# Mechanical Measurements and Metrology
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## Module -2
## Lecture - 10
## Overview of Thermometry

This will be lecture number 10 in our ongoing series on mechanical measurements. If we recall, in lecture number 9, we had introduced the fundamental ideas of thermometry. The basic idea was to describe how to come out with the temperature scale, which can be repeated or replicated. It required the use of different thermometers to cover different ranges and I was explaining that there should be some overlap between different instruments which are going to be used to measure the temperature, depending upon the ranges. What we will do now in this lecture is to start with, complete that discussion, and then we will go and look at, or take a look at, an overview of thermometry, which consists of just describing, without going into too much detail, the different types of thermometers that can be used. And then we will start taking up one by one for a detailed study. The first one which we will take up in this lecture would be the thermoelectric thermometer or the use of thermocouples to measure temperature.

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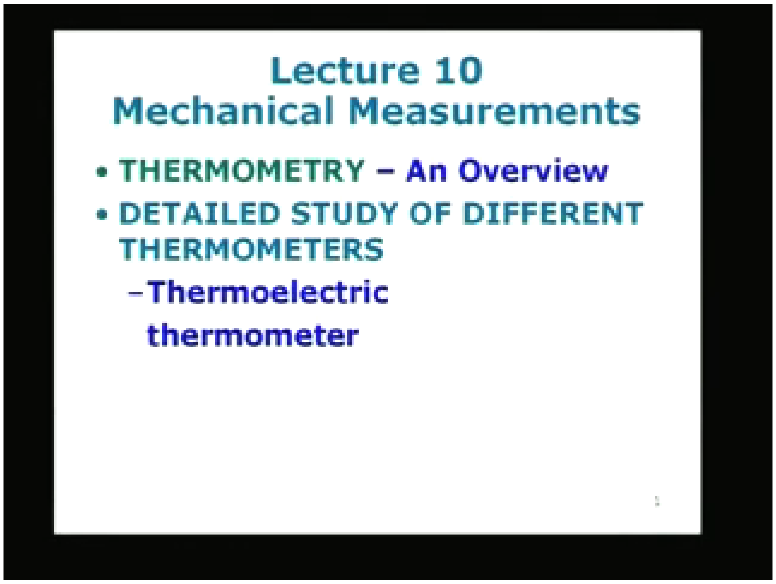

Lecture 10
Mechanical Measurements

- THERMOMETRY – An Overview
- DETAILED STUDY OF DIFFERENT THERMOMETERS
  - Thermoelectric thermometer

To just recall what we were doing in the last lecture we were talking about several different thermometers being used to cover the entire range of temperature from the lowest about 0.01 Kelvin to the highest about 100,000 Kelvin. That was the range which we mentioned, and we said that it is not possible to cover the entire range using the single thermometer. Therefore, it is essential to have different thermometers. We also introduced the concept of the single reference point or single fixed point for thermometry, which was the triple point of water. We also described in some detail the functioning of a constant volume gas thermometer and we indicated how it can be used to define temperature as the ratio of pressures. So we will start from there, and now what we are trying to do is to see how to use, or how to measure using different thermometers, and obtain values which are in agreement with each other.

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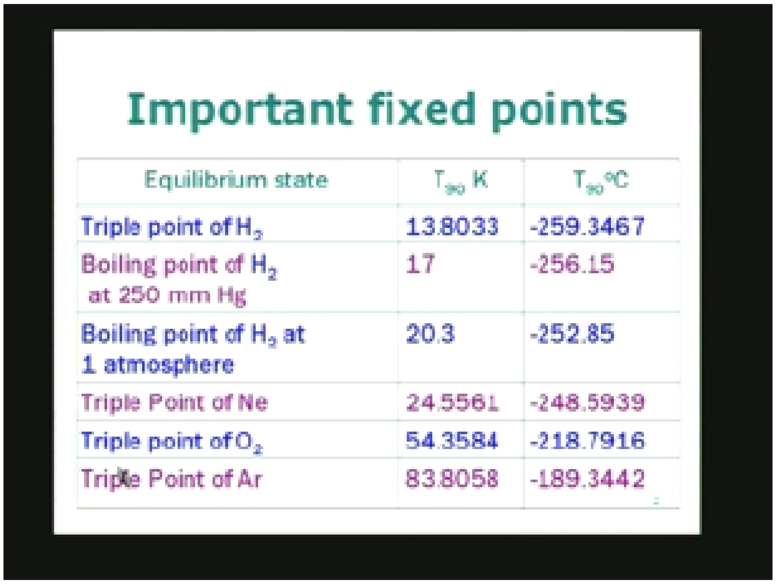

## Important fixed points

| Equilibrium state | $T_{90}$ K | $T_{90}$ °C |
|---|---|---|
| Triple point of $H_2$ | 13.8033 | -259.3467 |
| Boiling point of $H_2$ at 250 mm Hg | 17 | -256.15 |
| Boiling point of $H_2$ at 1 atmosphere | 20.3 | -252.85 |
| Triple Point of Ne | 24.5561 | -248.5939 |
| Triple point of $O_2$ | 54.3584 | -218.7916 |
| Triple Point of Ar | 83.8058 | -189.3442 |

This is what was explained in the last lecture. For doing this, we require a certain number of subsidiary or secondary fixed points, and those fixed points are given in the tables which you see now. The fixed points correspond to equilibrium states and the subscript 90 here—for example, I say $T_{90}$ Kelvin—that means this temperature, which is assigned in this table, is according to the international temperature scale 1990. The third column gives $T_{90}$ in degree Celsius. That means this is the corresponding temperature, corresponding to $T_{90}$ Kelvin in Celsius units. So the various fixed points which are used for thermometry are given by the triple point of hydrogen, which is assigned a value of 13.8033 corresponding to minus 259.3467 degree Celsius. Then we use boiling point of hydrogen at 250 millimeters of mercury, which gives a temperature of 17 Kelvin, or corresponding to minus 256.15 degree Celsius. Then, boiling point of hydrogen at 1 atmosphere corresponds to 20.3 Kelvin or minus 252.85 Celsius. Then as you go along, you see here several other fixed points like the triple point of neon—Ne stands for neon—which is 24.5561, triple point of oxygen is 54.3584, and triple point of argon is 83.8058. So if you look at the table, the temperatures are going in the ascending order. From the lowest value of 13.80, we have come up to 83.8058 Kelvin or minus 259.3467 degree Celsius to minus 189.3442, and we need more fixed points to cover the entire range of interest to us and they are given in the next slide. So the fixed points to cover the higher temperature range are given in this slide: triple point of mercury, triple point of water, in fact if we remember or recall 273.16 Kelvin triple point of water was considered as the first single fixed point for thermometry. Then we have melting point of gallium, which is about 29.7646, which is very close to the room temperature, normal room temperature. Freezing point of indium is 156.59; then, freezing point of tin, freezing point of aluminum and the freezing point of silver, which corresponds to 961.78 degree Celsius or 1234.93 Kelvin.

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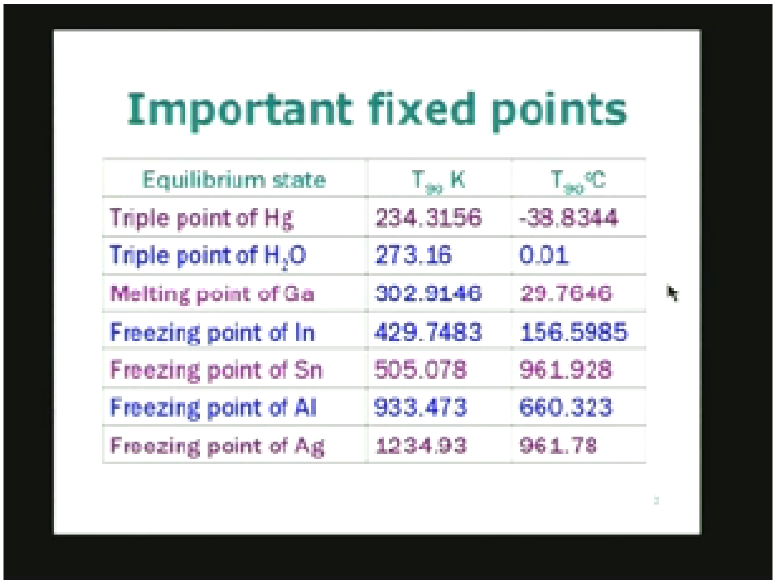

## Important fixed points

| Equilibrium state | $T_{90}$ K | $T_{90}$ °C |
|---|---|---|
| Triple point of Hg | 234.3156 | -38.8344 |
| Triple point of $H_2O$ | 273.16 | 0.01 |
| Melting point of Ga | 302.9146 | 29.7646 |
| Freezing point of In | 429.7483 | 156.5985 |
| Freezing point of Sn | 505.078 | 961.928 |
| Freezing point of Al | 933.473 | 660.323 |
| Freezing point of Ag | 1234.93 | 961.78 |

So if you look at the fixed point we have, there are several fixed points covering the range from the lowest temperature to the highest possible, and in fact, there are temperatures higher than this, which we have not included here. We will come to that in a little while from now. So the ITS 1990—which is a modification of IPTS 1968, the international practical temperature scale of 1968—ITS 1990, includes some modifications.

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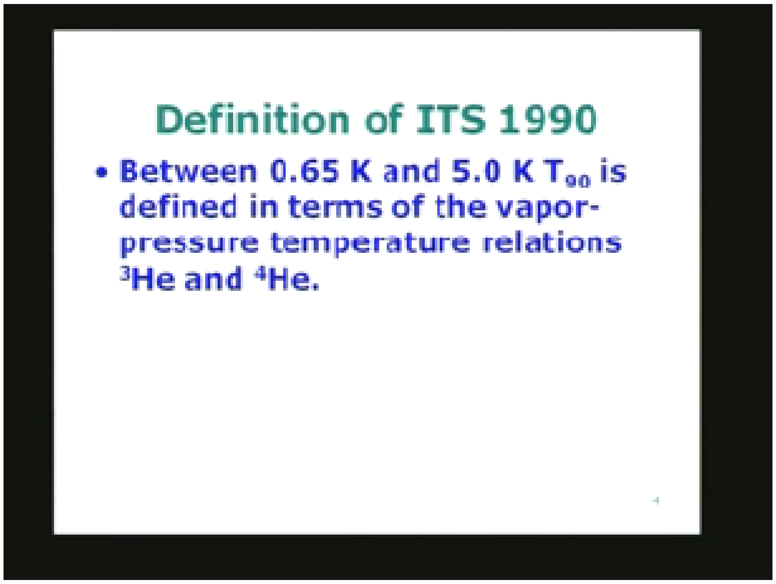

Definition of ITS 1990

- Between 0.65 K and 5.0 K $T_{90}$ is defined in terms of the vapor-pressure temperature relations $^3He$ and $^4He$.

Some of them are minor, some of them are more important modifications. Of course we will not go into the history of these things. What the ITS 1990 does is to provide the secondary fixed points, which are given, accurate, and acceptable values of temperature as given in the previous 2 slides. It also specifies how to use these fixed points and what kind of instruments you must use to determine the temperature. That is what the ITS 1990 is doing for us. For example, between 0.65 Kelvin and 5 Kelvin, $T_{90}$ is defined in terms of vapor pressure temperature relations for helium 3 and helium 4. So using this vapor pressure temperature relationship for helium 3 and helium 4, we will be able to cover the range between the 0.65 and 5 Kelvin. This is one of the lowest ranges.

And between 3 Kelvin and the triple point of neon, which corresponds to 24.5561 Kelvin, it is defined by helium gas thermometer, the constant volume gas thermometer, which uses helium as the thermometric substance and it is calibrated at 3 experimentally realizable temperatures having assigned numerical values. Actually, you will see if you go back to the previous 0.65 to 5, and here we have 3 to 24; therefore, there is an overlap between the two. That means, I can calibrate this with respect to the other one by using this overlap, which is going to cover the two ranges in a certain portion. It also specifies what kind of fixed points have to be used and the interpolation procedure that has to be used is also specified by the ITS 90.

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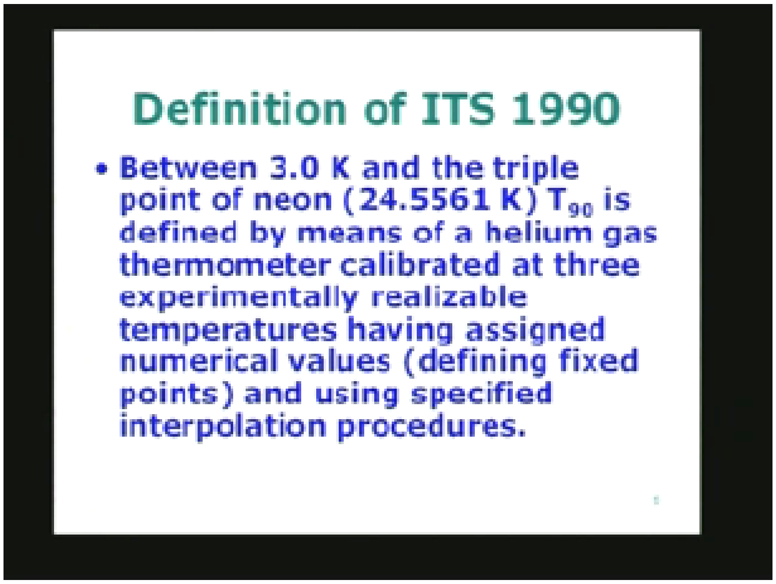

## Definition of ITS 1990

- Between 3.0 K and the triple point of neon (24.5561 K) $T_{90}$ is defined by means of a helium gas thermometer calibrated at three experimentally realizable temperatures having assigned numerical values (defining fixed points) and using specified interpolation procedures.

Between the triple point of equilibrium hydrogen, hydrogen occurs in Para and two forms, and the equilibrium hydrogen contains two forms of hydrogen in the equilibrium proportion. That’s what we mean by triple point of equilibrium hydrogen and it is 13.8033 Kelvin, and the freezing point of the silver, which is the highest temperature, we saw 961.78. This entire range is defined by means of platinum resistance thermometers, calibrated at specified sets of defining fixed points and using specified interpolation procedures. That means we are covering the entire range from the lowest, which was 0.65 Kelvin, to the highest, which is 961.78 degree Celsius by using different thermometers and different procedures for making them agree with each other by using suitable interpolation formulae.

And above the freezing point of silver, which is 961.78 degree Celsius, $T_{90}$ is defined in terms of a defining fixed point of that silver point and the Planck radiation law. Planck radiation law relates the emissive power of black body radiation with temperature by unique relationship and that becomes the definition of the temperature that is above the silver point.

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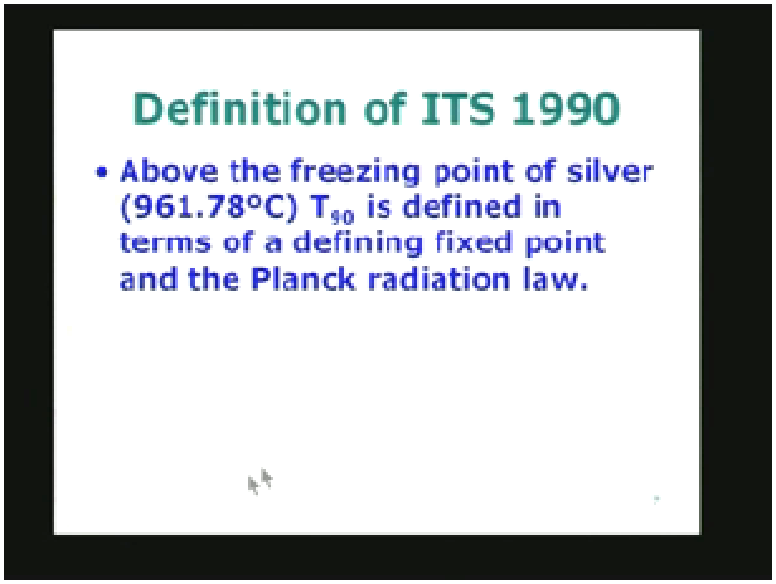

### Definition of ITS 1990

- Above the freezing point of silver (961.78°C) $T_{90}$ is defined in terms of a defining fixed point and the Planck radiation law.

So basically, ITS 90 is giving us a method by which thermometry can be practiced; it tells us about the fixed points which have been assigned some specific numbers which came out of a large amount of experience gathered over a long period of time, and it also tells us about the different thermometers that have to be used; and therefore, the legitimate question is, how do we make a thermometer?

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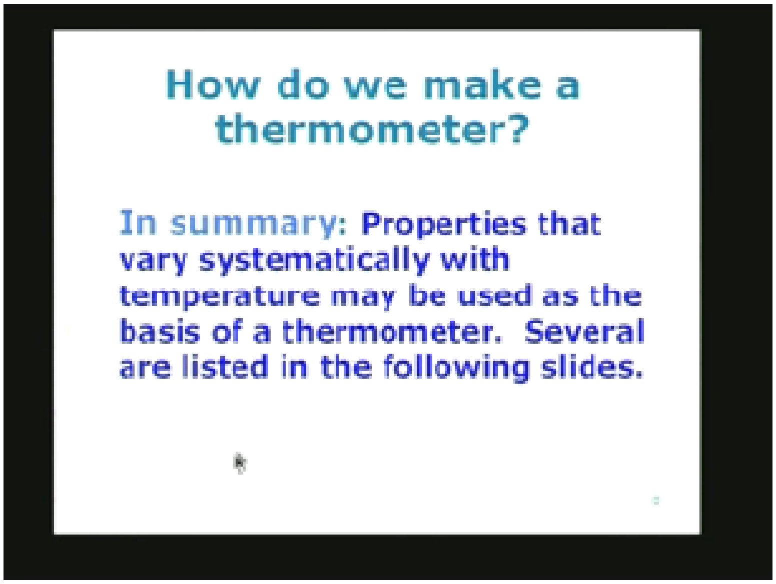

### How do we make a thermometer?

In summary: Properties that vary systematically with temperature may be used as the basis of a thermometer. Several are listed in the following slides.

What kind of a process is involved in that? And in summary, by looking at what we have discussed in the last 1 hour, we can immediately say that properties that vary systematically—in fact that is how we started the whole thing—properties that vary systematically with temperature may be used as the basis for any thermometer. And in fact, several are going to be listed below, and we will look at them one by one in the order of their importance, order of their applicability, and the order in which they are found to be used in practice. More often we find thermoelectric thermometers used, and therefore, I have taken that as the number 1.

So the thermoelectric thermometry consists of the use of thermocouples. We'll come to it a little while from now; it means use two wires of different materials, make a junction between them, and subject the junction to a different temperature. You have essentially two junctions, one which we call as the measuring junction and other is the reference junction. As the temperatures are different you generate an emf and that emf is the thermometric property and there is a unique relationship between this emf and the temperature. The next one is the use of electrical resistance as the thermometric property.

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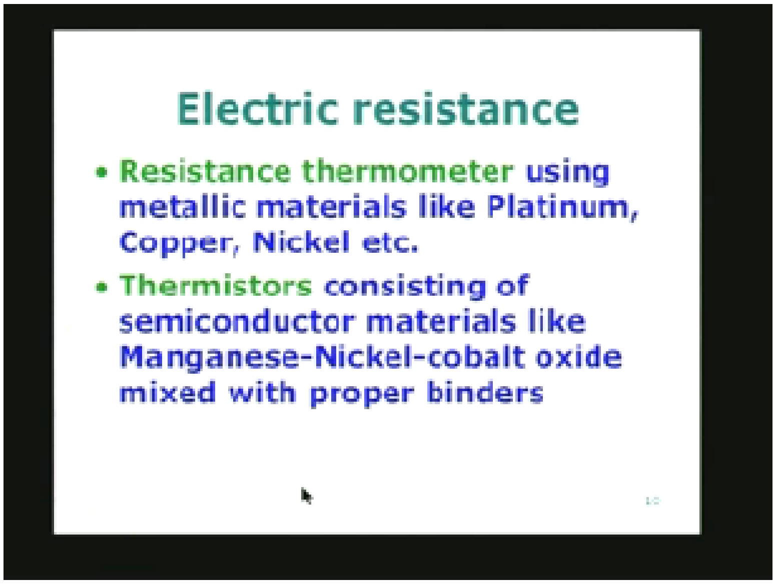

### Electric resistance

- Resistance thermometer using metallic materials like Platinum, Copper, Nickel etc.
- Thermistors consisting of semiconductor materials like Manganese-Nickel-cobalt oxide mixed with proper binders

There are two types of electrical resistance thermometers: the number 1 is resistance thermometer which uses materials like platinum, copper, nickel, etc. If you take an element of copper or platinum in the form of a coil and change the temperature of the coil by subjecting it to an environment of whatever temperature we want to measure, the resistance changes with temperature. Actually, the resistance increases with temperature and this increase with temperature is characterized by a regular relationship, and that relationship itself becomes the basic interpolating formula for the platinum resistance thermometer.

In principle, we can also use copper, nickel and other materials, as long as we standardize them and we characterize the relationship between the resistance and the temperature unequivocally. The other kinds of electrical resistance thermometers are called thermistors. These are actually semiconductor materials, which consist of manganese, nickel, cobalt oxides, and mixed in proper proportion with proper binders and these can be made into very small elements and the resistance of the thermistor element is a very strong function of temperature. If you remember I said the resistance thermometer which uses platinum as an element, the resistance actually increases the temperature. In the case of thermistors the temperature variation is actually the opposite, so if we increase the temperature the resistance decreases rapidly. The difference between the platinum resistance thermometer, or the first type, and the thermistor is that the thermistor is extremely sensitive and it can be used over a small range because the resistance change is very rapid over a small range of temperatures, it is very sensitive, but the range is limited. In the case of platinum resistance thermometer, it is exactly the opposite. The resistance change is very slow or very small. The rate at which it changes the temperature is very small, but it can be used over a very large temperature range.

If you recall in the ITS 90, we used essentially the platinum resistance thermometer to measure the temperature from triple point of hydrogen. You can go back to the slide and see what we did there between the triple point of hydrogen 13.8 and the silver point 961.78. Actually we are using the platinum resistance thermometer as the specified thermometer, to cover that range. Therefore, you see that the range is very large: 13 to 961, but the change resistance is not very large. Therefore it is not very sensitive in that sense of the term, but you can use repeatedly to cover a large range of temperature.

The third one is that which uses thermal expansion. In fact we know that most materials will expand on heating and the change in the volume can be used as the thermometric property. In fact we have several of them. The liquid-in-glass are very common; for example, we use mercury, another organic liquid—for example, some organic liquids are used and it is taken in a small capillary with a bulb and the bulb is a reservoir to hold the material, and the capillary gives an amplification to the change in volume in terms of the length.

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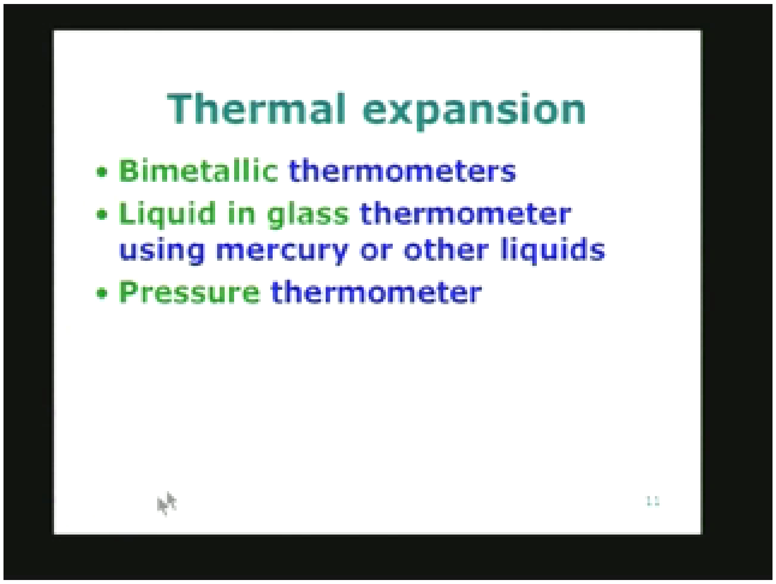

## Thermal expansion

- Bimetallic thermometers
- Liquid in glass thermometer using mercury or other liquids
- Pressure thermometer

So liquid-in-glass thermometers or liquid-in-metal thermometers are very common, and these are used in day-to-day practice. We will not be considering too much details of this because there is not too much of detail one has to know. There are other thermometers like bimetallic thermometers, which actually are made by taking two different materials having two different coefficients of expansion, and placing them together intimately. When you subject it to temperature, one of them will expand more and the other one will expand less. Therefore the element, if it is straight to start with, will start bending. And when it bends, the amount of bend can be a measure of the temperature. It can also be used in many applications; it can be a thermal switch. When it bends it will make or break a contact and therefore it can be used as an electrical switch also. That means at a preset temperature, it will open the switch and yet again when it cools back, it will remake the contact and so on and so forth.

We can also measure, instead of measuring the volume directly, we can confine the material to a constant volume or take it as a rigid vessel and when you increase the temperature the pressure is going to increase because we are not allowing the expansion of the liquid to take place. It means you are converting it to a pressure and pressure can be measured using a pressure transducer, or some such arrangement, and therefore, thermal expansion basically consists of three different types of instruments, which are given here.

And if you remember or recall the ITS 90, we said that above the silver point, temperature is measured using the radiation of the black body radiation law, which is basically Planck’s radiation function.

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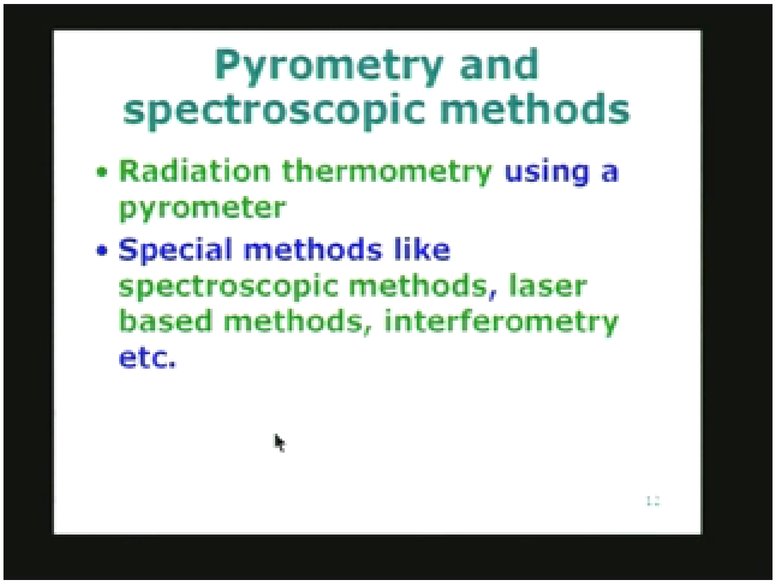

## Pyrometry and spectroscopic methods

- Radiation thermometry using a pyrometer
- Special methods like spectroscopic methods, laser based methods, interferometry etc.

In fact, the radiation thermometer uses what we call as the pyrometer. In fact later on, we will describe more fully the pyrometer and how it works, but right now, it is sufficient to understand that the basis of the pyrometer is comparison between the source and the target whose temperature I want to find out, and the temperature of the black body, which has got a certain match between the two. I will describe it more fully later on. The other advantage of radiation thermometry is that there is no probe, therefore, there is no contact between the measured quantity, whatever we are measuring, temperature and the system whose temperature we want to measure. That is an advantage in this particular case. There are also special methods like spectroscopic methods, laser-based methods based on fluorescence, for example, interferometry and so on. These are somewhat advanced and may be we will just have a little bit of time to touch up on these. We may not be able to consider them in great detail in this particular course. I am just labeling, I am just giving it here for the sake of completeness.

So with this background we can now look at different types of thermometers. The first type, which we are going to describe in some great detail, is the thermoelectric thermometry, or the use of thermocouples. Let us just look at what we are going to do, the way we are going to arrange this particular topic (Refer Slide Time 19:43 min). The thermoelectric thermometry uses three distinct effects, which have been found to occur, and these are given here. The three effects are labeled as Seebeck effect, discovered by T J Seebeck in 1821, long, long ago. Then we have the Peltier effect discovered by Peltier in 1824, and then we have the Thomson effect discovered by William Thomson, who later became Lord Kelvin in 1847. Therefore, thermoelectric thermometry uses these three effects as the basis of the thermometer.

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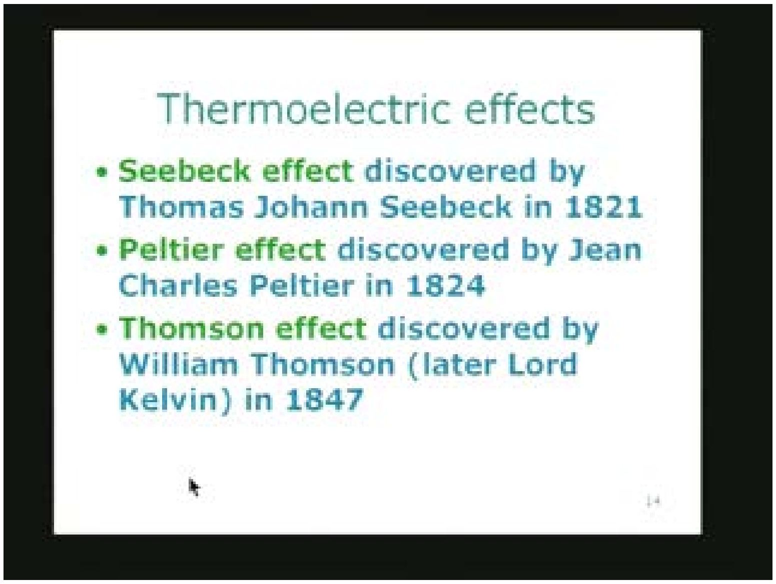

### Thermoelectric effects

- Seebeck effect discovered by Thomas Johann Seebeck in 1821
- Peltier effect discovered by Jean Charles Peltier in 1824
- Thomson effect discovered by William Thomson (later Lord Kelvin) in 1847

So what we would like to do is to describe the fundamental effects or the basic effects that take place, and we call it thermoelectricity fundamentals. I will work this out on the board, then we will describe or define or try to tie all these three effects together by what are called Kelvin relations.

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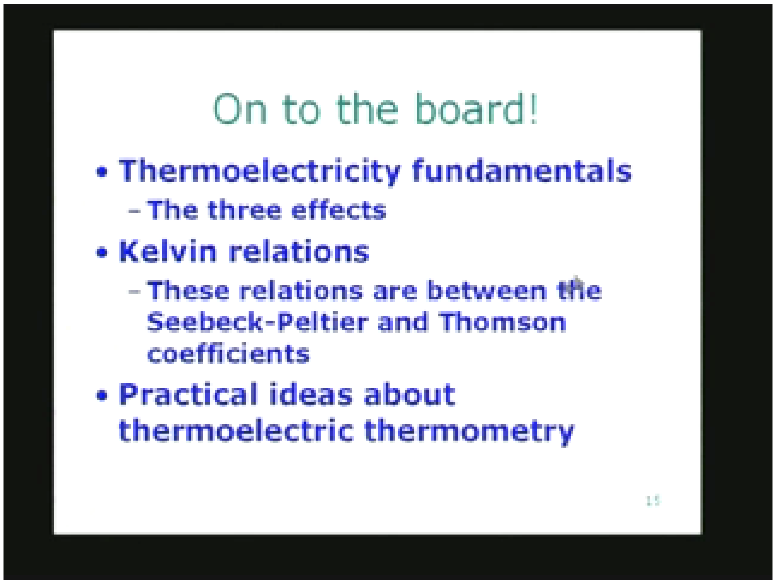

On to the board!

- Thermoelectricity fundamentals
  - The three effects
- Kelvin relations
  - These relations are between the Seebeck-Peltier and Thomson coefficients
- Practical ideas about thermoelectric thermometry

These relations are between the three effects: Seebeck, Peltier and Thomson effects, and the 3 effects are described in terms of three coefficients, the Seebeck coefficient, the Peltier coefficient, and the Thomson coefficient, and the Kelvin relation—what they do is to give relationship between these three quantities. After understanding these Kelvin relations, we will be able to actually look at how to construct a thermometer using a thermocouple, and that is what we are going to do, we will follow practical ideas about thermoelectric thermometry. Once we have understood the Kelvin relationship and how to construct a particular thermocouple circuit, the basic ideas will follow from that. So what I will do now is to terminate the show and go to the board, and let us look at the fundamentals of thermoelectricity.

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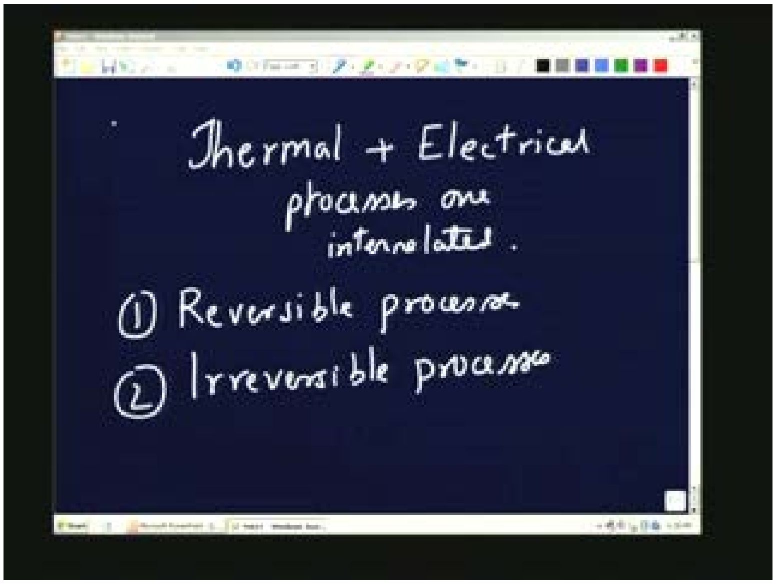

Thermal + Electrical processes are interrelated.

① Reversible processes

② Irreversible processes

Thermoelectricity basically involves the three effects I mentioned earlier, and before we go into the details let me just mention what are the things that happen in the case of thermoelectricity. Thermoelectricity means thermal and electrical phenomena or processes which are interrelated. Let me just classify these into two types of processes: one, we will call reversible processes. The term reversible is being used in the thermodynamic sense, and there could be an irreversible process. I am just going to label them first. I am going to discuss what these different processes, and then look at the details. So let me just recapitulate or talk about the reversible process. I talked about the reversible process of the three effects earlier, so the reversible ones are the Seebeck, Peltier and Thomson effects. The irreversible ones are: we have production of heat due to a current flowing in a conductor. This is called Joule heating. This is because the conductor will impose a resistance on the flow of electricity, and therefore, there is an irreversible conversion of electrical energy to heat. This is called Joules heating.

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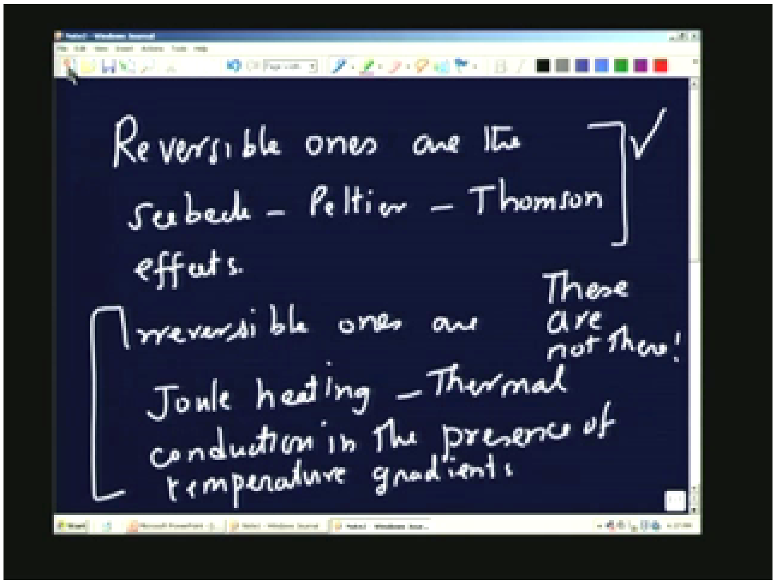

Reversible ones are the Seebeck - Peltier - Thomson effects. ✓

Irreversible ones are Joule heating - Thermal conduction in the presence of temperature gradients

These are not there!

And the second one is thermal conduction or heat conduction, thermal conduction in the presence of temperature gradients. That means if the temperature is varying within a material from position to position, there would be irreversible transfer of heat from a high-temperature region to a low-temperature region. So these are the irreversible ones and these are the reversible ones. What we are interested in studying are these effects assuming that these are negligible. Not there, ok. Of course, in practice, the irreversible effects are always going to be there and they will of course create some problems for us as we will see later on. So let us look at these three effects: the Seebeck, Peltier and the Thomson effects.

Let (Refer Slide Time 29:29 min) us consider two materials, A and B. They may be either metal or alloy, but they have to be two different materials; and let me just consider a simple circuit where we have the two materials connected to form two junctions. Let me just write the two wires, this is wire A, and I have shown a junction deliberately by a blob there, and this is material B. This is temperature “$T_1$,” this is temperature “$T_2$.” $T_1$ not equal to $T_2$. And if I maintain a current in this loop by having some source of power like a battery connected, which I am not showing because, it is of no importance to us right now. I have maintained a current through this circuit and some interesting results take place. So let us look at the direction of the current. It is shown by the arrow which is shown here. Actually, the electrons are moving in the opposite direction—this is the flow of electrons.

Conventionally, the current direction and the electron’s flow direction are opposite to each other. Suppose the temperature $T_1$ is less than $T_2$;, the electrons are moving here like this and in this case they will be moving in the opposite direction. So in the case of flow of the electrons in this direction, heat is either absorbed or rejected. Heat is absorbed at the junction here and heat is released. This is called the Peltier heat. So if you have electrons crossing over from one material to other material at a junction, the heat is absorbed at this junction, for example, and it is heat, some heat is released at the junction, the temperature is equal to $T_1$.

Of course if I reverse the direction of the current that means instead of this direction if I change the current direction to this direction the electrons will move like this and move like this, and the heat rejection and absorption, the reactions, will be interchanged. That means, wherever heat was absorbed in the first case it will be released there, and in the other case, it will be rejected, ok? So the heat absorption of the Peltier heat which is absorbed at this junction as the current is maintained, the direction of the current determines whether the absorption or the release of heat is going to take place. That is why we call it a reversible process, reversible absorption, or rejection of heat, or rejection. This is the Peltier effect. So the Peltier effect, or the Peltier heat, depends on the materials A and B.

So, the Peltier effect depends on the two materials A and B, and it is found that the rate at which heat is absorbed or released is proportional to the current and proportional to a coefficient called the Peltier coefficient or Peltier emf, so, the power absorbed or rejected. I have already explained that absorption or rejection will be dependent on the direction of the current and their work. The power absorbed or rejected is given by dQ Peltier by d theta. I am going to use theta for time because t will be used for temperature. I am going to use theta as the time quantity. This will be given by pi, which is a function of the two materials A B, and the current I, ok? We can go back to the previous drawing here. The current is moving from,or the electron is moving from B to A here and here it is moving from A to B.

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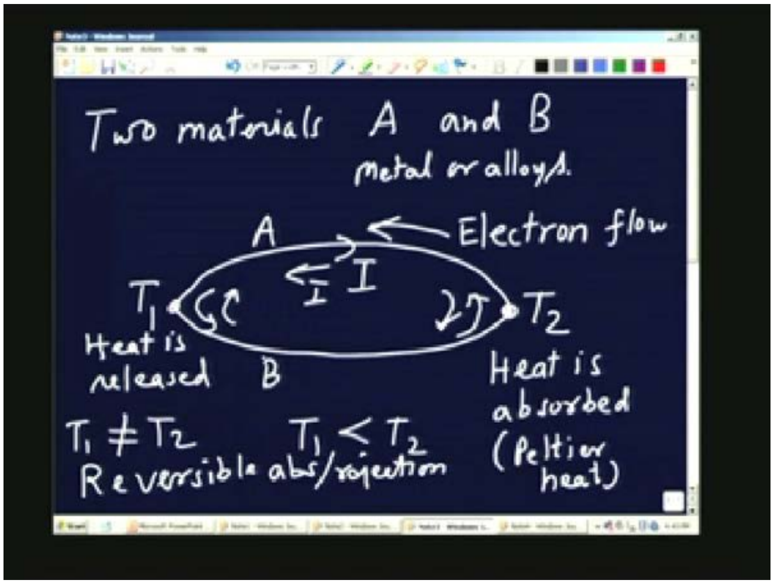

Two materials A and B
metal or alloys.

$T_1 \neq T_2$ $T_1 < T_2$
Reversible abs/rejection

So at one junction it is going to move from the material B to A, and at the other junction is going to go exactly opposite. Therefore at one junction there will be absorption of heat, at the other junction, it will be rejection of heat. Of course that depends on the temperatures. So, keeping this in mind we can say that if we have two junctions the potential,as you can see, this unit is voltage, because the rate of doing work, $dQ_p$ by d theta is nothing but the rate at which heat is absorbed or rejected, is equal to I into this much I have units of voltage, so this is called the Peltier voltage or Peltier emf.

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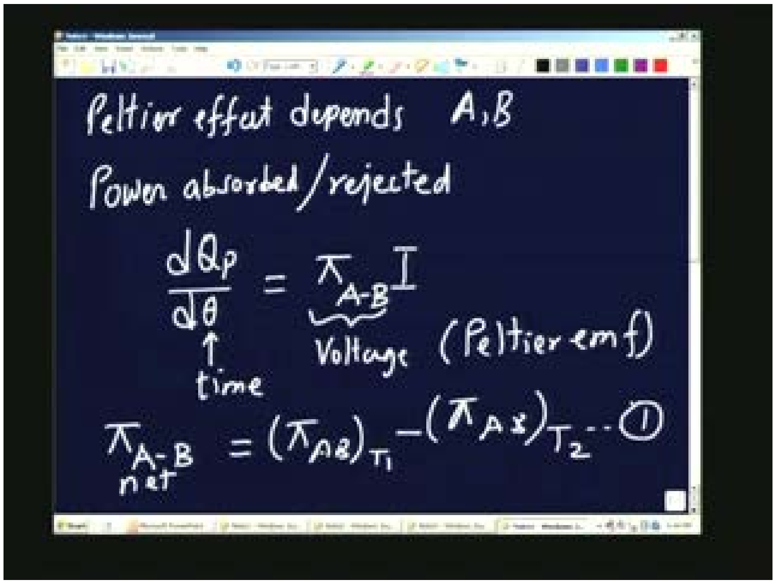

Peltier effect depends A, B

Power absorbed/rejected

$$\underset{\substack{\uparrow \\ \text{time}}}{\frac{dQ_p}{d\theta}} = \underbrace{\pi_{A-B}}_{\text{Voltage}} I \quad \text{(Peltier emf)}$$

$$\underset{\text{net}}{\pi_{A-B}} = (\pi_{AB})_{T_1} - (\pi_{AB})_{T_2} \cdots \textcircled{1}$$

Therefore, I can say that the Peltier emf net (I’ll call it as “net”) must be equal to pi $_{AB}$ at the junction at temperature $T_1$ minus pi$_{AB}$ at junction $T_2$. Whichever is positive, whichever is negative will automatically take care of the signs. So, the net Peltier emf generated or involved in this process of absorption of heat, and rejection of heat is given by a simple formula, I will label it as 1. So, let us look at what else is going to happen. So we have considered the Peltier heat. Now suppose I take the single element A—of course in the previous sketch we had a B material also—connected to form two junctions. I am taking one of them now and I am maintaining a current through that and this end is the $T_1$, this end is $T_2$, and if you just go back you will see that $T_1$ is less than $T_2$; the direction of the current is this. That means the electrons are moving in this direction, electron flow, this is a material, material of this is A, which is assumed to be a homogenous material and it is in the form of wire for our particular purpose. Now, the electrons are moving from a region of higher temperature to a region of lower temperature. So they will release heat, give out heat.

When the electrons move from a region of higher temperature to a region of lower temperature in a homogenous and renegade material, heat is rejected if it is from a higher temperature to a lower temperature, whereas, heat will be absorbed if it moving, if the electrons are moving from this side to that side.The same amount of heat will be either rejected or absorbed depending on the direction of the electron flow. Of course the electron flow direction is exactly opposite the direction of current. So, in this case, we will say that the heat rejection $dQ_{Thomson}$ divided by d theta, and it depends on the entire length of this conductor.

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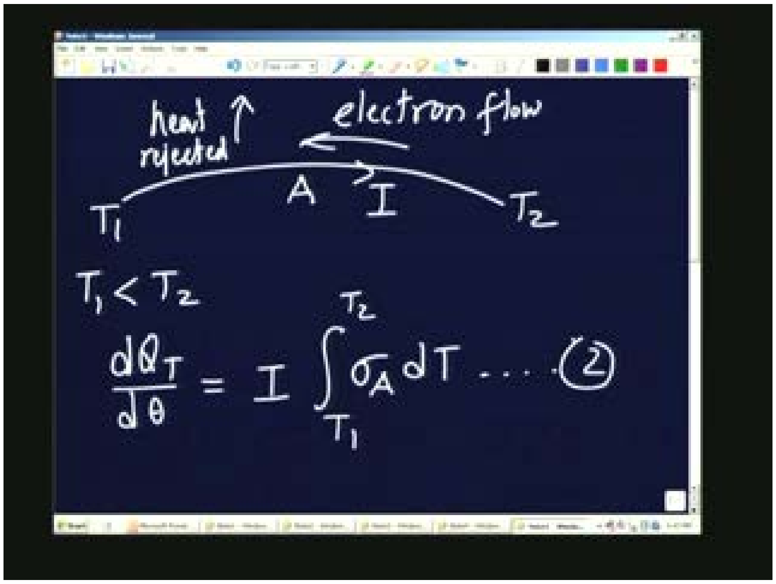

$T_1 < T_2$

$$\frac{dQ_T}{d\theta} = I \int_{T_1}^{T_2} \sigma_A \, dT \quad \ldots\ldots ②$$

So the sum is the integrated effect over the entire length of the conductor, or I will say it is I times integral $T_1$ to $T_2$ or $T_2$ to $T_1$, some coefficient which I will call sigma A, which is the characteristic of the material of the wire A. This is called the Thomson co-efficient. I will call this equation 2. Now I can in fact go back to the earlier sketch. In the earlier sketch I had two materials, A and B, to form a complete circuit, and now we can see that it is for this part of the circuit is the amount of Thomson heat, which is involved.

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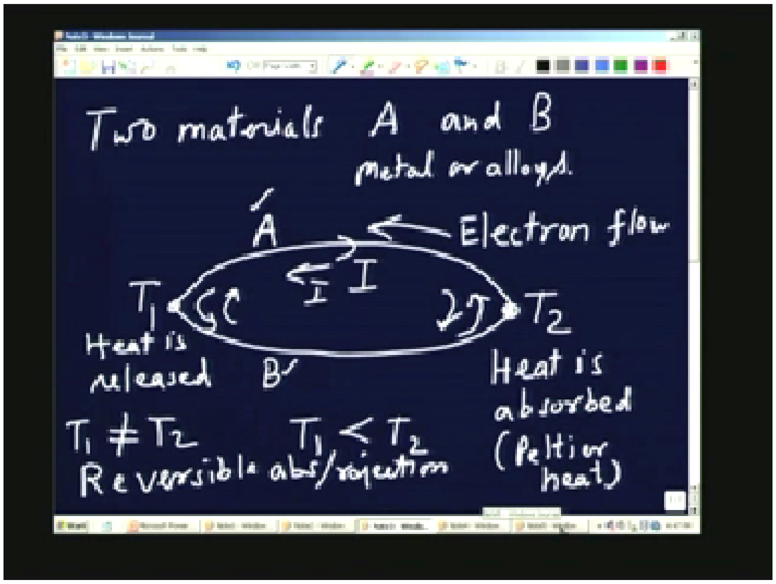

Two materials A and B
metal or alloys.

$T_1 \neq T_2$ $T_1 < T_2$

Reversible abs/rejection

Similarly the amount of Thomson heat will be involved in the other leg of the circuit. Now you notice that if the current is moving in this direction, in this A, if I complete this one, the current will be moving in this direction; that is, in one material it is moving from a low temperature to a high temperature, or a high temperature to a low temperature. In the other part of the circuit it will be doing exactly the opposite. Therefore just like what we had in the case of Peltier effect if heat is injected along the length of the conductor A, it will be rejected along the conductor B. So it will be a difference of similar terms and therefore I can write the second equation as $dQ_T$ by d theta. I will call it, net is equal to the difference between the two, I into integral $T_1$ to $T_2$ sigma A minus sigma B.

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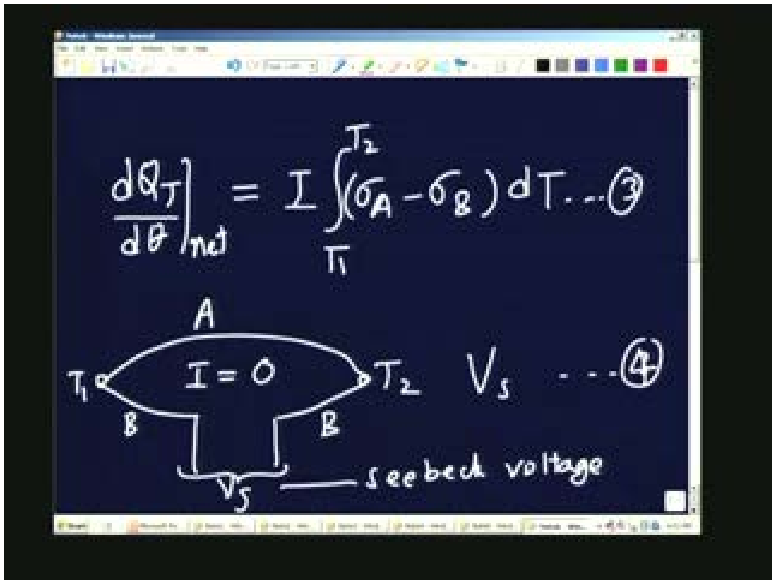

$$\left.\frac{dQ_T}{d\theta}\right|_{net} = I\int_{T_1}^{T_2}(\sigma_A-\sigma_B)\,dT \ldots ③$$

$V_s \ldots ④$

This we can call 3. So we have actually described the two effects when we have a circuit made up of two dissimilar materials connected in this form shown here, and there is a current maintained in the two circuits, here there is a current flowing through the circuit. Now let us look at what is going to happen if I do the following. Suppose I take the same circuit. This is made of material A and this is made of B and I am going to introduce a cut. That means there is no current; it is observed again $T_1$ and $T_2$ are the two temperatures. It is observed that a potential difference will appear across the cut. This is called the Seebeck voltage and that is going to appear across the cut. So we can call this $V_s$ as the third quantity or fourth quantity; so $V_s$ is the fourth quantity, which is going to be marked. Now let us go back and just see what happened. In the first case, I had a current and I had the complete circuit completed. Some amount of heat interaction took place at the two junctions as well as along the length of the two wires.

Now, when I introduce a cut in one of the limbs of the circuit, it may be either here or in the wire B or wire A, whichever you cut it doesn't matter. Only the direction of the voltage may be different. A voltage will appear and therefore this voltage must be related to the two effects we had earlier. Those two effects are not there now, manifesting as an appearance of a voltage across the cut, and therefore, I can just write using the conservation of energy principle that the voltage, which is due to Seebeck effect, when I make a cut, the voltage which is appearing as the Seebeck voltage, this must be the sum of the Peltier voltage and the Thomson emf or the Thomson voltage. We already have expressions earlier; so this will be pi $_{AB}$. I will remove AB subscript; I will not use it now because we know what we are taking about. So I will say pi$_{T2}$ minus pi$_{T1}$. This is a Peltier plus integral $T_1$ to $T_2$ sigma A minus sigma B dT. We will call it equation 5.

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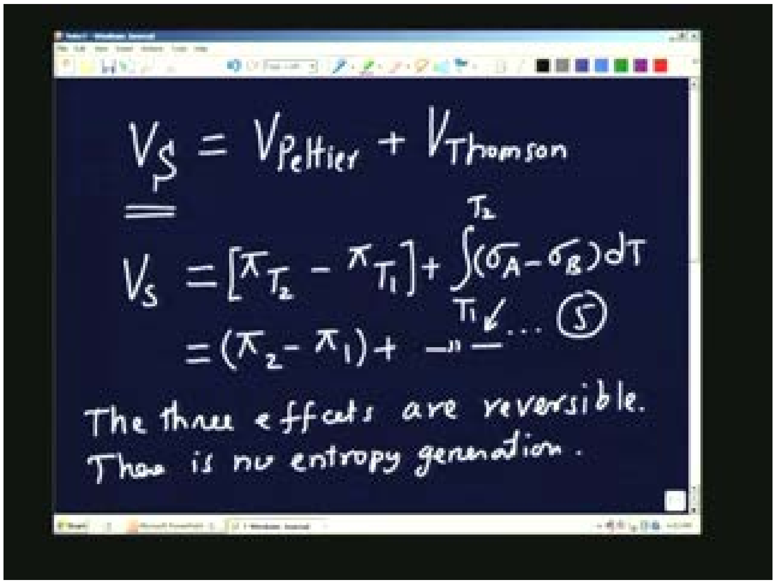

$$\underline{\underline{V_S}} = V_{Peltier} + V_{Thomson}$$

$$V_S = [\pi_{T_2} - \pi_{T_1}] + \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT \quad \ldots ⑤$$

$$= (\pi_2 - \pi_1) + \text{—"—}$$

The three effects are reversible.
There is no entropy generation.

So we can just write here the Seebeck voltage is related to the Peltier emf and the Thomson emf, which would have occurred had we made the complete circuit and allowed a current to pass through that. So this is an important relationship and it is a starting point for our discussion. In fact I am going to write it in a slightly different fashion. I will just say pi$_2$ minus pi$_1$ to avoid the big subscript and plus, of course the same equation sigma A minus sigma B into dT—this is equation 5.

Now I am going to invoke the fact that the three effects are reversible, and what it means is that there is no entropy generation, or the change in the entropy is 0. Because these are all reversible processes, there can be no entropy generation and in fact all I have to do is to find out what entropy generation is and then put it equal to 0 and find out what that particular relationship is going to give me. Before we do that, let me just introduce another quantity, which I call the Seebeck coefficient. In fact it is known by different names; it is also called the thermoelectric power. It is nothing but the derivative of $V_s$ with respect to temperature. So we will say that d$V_s$ by dT is equal to alpha, which is a function of A and B, and in general, I simply write alpha because we know that is a function of AB instead of explicitly mentioning just simply say alpha. d$V_s$ by dT is equal to alpha. This is equation 6. So actually what I can do now is define d$V_s$ by dT as alpha.

Let me just digress a little bit and try to give a meaning to this. Suppose I have the following circuit: this is A, this is B. The Seebeck of voltage is going to appear there. Let us say this is T and this is T plus delta T. So d$V_s$ by dT is nothing but the elemental, you can see that there is a very small difference in temperature between junction one and junction two. The voltage which appears will be delta $V_s$, and the ratio of delta $V_s$ to delta T as delta T tends to 0, is your alpha. So we say that the Seebeck potential per unit temperature difference is nothing but your alpha, it is just a useful way of describing the Seebeck effect.

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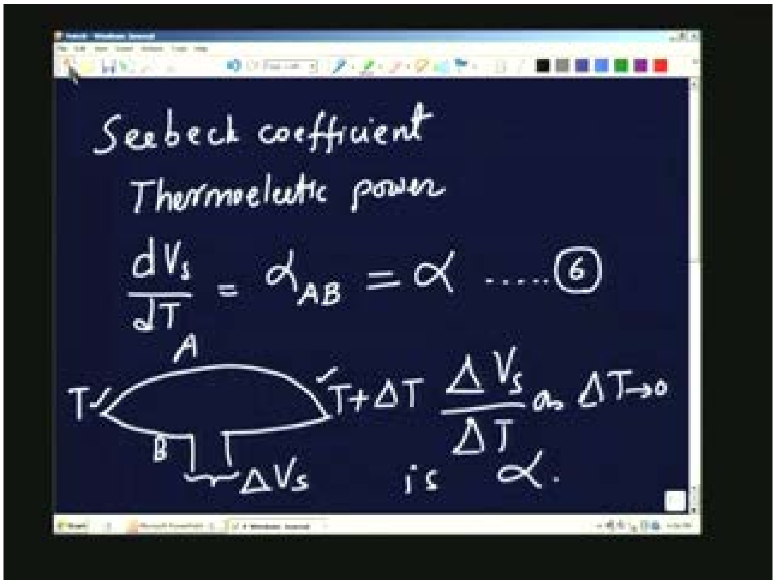

Seebeck coefficient

Thermoelectric power

$$\frac{dV_s}{dT} = \alpha_{AB} = \alpha \quad ..... ⑥$$

$\frac{\Delta V_s}{\Delta T}$ as $\Delta T \to 0$ is $\alpha$.

Now let us look at the entropy generation, which is going to take place in such a system. If it were to take place we are going to, of course, say that entropy generation is 0. Entropy change is due to all the heat interactions, ok? So I can find out what the heat interactions are. I know the Peltier heat at the two junctions, and the Thomson heat all along the two conductors. These are the ones which are going to give rise to the change in entropy and I am going to require that the change in entropy, the net change in entropy, must be equal to 0.

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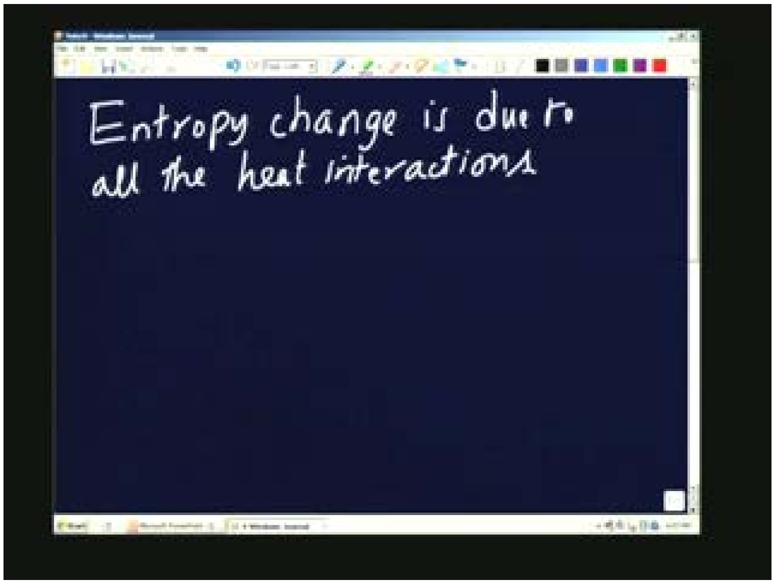

Entropy change is due to all the heat interactions

So we will write the expression for that. Before we do that, let me write the previous equation also. I think it was given in 7, I think. Equation 5, I will write it for a small difference in temperature between $T_1$ and $T_2$ just like what we did just now with respect to the Seebeck options.

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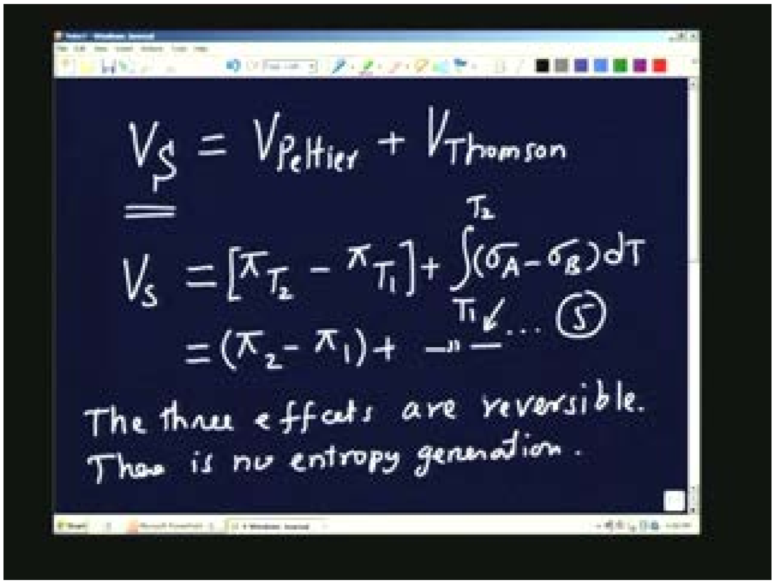

$$\underline{\underline{V_S}} = V_{Peltier} + V_{Thomson}$$

$$V_S = [\pi_{T_2} - \pi_{T_1}] + \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\,dT \quad \ldots \text{⑤}$$

$$= (\pi_2 - \pi_1) + \text{—''—}$$

The three effects are reversible.
There is no entropy generation.

So what I will do is I will rewrite the equation in the form of, before we do that, I will rewrite equation 5 in differential form. If you remember, there is an integral appearing there. I want to write it in the form of a differential. That means I have a small difference in temperature between the two junctions. That means, the Peltier effect is due to small temperature differences at the two junctions, and the Thomson effect is also due to the small temperature variations along the length of the wire. Therefore the differential form will be given by dV$_s$ by dT. I have to take the derivative of that expression with respect to temperature which we have defined as alpha. If you want you can explicitly say, alpha $_{AB}$ is given by dpi by dT plus integral when you differentiate with respect to T, it becomes sigma A minus sigma B, or dV$_s$ equal to dpi plus sigma A minus sigma B into dT. I think this is 8. So we have the equation and let us now write the appropriate expression for the change in entropy.

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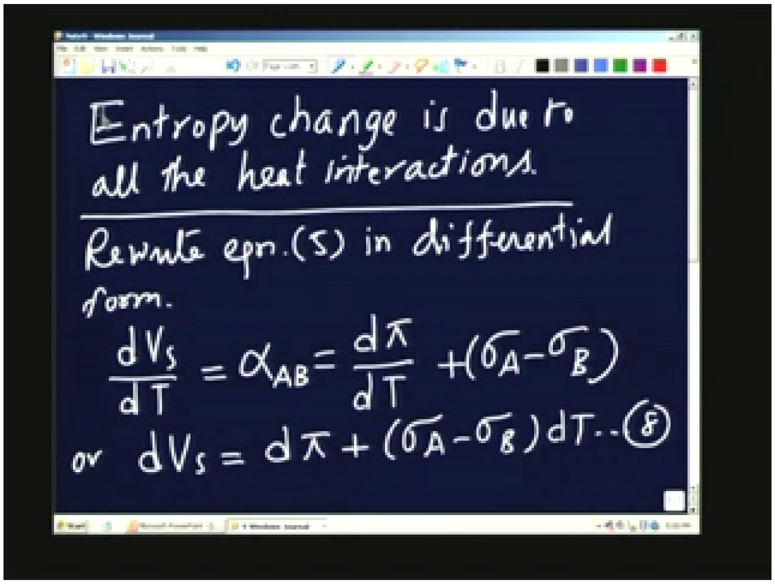

Entropy change is due to all the heat interactions.

Rewrite eqn.(5) in differential form.

$$\frac{dV_s}{dT} = \alpha_{AB} = \frac{d\pi}{dT} + (\sigma_A - \sigma_B)$$

or $dV_s = d\pi + (\sigma_A - \sigma_B)\, dT$ .. ⑧

I am invoking the fact that these three are reversible processes; dS equal to 0 and dS itself can be written as the heat is absorbed at one of the junctions at temperature T, so the entropy change is pi by T. y pi is absorbed or rejected divided by temperature at the junction and other junction is T plus deltaT dt. Therefore I can say that the entropy change is d of pi by T for a small temperature difference between the two junctions plus sigma A minus sigma B into dt divided by T multiplied by I is equal to 0. This is because we have a junction at T, T plus delta T pi divided by T pi divided by T plus delta T. Therefore, the difference between these two is nothing but???? ok? So that is how it comes out like that.

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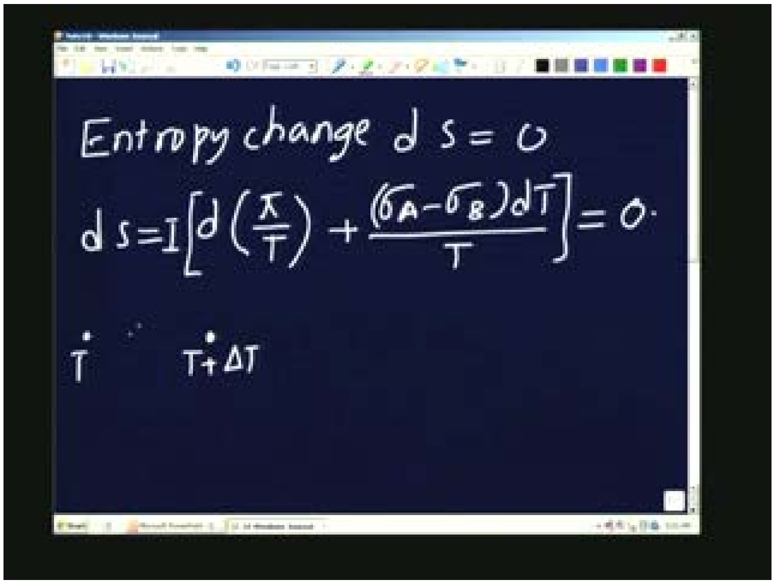

Entropy change $ds = 0$

$$ds = I\left[d\left(\frac{\pi}{T}\right) + \frac{(\sigma_A-\sigma_B)dT}{T}\right] = 0.$$

$\dot{T}$ $\dot{T} + \Delta T$

So now there are two equations. I think this is equation number 9. From here we notice that sigma A minus sigma B into dT is equal to minus T into d of pi by T, and I will put this into the earlier equation, which related the Seebeck emf to the Peltier and the Thomson emfs.

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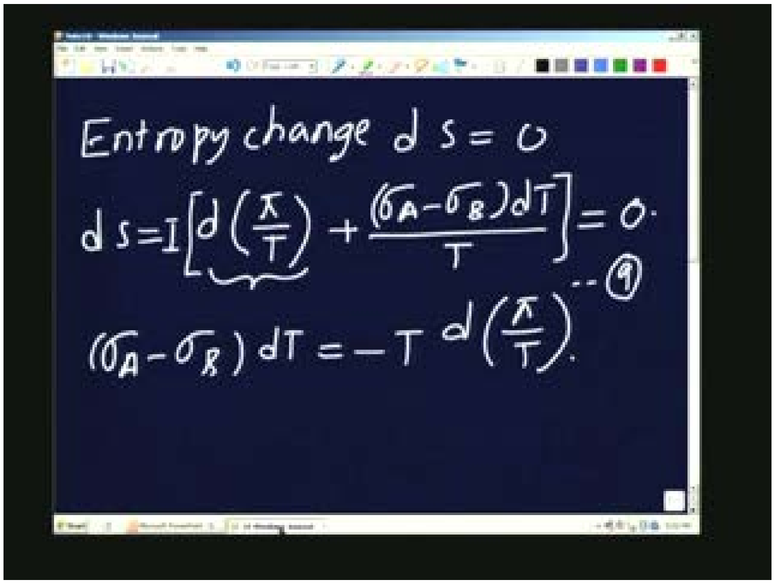

Entropy change $ds = 0$

$$ds = I\left[\underbrace{d\left(\frac{\pi}{T}\right)}_{} + \frac{(\sigma_A-\sigma_B)dT}{T}\right] = 0. \quad \text{--⑨}$$

$$(\sigma_A-\sigma_B)\,dT = -T\,d\left(\frac{\pi}{T}\right).$$

Therefore, for example, if I go back, I have got the equation 8: $dV_s$ equal to d pi plus sigma A minus sigma B into dT, and what I have now done is, I have got sigma A minus sigma B dT from here. So I am going to just substitute there and after some simplification, which can be easily done by the student, we can see the following: minus

T d pi d of by T,I can say minus T 1 over T d pi plus  pi d of 1 over T will be minus  dT by T squared. I am differentiating it like that: sigma A minus sigma into dT equal to minusT into 1 over T d pi minus pi into dT by T squared. I am going to put this into the other expression, the earlier expression.

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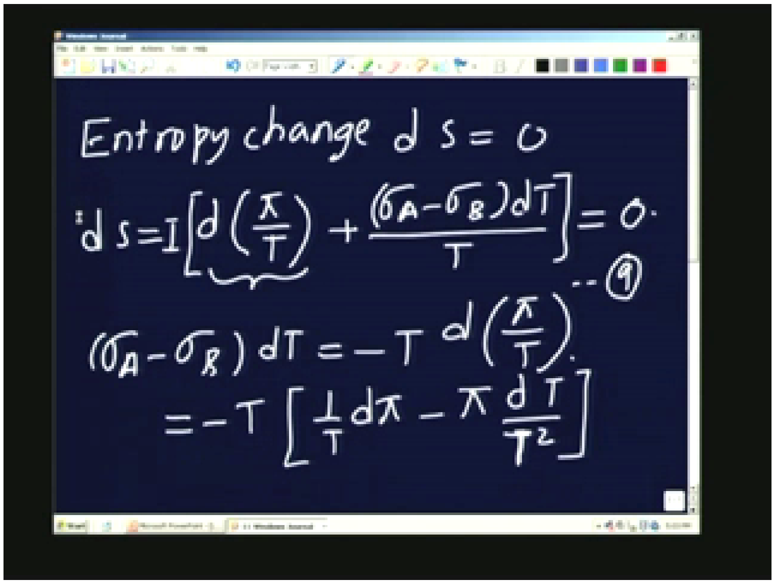

Entropy change $ds = 0$

$$ds = I\left[\underbrace{d\left(\frac{\pi}{T}\right)} + \frac{(\sigma_A - \sigma_B)\,dT}{T}\right] = 0 \quad \cdots ⑨$$

$$(\sigma_A - \sigma_B)\,dT = -T\,d\left(\frac{\pi}{T}\right)$$

$$= -T\left[\frac{1}{T}\,d\pi - \pi\frac{dT}{T^2}\right]$$

If you do that I will just write the final expression. So I should get d pi by T minus pi dT by T squared. This is what we have just now: plus $dV_s$ by T minus dpi by T equal to 0. Therefore these two will cancel out and one T will cancel out with this T and you see therefore pi is equal to T $dV_s$ by dT; it is nothing but T times alpha. I think this is the equation number 9. That means that the Peltier coefficient, or Peltier emf, is the product of the absolute temperature multiplied by Seebeck coefficient.This is called Kelvin relation number 1. This was derived by Kelvin, and therefore, the name has come.

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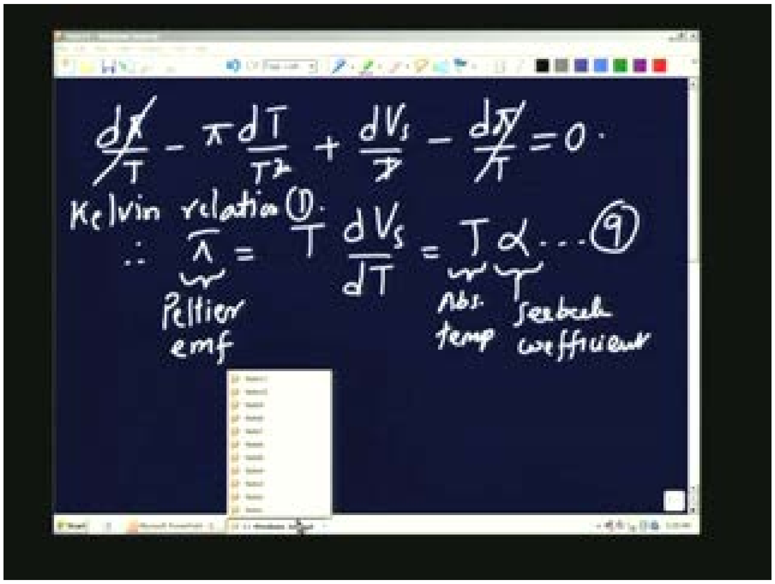

$$\frac{d\cancel{\pi}}{\cancel{T}} - \pi\frac{dT}{T^2} + \frac{dV_s}{\cancel{T}} - \frac{d\cancel{\pi}}{\cancel{T}} = 0.$$

Kelvin relation ①.

$$\therefore \underbrace{\pi}_{\text{Peltier emf}} = T\frac{dV_s}{dT} = \underbrace{T}_{\text{Abs. temp}}\underbrace{\alpha}_{\text{Seebeck coefficient}} \ldots ⑨$$

In fact if you can go back and see that if you go to the previous expression, sigma A minus sigma B dT equal to this, and now again, I have already got the value of pi in terms of alpha. So all I have to do substitute here and if I simplify further, I should be able to get it. So Kelvin relation 2 will come and I will just give the answer sigma A minus sigma B equal to minus T d alpha by dT. So we have 2 relations, Kelvin relations 1 and 2. What they do is to relate the Seebeck coefficient alpha, or the thermoelectric power alpha, then the Peltier coefficient or Peltier voltage pi, and the Thomson emf through these 2 relations. That means the three are essentially interrelated processes and they are related through this expression.

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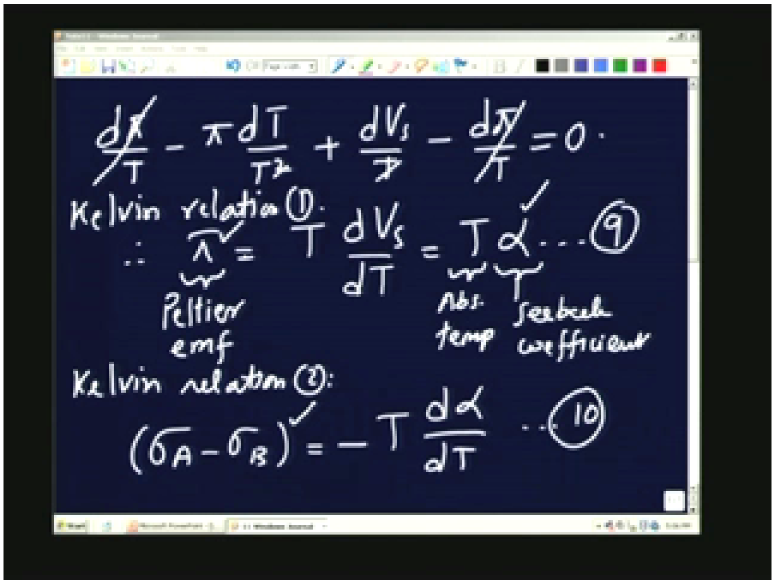

$$\frac{d\cancel{\pi}}{\cancel{T}} - \pi\frac{dT}{T^2} + \frac{dV_s}{\cancel{T}} - \frac{d\cancel{\pi}}{\cancel{T}} = 0.$$

Kelvin relation ①.

$$\therefore \underbrace{\pi}_{\text{Peltier emf}} \checkmark = T\frac{dV_s}{dT} = \underbrace{T}_{\text{Abs. temp}}\underbrace{\overset{\checkmark}{\alpha}}_{\text{Seebeck coefficient}} \ldots ⑨$$

Kelvin relation ②:

$$(\sigma_A - \sigma_B)^{\checkmark} = -T\frac{d\alpha}{dT} \ldots ⑩$$

In fact, we will notice that we expect all these to be functions of temperature alone, as soon as we have chosen A and B. This is a very important observation; it looks like it is a very simple and straightforward statement. That is very important, because once you have two materials, A and B, then, there is a unique relationship between these coefficients and the temperature. Let me just take a simple expression for $V_s$. Suppose I take $V_s$ equal to some constant a plus b times t plus c times t square. Here I will take t in degree Celsius. In the earlier case $T_1$ and $T_2$ were there. I will take $T_2$ equal to t and this I will take 0 centigrade or the ice point. If I take the temperature, $T_2$ as equal to t, and I take the other temperature, as the reference value at ice point, then I can write something like this, and the coefficients a, b, and c are related to the three effects. So a, b, and c are related to the three coefficients or three effects.

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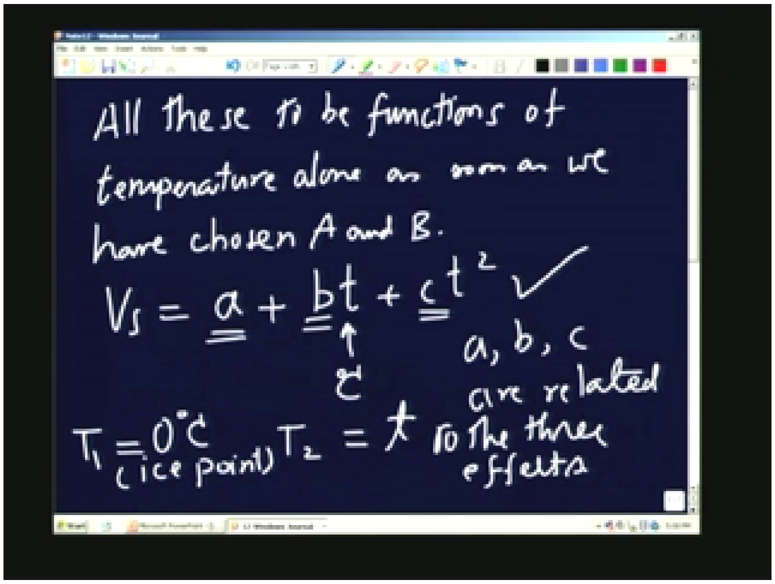

All these to be functions of temperature alone as soon as we have chosen A and B.

$$V_s = \underline{\underline{a}} + \underset{\underset{t}{\uparrow}}{\underline{\underline{b}}}\,t + \underline{\underline{c}}\,t^2 \checkmark$$

a, b, c are related to the three effects

$T_1 = 0°C$ (ice point) $T_2 = t$

So I will start lecture number 11 from here, and find out what is the correspondence between a, b, and c and the three coefficients or three effects we have already talked about, and I will also take an example and find out what these numbers are, what is the magnitude of these numbers, and how whatever we have described till now can be used to construct the thermometer. That is our basic reason for talking about these different materials, and junctions, and circuits, and so on. We will get back to that in the next lecture. Thank you.