Introduction to Finite Element Method
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Lecture - 30

Yeah, in the last class we defined or discussed the contact element and we said that the key concept in the contact element definition is master and the slave nodes.

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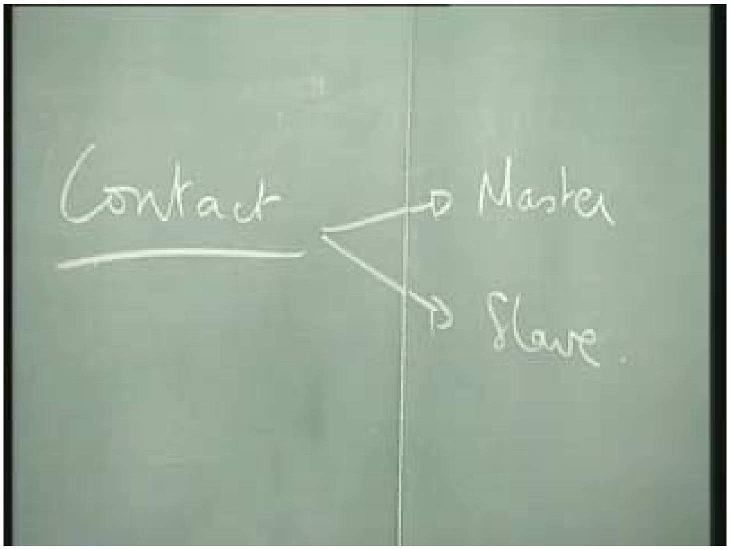

There are some questions as to how this can be used in practical software. So, let us now look at practical software like Abaqus (pl check the spelling), which has a good contact definition and see how it can be used here.

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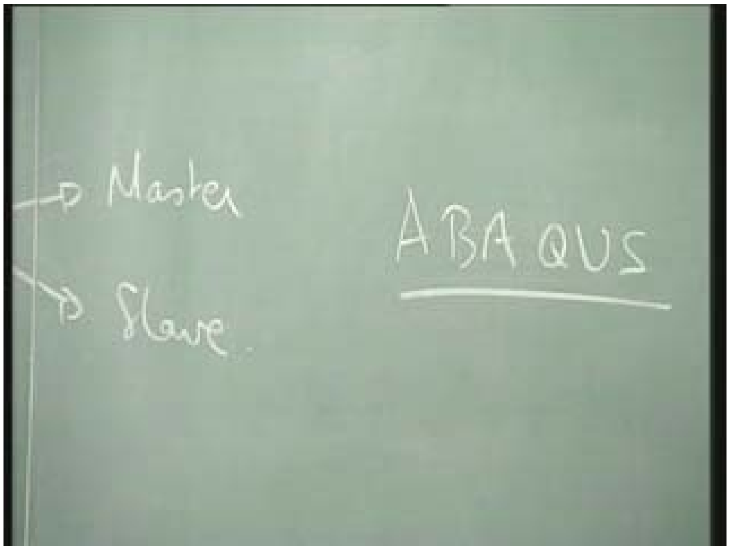

As I told you in the last class, this master has or can be defined using a rigid surface or a rigid element or a deformable element. The slave will always be a deformable element. For example, we defined the dye. In our dye material, we said the dye to be the master and the material which is deforming to be the slave. For example, if there are two gears, two gears which we are mating, we have seen this example before; if there are two gears, then both the gears are deformable.

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So, the contact is between deformable and deformable. There is one set of another set of contact called self contact. For example, a sheet can buckle and come into contact with itself.

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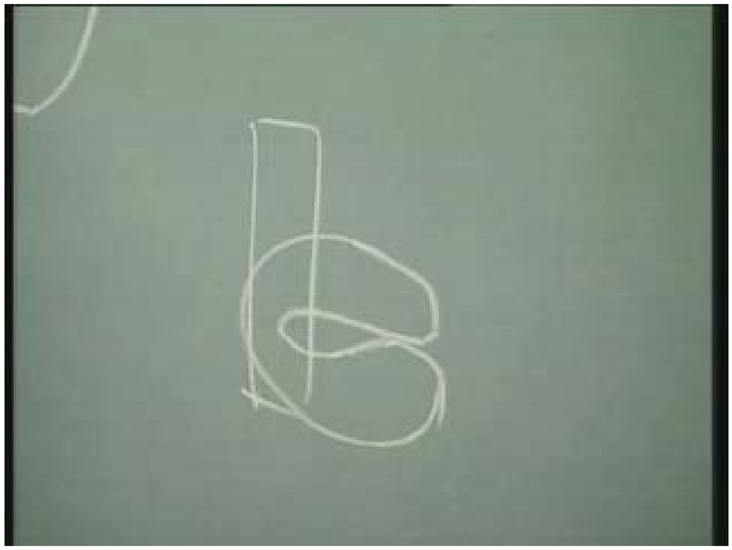

Suppose I have a sheet like this. It can buckle and say, come into contact like that; it can contact, it can have a contact with itself. This is called self contact. This is used for example, in crash worthiness problems and other things. Let us not go into this, this is quite advanced. There are few packages which can support this kind of self contact. Now let us come back to our problem of how to define it in Abacus. Let us now look at how we define contact between rigid and deformable. There are special elements for example here called R2D2 element.

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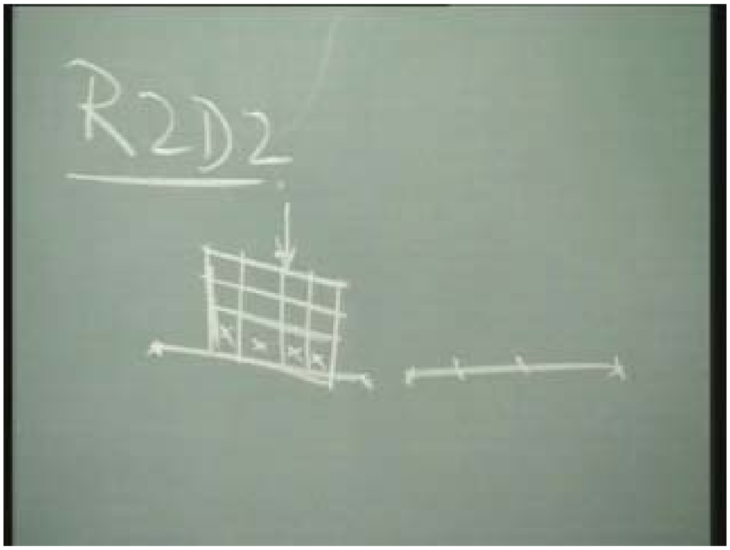

This is a rigid element which can, for example, be used with respect to or with the plane strain problem. Similar rigid elements are available with axi-symmetric and so on. Since we are talking right now about, as an example about plane strain, this element happens to be just a, say a line; just a line. Now this element can be used to define the master surface and then you can have one more set, say for example, this is the material that we are going to say compress; that is what we are going to compress. Then we can say that these elements are in an element set and the surface, the bottom of it, bottom of this element, defined as a surface is the one which is going to be the slave surface or in other words, when I say this is the slave surface, automatically the nodes are extracted. So, number 1, you define a master surface with respect to some elements or in fact you can have a master surface defined analytically also. So, that is one thing. Then you define a slave surface and then define the friction conditions between them and combine them together.

There are about four commands, for example in Abaqus which goes to define the contact condition. It is very, very simple but it varies from one software to another software. How you are going to define contact varies from one software to another software. Nevertheless these two are the ones which we are going to define. We will leave contact for the time being. Now that we know what types of contact are, how important it is, let us get back; let us get back and define a more important problem which is a part of contact, which is non-linear aspects of finite element analysis.

Non-linear finite elements as a topic, is a very big topic. I am not going into the details of it, because there are lots of things that you have to know in non-linear finite elements. But I am going to take you to a level where you can appreciate what is non-linear finite element and if you have a software you can use non-linear finite element codes. In other words, I am going to define all the important terms that we are going to use in non-linear finite element analysis. The linear part, whatever we have done, has been quite rigorous, but we cannot take to that extent the non-linear part of it. So, we are going to just define certain things which are of importance.

What is non-linear finite element or what is non-linearity and why do we use it? There are various reasons why a problem becomes non-linear. Watch this carefully, there are various reasons. Basically that the material that you are using may become or may leave the elastic range, if you are using metal, and go into the plastic range, which results in what is called as material non-linearity or the deformations can be very or the displacements can be very large.

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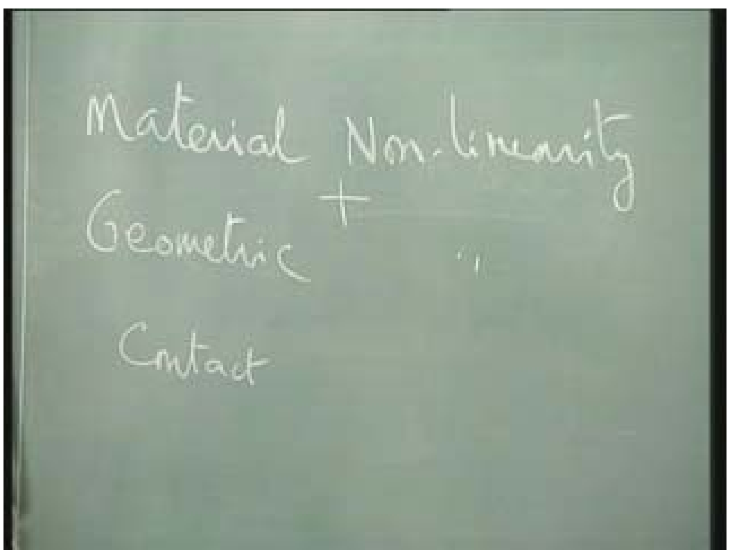

Though the strains may be small, the displacements can be very large in which case the definition of stress is no more what we have defined here in the Cauchy’s stress and so on and the way you define stress and strain have to be different or in other words you can get into what is called as geometric non-linearity. When the displacements are very large, then you get into what is called geometric non-linearity.

The non-linearity is also due to reasons like contact. In metal forming problems or many of the manufacturing problems, we have to combine material as well as geometric non-linearity. When we use material non-linearity our geometry does not change that much. Our original geometry is retained.

There is always a question. These are what are called very practical things which I am going to talk about. There are always some questions as to what do you exactly mean by material non-linearity? Because, for example, if there are going to be plastic strains, if there are going to be plastic strains, immediately I know that the strains are much, much larger than the elastic part. Then what do you mean by material non-linearity alone?

Yeah, let us take our simple example.

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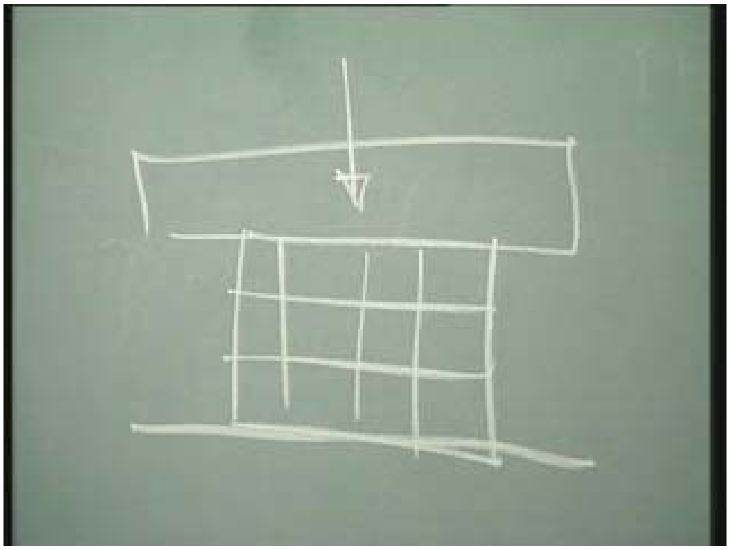

The simple example is very useful to explain things and then this is some flat end and then you are applying the load. Now, this results in, say for example, plastic deformations. There are zones where the yielding has taken place and when I release it, it will get into a permanent set. The strains during this permanent set will be definitely larger, larger than what you will have in the elastic strain; elastic region strains will be very, very, very small. It is usually the practice to say that if the strains are less than 5%, then the material can be analyzed or this problem can be analyzed using only material non-linearity. So, what is that? 5%; it is not God given figure. It is just that by experience we feel that about 5% strains would be a plastic strains, if you are going to do plastic problems. 5% of strains would be good enough to do a problem, say material non-linear problem.

Many of the problems, for example the problem which we saw for the heat transfer and distortions or the welding problem, the strains were such that it can be considered to be a material non-linearity problem. So 5% is the usual limit; above 5% then you cannot use material non-linearity alone. In other words when do we use geometric non-linearity? We use geometric non-linearity when the deformation, the deformation suppose I am pushing it. So, it has come to this level, just that level; very, very close.

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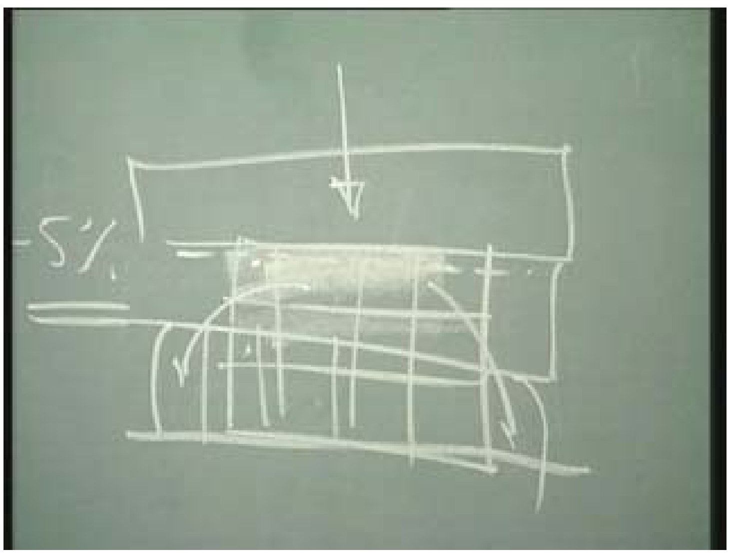

The original mesh and the deformed mesh are not very different, say for example. Whatever reference I take, they are the same. Then what do I say? I say that problem is not geometrically non-linear. On the other hand, the original mesh and the final mesh are quite different. For example, I move this flat end to this position. Now, what will happen to my new mesh? The new mesh now will become something like this. The whole, all this guys who are here have to now flow here and the mesh will be totally distorted. The element which was here would have come to this place. You definitely cannot say under this condition that the original mesh closely resembles the final mesh; no way. When we say resembles, it is not only the geometry, but also the positions; co-ordinates.

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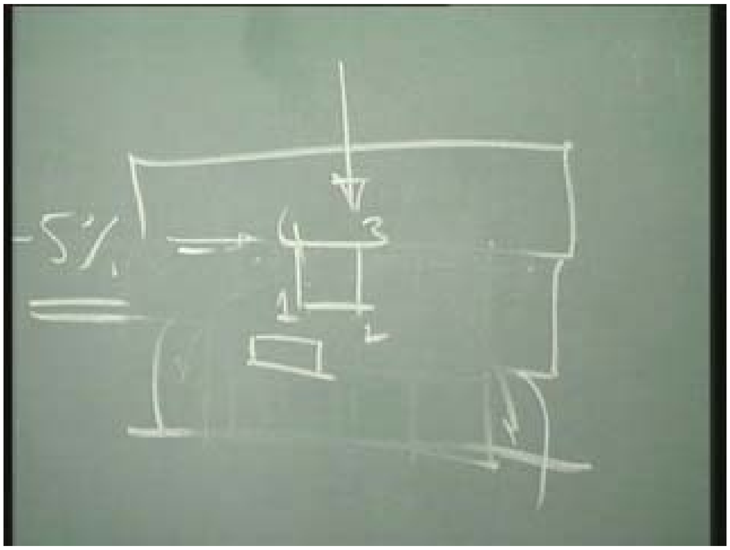

If I define the original mesh just take one element, suppose I take only this element, say, let us say 1 2 3 4; I have four nodes defining the element. Under now compression this element goes to somewhere here and occupies this space; under compression. Definitely it is obvious that geometrically they are not, no where nearby. Position wise also, note that carefully, position wise also they are not, cannot be considered to be the same and hence say that the problem is geometrically non-linear when the final deformed mesh and the original mesh they are not at the same position. Please note that it is not necessary that there should be lot of strains in order that there has to be a geometrically non-linear problem.

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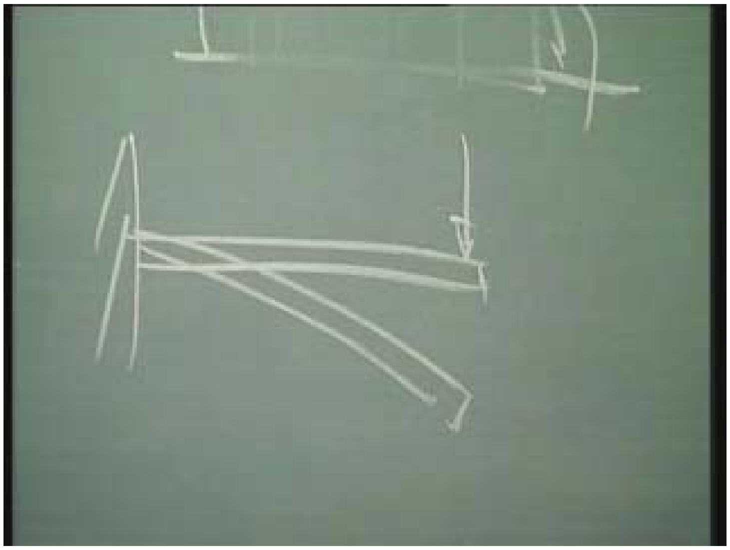

For example, if I have a cantilever beam and if it is very, very slender beam and if I just apply the load, the deformations may be larger or in other words the displacements may be larger. It may go like that, but still I might not have yielded; many times it may still be elastic, but the original and the final positions may be so very different that I may not be able to consider it as a linear problem and I may have to invoke what is called as geometric non-linearity. Is this clear? So, geometric non-linearity is basically because you are not able to identify the deformed or displaced, however you want to call it, mesh with the original mesh positions as you see it in the result.

Material non-linearity is of course due to, for example, plasticity. Plasticity is one of the examples why materials non-linearity is there. There are lots of other examples, for example, non-linear elastic materials like rubber. We saw a tyre analysis. A rubber has a very peculiar behavior under load. In other words, it has a non-linear stress strain curve. The stress strain curve for rubber may go something like that.

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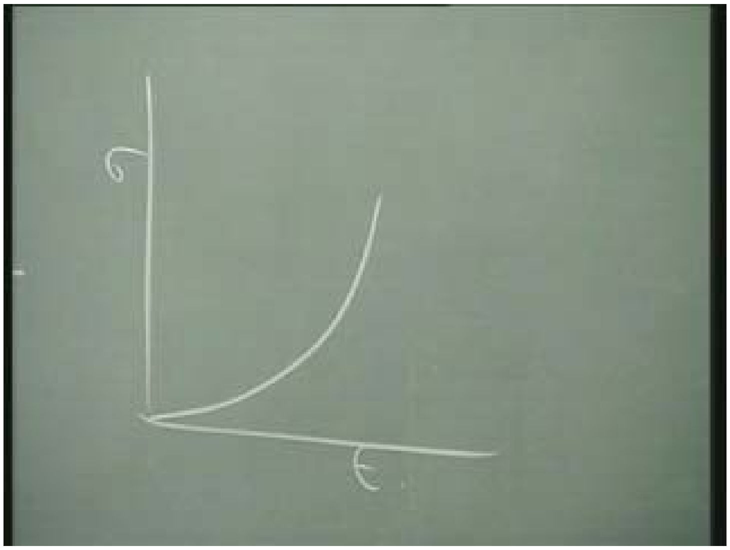

This is sigma and epsilon; sigma and epsilon and hence this material, this rubber is also supposed to be materially non-linear. There is non-linearity in stress and strain. Of course many problems in rubber would involve both geometric as well as material non-linearity, because many times you do not just strain rubber to 5%. It will be much, much more than 5% and hence we will have both material as well as geometric non-linearity. Is this clear? Even if you do not have both material and geometric non-linearity and if you have contact, then also the problem will be considered to be a non-linear problem and its approach is going to be very different. Is it clear? So that is the definition for non-linear problems, but unfortunately in this world, most of the problems are non-linear. Especially in mechanical engineering many, many problems are non-linear problems, but usually we make some assumptions so that problems are handleable and still do it using the linear analysis. Is it clear?

Now having defined this kind of broad set up, let us see what the approach is that we are going to take to do non-linear problems. Please note that the element types that are used are exactly the same, but the approach is going to be slightly different. The approach that we are going to take is what is called as an incremental iterative approach; incremental iterative approach.

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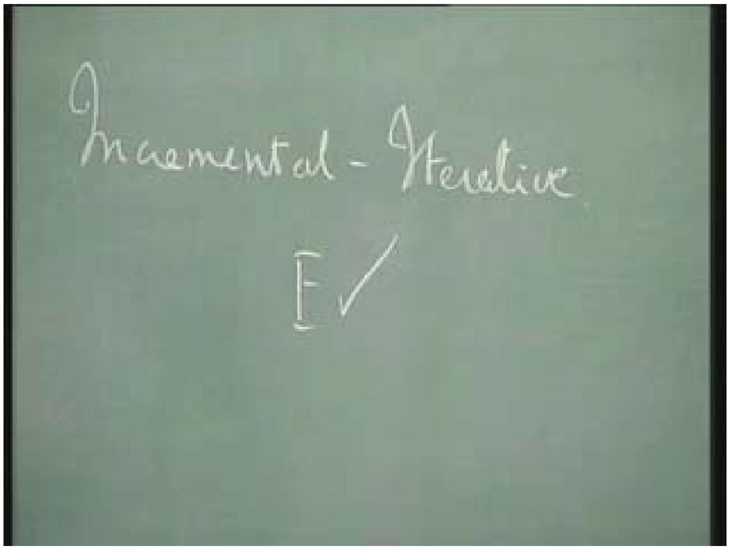

What is incremental iterative approach? Now let us look at this curve here. What do you get out of this curve? You just see the curve and what do you think is the major difference between say this curve and a linear curve? I am not going to draw the linear curve in that, but can you tell me what is the difference between this curve and a, say a linear stress strain curve? Slope; correct. So, the slope keeps varying. In the linear case, we calculated stiffness matrix k. Remember how we calculated it?

We calculated stiffness matrix k with the Young's modulus E and of course Poisson's ratio and we used only one Young's modulus which is the initial stress strain curve and so E was good enough for me to being used. Is it clear? On the other hand, if I look at the stress strain curve for this new rubber material that I have drawn, then obviously this is not enough. So, what do I do? Just look at that curve or I mean better still I do not want to go to rubber. So, better still let us take a curve like this, because there are other difficulties with that. Let us take a curve like that.

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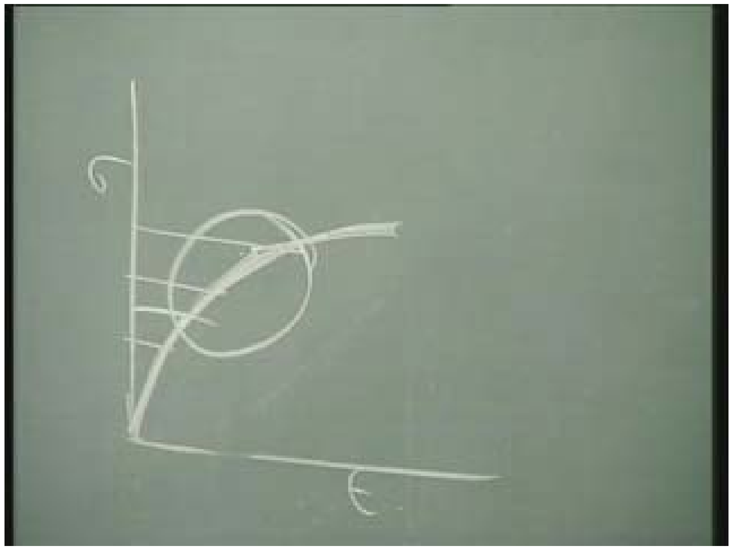

For example, stress strain curve for a material, say metal and then say let us say that this is yielding. This is straight and then this hardening and so on, that is the yield point. Let us see how to do this problem. Do you understand? The issue is E for the elastic material which we have done so far is what? Is constant and now the slope varies, as you correctly said, with respect to the strains. So, my approach has to be entirely different. Is that right? Now tell me, can you tell me what best you can do in order to arrive at the same type of approach? Yeah; divide it into linear parts. That is nice. So, that is the first very typical solution; divide it into linear parts so that I will have some linear positions, linear portions rather, of the curve. That is nice.

In other words, what he has essentially done? See, you can always look at this as a load deflection curve. You know, it is very similar. Does not matter, you know. All of you know. But what is that exactly he has done by saying that I will divide this into a number of linear parts? What has he done? No, no. Look, yeah, that is true. So, he has said that essentially we can increment the load, instead of giving the complete load. He can increment the load instead of giving the complete load at one go. Is it clear? Have a look at that. Say for example, I can give load up to this, then up to this, up to this and so on.

The first concept that you get out of it is that the loads are not given completely in one shot, but is given in an incremental fashion. What do I mean by that? Let me again tell you, say for example I have 1000 Newtons, just for example. I will not give 1000 Newtons in one step. I will run the problem for a number of steps. I will first give 100 Newton, look at the result; 200 Newtons, 300 Newtons. Now, 100 Newtons I will look at the result. What do I mean? I will look at the equilibrium. Then what I will do is I will add one more 100 Newtons so that I go to 200 Newtons. Then I will go to look at the result at 200 Newtons; things are okay. What is things are okay? Equilibrium; if equilibrium has achieved in 200, then I will go to another 100, say 300. If things are working out very well, from 300 I will jump to say 500 and so on. So, the first concept is that I will apply the load in incremental fashion. I will apply the load in an incremental fashion; that is the first thing.

The next concept that becomes important is iteration. What is meant by iteration? Yeah; wait a minute. I mean that is the regular thing, but why do we need it? Let us just look at that curve. I will just expand this curve; this part of the curve. Say for example, here I will just draw that part of the curve alone.

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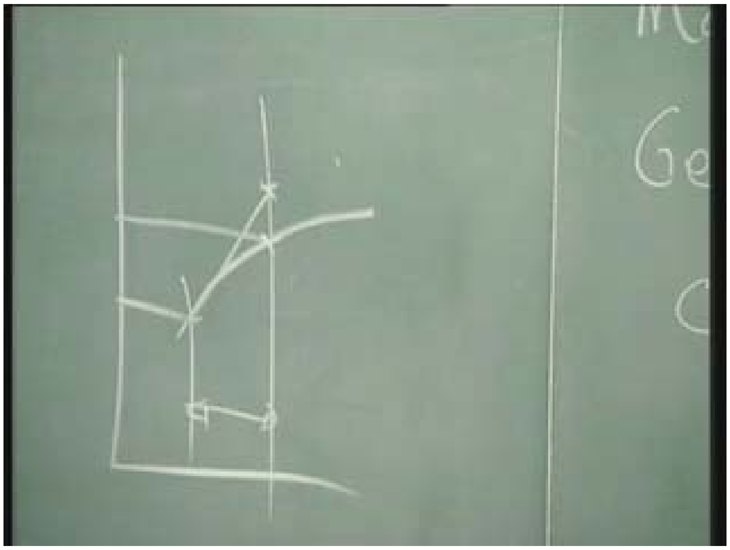

I have just exaggerated that curve; that point. Let us say that that is the slope at this point and I want to go further in this curve. Is that clear? The curve extends on either side. At this point, let us look at this as a load deflection curve for the time being, load deflection curve. The slope of the curve is what I have taken, from which we can easily work out tangent and the hence the stiffness. Now what happens? Like our friend said, if I take, if I divide it into number of say linear part just exaggerated that, so, what happens here? Error; so, what is the error? See, suppose I have to go to this point. For this load, suppose that is the load that for this load I have to go to that point according to the actual load deflection curve.

This is 1d problem, just to understand things. If I had taken the slope at this point, where would I go? I would have gone here, which would be outside the load deflection curve. So, I would have introduced error. Now, this is one step, error in one step. So, if I keep doing this in every step, then what will happen? I will have error in every step and there will be a total drift in the solution and hence what should I do? I have to somehow adjust this, so that my error is reduced or minimized so that I will somehow get to this point. Is it clear? Have a look at that for a minute now, understand what is happening at a micro level in one step; one step what is happening as you proceed in the load deflection curve? Is it clear? So, you get into error. What is the lesson? Though you are using an incremental approach, though you are using an incremental approach, until and unless you check for what is called as convergence until and unless you check for convergence there will be error in your solution and the error is bound to explode or become bigger and bigger. Is it clear?

Now, what is this error what is this error, what is this error? I have defined now. No what do I mean by error? I said this error, you also saw that there is an error, all of you agreed. But what exactly is the error? You have a look at that, look at that and tell me what exactly the error is? No; no, no. I want it in physical terms, what is this error? Drift; much more clearly. I know all of you know it. I know, because all of you have an intuitive feeling what is an error but still not defined what exactly it is. What is happening, when you apply the load what will happen? There will be an internal force that is created. The stresses are the internal forces. These internal forces have to equilibrate the external forces. Is that clear? So, the error is the difference between the external applied force and the subsequent internal force that is generated. So, that is the error.

What do I mean by equilibrium? I mean to say that the internal forces and the external forces should equilibrate and then only I say that there is no error or in other words, in other words I say that there is convergence of result.

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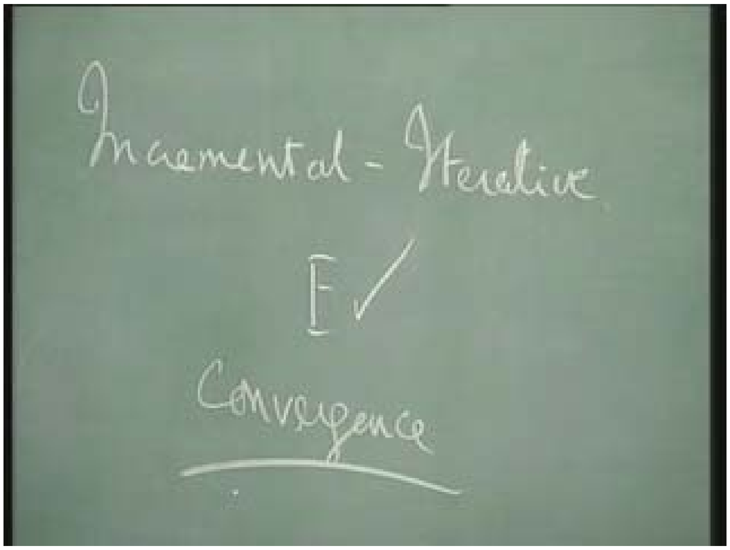

I say that there is convergence of result. Note the term convergence. We say that there is convergence when the externally applied forces and the internally applied or internally developed stresses which would result in internal forces, both of them are under equilibrium. Is it clear? Now, let us go back and look at this curve, this example.

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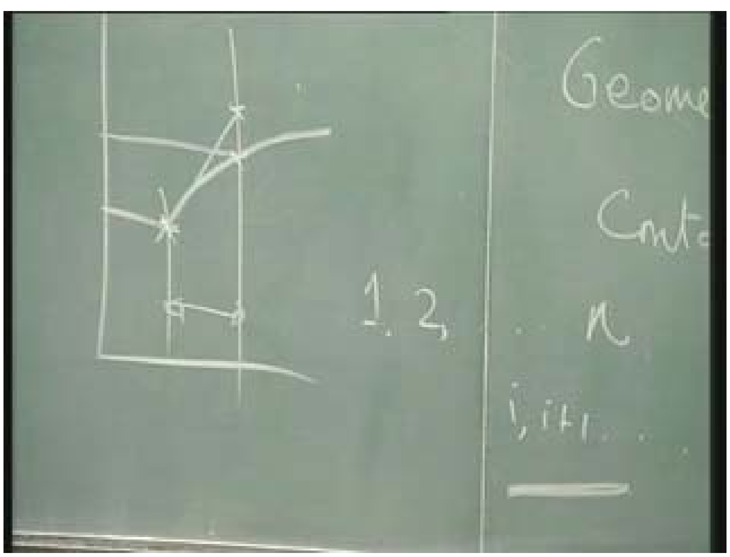

Let us say that, first let me introduce some notations. Let us say that I will apply the loads in increment. Let me call this as increment 1 2 and so on, say up to n. Is it clear? Now, let me say that some ith increment, i plus 1 is the way I will define the increments. i can be 2 at a point; then, i plus 1 is 3, so, I will have to go from 2 to 3. Usually the loads that are applied may not be a constant. For the time being you can even assume it to be a constant to understand it better, but please note the loads that are applied during this increments are not necessarily constants.

In the first increment, I may apply 100, 200; second increment, I may apply another 100, then fourth increment I can apply 300. If things are well, proceeding in a good fashion I may increase the amount of load that I am going to apply and so on. Let us not worry about that right now. The first thing is that I am going from one increment to the other increment. So, first I will apply 100. That means that I would have completed the first step. Let me say that I am at the ith increment, at the ith increment and I am now proceeding to the i plus 1th increment. Next increment, I am proceeding. Is it clear? No?

See, let me say for example that I am applying a load of 1000 in 10 equal increments for the time being. That means that if i is equal to 5, I am in the fifth increment. That means that I have completed 5 increments. Then I have already applied 500, say Newton. Now, from 5, I am going to 6th increment. That means that from 500, I am going to apply another 100 to go to 600. Is that clear? When do I go, when do I start going? Only after I make sure that at 500, I am under equilibrium. What does it mean? It means that What is equilibrium as far as this load deflection curve is concerned? That I am sitting right on this curve. If I am here, then the load versus the corresponding deflection is right and I have the corresponding deflection, they are correct. So, I will be under equilibrium. As long as I lie in this curve, what does it mean? It means that I am in equilibrium.

There is a displacement or deflection, which is the result of equilibrium and that is what is depicted in this load deflection. From this point I am moving now to a higher point, say from i to i plus 1; that is what I said 5 to 6. So, I apply more load.

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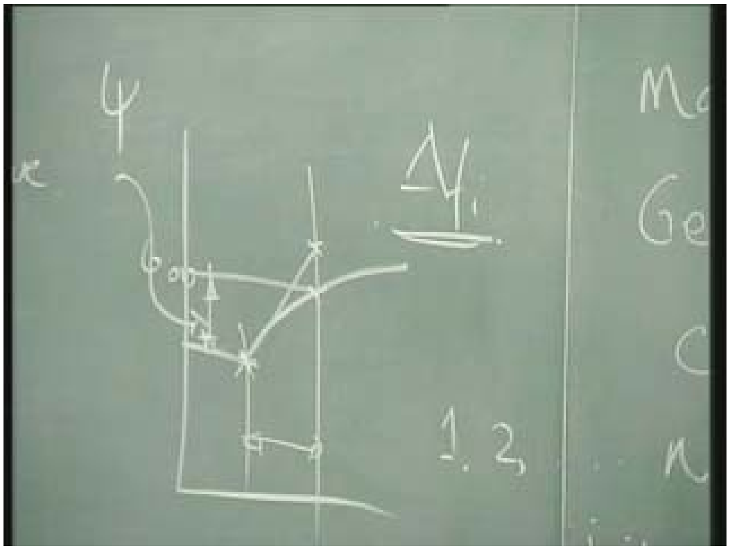

Let us say that I apply a load of say delta f_i. I apply load of delta f_i. What is delta f_i in this case? That is delta f_i. Is it clear? That is delta f_i. So, I apply more load. To start with, immediately I will have an error. What is the error? First step as I immediately see it, as soon as I apply here there will be an error, there is an error. The error is that at i, whatever stress I have developed is good enough to equilibrate; good enough to equilibrate that 500, but not good enough to equilibrate 600. So, now I have to calculate the stress for 600. This is 500 and that is 600. Is it clear? So, what is the error now, first as soon as I apply delta f? Delta f itself, 100; fantastic. So that is the error; first step that is the error. So, let me call that error as say psi.

To make this clear, let us look at the error term, how it looks like? Yeah, what is the question? No, this is the error. I called this, one second, one second.

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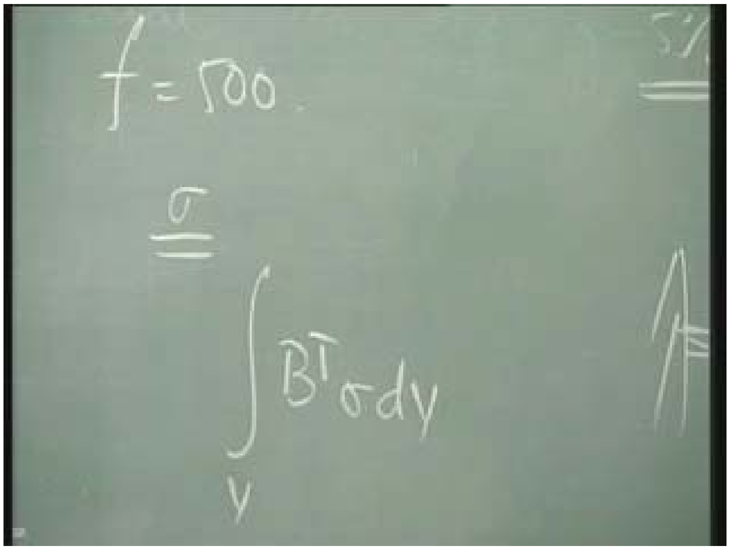

See, we are at 500. At 500, whatever load I apply, I applied the load of, let me call this load as f, f is equal to 500 and due to 500, I develop some sigma, stress; some sigma. You can easily see, for example in a finite element situation, the internal forces are given by integral B transpose sigma dV. You can very easily see that that is the force. We will derive that in the next class. Let us first understand the conceptual aspects. So, that is the force.

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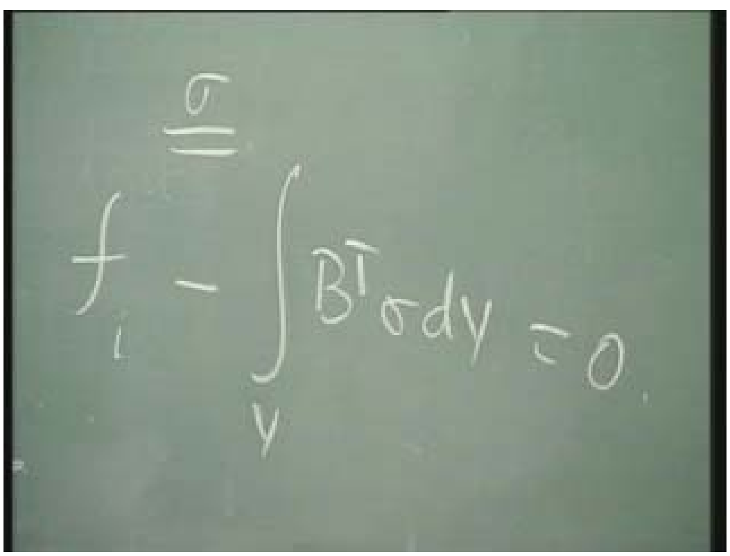

I have on one hand the force that is to be applied externally with respect to the increments, say i and on other hand I have an internal force that is generated. Sigma, stress is not the force. Stress results in a force, stress results in a force. That force for example in a finite element situation is B transpose sigma dV. I apply a force, I get an internal reaction to it; material reacts to my force, because of which I develop a force which is B transpose sigma dV. That has to be equal to zero for equilibrium. Have a look at that. Is this clear? Is it clear? Now, I will give you a second or even more. Now, tell me why what I have pointed out is psi?

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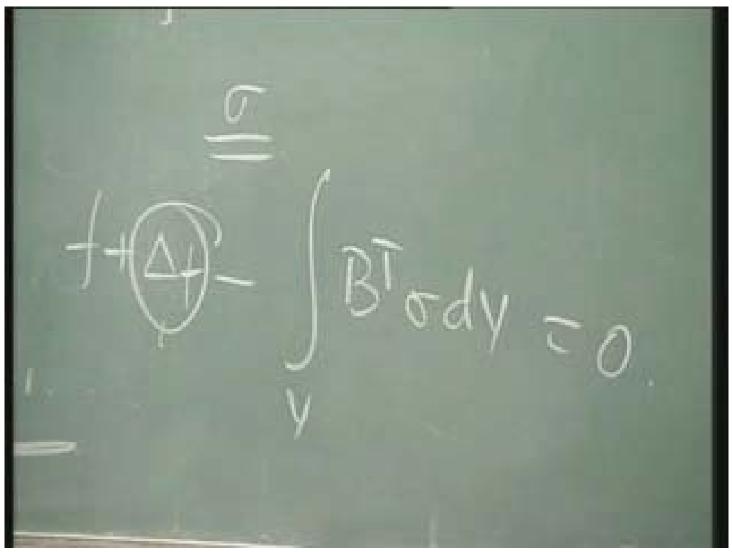

When I apply now f plus delta f, obviously this fellow here will now have f plus delta f, delta f being the 100 which we have been talking about and this we have not yet calculated for the new guy, for this new delta f or f plus delta f, we have not calculated and hence the error will be this, because this and this will be the same. Is it clear? So, this will be the error. How many of you understand? Is there any question? Yes, you do not understand? First of all you understand that what equilibrium is? There is an external force, there is an internal force. Internal force is due to stress, you understand that.

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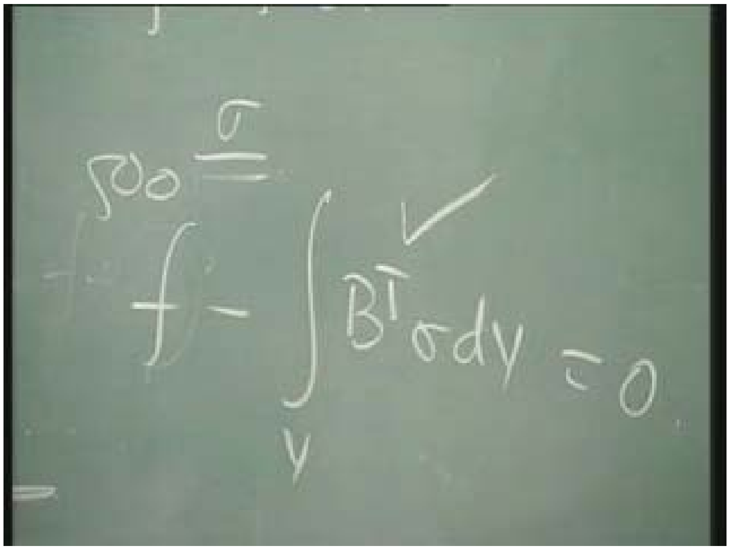

Let me say that external force is say, f. This force I told you again and again that we are applying it in an incremental fashion. After 5th increment say, the force can be say 500, for example. This is the force, external force. This is the internal force. I will show how this is internal force in the next class. Let us first understand physically what we are trying to do. Now B transpose sigma dV is the internal force. So, this minus this should be equal to zero. This force can be, both of them can be, a vector in a finite element sense. Let us say that we are only looking at it, for the time being, as a unidirectional or a uniaxial problem. So, B transpose sigma dV will just become sigma a. If I just take a bar, it will just become sigma a. So, f minus sigma a is equal to zero.

Now, what is that I am doing? I am going from 500 to 600. For 500, the sigma was okay. Now, I am applying 500 plus 100. That is what I called as delta f. Now, I have not yet calculated sigma. I have the old sigma only, I have to calculate now. When I immediately go to 600, my body's equilibrium position is changed now, because I have 100 more …… to the system. Now, I have to calculate new sigma for it, new sigma to compensate for this 100. That compensation initially is the delta f. Is that clear and that is what I called as error.

What is error? The difference between what is applied externally and what has been calculated internally becomes what is called as the error. Is it clear? When the error becomes smaller and smaller, I am approaching equilibrium. At one point of time, I say that, fine, this is what I want. The error becomes very, very small. It will never become zero exactly, but it can become very, very small, say naught, naught, naught, naught 1. Then I would say, fine, be done with it, problem is over. I go to the next step. Is it clear? Now, concentrate on this. So, initially this will be my error. Now, even before we write down what is called as Newton-Raphson formula, let us just worry about that in the next class. Immediately you can say that, as our friend said, why not I calculate the slope at this point, at this point.

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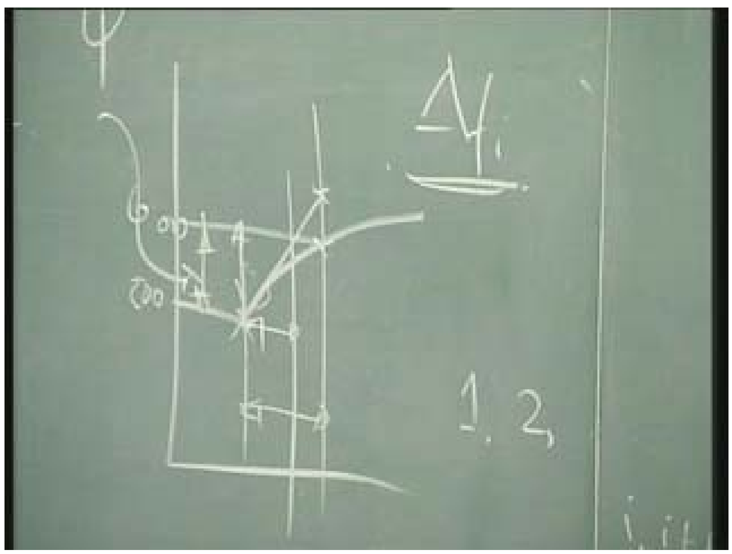

Let me say that the slope at that point is, say et and et I use to calculate K or what I call as K_t. B transpose dB, there for example, instead of d let me say that instead of e and d, I put et. So, I calculate. In a one dimensional problem what will this lead me to? It is ae by l; remember that. So I will just substitute instead of e, et. In a one dimensional problem, this is the force. The slope is the displacement, because this is p dp by d delta this force versus displacement. That is the force. The slope is actually the stiffness. So, where will I go? I will go, I will calculate that as the deflection; I will calculate that as the deflection. So, this is my first calculation or in other words iteration, after I had gone to 500 trying to move towards 600. Is it clear?

Now with this displacement, I calculate the stress and look where I am in the curve.

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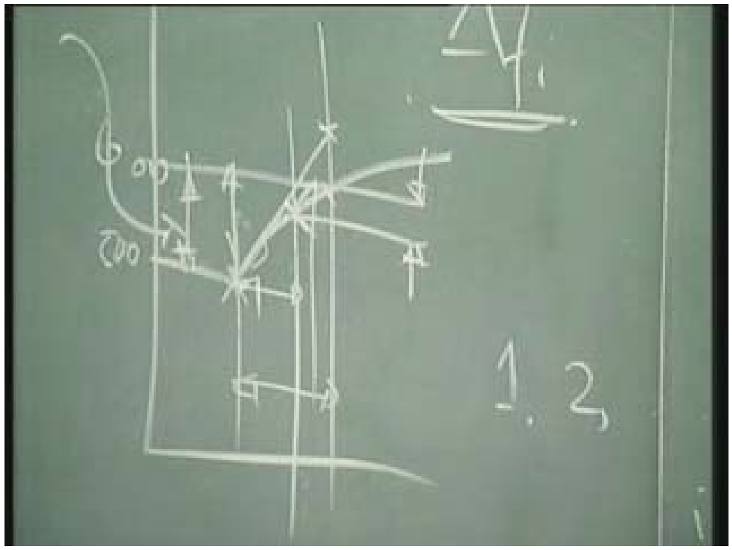

I am still here, I am still here. Why, because I have essentially linearized. This is called linearization. I have essentially linearized a non-linear curve. This is exactly what would result in a Newton-Raphson form. Psi plus dow psi by dow d into delta d is equal to zero is what would give you a Newton-Raphson scheme. Am I now devoid of errors? No, because I still have that much as the error. I repeat that this error is due to linearization at that point. What do I do? I go to that point.

I have new delta d. I calculate the stress. Find that the stress is not good enough to compensate for the external load. This is what I find. So, I go to now the new point. What is the new point? The new point is this. So, that is the new point. So, I again calculate there one more et. Is it clear? I am there after the first. This is called iteration. I am in the same increment from 5 to 6, but I am iterating between 5 and 6. Then, again I look at the result. I keep on doing this.

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So if I exaggerate that figure, again I calculate like this and come back and calculate like this, come back, yeah, sorry, then calculate like this, come back and then ultimately I will have the error to be extremely small. I say that this point I have converged. After 4 iterations, I say that I have converged. Is this clear? We follow what is called as an incremental iterative approach in order to do a non-linear problem and we always look for convergence. A convergence is one which makes my stress, the internal stress and hence the internal force equal to the externally applied load or force. Is that clear? So, I keep on doing this for every load.

In other words, what is that final equation? We will come to the derivation later. What is the final equation I would be looking for?

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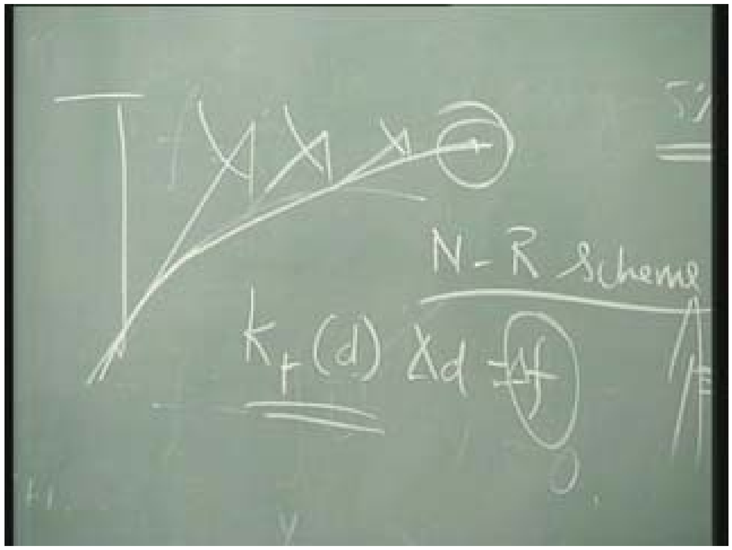

Final equation is something of the form K_t a function of d delta d is equal to some say f. K_t is the tangent stiffness that I will find out. That is a function of displacement, because as I just now saw or just now showed that the slope of this curve is a function of displacements. So, K_t becomes a function of displacement. So K_t a function of d into delta d is equal to f or the error or I would say rather delta f. So, this K_t is obtained by what is called as Newton-Raphson scheme, generally by Newton-Raphson scheme. There are lots of schemes for solving a non-linear problem. Here actually we are not strictly in finite element. We are leaving the field of finite element analysis to look at all sorts of techniques that are available in numerical analysis in order to solve this non-linear problem. Is that clear? In order to solve this non-linear problem, we may move away from the boundaries of finite element analysis. There are lots of techniques to solve a non-linear problem. Let us for this course worry only about Newton-Raphson scheme and probably about 90% of the problem that is solved in the non-linear finite element world is through or is using Newton-Raphson scheme. Is that clear? So, we will concentrate on Newton-Raphson scheme.

Our major aim here is to find out an equation for K_t. My major aim here is to find out how to calculate K_t, because once I do that, I can keep on going further. Before we go to that, you look at the magnitude of the problem. Just take a very, very simple case. Now, I have a load of 1000 Newton, 1000 Newton. I apply it say for example, in 10 steps. Just for small calculation let us say that I take about 4 increments to converge.

What is increment? 1 2 3 4; 4 increments to converge. In each of this step, I solve this equation. So, first step that was the error, second step that was the error; that was the error, that was the error and finally the error is very, very small. So, the right hand side, I have the errors being shifted from left to right as I move in the increments. Now, what does this tell you? What does this tell you? This tells me that in the elastic case I solved the same equation only once K d is equal to f. That is all, only once.

Now, how many times I am going to solve? 10 into 4; 10 into 4, 40 times I am going to solve the same problem, the same equation. Every time, I am going to calculate K_t, now. What we call as a full Newton-Raphson approach, I am going to calculate K_t and then solve the problem not once, 40 times. So, the problem just explodes. What you can do it in a minute is now going to take 40 times more time than what you did for the simple linear case. So, non-linear analysis hence require lot more powerful computers in the sense of calculating, the power to do number crunching to calculate this K_t as well as good storage capacity, because many times we have to store lot more things, intermediate steps and other things as we go along has to be stored and so the amount of space that is required also is going to be very high. But, interestingly except for these things, the concept of finite element is going to be the same.

The stiffness matrix calculation you are going to see is going to be exactly similar to what we did in linear case, except that there are terms which are going to be slightly different. In other words, the isoparametric element concept that I used will also be used or followed in the non-linear case. Is the general non-linearity clear? How we solve it, incremental iterative approach and so on. In contact analysis also this is exactly followed, because there also the penetrations have to be made to zero and hence the forces are every time we calculate see that there is no penetration or the calculated forces are in equilibrium with the applied forces and so on and hence this kind of approach is what is followed.

In the next class, we will derive some fundamental equations for Newton-Raphson scheme and then we will take up one method of using large deformation plasticity for solving metal forming problems. So, we will complete it may be in the next three classes.