this side to the ambient. So this is actually loss from the surface.

So the conduction coming in, conduction leaving, the loss from the surface, so the energy balance will give us, the appropriate equation to be solved and I am not going to even derive the equation or even solve the equation, I am just going to directly give you the expression which we have for the heat transfer from the surface. So the heat transfer is proportional to the heat

transferred, heat loss by conduction equal to T_t minus T infinity. This is the temperature difference available for heat transfer multiplied by a factor which we get from the theory of basically solving the equation governing the problem. And I am not going to solve the problem.

I am just going to say that this can be shown to be equal to the product of the thermal conductivity area for the thermocouple wire, the two wires together divided by the resistance which was again obtained earlier. So, heat loss by conduction is equal to, this is like the potential difference, this is like the conductance. So it is a product of two things. This kA product represents the sum of $k_1 A_1$ plus $k_2 A_2$ for the two wires which make up the thermocouple pair. This expression basically comes from the solution to the heat transfer problem. So what we should notice is that, the heat loss due to the wire which is attached to the object whose temperature I want to measure is given by a simple formula.

If this heat transfer comes from somewhere, that amount of heat transfer from whatever the thermocouple lead is attached to, the amount of heat it acquires must be exactly equal to the heat lost by conduction. That will give some kind of a relationship between the various temperatures which are involved. Let me take a just a simple example which was earlier shown in the previous lecture.

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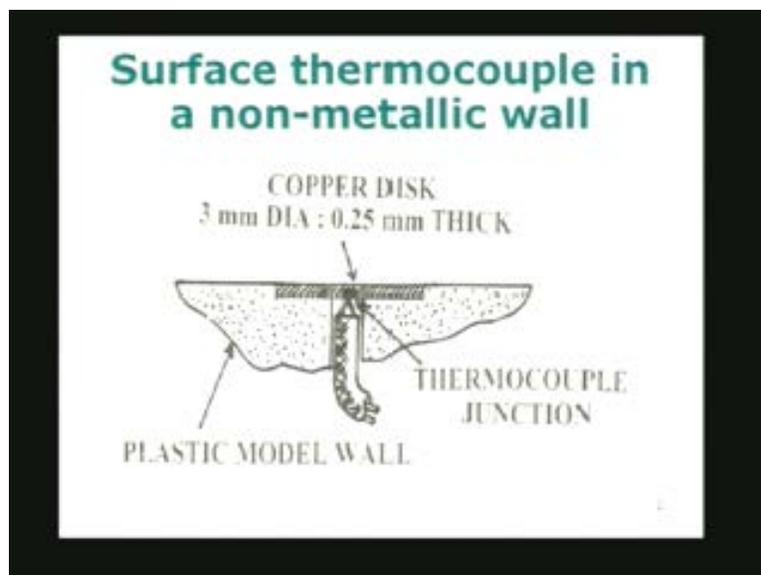
So let me just take a typical example and see what is happening. So we have a heat conducting, heat collecting pad, I indicated the copper disk in this case, a thin copper disk to which the thermocouple junction is attached. So the thermocouple conduction due to thermocouple which is attached to the back is modeled according to the equation which is proportional to the temperature of the junction minus the temperature of the ambient here multiplied by that factor. Now suppose I look at the other side of the copper disk in this particular case, the copper disk has got some area of the surface equal to S which is exposed to the fluid which may be flowing on this side.

Suppose I assume that the heat transfer coefficient for heat transfer convection from this fluid to the surface is given or known, then I can find out what is the heat transfer to this surface from the medium which is flowing by simple expression which is given by the heat transfer coefficient times the surface area exposed to the flow multiplied by the temperature difference between the fluid which is flowing and the temperature T_t. So I will then equate this to the heat loss through the thermocouple junction, through the thermocouple wire and I will get an expression between the temperature of the fluid T_f, the temperature of the junction which is now indicated by the measurement, and the ambient temperature T infinity. Therefore if two of them are known, the third one can be determined. So let us just look at this situation by going to back to the board.

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So I will make a simple sketch so that we have the heat collecting pad. Let us assume there is a surface area is S and there is a fluid which is flowing here at a temperature equal to T_f and heat transfer coefficient is h_f so it is demarcated from the other one and now I have the thermocouple attached to the back. This is thermocouple and this is the model T infinity and of course a resistance R, we have already done that earlier. Therefore now I can write a heat balance under the steady state when nothing is changing with respect to time. Steady state means no changes with respect to time.

The heat acquired by the surface is h_f multiplied by the surface area. This is by definition of the heat transfer coefficient, h into area into temperature difference T_f minus T subscript t which is the temperature indicated here that is T_t, this must be equal to the expression we wrote down just a little while ago. It will be given by kA product divided by R multiplied by T_t minus T infinity. So now I can solve for the temperature indicated by the thermocouple or a 2 factor. So you can see that I can transpose 2 T_t multiplied by $h_f S$ plus square root of kA by R is equal to $h_f S$ into T_f plus square root of kA by R into T infinity. Or I can take this to the denominator and I can therefore write T_t equal to finally h_f into S into T_f plus square root of kA by R into T infinity whole divided by h_f into S plus square root of kA by R.

This expression indicates that the temperature indicated by the thermocouple, that is the left hand side, this is what is measured, is a weighted mean of the temperature T_f, the weighting factor being h_f into S, so this is the weighted mean of T_f multiplied by the weight factor $h_f S$ plus the weighted mean of T_f with this factor, T infinity with the factor square root of kA by R. That means I am not going to get either T_f or T infinity. Depending on the particular case, I will get a temperature which is the mean of these two with the weights being given by $h_f S$ on one side and square of root kA by R for the other side. So, you can immediately see that, if this term $h_f S$ is very large compared to this term, T_t will be close to T_f, so that is what we would like guarantee or make sure that the temperature measured by the thermocouple attached to the surface is going to actually indicate more closely the temperature of the fluid rather than the temperature of the ambient which is at the back.

That means that if I reduce this term by the proper choice of the insulating material and so on and also making sure that there is not too much of convection at the back side and if I make sure that this product of h_f into S,

if h is not very large and if I make S large enough then this product becomes large enough. Therefore the idea is to increase the product of the heat transfer coefficient on the fluid side and the area of contact between the fluid and the solid. If I increase it, then T_t will approach T_f. And in fact we can say that, the temperature difference between T_t and T_f is actually the error in this particular arrangement. So, if we want to minimize the error you have to increase this factor as compared to this factor. That is what the analysis shows. I am not going to work out any problem but just to get a physical idea we have to look at this problem.

Now we will go back to the slide which we were looking at and take an example which is very common in practice.

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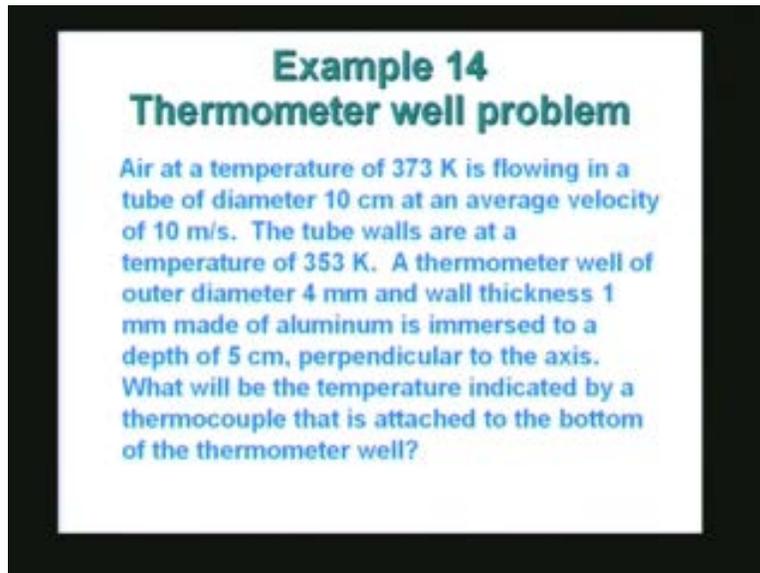
In fact I have already referred to this earlier that we use a thermometry well or a thermo well in applications where we are measuring the temperature of fluid which is flowing inside a tube and we would not like to have a contact between the thermal sensor which we are going to use, may be a thermocouple or may be some other thermometer. We do not want to have a direct contact for some reason or the other. Then, we have no option but to use what is called the thermometer well and the design of the thermometer well is an important aspect when you want to minimize the error between the temperature indicated by thermocouple and the temperature of the fluid whose temperature you want to measure.

A typical example is, a steam carrying pipe which is very common in process applications. The steam may be at a high pressure or may be 100's of bars. So we cannot have direct insertion of the thermocouple because that will lead to leakages and other problems. Therefore a thermometer well is built in to the system and the thermocouple is attached to the bottom.

In fact we have already seen this kind of an arrangement and we indicated the mean, physical and thermal parameters which are going to play a role in this. So, I have taken a simple example, through this example I am going to just look at how we are going to get the thermometry error in this particular case. And also we will able to decide what are the parameters which are important or which have to be chosen properly so that the temperature error is minimized.

We have air at a temperature of 373 Kelvin flowing in a tube of diameter 10 centimeters and its average velocity is measured and is equal to 10 meter per second and the tube walls are at a slightly lower temperature, in this case it is 353 Kelvin and a thermometer well whose outer diameter is 4 millimeter and whose wall thickness is 1 millimeter and made of aluminum which is just an example, aluminum may not be the most suitable material. We just to want indicate how the calculation can be done. It is immersed to a depth of 5 centimeters that is 1/2 the diameter of the pipe and it is inserted perpendicular to the axis and we would like to find out what is the temperature indicated by a thermocouple that is attached to the bottom of the thermometer well.

Ideally speaking, we would like to have the temperature indicated equal to the temperature of the fluid which is 373 Kelvin and anything different from that is certainly an error. And let us look at what is going to happen in this case. We have 373 Kelvin which is the temperature of the fluid and the wall is at 353 Kelvin and the thermometer well is connected between the fluid and the wall and therefore there is heat transfer from the fluid to the wall through the well. This is basically a conduction error problem and the heat transfer to the well is by convection from the fluid and the heat loss to the wall is by conduction through the well material. So, with this background let us just look at what are the things involved. In fact I am going to go back to the board and indicate some of the details.

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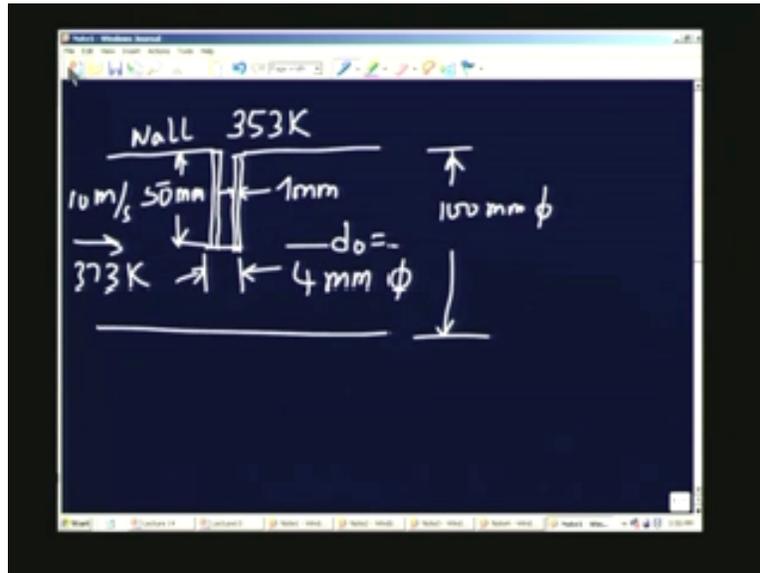
Just to recapitulate we have the wall of the tube and this is the axis and as the well right down into the middle, it is made of the material which is certain wall thickness 1 millimeter and this is 4 millimeter diameter if I call it d_0, outer diameter and the wall thickness is 1 millimeter and this depth of immersion is 50 millimeters, the tube diameter is 10 centimeters and therefore indeed it is equal to 50 millimeters, this is 100 millimeter diameter. And the wall is at 353 Kelvin and the fluid is at 373 Kelvin and the velocity is 10 meter per second. So let me just draw a simple sketch of the well which is slightly magnified so that we can look at what is going to happen.

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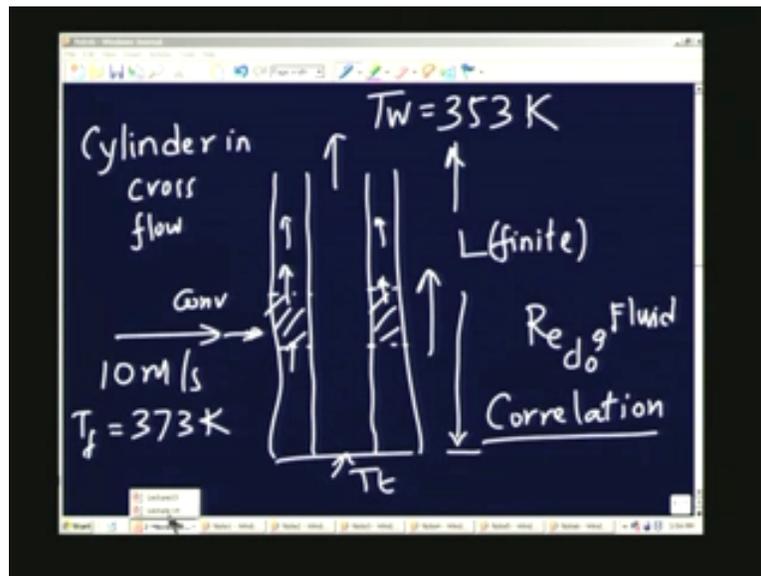
So I will just draw, this is the well with its thickness as shown here. The fluid is flowing at the rate of 10 meter per second and the thermocouple is attached here T subscript t, and this side is connected to the wall at T_w equal to 353 Kelvin, so here T_f is 373 Kelvin. So this is a 20 degree difference. So the heat, the surface of the tube, the outer surface of the tube, suppose I will take a look at a small piece of the tube again, I look at this small piece of the tube. The heat is gained by convection, so this is convection and because the temperature is varying in this direction being higher here, T_f is higher possibly the temperature of the well is in between these two, that is in between 373 and 353. Therefore the heat transfer from here to the well material and then there is conduction along in this direction.

The model is exactly similar to what we had earlier in the case of the lead wire model, there is no difference really. The only difference between the lead wire model which we had earlier and this case is that the lead wire was of infinite length or very large length therefore we had a simple solution to the problem. Here we have a finite length for the immersed well, this L is given from the wall to the depth here, this length is the finite quantity, finite length. So the governing equation is somewhat similar to the previous case and again I am not going to go through the calculation procedure but what I am going to do is just to indicate how to make the calculation.

The heat transfer coefficient or heat transfer from the fluid to the outside of the cylinder is treated as cylinder in cross flow and the heat transfer coefficient depends on some specific parameters. The heat transfer depends on the Reynolds number based on the diameter of the tube, the outer diameter of the tube, outer diameter of the well and then it depends on the fluid properties. In this case we have air flowing across the tube, therefore the properties of the air are going to come into the picture.

Of course Reynolds number itself contains the property of the fluid going into this calculation. This relationship between the convective heat transfer coefficient and these properties or these number parameters comes from knowledge of fluid mechanism and the student will certainly learn these things in a little later stage in the course on heat transfer.

But right now what we will do is simply assume that this amount of knowledge is available in the form of a simple recipe. So recipe will be usually represented as a correlation. Just for calculation purposes, it is not necessary to know exactly how the things come out, but it is enough to understand the correlation and get a working knowledge so that we can just make some calculation. That is the idea with which we are presenting these materials now. So with this let us look at the details of the problem. So we have to calculate the heat transfer coefficient on the outside which is calculated based on the properties of the fluid and the velocity of the fluid which is given, the temperature and so on.

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So what are the properties we require? We require calculating the Reynolds number. It is defined as the velocity U multiplied by the diameter of the tube, when I say tube it is the well I am talking about, outside diameter of the tube which makes out the well and the viscosity of the fluid which is moving. This is the kinematic viscosity mu and from the tables of air properties, we take these values at a temperature. In this case the temperature of the fluid is 373, the wall temperature is 353.

So, for the sake of taking the values of the properties at the intermediate temperature in between the average temperature between these two, that is good enough for this purpose. So the value of viscosity is 23 point 02 into 10 to the power of minus 6 in meter square per second, then the thermal conductivity is point 03127 watts per meter Kelvin. The prantle number is a parameter which describes one of the properties of the fluid which comes into the picture when you have heat transfer calculations being done. So it is equal to point 7 in the case of air and the diameter outside is going to come in to picture is 4 into 10 to the power of minus 3 meter. I am converting millimeters to meters. U is 10 meters per second, therefore I am able to calculate the Reynolds number as the product of Ud_0 divided by mu which is 1 point 738 10 to the power of 3 or 1738.

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Zhukauskas correltion

- For cylinders in cross flow:

$$Nu_D = C\,Re_D^m\,Pr^n \left\{\frac{Pr}{Pr_s}\right\}^{0.25}$$

- All properties at film temperature except Pr_s which is at surface temperature.

The value of the Reynolds number, the prantle number and so on, we will have to put into this recipe. I talked about the correlation, it is called the zhukauskas correlation, I think there is a spelling mistake here zhukauskas correlation. For cylinders in cross flow, the information is represented in the form of a relation of the form Nu_D, this is called the Nussle number which is given by a constant C multiplied by Reynolds number to the power of m where d is the characteristic dimension, in this case it will be d_0, prantle number to the power of n and then the ratio of Pr by Pr_s to the power of point 25 where S stands for the surface temperature.

In the present case, these two are close to each other therefore this factor is not important. Only for those fluids where the prantle number is a strong function of temperature we have to worry about this factor and therefore for the present this factor is not going to come to picture. That is, all properties are taken at the film temperature which is nothing but the average between the surface temperature and the fluid temperature excepting the one here which is taken at the surface temperature. But in this present case we need not worry about this factor. The values of C,m and n are specified in the table which is given below.

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Coefficients in the correlation

- Coefficients C and m are given in the following table:

Re_D	C	m
1 - 40	0.75	0.4
40 - 10^3	0.51	0.5
$10^3 - 2 \times 10^5$	0.26	0.6
$2 \times 10^5 - 10^6$	0.076	0.7

These are all based on experimental data. Experiments have been conducted, the cylinder in cross flow with different Reynolds number ranges and the relationship has been plotted and using the methods we have already discussed earlier, curve fitting and so on based on the curve fit values of C, m and n have been obtained. So there are several ranges of Reynolds number 1 to 40 the values are point 75 and point 4, 40 to 1000, point 51 to point 5, 1000 to 2,00,000 is point 26 and point 6 and 2,00,000 to about 1,000,000 it is .076 and .7.

Why do we have different values for different ranges for Reynolds number? It is because the range of the Reynolds number determines the regime of flow which is taking place. We have essentially laminar flow for very low velocities or very low Reynolds number and as the Reynolds number increases the flow becomes more and more it goes towards turbulent flow and there are variations from laminar flow to turbulent flow through a series of steps like this. So the reason why we have different values for C and m because the regime is changing slowly from laminar flow to fully turbulent flow and that is represented by this kind of a data.

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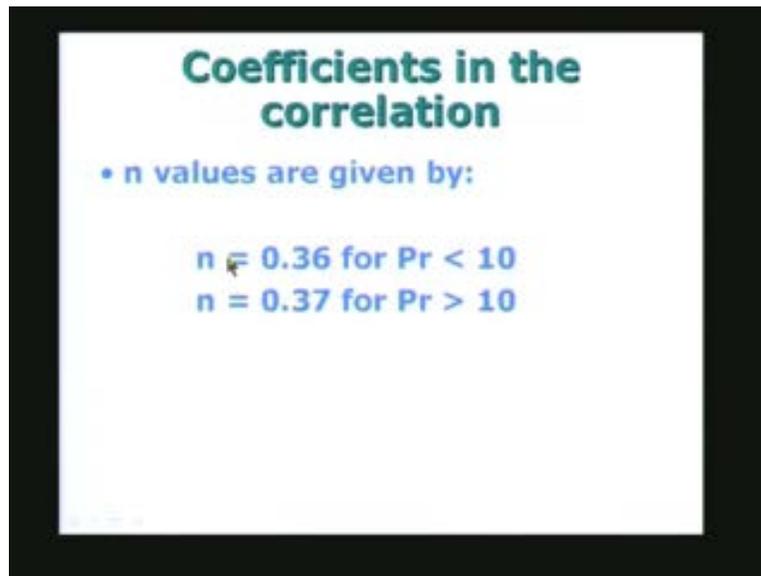
And if you remember in the last slide, we calculated the value of the Reynolds number to be somewhere in between it was 1700 and something, so it comes in the third row here and the n values are given by for prandtl number less than 10 and greater than 10 and this is appropriate for the present case because prandtl number is point 7 in the present case and for the range of Reynolds number, the Reynolds number being 1730 or something like that, the value of C and m are given here are .26 and point 6, n is point 36 because prandtl number is less than 10.

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Calculation of h

Zhukaskas correlation is used now.
$C := 0.26 \quad m := 0.6 \quad n := 0.36$

$Nu_{do} := C \cdot Re_{do}^{m} \cdot Pr^{n}$ or $Nu_{do} = 20.1$

Hence $h := \dfrac{Nu_{do} \cdot k}{d_o}$ or $h = 157.13$

:W/m²-K

So the Nusselt number based on the outer diameter of the well is given by the product of C point 26 multiplied by the Reynolds number which was already calculated to the power of point 6, prandtl number point 7 to the power of point 36 and that gives you 20 point 1. This Nusselt number is a non-dimensional parameter.

It is also called as non-dimensional heat transfer coefficient, and the relationship between the heat transfer coefficient and Nusselt number is such that heat transfer coefficient is equal to Nusselt number multiplied by the k of the fluid divided by the diameter of the characteristic dimension in this case which is equal to d_0 and, if I substitute 20 point 1 here, k is point 03 something that was given in the first slide divided by point 004, I get the value of h equal to 157 point 13 watts per square meter Kelvin.

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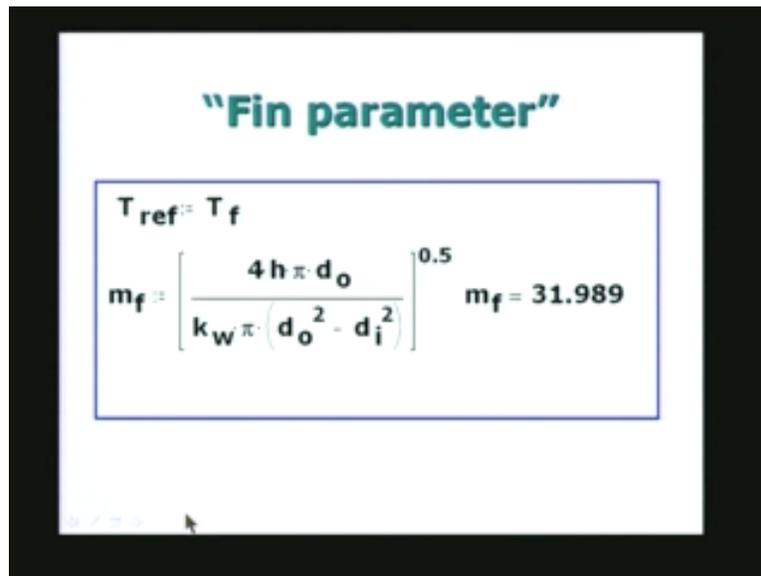
So, we have calculated the value of the heat transfer coefficient. Now, what we have to calculate is what is called the fin parameter. The fin parameter appears in the solution to the problem when you model the heat transfer in the wall of the well, the heat transfer conduction is in the axial direction and the heat transfer by convection is in the direction normal to the axis of the cylinder or axis of the well. In this case we can show that the governing parameter for heat transfer in the fin is given by m subscript f, it is called the fin parameter which is given by h into the perimeter which is pi d_0, pi d_0 is outer perimeter of that well material divided by the thermal conductivity of the well and the area of cross section of the material of the well.

It is given pi into d_0 square the outer diameter square minus d_i square which is the inner diameter square divided by 4 actually comes from pi d square pi into d_0 square minus d_i square by 4. This 4 is coming from this. So it is actually h into the perimeter divided by k into area of cross section, whole rise to the power of .5 and if I substitute the values I have obtained earlier, I will get a value of 31.989.

And the non dimensional governing parameter of the fin is given by the product of the fin parameter we calculated earlier at the depth of immersion. So the depth of immersion is making its appearance in this parameter and if I put L equal to 50 millimeters point 05 meters, so 05 multiplied by the value which was there about 31 in the previous case, I get a value of 1 point 599 and again it can be shown from the heat transfer calculation that the non dimensional temperature theta t. So, let me just go back to the board and discuss the temperature.

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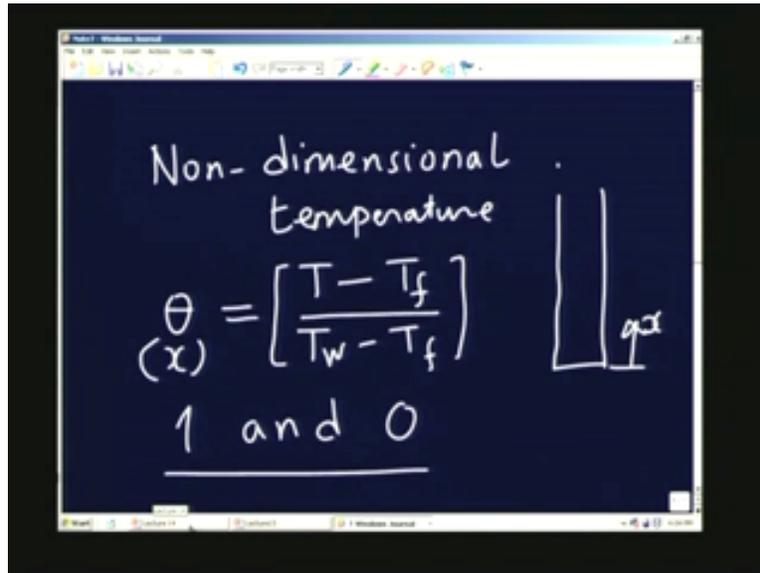
So, we have a non-dimensional temperature function which we can define in various ways, one of them is equal to T minus T_f minus T_w minus T_f. That means the value of this theta is equal to 1 at the place where the value is attached to the tube and it is equal to 0 as for the fluid is concerned. This varies between 1 and 0. So what I want is this theta is the function of the position along the fin. So there is a function of x measured, in this case the L is like this, I am measuring x, either I can measure it from here or from there, it does not matter. So it is measured like that. So, with this background, we can prove that the value of theta subscript t. This is the temperature at the bottom of the well is going by 1 over cos hyperbolic mu_f and if I put this 1 point 59 here, I get a value equal to point 388 and using the information which I just presented on the board the temperature indicated by the thermocouple attached to the bottom of that well is T reference which is taken as the temperature of the flowing fluid T_f plus $theta_t$ into T_w minus T_f.

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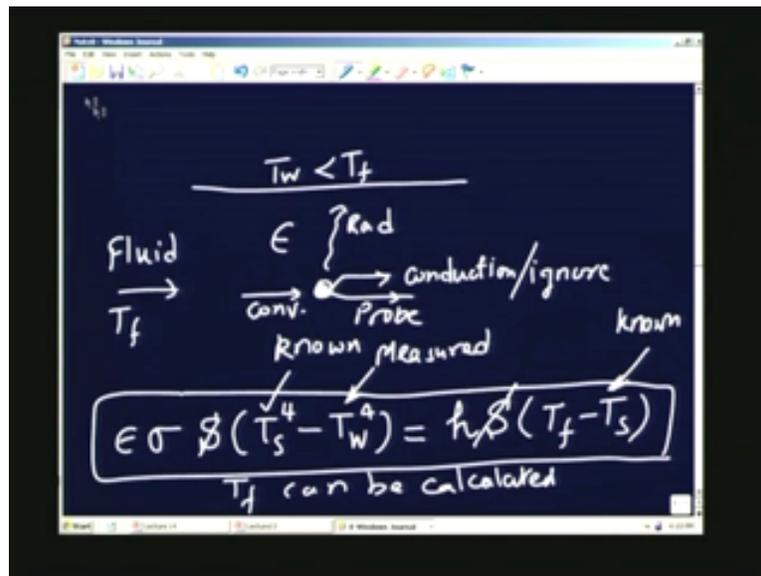
So, if I substitute these values of theta$_t$ point 388 here, T_w is 353, T_f is 373, I get a value of T subscript t equal to 365 point 24. So the temperature indicated by the thermometer which is connected or the thermocouple connected to the bottom of the well is not equal to the temperature of the fluid but it is 365.24 almost about 8 degrees different.

So we can now say that, when we measure the temperature of 365 point 24 the actual temperature of fluid is 373. The thermometric error or the systematic error due to the measurement is 365 point 24 minus 373 which is the difference between these two. So if I want to use this particular arrangement for the measurement of temperature of a fluid flowing through a tube, I will have to use this as a correction to the measured value because I am going to measure this value, these are the temperatures indicated by the thermocouple. So I am going to add this thermometer error which is calculated based on this kind of an analysis and correct for the temperature.
So the next example I am going to take, is going to be slightly different and I am going to look at the error due to radiation. So, the problem occurs like this, we have already indicated this earlier in the lecture number 13 in a sum but not too much in detail.

Suppose I have a tube carrying a fluid, let us say this is the probe, this is the wall which is at a temperature less than the fluid temperature, radiation loss will be there from the probe to the wall and there will be convection from

the fluid to the surface of the probe. Again you will see that, if I ignore there will be conduction here in general. So ignore this because the wires connecting the probe are usually very small in diameter, therefore the conduction may not be very important. Then all I have to do is, the analysis is going to be balancing the convective heat transfer from the fluid to surface of the probe by the loss due to radiation from the surface. And the loss due to radiation from the surface depends on the emissivity of the surface which I will indicate as epsilon.

Actually, if you write the expression for the radiative loss, it will be given by epsilon sigma, sigma is a stuffen boltzman constant multiplied by the surface area which we will say S is the surface area of the probe multiplied by T to the power of 4 of the sensor minus T_w to the power of 4. This must be equal to the heat transfer coefficient by convection multiplied by the area of surface which is again equal to S multiplied by T_f minus T_s , and this S will cancel off and you see that the temperature of the sensor now is going to be governed by this expression. So the sensor temperature is given by this expression.

Now, in actual practice what is going to happen? I know the reading of the instrument, therefore T_s is known, this is also known, this is also measured by possibly embedding a sensor in the wall of the tube. So this is also measured and known, therefore I can obtain T_f can be calculated. Actually what will happen is, I will show it in the next slide here.

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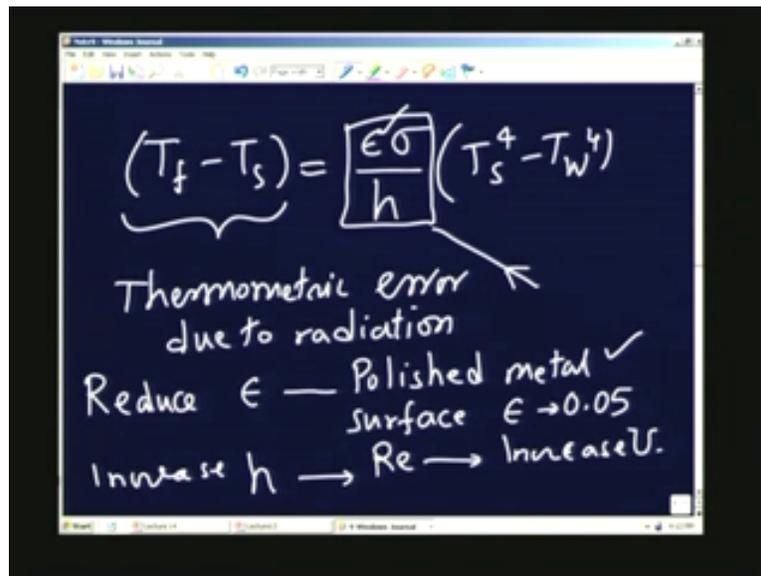
T_f minus T_s equal to epsilon sigma divided by h. I am just rewriting the previous expression. T_s to the power of 4 minus T_w to the power of 4 and this is your thermometer error, thermometric error due to radiation. So, it is a straight forward problem. So the magnitude of this term, the larger the value of epsilon sigma by h, the larger is the thermometric error. So how can I decrease the error? This is the question I would like to ask. The way to decrease the error is reduce the epsilon. So I will say reduce epsilon, epsilon is the surface property.

It is well known that if you have a highly polished metal surface, for example, aluminum, epsilon will be something like, tends to point 05. So I would be well off if I have a polished surface metal surface for the sensor. I would also like to increase h, and h as we know already depends on the Reynolds number and how do we increase Reynolds number? Increase U.

In fact if you remember in lecture number 13 I indicated that U can be increased by actually having some kind of an arrangement in which the fluid is accelerated and flows past the sensor at a higher velocity than it would do otherwise. You could have some kind of a small converging diverging nozzle and the sensor can be within that and in fact, the advantage of such on arrangement is that there is a radiation shielding also which is going to come in to the picture because the converging diverging nozzle can be made out of a material which is a highly polished material made of metal so that

the emissivity is small and at the same time it accelerates the fluid to a higher velocity and therefore the Reynolds number increases and when the Reynolds number increases the heat transfer coefficient increases. That means either we decrease epsilon or increase h, we are going to have an effect of reducing the thermometric error by radiation. So I think I just have enough time to take the example and look at that.

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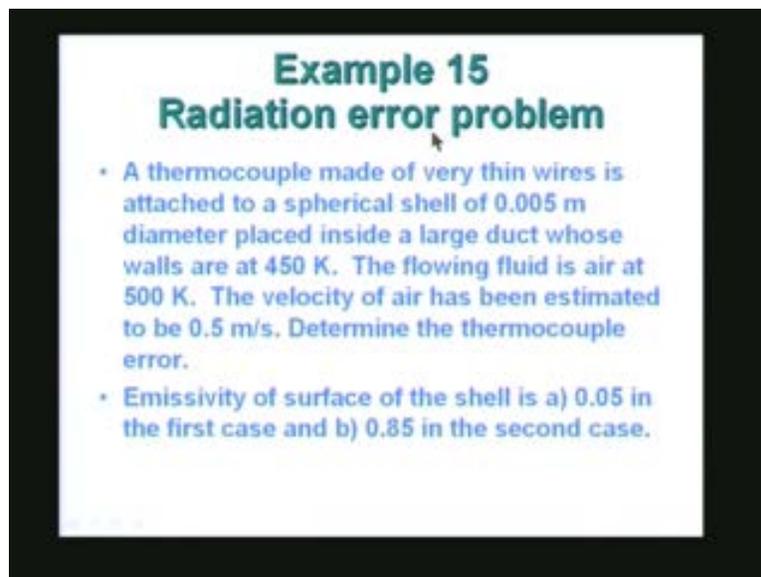
Example 15
Radiation error problem

- A thermocouple made of very thin wires is attached to a spherical shell of 0.005 m diameter placed inside a large duct whose walls are at 450 K. The flowing fluid is air at 500 K. The velocity of air has been estimated to be 0.5 m/s. Determine the thermocouple error.
- Emissivity of surface of the shell is a) 0.05 in the first case and b) 0.85 in the second case.

So the example number 15. A thermocouple is made of very thin wires, very thin wires are attached. I have said very thin wire so that the conduction through the lead wires can be ignored and it is to a spherical shell of point 005 meters placed inside a large duct whose walls are at 450 Kelvin. The flowing fluid is air at 500 Kelvin. The velocity of air has been estimated to be point 5 meter per second. So we want to determine the thermocouple error and two cases are taken; one in which the emissivity of the surface is point 05 and in the second case it is point 85. So we will quickly go through this example.

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Again the air properties are taken at the mean value as we did in the previous case and the values are taken at 475 Kelvin, 450 and 500, these are the two limits. mu is 34 point 09 into 10 to the power of minus 6 square meter per second, k is point 0489 watts per meter Kelvin all in SI units, U is point 5 meter per second that is given and the diameter is point 005 meter and it is a spherical bead like structure, a spherical sphere over which the flow is taking place and we can use a simple relationship between Nussle number and Reynolds number just like what we did earlier, this relation is recommended by McAdams. It is valid from 17 less than Reynolds number to 70,000 and it is taken from the book by Holman which is a very popular book.

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Calculation of h

Using relation recommended by McAdams:
(valid for 17 < Re < 70000)
Taken from the book Heat Transfer,
Seventh Edition by J.P.Holman, p.305.

$Nu = 0.37 \cdot Re^{0.6}$ or $Nu = 4.87$

Heat transfer coefficient then is:

$h = \dfrac{Nu \cdot k}{D}$ or $h = 47.61$:W/m²-K

Nussle number is equal to point 37, Reynolds number is point 6 or if you put the value of Reynolds number calculated earlier you get a value of 4 point 87. And from Nusselt number, we again calculate the heat transfer coefficient Nu k by D. This is the definition. h is equal to 47 point 61 watts per square meter Kelvin and we will go to the two cases.

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Data for case (a)

a) The thermocouple indicates the equilibrium temperaturewherein the heat transferred from air to sphere is equal to the heat loss from the sphere to the background.

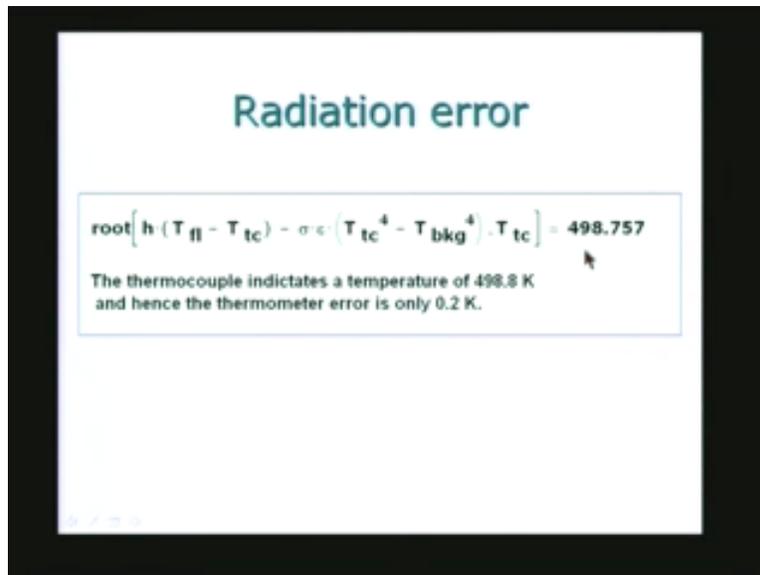$\varepsilon := 0.05$ $\sigma := 5.67 \cdot 10^{-8}$ W/m²-K⁴

Let $T_{tc} := 470$ $T_{fl} := 500$

and $T_{bkg} := 450$:K, respectively.

Data for case (a) the thermocouple indicates the equilibrium temperature wherein the heat transferred from air to sphere is equal to the heat loss from the sphere to the background is just what we explained on the board, epsilon is point 05, sigma is 5 point 67 into 10 to the power of minus 8 which is the Stephen Boltzman constant.

Let the thermocouple temperature…….., we do not know, we have not given the temperature indicated by the sensor. In this case the sensor temperature is to be determined, I will say T subscript tc is equal to 470 as a starting point, then the fluid temperature is 500 and the background temperature 450, these are already given, all temperatures in Kelvin and when we are calculating anything to with radiation you have to use the temperatures in the Kelvin degree. And I have to solve the equation h into T_f fluid minus T_{tc} minus sigma epsilon into T_{tc} to the power of 4 minus T background to the power of 4. This solution can be done by Newton Thompson method and I get a value of 498.757 which is rounded of to 498.8 and the thermometer error is only 2.

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Radiation error

$$\text{root}\left[h \cdot (T_{fl} - T_{tc}) - \sigma \epsilon \cdot \left(T_{tc}^4 - T_{bkg}^4\right) \cdot T_{tc} \right] = 498.757$$

The thermocouple indictates a temperature of 498.8 K and hence the thermometer error is only 0.2 K.

When epsilon is equal to point 05, the error due to radiation is negligible. However in the case (b), all I had do is to change epsilon to point 85, all other things are the same and the I solve the same equation to get a value of 485 point 34 and the error is almost equal to 15 degree 14 point 7 Kelvin.

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So the radiation error is very significant. So, when do you get a value of point 85. For example, in practice if you start with a very highly polished material, and if it is in service for a long time and the surface gets oxidized because of the flowing fluid and the impurities and so on, the emissivity will degrade with time. Of course, it may not degrade up to point 85 but certainly there will be a degradation and therefore if you want to keep the radiation error under check periodically, the surface must be cleaned and polished again and again and put back to service. So the observation from this particular example is that surface emissivity of the sensor plays a very important role in this particular case.

Therefore, in the last two lectures, lecture number 13 and lecture number 14, we have looked at systematic errors in a measurement of temperature and various situations. In lecture number 15, we are going to look at the measurement of temperatures which are varying with respect to time, the transient temperature measurement is what we are going to look at in the next lecture. Thank you.