Introduction to Finite Element Method

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Lecture - 33

In this course, so far we have seen a number of elements which can be used in various situations. We have talked about how these elements are formulated and how it can be used to solve various practical problems. Though we have covered majority of elements, for example we have seen how plane stress, plane strain, axi-symmetric, three-dimensional solid elements are used, two of the elements which we have not concentrated in this course are what are called as plates and shell elements. In fact, in today's atmosphere of the softwares that are available, commercial softwares that are available, they do not distinguish between plates and shells separately; many of them at least and they call these as plate shell element. There are reasons to it. We will see why this is so, but plate shell elements are one of the most difficult elements to formulate.

In fact, lot of research has gone into formulating these elements and if you really look at the number of papers published over a period of time, may be about 10 to 15 years, past decade or so, you will see thousands of papers; not hundreds, thousands of papers are published in shell elements formulations, its use and so on. But, still there are number of issues which are to be solved, before we get very correct and perfect shell elements. Now is it possible for us to go into these issues? No; it is difficult, because the background that is required in order to understand the formulations completely are quite involved. Hence it may not be possible for us in this course to go into the depth of element formulation. Nevertheless, we will understand certain aspects of the shell or plate formulations or plate and shell formulations which will be useful to us when we use these elements in practice. In other words let us look at the physics behind the shell theory, rather the plate theory and see how exactly we can understand this and use it when we require them.

Remember, right in the first class, we had seen some examples on how the shells were used. Remember that the side wall sheet of that coach which we had seen quite often in this course was used or was modeled using this shell element. Let us now look at the fundamentals of plate theory; very, very raw details. We will not go into the sophisticated approaches that have come into picture or that has evolved over period of time, but we would at least look at what are the things that we have to know. Please understand that what we are looking at is mechanics; it is not finite element which will form the basis of finite element analysis.

Now, let us look at a plate. This particular figure here gives you or shows a typical plate in action.

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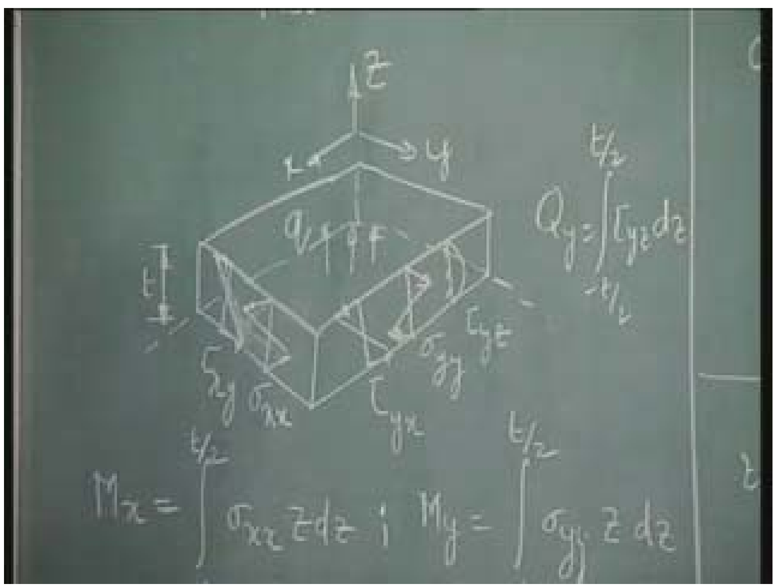

See that there are a number of stresses. We will see each of them in a minute, that the number of stresses that act in the cross section of the plate and that the plate is supporting, say distributed load q. Immediately one of the things that comes to your mind is that looks like plate is nothing but a beam, which is say extruded in the other directions. You can look at this as some sort of a beam extruded in the third direction. Especially, mechanical engineers would find this kind of approach easier, because usually in many of the courses we do not go through plates and shell theories. The civil engineers may be slightly more familiar with plate and shell theory, but nevertheless it is easier to understand it that fashion, because it would give a clue as to what are the stresses that are acting on the plate in order to support, say this kind of a distributed load.

One is the normal load. Look at that here. You can immediately recognize from the pattern or from the way this normal load distribution takes places.

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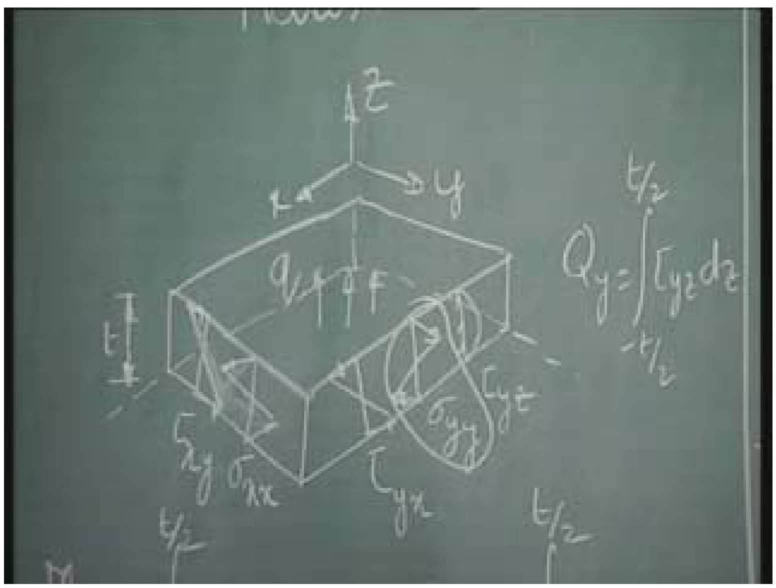

Look at that closely, you can see that this normal load distribution, a linear distribution, is very similar to that what you would have when you have or when you have a beam in bending. Look at the shear stresses. Again, shear stress distribution is very familiar to you. The shear stresses and this is what we call as transverse shear stress, which has a parabolic distribution. These stresses are there in order to support or in order to produce moments which would give an equilibrium or which would equilibrate the plate against the loads that are acting externally. In the same fashion, if you look at the plane which is say normal to the x direction you would have similar stress distribution as we can see here. We have again the normal the stresses which is sigma$_{xx}$ stress and we have the shear stresses and of course, I have not drawn the transverse shear, which can again be, parabolic distribution can be drawn in this side of the plate as well.

These stresses, how are they related or how do they produce the moments and the shear forces? The approach is exactly similar to what would happen or what you would see in a beam. Have a look at this. Now, this is exactly the moment.

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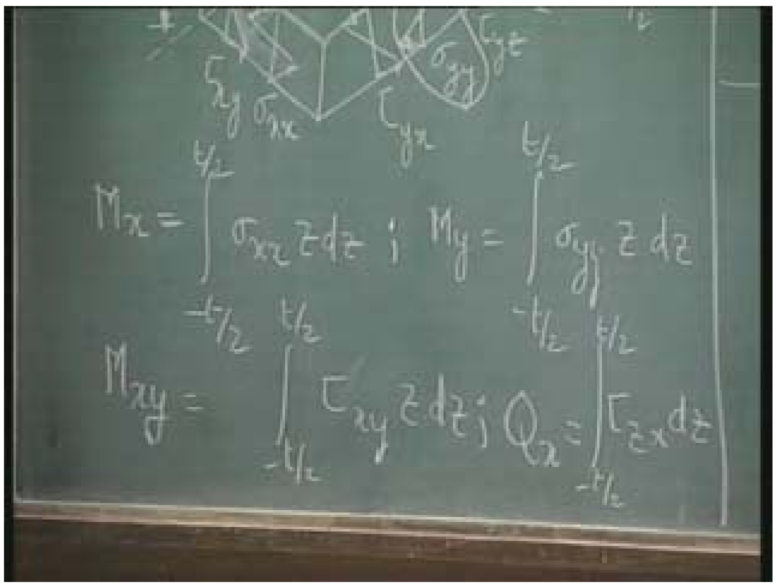

Please note that we are integrating between minus t by 2 to t by 2, where t is the thickness of the plate. So, you can see that this would produce a moment which we call as M_x. Note that M_x is a moment, which is actually, actually the plate is bending like this; plate is bending like this and the moment is acting perpendicular to the x-axis. Please note that this is the x axis what we have say seen here and the moment M_x is acting like that; that is the moment.

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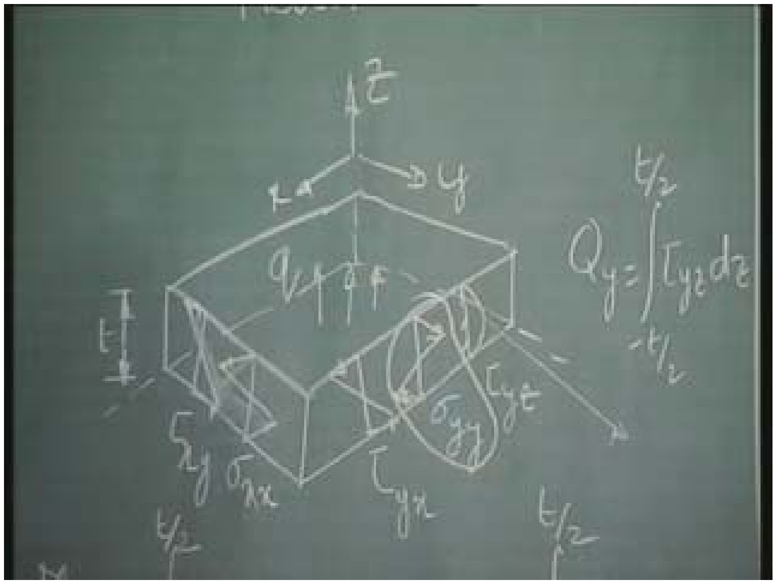

So, it is actually a moment that is produced because of the stress sigma$_{xx}$. So, M_x is equal to sigma$_{xx}$ Zdz integrated between the limit minus t by 2 to plus t by 2.

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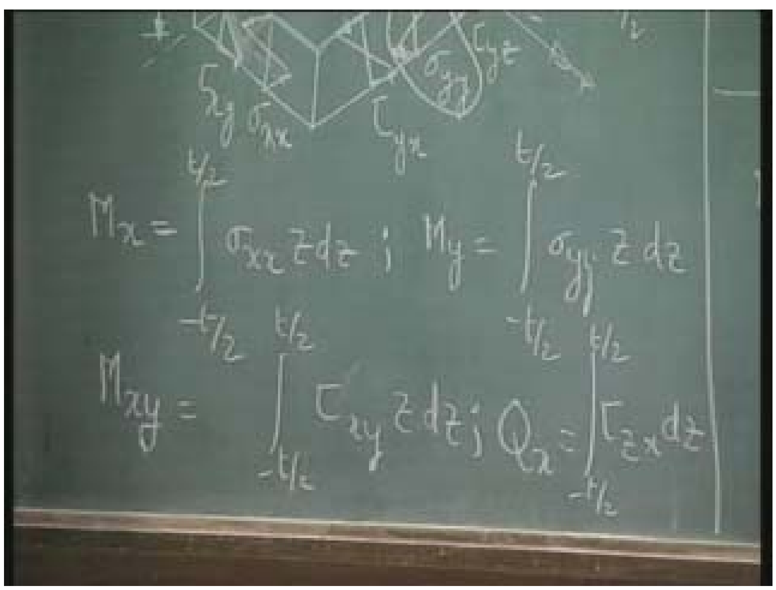

Now, let us look at M_y. M_y is a counter part of M_x. Again, here you can see that sigma$_{yy}$ Zdz produces this moment, M_y and that again acts in a direction perpendicular to y. That is in other words, it is in the other plane which is perpendicular to x. These two are the M_x and M_y moments produced by sigma x and sigma y and we have M_{xy}, which is again produced, because of the distribution of tau_{xy}. That tau_{xy} distribution we have already seen, how it looks like and that produces a moment which we call as M_{xy}. Apart from these moments, we also have what we call as the shear stresses or the shear forces.

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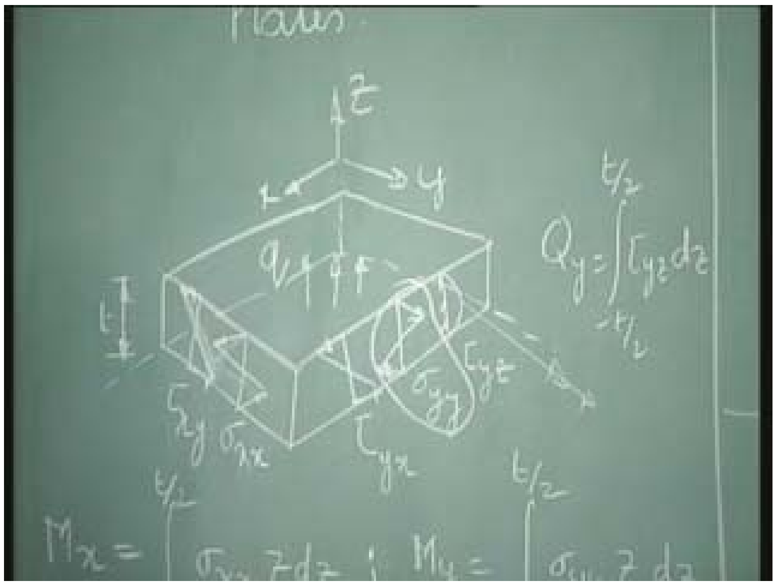

This shear forces are produced by say tau_{zx} and tau_{yz} and these shear forces we designate as Q_x and Q_y. Look at all these things carefully. That is in other words, we have M_x, M_y, M_{xy}, Q_x and Q_y, these being the forces or moments and forces which are produced due to the stresses; normal stresses and the shear stresses. That gives an overall picture. In fact, this whole picture gives us as to how plates are equilibrating or are the stresses that are developed in plate how do they equilibrate the external forces that are applied to them. Whatever be the forces, we have shown here what we call as distributed loads. It need not, I mean, generally be only distributed load. It can be say unit or concentrated load or whatever it is this is the way the stresses are developed.

From this point of view, we can also determine what the maximum stresses are that develop at various points or various stress fields. It is very clear from here that the maximum stress, say for example, sigma_{xx} or sigma_{yy} is happening at the top most fiber or top most layer, whereas on the other hand, say, tau_{yz} the maximum stresses are happening at the meridian or at the neutral surface. Now, these maximum stresses can be written down in a simple fashion as shown here. You can see that the maximum stresses sigma_{xx}, sigma_{yy} are very similar and are related to the moment M_x and M_y as show here.

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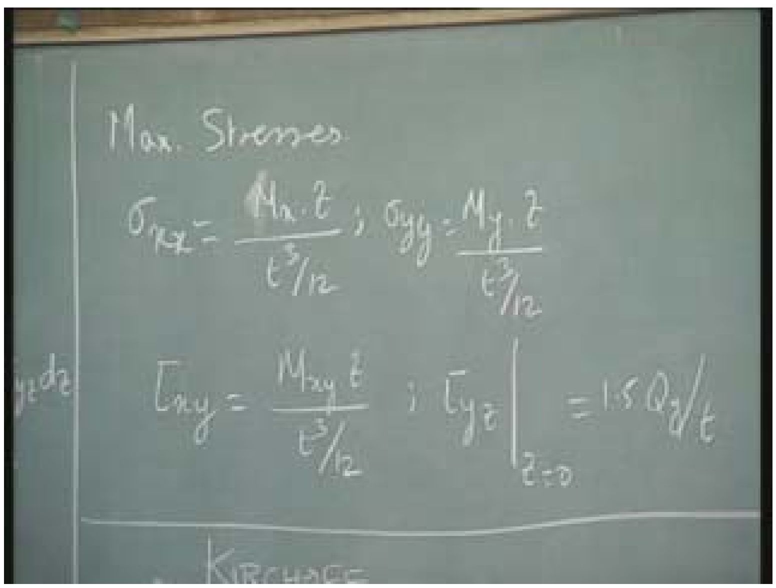

How do I get this? It is very simple. We can easily work out from here or if you have any doubts, you can apply these things back here and see that these equations are satisfied. The second set of equations can be substituted into the first set of equations. See, generally that that is the way the maximum stresses happen. Here these are the fundamentals. Now from here we deviate, deviate a bit or make some assumptions, in order to carry out the or in order to develop what we call as the plate elements. Now please note, please bear in mind that after we finish the plate element we are going to make some modifications or some additions, in order that we can get shell elements.

So, plate elements, understanding plate elements help us to understand the shell elements and apply them in a certain situations.

One of the theories, in fact the plate theories can be broadly classified into what we call as Kirchhoff's theory and Mindlin theory. What is the difference between them?

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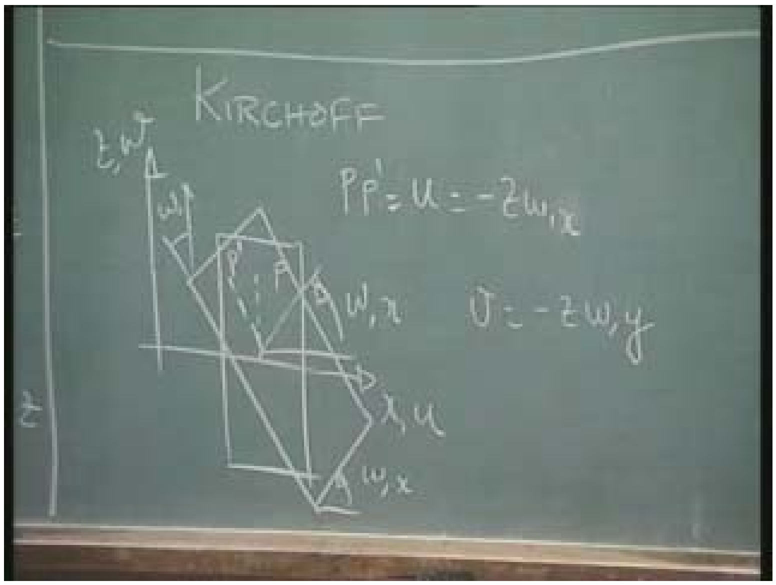

Kirchhoff's theory does not take into account what we call as shear deformation or in other words, the shear forces Q_x or these two shear forces are absent. These two shear forces are assumed to be zero, because the shear stresses and hence the shear strains are not supposed to be acting. So, that is what is the fundamental for Kirchhoff's stress or in other words this is very similar or people who have done some beam theory would realize that it is very similar to some of the assumptions that we make in the beam theory. In other words, the deformation of the plate is such that these two lines, the sections, is an infinitesimal small piece which we have taken from the plate and these two lines are perpendicular to each other and remain perpendicular when the deformation takes place or in other words this w x and this w, sorry, w comma x that is dow w by dow x that is the slope, that of this line is the same as that of the slope of this line.

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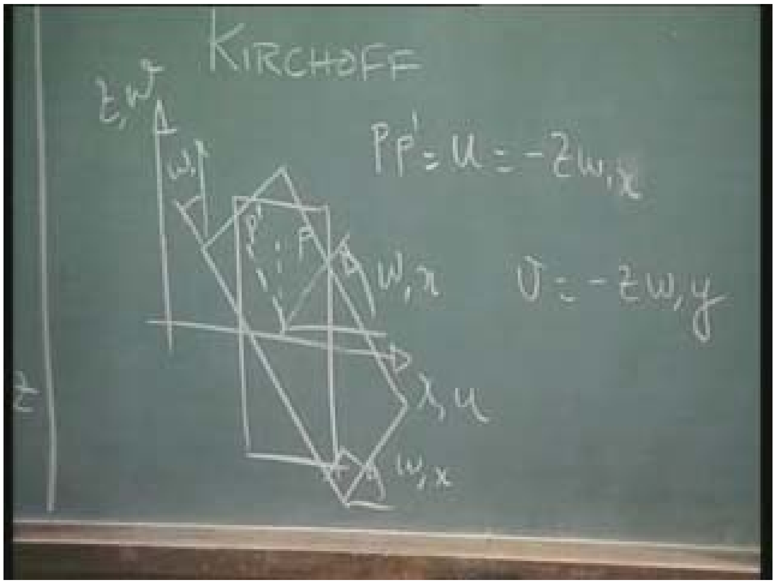

So, that is the assumption; that in other words this perpendicular is, perpendicularity is maintained. Plane sections remain plane; probably many of you are familiar with it, plane sections remain plane and not only that, they do not, that is not the only condition, but also that they are perpendicular. What is the result of this? The result of this is that the displacement of say a point P in the u direction can be written as minus z w comma x and that in the v direction can be written as minus z w comma y. So, that is the Kirchhoff's theory. Where is this applied? When the plates are not very thick, may be thick in the sense that when compared to the length and breadth it is less than say one-eighth, one-tenth and so on, then it is possible to apply this assumption that the shear deformations are negligible or small and hence we can apply Kirchhoff's theory, in which case we will get, what are the things we are going to get? We are going to get only M_x M_y and M_{xy} and these two guys will go to zero.

On the other hand, when shear deformations become important we go over to what is called as Mindlin theory. Now, what is the Mindlin theory? Have a look at this particular figure here.

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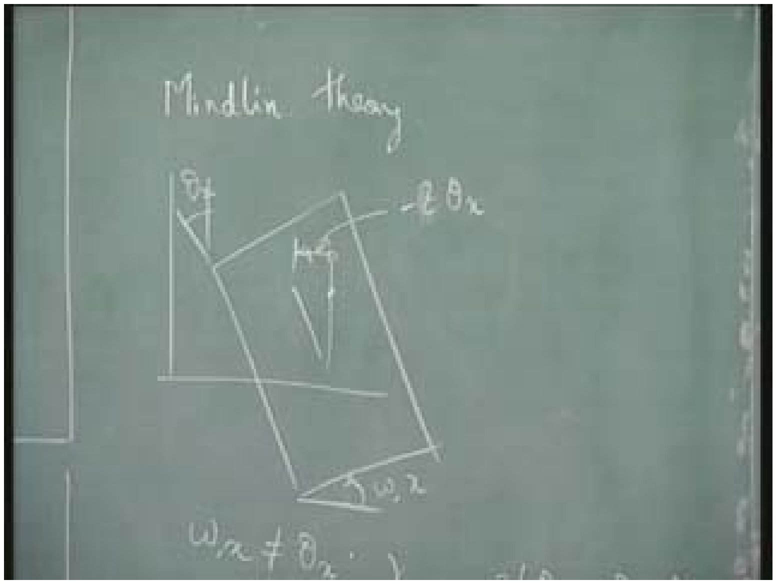

You can see that the figure is quite different from what we had seen some time back or the previous figure. What is it? We now see that though the plane sections remain plane, this section is plane, but this fellow here, this perpendicularity is not maintained.

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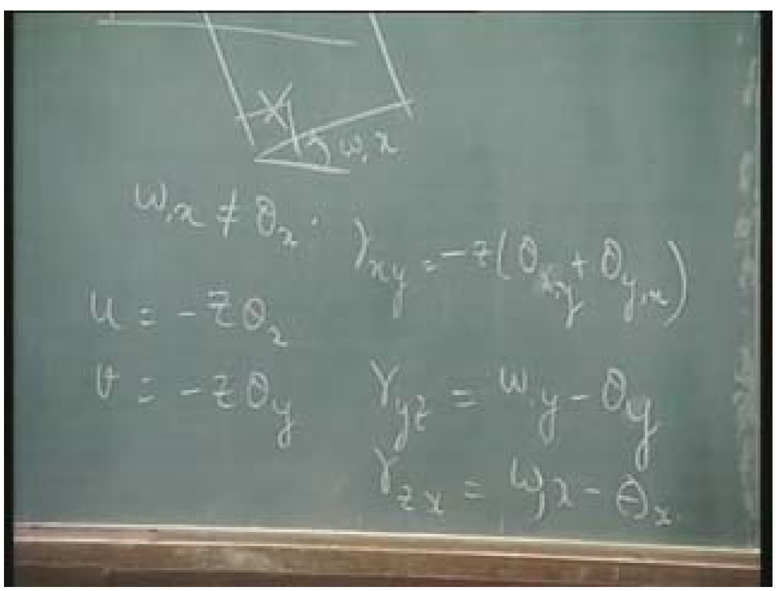

Please note that θ_x is not equal to w comma x, which was the case in the previous figure. So, we have a situation where a shear strain is developed. So, when this, when this particular perpendicularity is not maintained, what does it mean? It means that a shear strain is developed. What is the shear strain? The shear strain can be written as, this γ_{yz} is equal to w comma y minus θ_y. Same way, we can write down shear strain in zx as w comma x; w comma x, please note is nothing but dow w by dow x; w comma x minus θ_x. What is the result of this formulation or this assumption?

I would say that this is such that, we do not have only three, M_x M_y and we do not have only three moments, three moments M_x, M_y and M_{xy}, we also have the shear forces taken into account, Q_x and Q_y through the shear strains and hence the shear stresses and so on. In other words, what is the next step that we require? Of course, we require the stress strain relationship. But before we go further, have a look at them. That Kirchhoff's theory is one which is applicable for thin plates, where shear deformations are not important. Mindlin theory is one where shear deformations are important. There are elements which are based on Kirchhoff's theory; Kirchhoff's elements. There are elements which are based on Mindlin theory or Mindlin elements which are based on Mindlin theory called Mindlin, say elements. Now, the question is that, is it that Mindlin elements are much more broader than say Kirchhoff's elements and would it be that these elements work better than Kirchhoff's elements in most situations?

Ironically when the plates become thin, Mindlin elements some of them, some of the elements get into some trouble. We may not have time to discuss all that, because as I told you it requires a background. But ironically many elements, when the element, when the plate becomes thinner may not give as good a result as some of the elements, which are developed on Kirchhoff's theory. Hence many softwares may have different elements itself; element formulation itself, one set taking into account shear deformation, another set which may not take into account shear deformation. But, it is a good idea to have some idea of what should be the, or what is the thickness and then use the elements appropriately. If you notice in this figure, one of the things, one of the assumptions that we have made is that there is no stress perpendicular to the plate. That is in other words, epsilon z is equal to zero.

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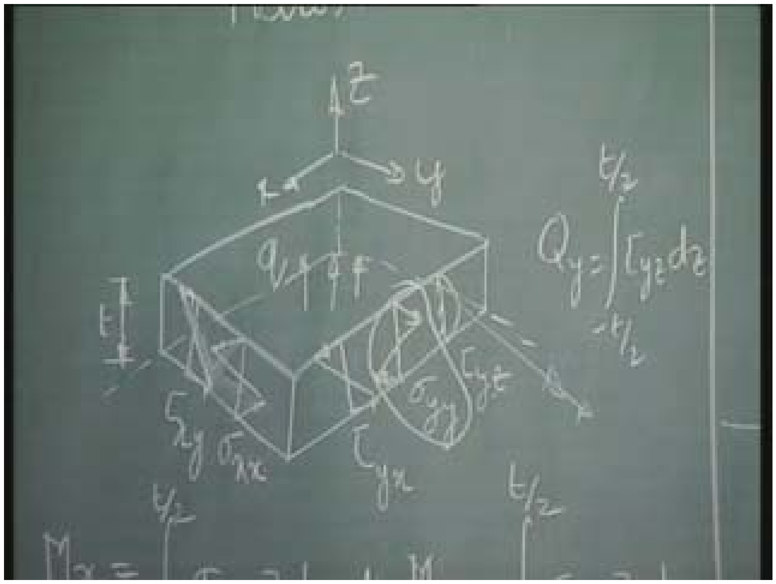

Please note that epsilon z does not enter into any of these derivations. So, what is the effect of it? This has an effect in our stress strain relationship. Now, look at the stress strain relationship carefully. What does this tell you or what does this show you?

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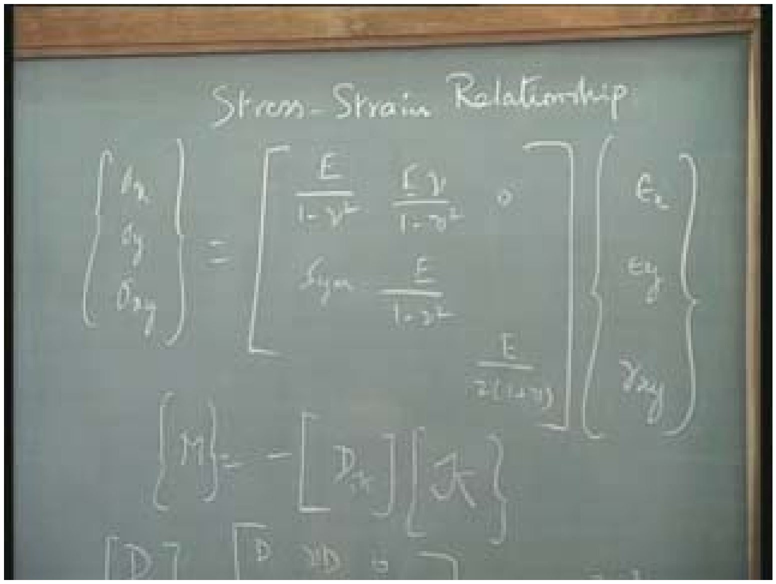

See that, see the relationship, which is brought about by this particular matrix. Yes; does it ring a bell? What is the type of element? Yeah, this is very similar to what we had seen in the plane stress case; these elements are very similar to the plane stress case. Please note that this stress strain relationship, this stress strain relationship what you see here is ones which we use, which use for Kirchhoff's theory. Now, if I want to use this for Mindlin theory, then the right hand side has to expand down. I have to include two more shears and of course, the left hand side also has to go down and the relationship would go through or I have to include two more terms or in other words I have to expand this further down in order to include or in order to write this down for a Mindlin plane.

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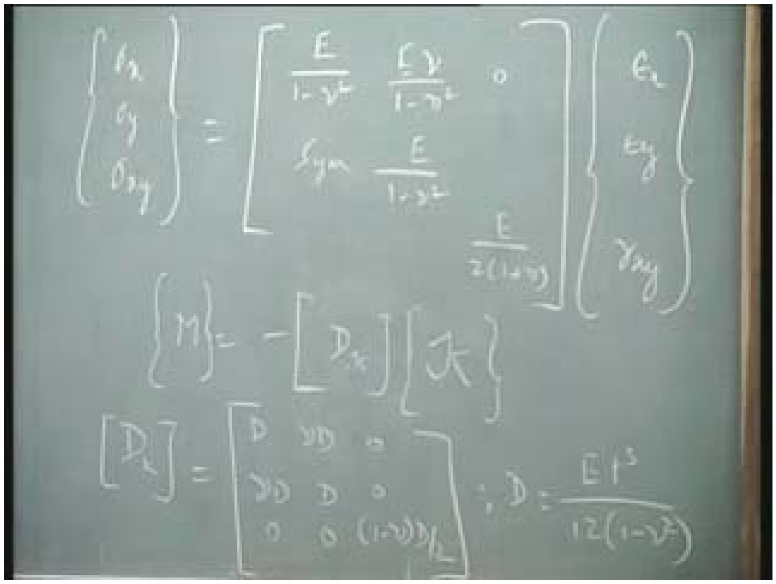

It is customary to write this equation in terms of moments and curvature. From where do you get the curvature? By writing down the expression for epsilon x from the displacements u, with the familiar equations which we know, we get curvatures. In other words there epsilon_x would get into, when I substitute u, we would get terms like u, w comma xx or dow squared w dow x squared and so on and that curvature is the one which makes the formulations of this problem quite difficult. Please note that now we are dealing with curvature, say for example, in Kirchhoff's theory, then we have to have C_1 elements. C_1 elements are not going to solve the problem, because continuity becomes quite complex. In other words, before we go further it is important to realize that it is possible to formulate a plate element, something like this.

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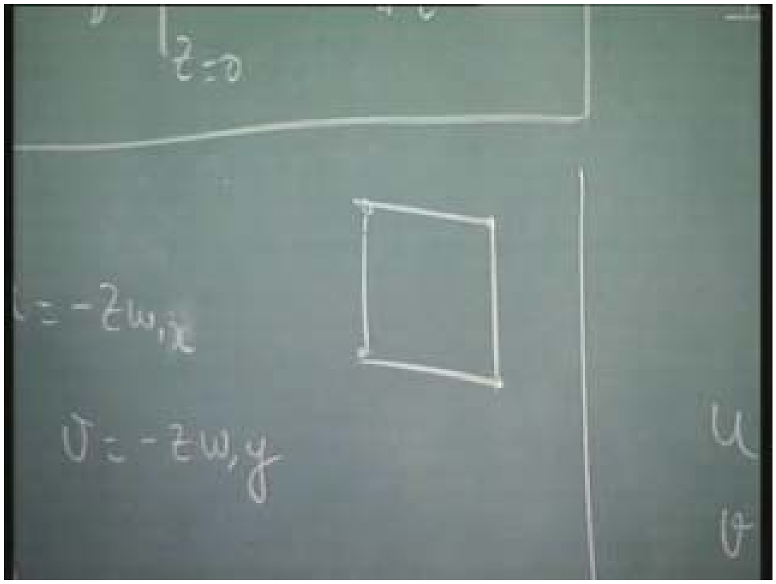

Now, the plate element becomes something like this, with the nodes sitting on these corners, with three degrees of freedom. Interestingly, whether it is a Kirchhoff's element or Mindlin element, we have three degrees of freedom. The degrees of freedom, for example, in the case of the Kirchhoff elements being w perpendicular to the board or in the z direction and w comma x and w comma y. In the case of Mindlin theory, these three degrees of freedom are now w $theta_x$ and $theta_y$.

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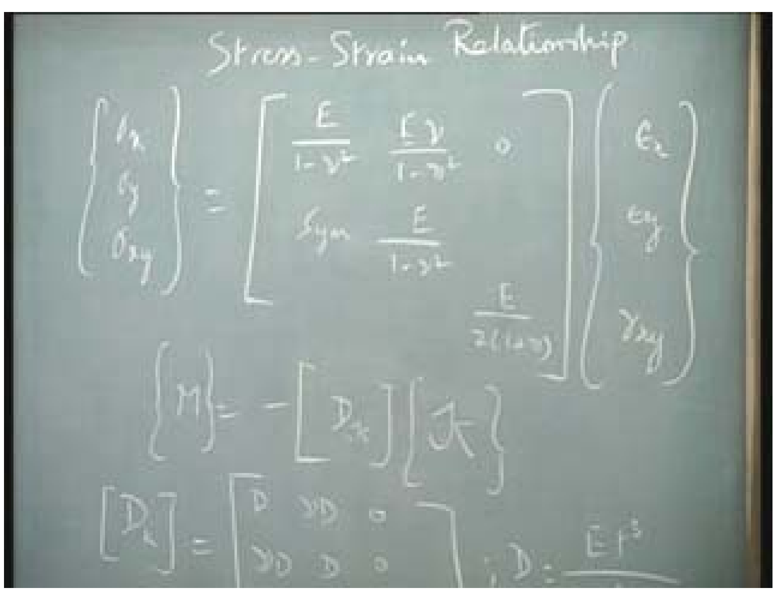

So, the formulations now take off from here and approach is very similar. Many of the elements, older elements at least, are very similar to what we had done in our earlier classes with all other elements. In other words, we go into our variational approach. We can write down, for example, the potential energy, the strain energy, using these functions. We can write down strain energy using these functions and then the formulations can proceed in a very similar fashion with interpolation function and so on, a very similar fashion as we have done earlier.

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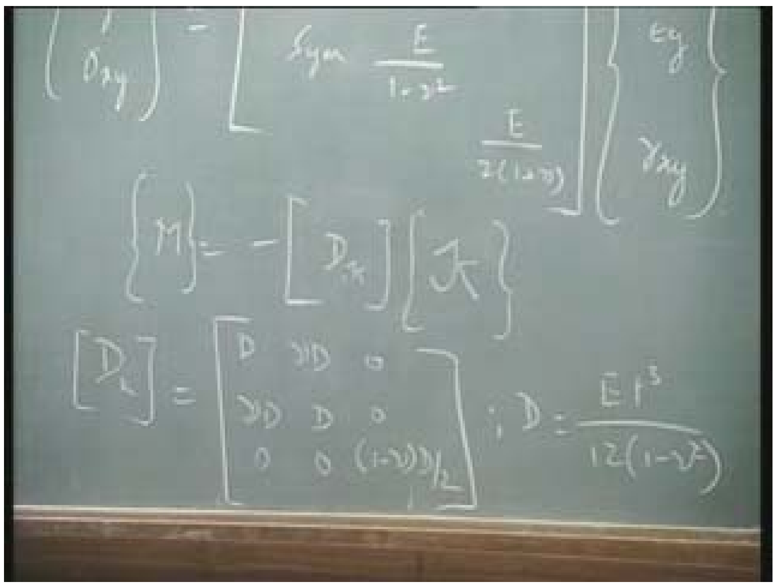

Yes of course, the problem as I told you, as I indicated to you especially for Kirchhoff's element, it has to be C_1 elements where as if it is theta, if theta_x is taken as a degree of freedom, in that case we are going to get theta x comma x and so on or in other words with Mindlin element, C_0 elements can be used and so on. This is a very raw statement. There are other difficulties when we use C_1 elements and it is not very straight forward. Nevertheless, I want to bring out the difference in the Kirchhoff's and the Mindlin theory. Lot of people who use plates and shells have one question. We will move to the shell in a minute, but before that let us answer this question. What is it?

They feel that 3D, everything is in this world is 3D. Now, why do I need to use plate elements? Computers are powerful, I can have lot of space, my systems are very fast, so, why do I need to use plate elements or shell elements? I can as well use a 3D solid element. Why not I use a 3D solid element? This is a question which many of you may have. Many of the users of this software are quite confused and many people use in the place where plate elements have to be used directly solid elements. The answer lies in the simple fact that in solid elements apart from the saving, we are not talking about saving of time; yes of course, that is very, very important. For solid elements, the amount of time you are going to spend is going to be very large and the nitty-gritty details of producing the model is again quite difficult. But, we have one more important problem which creeps up when we use for example solid element in the case of plate element.

What is that problem? The problem comes from the fact that epsilon_{zz}, zee zee or sigma_{zz} is such that it is equal to zero in this plate elements.

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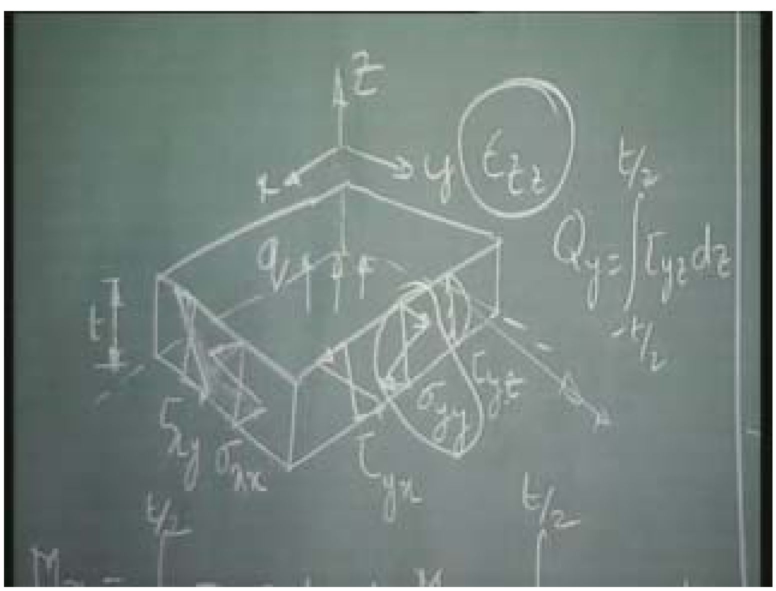

In other words, the plate is supposed to be very stiff in the z direction. When I now replace this with our familiar 3D solid element, then this z direction stresses are going to be present. When z directions stresses and hence epsilon_{zz} is going to be present, what would happen? Now, that stiffness, the terms which go to indicate the stiffness of epsilon$_{zz}$, those terms are going to be quite large, quite large when compared to other stiffness terms. So, when we want to now solve the problem or when we want to solve, put down the solution, we are going to have difficulty, because one of the terms associated in the stiffness matrix with one of the say epsilon$_{zz}$ is going to be very large when compared to other stiffness terms with the result that we may end up in numerical difficulties. So, it is not possible to do a problem straight away replacing plates and shell element by means of 3D solid element. That may not work many of the time or most of the times and the results may be totally wrong as well.

With this assumption or with this background, let us see what we mean by shells. What are shells? Let us keep that figure there. Let us just replace these things. In a very simple fashion we can look at shells.

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Shells are of course, I mean geometrically they are curved shapes. We know, all of us know, what a shell is. We have seen, lot of us, we are fond of picking up sea shells when we were young and maybe we do it even now. So, shells have a structure which is geometrically quite different from plane. What is the effect of these structures? The effect is that in plate, the whole load is taken by bending. In shells, we have plates, we have plate behavior plus plane stress behavior; we have bending and what we call as membrane behavior. So, plate plus plane stress are the ones or combined in order to give the behavior of a shell. Immediately one thing comes to our mind; what is it? Immediately we see that sigma_y on top of this kind of distribution, linear distribution, we have one more distribution on sigma_y; one more distribution of sigma_y which comes out because of the plane stresses. Yes that is the difference between the two.

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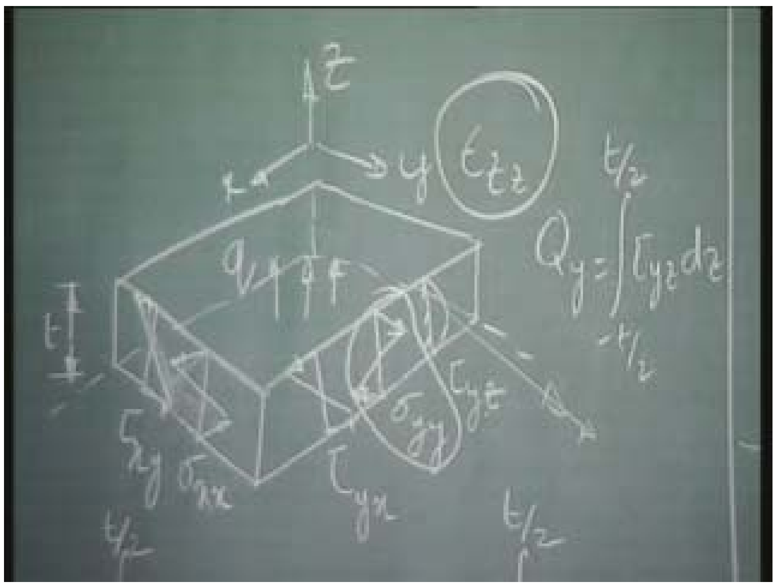

In other words, look at that that zero here, right there at the neutral point, which may not be zero, when we include for a shell, a plane stress behavior as well. So, the plane stress in fact or in other words this plane stress behavior most of the time may even dominate the bending behavior and would take the load. Now, when I say that plane, so, first thing please note in plates we do not have plane stress behavior and that gives us an interesting possibility which is exploited by many of the packages. So, they do not have a separate plate element. What do they do?

They use shell theory or shell elements in order to model plates, simple, because as I told you just now, since we have plates and plane stress behavior we can use plate alone, behavior of plate alone in order to model a plate and in fact many of them you may not even have plane stress because, plane stress element separately, because we can use this as well for plane stress and so on. With that background you can immediately pick up and tell me that, look the formulations are quite simple. We can put the equations in such a fashion that I can superimpose plate and plane stress behavior in order to get shell. Yes, that is a good beginning and many elements are formulated by some sort of that kind of superimposition, but generally they have some difficulties and they do not perform that well under many circumstances. They are basically flat elements; they do not take into account curved shapes and so on. They do not perform that well in many circumstances especially when the plate or the shell is quite curved. On the other hand, generally shell approach today, in the past decade or so has undergone lot of research or a lot of people have worked on the shell problems or can broadly be classified into two categories. What are these two categories?

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One we call as what is called as the degenerated shells, which are basically some sort of degenerated 3D elements where the element is brought down in the thickness direction or elements which are based on theory of shells. The theory of shells is quite involved; lot of things has to be understood here. For example, these theories are based on what are called as Cosserat continua and are and their fundamentals, basis, is what is called as a director theory. Degenerated shells on the other hand are simpler to understand and the major difference between these two is that the degenerated shells, for degenerated shells, we have to integrate across the thickness whereas the thickness effect, the thickness directions and change in thicknesses and so on are taken into account in the theory of shells through what is called as director approach. As I told you, the mathematics behind it is very complex, because of the fact that the shells cannot that easily be used or cannot be that easily understood by using our regular approach. So, people have gone into what they call as local coordinate systems or in other words, co-rotational approach and so on and hence the mathematics, the tensor analysis that is involved for shells are very complex.

The last word is still not yet said about the shells, but it is important to realize that when you use a package on what is this based on and may be it will be a good thing to do a few examples for shells in order to understand how shell elements work. In fact, I would say that the real test of good software is to look at shells. How does the shell behavior or the shells work? Especially, if you are looking at a non-linear package, then it is important to realize how the shells are formulated and implemented in that package would give us the good idea as to how the package itself is. But, generally in order to understand the shell behavior and in order to use the shells, what we can do is to look at shells as some sort of a plate plus plane stress. That will give us an idea, a physical picture, as to how we can use it under various situations.

There are number of situations for example in metal forming, sheet metal forming, where lot of work, for example, door of a car has to be modeled, then, you many times you use a sheet element sorry, a shell element. Sometimes people use what is called as a membrane element. Again this is a modification of the shell element. We will not go into the details of it, but many times you use for metal forming, sheet metal forming work you use what is called as the shell element.

A variant of the shell element is what is called as axi-symmetric shell.

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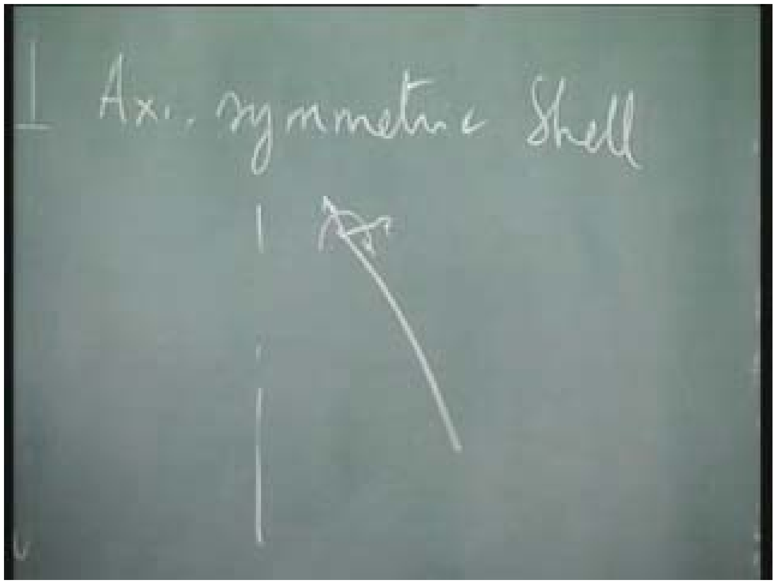

Axi-symmetric shell, yeah, I know that the name is very familiar. The axi-symmetric shell is very similar to our axi-symmetric element. What does it mean? It means that there is an axis of symmetry about which you can rotate the shell, rotate an element and get the complete element. Again, these elements are based on Kirchhoff's and Mindlin theory. For example, locally each of these nodes have three degrees of freedom and then rotation. So, you can have three degrees of freedom, say u, w and say beta.

What is an example? Good; civil engineers would immediately pounce and say that say, cooling towers can be looked at as an axi-symmetric shell. Whether it is or not, it is a different issue, but at least looks like you can at a first instant, you can use it for cooling towers. Say, suppose you want to do some, again sheet metal work, many places axi-symmetric shell would be useful. But, please understand that whenever we talk about this kind of sheet metal forming and so on, the shell elements that we are going to use are highly non-linear elements.

Please note that they are all highly non-linear elements, which we have to take into account, not only geometric non-linearity but also material non-linearity. But nevertheless, the behavior per say is not going to change to that extent. That in a very brief fashion what we mean by shells and axi-symmetric shells and since we are coming to an end of this whole course, it is important to realize as to where we are going to put these in place and use them. The emphasis here is only what the fundamental is and how you can use these in different situations, understanding the physical behavior, the assumptions that are made and you can use them with that kind of confidence.

One of the things which I wanted to emphasis here is that when we look at these shell models, whether it is a degenerated shell or theory of shells, you know based on theory of shells and so on then, the inputs that are required can become quite vague sometimes. If this is, for example you may be asked to input what are called as directors, if that is the case then, input becomes very difficult.

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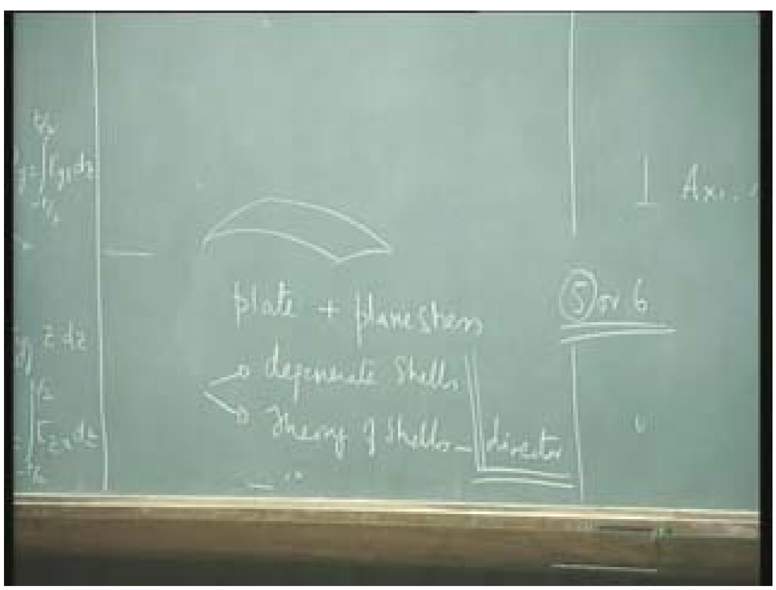

Many of these shell models in fact are based on 5 or 6 degrees of freedom. In fact, many of them are based on 5 degrees of freedom. If you really look at theory of shells, say for example, formulation of Prof. ….. and his co workers, then many of these elements are based on 5 degree of freedom per node; 5 degrees. It is important that you realize when you use a shell element what the degree of freedom is and how many degrees of freedom are there? For example if the formulation is based on theory of shells, then the 5 degrees of freedom may not indicate the degree of freedom with which you are familiar with. This may be of course, displacements in the x y and z direction; the other 2 degrees of freedom are not pure rotations. They are related to the director theory.

On the other hand, in order to make it palatable, in order to push lot of problems under the carpet, many of the softwares have 6 degrees of freedom. They have 6 degrees of freedom and that is through what we call as degenerated shells. What is the problem? Why is that we are harping on say, 5 and 6 degrees of freedom?

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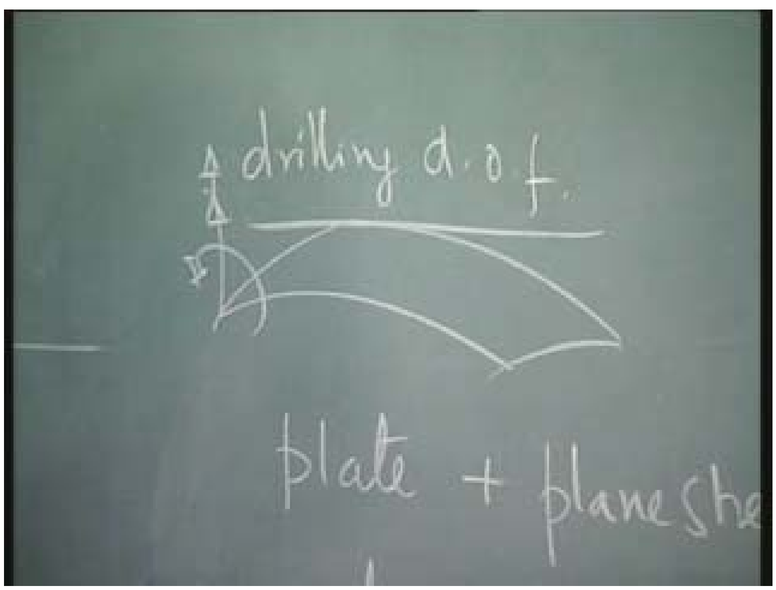

The problem comes about because of the rotation; the rotation that is this rotation, what we call as drilling degree of freedom. It is important that we realize how this drilling degree of freedom is actually handled. Many softwares handle this by means of some pseudo approach, where they say, average out the stiffness' in the other two degrees of freedom and then put that out here and so on. But, this would vary because I mean, it is impossible to cover this in this course, because I mean actually till about say may be 2 to 3 years back there are about 140, 130 to 140 different formulations. So many formulations are there, so it is impossible to deal with it on a canvas like this. But, it is important that when we use a software, you understand how exactly drilling degrees of freedom are dealt with; it is a 5 degree of freedom or 6 degree of freedom formulation and these things will also become important in order that you can model using the shell elements. That in a nut shell is the theory of the plates and shells.

In this course, we are coming to an end of this course and so, in this course what are the things that we have done. We have started with the stress strain relationship. We defined what strain is, what its relationship with displacement is and what the relationship between strain and stress is. Then, we defined what is called as the equivalent stress and that would form, we said that that would form the basis for interpreting the results when we get the finite element or when we do the finite element analysis. Then, we looked at the formulations. We looked at various theorems, energy theorems that are available to us from where we can draw and we can write down these element formulations. If you remember we saw the functional approach, we saw what we called as the virtual work principle and we saw its relationship with say potential energy theorem and so on. That forms the basis or that is the one which we are going to use in order to formulate elements.

Then, what we did was to break the complete continua into number of elements. We designated certain points as nodes and if you remember we wrote what are called as interpolation functions and we brought down the infinite degrees of freedom into finite degrees of freedom. Then, using these fundamentals we developed what is called as the stiffness matrix and the stiffness matrix form the basis for us to proceed further, to have a relationship between force and displacement. We determine the displacement after assembling the stiffness matrix and of course after putting down the boundary conditions, we determine the displacements. From the displacements, we went ahead and determined what we called as strains and ultimately we were able to predict what the stresses are.

Once we know the stresses, we were able to interpret them by looking at what we call as the equivalent stress or Mises stress. The Mises stress would form the basis to understand whether a material would yield or not. In the process of dividing it into a number of nodes and number of elements, we realize that not only geometry is important, but the structural behavior is important. From that point of view we had a number of elements to mimic the structural behavior. We said that the structural behavior can be looked at as plane stress, plane strain, axi-symmetric, plates and shells, beams, axi-symmetric shell and so on and number of them. We said that depending upon the structural behavior we would choose an element and put it in place.

In the process of looking at the whole issues of finite element formulations, we also saw that there are one or two issues which become very important, especially in mechanical engineering. These issues are contact and we said that it is possible to develop elements which can or which can take into account contact, in which case we are going to move away from linear to the non-linear region. So, we move away from linear to non-linear region. So when we go to non-linear region, we saw that there are several ways in which you can solve these problems.

One of the most popular approaches for solving this non-linear problem is by Newton-Raphson method. We saw how exactly we can use Newton-Raphson method and we were keenly watching as to what this convergences and so on. It should be realized that the non-linearity which is driven by geometry, material and contact are the essence of using finite element analysis for process modeling or in other words in order to model manufacturing processes. We saw some of the niceties of these approaches, may be we did not go into the details of every formulation that is not possible in an introductory course like this. But nevertheless, we saw how exactly we use this kind of contact elements and so on.

That part we covered and we also looked at how to model heat transfer problems, both transient as well as steady state heat transfer problems and there again we had an approach, which we called this as a semi-discretisational approach and we used, if you remember, we used some of the well known principles from finite difference in order to model or in order to work out problems in finite elements and lastly we have just given or we have seen an overall picture of the shell element formulations.

I am sure that this kind of background would give you an idea as to or at least you can, see, I do not claim that we have gone into the depth and breadth of finite element, because it is vast. There are about 20 to 25,000; I do not know how many, tens or thousands of papers that have been published in this field. It is an ever growing field. There are lot of still research issues that need to be tackled. Especially, we have not seen much on what we call as explicit finite element analysis which is becoming popular now-a-days, because of its ability to be used without much problems like convergence in the metal forming cases and for example, these are used for impact analysis and so on; we did not have time. But, what I want to tell you is that in this course we covered at least the fundamentals which will be useful for you to use a package and to grow further; to grow further or to understand further the niceties of finite element analysis. Thank you.