Introduction to Finite Element Method

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Lecture - 29

In the last class, we were talking about contact and we were really getting into the modeling aspects of the problem which we had carried out.

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There are lot of things that we learnt as far as boundary conditions were concerned and we realized that things like contact is very, very important in modeling, especially in mechanical engineering and that the results may not be correct, if you make assumptions, drastic assumptions in order to avoid contact. We will come back to this contact and I am going to right now and I am going to discuss more in detail about contact nonlinear finite elements. But before we do that, let us have a look at one of the things which we left out. If you remember, I had asked about the axi-symmetric element, which we just left out and we entered into certain discussions on contact, it is good. We will come back to that and we will just finish that part of the axi-symmetric element, sort of niceties regarding the axi-symmetric element, before we come and again talk about the nonlinear finite element analysis, where contact is going to play a major role.

Let us just have a look at this. You know, all of you have seen it, I have written this equation before in one of the earlier classes.

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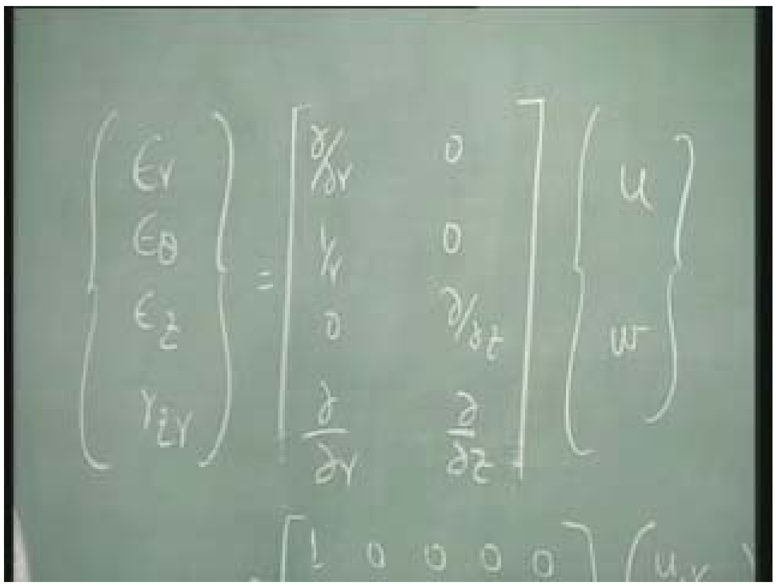

You know, for example, how the strain displacement relationship is given for an axi-symmetric case. We had already discussed what an axi-symmetric case is? We said that there should be an axis of symmetry and we had already discussed about the stress state, strain state and so on, in the beginning of this course. I hope you all remember that or else have a look at your notes, but nevertheless we had done this before. What I want to just point out is that there is a small change, when you go to axi-symmetric element with respect to this ones and zeros matrix.

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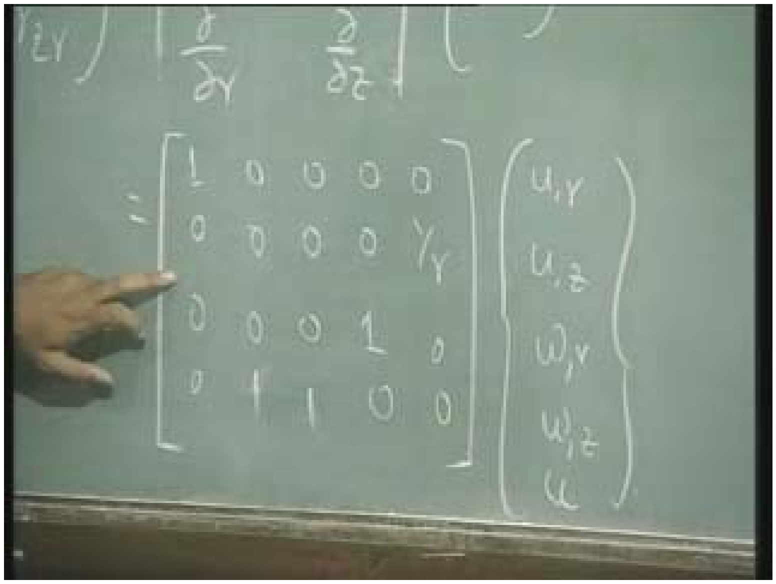

You remember that we had called this matrix as what? H matrix, with ones and zeros matrix or this is one of the earlier matrices that we had seen. In the earlier version of this matrix, we had how many rows and columns? 4 by 4. Now, because of the fact that $\text{epsilon}_{\text{theta}}$, $\text{epsilon}_{\text{theta}}$ which has a 1 by r in it, there is a small change in this matrix for axi-symmetric element. Actually, how does the axi-symmetric element look like? The axi-symmetric element looks something like this.

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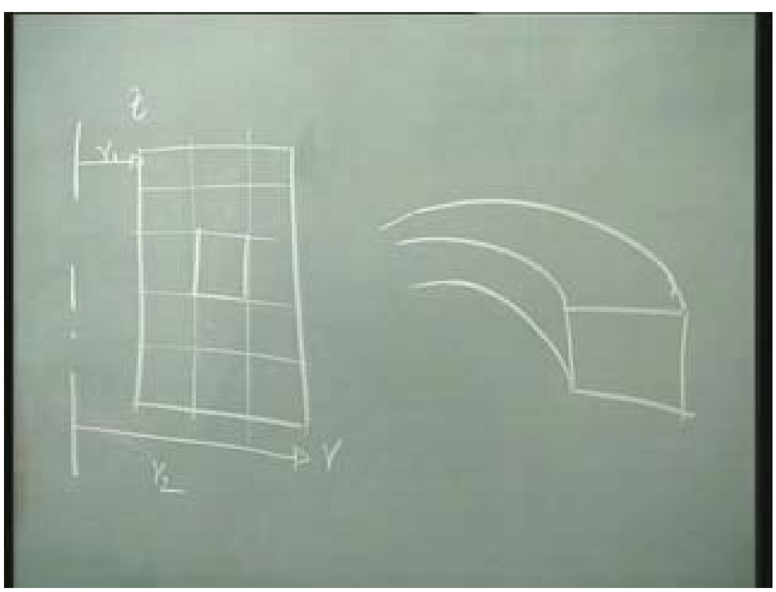

Suppose this is the axis and say this is the complete body which we are analyzing. Say, there is a cylinder, a hollow cylinder with a radius say r, r_1 and r_2. Suppose we are analyzing like this and we are dividing this into number of elements and one such element happens to be here, suppose we are dividing into a number of elements and so on, the type of element we are using is axi-symmetric element. Please note this carefully, though we are discretizing it in two dimensional regime or two dimensional plane or in a plane with say r and z, r and z plane, as the two axis, r and z being the two axes which define this plane, this element is actually a ring element. It is actually a ring which goes to the other side. So, the element is formulated, it goes like that. The element is formulated taking into account this complete ring. That is the first point.

Where do we bring this in? We will see that in a minute. First point is that, though we are discretizing it as an axi-symmetric element, in the sense that in the r z coordinate system, what we are considering as an element is actually a ring element. That is the ring element. Is that clear? Now, we will come to that in a minute as to where we are going to put it, but concentrate on this right now that this H matrix what we have has now one more column added to it because of this $\text{epsilon}_{\text{theta}}$ and $\text{epsilon}_{\text{theta}}$ is u by r, you remember that we had done this before, u by r.

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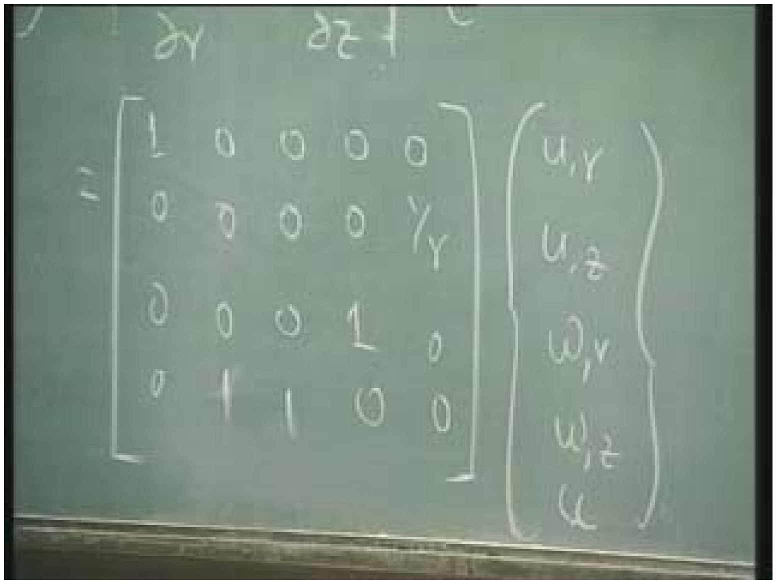

This is because of the change in the radius. As the radius moves out, remember that that is what we said that there is a change in the dimension of a ring and hence there is u by r that comes into play and that is epsilon theta and hence you see immediately that there is one more column that is added here as zero 1 by r zero zero. In other words, there is a small change in this one zeroes matrix which we have seen. This is similar to u comma x u comma y w comma x w comma y, that we have seen already. The only addition now is this u, to take care of this u by r.

What is the next step? Remember the next step. What we did was to replace this by an isoparametric formulation or in other words to replace r and z by our natural coordinate systems psi and eta. This u comma r u comma z and so on is now replaced by means of u comma psi through that gamma matrix; remember that gamma matrix.

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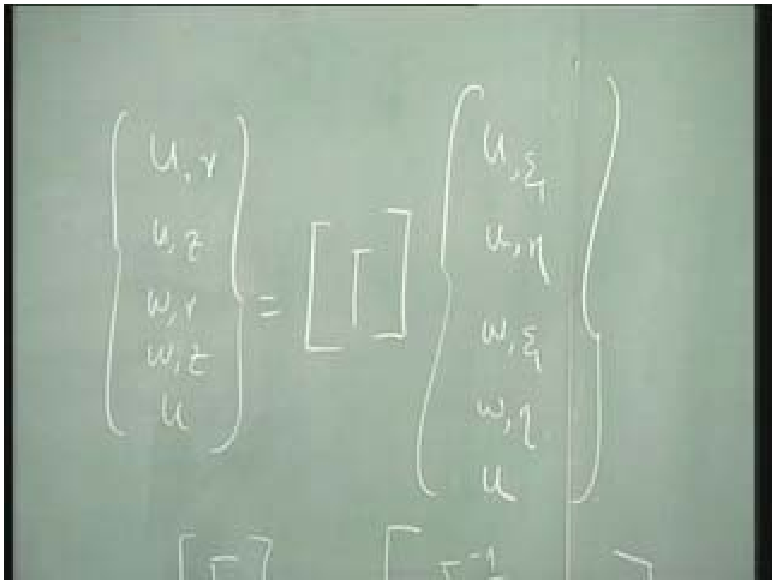

Again remember that in the plane strain or plane stress axi-symmetric problem, we had only a gamma matrix to be just j inverse. This is 2 by 2 then zero zero 2 by 2; 4 by 4 matrix.

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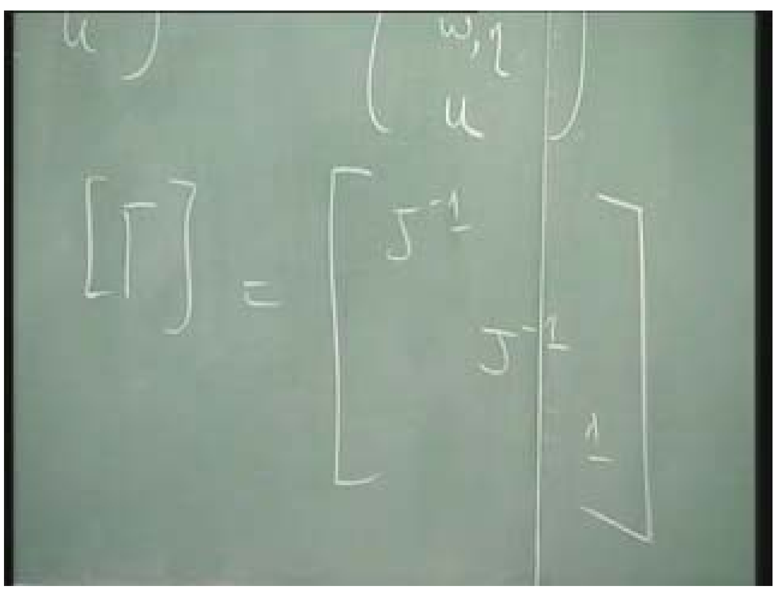

Here there is only one addition that we have 1 here. Is that clear? The calculations are very similar, exactly the same. There is no change. Only thing is that these matrices now change a bit. Is it clear? Have a look at that. If there is any question, just have a look at this and this. If there is any question, I will answer that. What is now B matrix? How do you get the B matrix? Remember how we get the B matrix? Yes, now what I do is I substitute; from here I substitute onto this equation. This I substitute it from this equation. When I substitute it there, then I get here, for example, let me remove that and put say along with it, let me call this as H matrix just for brevity that gamma, gamma I had defined here already and then now what happens to this u comma psi u comma eta w comma psi w comma eta and u, they go here.

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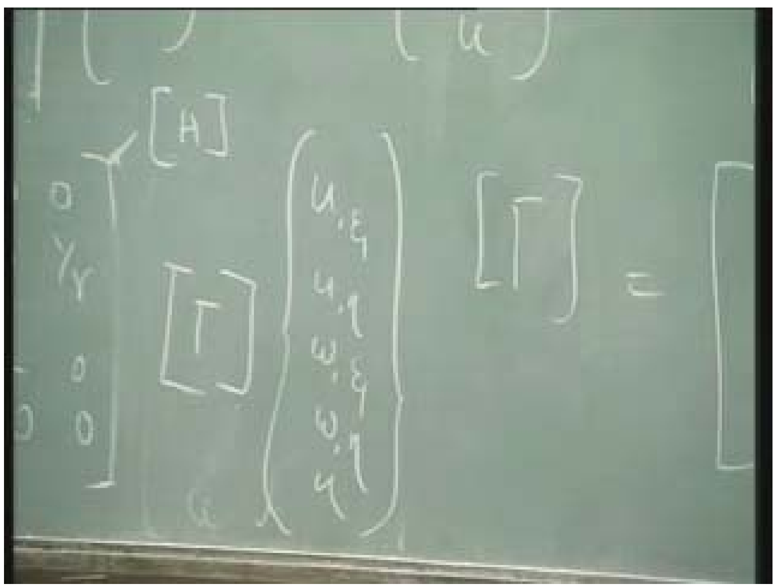

Is that clear? Now, what is the next step? Next step is this u comma psi u comma eta and all those things have to be replaced by N_1 comma psi and N_1 comma eta and so on. Is that clear? That is what we did. You remember that that is what we did. But, the only difference now is that, what is the difference? I have a u here. So, my matrix will have another row with N_1 N_2 and so on. Is it clear? What will be the size of that matrix? Correct; 5 by 8, beautiful. So, that matrix will be a 5 by 8 matrix. You know how to substitute it, straight away. My B matrix will be this multiplied by this multiplied by the substitution for this which is a 5 by 8 matrix and that would be my

B matrix. Is that clear? So, u comma psi u comma eta all these things would be replaced by N_1 comma psi zero N_2 comma psi. I am not going to write it, because it is exactly what you have written in you previous classes. So, that will be my B matrix.

Having done that what is my next step? I will just substitute it in my K. What do I do there?

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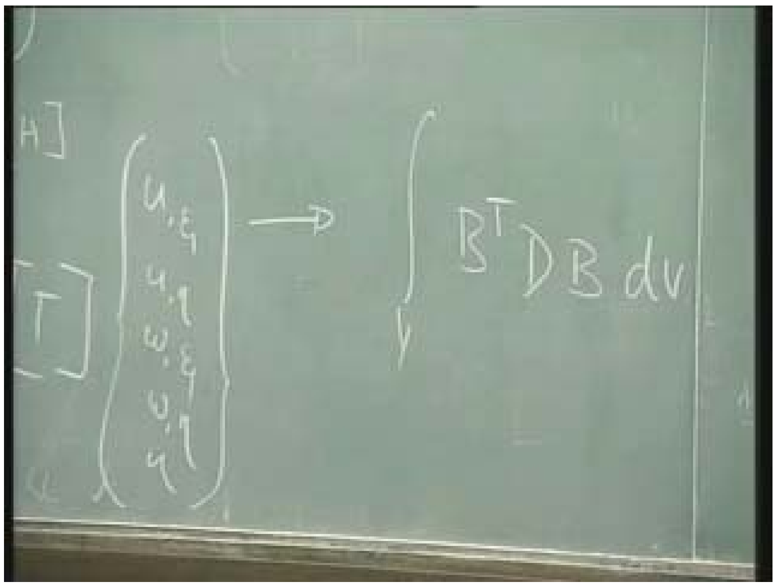

This is now substituted into B transpose D B dv. I have to now get this in isoparametric form. How do I get that? How do I get that? I now substitute it for dv. I am substituting for dv. How do I substitute for dv?

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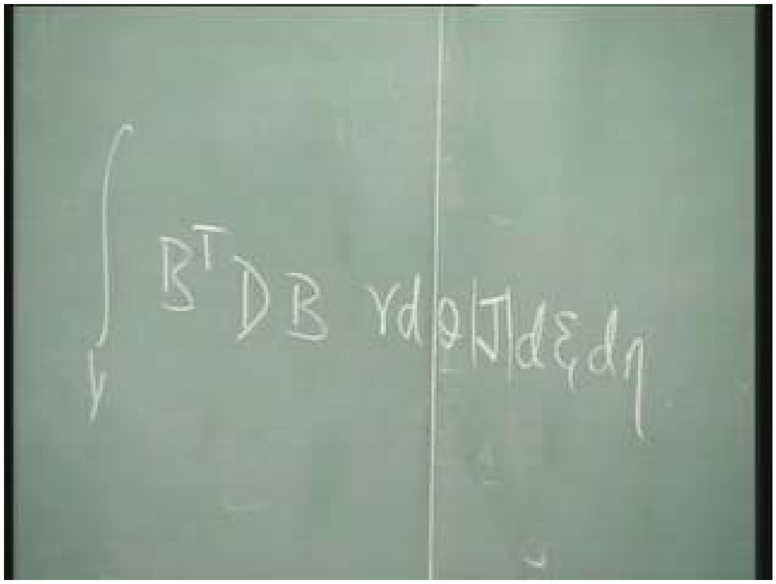

I substitute for dv as r dtheta J dpsi deta. J is, of course, the determinant of my Jacobian. Because I am taking 1, I already told you that I am going to take the complete ring.

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In order to analyze this problem, first I take a small part of it as r dtheta, so, r dtheta. dv is r dtheta into dx dy. So, r dtheta dx dy or in other words, dr dz now becomes J dpsi deta. This is dr into dz into perpendicular to that r dtheta. What will happen to my integral sign here?

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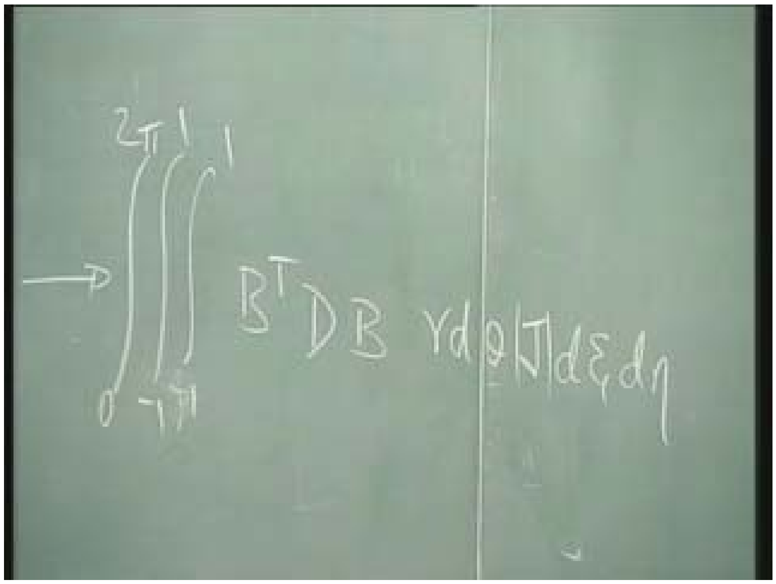

That will be my minus 1 to plus 1; another fellow minus 1 to plus 1, another chap from zero to, yeah, 2 pi, fantastic. Do you understand? Zero to 2 pi. Is it clear? dtheta, this is independent of dtheta. So, that fellow can come out, so, 2 pi can come out.

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So, you can have 2 pi out and then you have an equation of this form. Now, the only question is r. r is the radius. Where do we get this radius now? r is the radius, where do we get, how do we get this? Remember that we are going to integrate it with respect to or we are going to find out the values at the gauss points and so we are going to get this values at the gauss points and so we need r.

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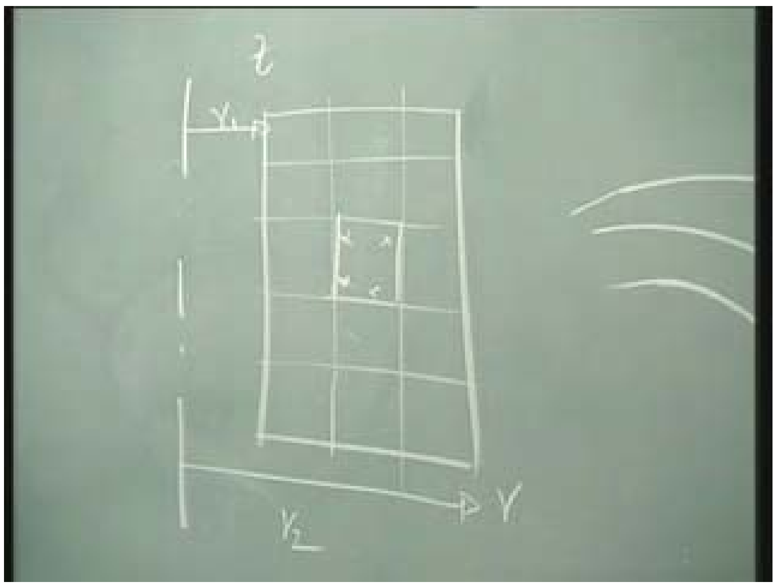

We have to substitute for r at the gauss points, at these four gauss points, say for example, if I am going to use 2 by 2 rule, I am going to get these radiuses at the gauss points. How do I get the radius at the gauss point? The rule is very simple, because I am using an isoparametric element and the shape of the element is what? Interpolated with respect to the nodes or in other words the coordinates are interpolated from the nodal values through the shape functions defined in terms of psi and eta. So, problem becomes very simple. I need to get the shape functions at these gauss points and say that r at gauss points can be written as N_1r_1 plus N_2r_2 plus N_3r_3 plus N_4r_4.

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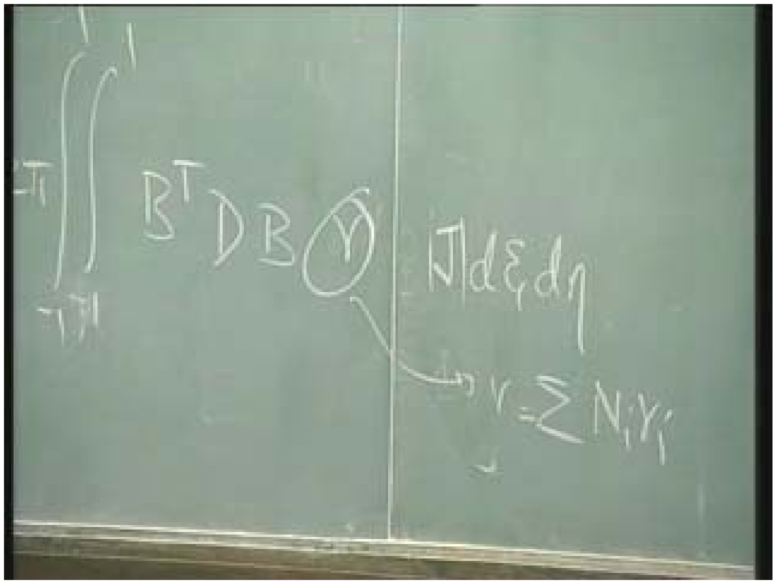

This r is nothing but sigma $N_i r_i$. Because psi and eta values of the gauss point are known, substitute it here. You will get sigma $N_i r_i$. You do the same thing. Please note that similar thing is done in the case of force and other things as well. For example, if there is a pressure that is acting, then the pressure will be acting say internally. 1530

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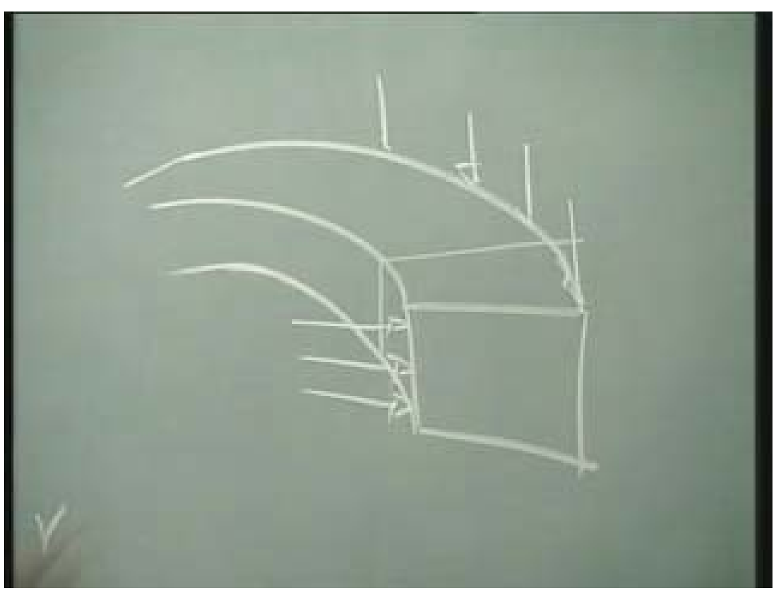

The pressure will be acting internally and you have to now integrate it throughout the ring. Again, there will be one 2 pi term, which will come out. Is it not and in a similar fashion even if you have a line load, a load that is acting, that load makes the problem axi-symmetric only if that load acts throughout the line like that. Is that clear? What does it mean? It means that this 2 pi term which we have here, this 2 pi term is common to the stiffness matrix, to all the force terms, because we are going to integrate the force throughout; zero to 2 pi, we are going to integrate. Is it clear? For example, if you are looking at pressure, N transpose p into dS. What is dS in this case? Say, length of the element into r dtheta, integrated from zero to pi; so, r l it will become, so, 2 pi will go out. Everywhere the 2 pi will come, because we will take an infinitesimal element, an infinitesimal volume and then integrate it from zero to 2 pi. Is that clear? Everywhere, we will integrate it from zero to 2 pi. Go and have a look. Deliberately I am not writing it. I want you to write it, because go and take every term and integrate like this. You will get one 2 pi term standing outside. So, 2 pi will get cancelled.

What is the significance? Why am I saying this? This becomes very important because, when you give an input to many of the packages, you have to be very careful as to what that package asks as input, because some of the packages would require the total load that is acting on that line, some of the packages would require the total load that is acting on the line, some of the packages would require the load acting per unit length and so on. It is very important that you check up what the load is that this software requires. What do I mean by that? What I mean to say is that, suppose there is a load that is acting at this point.

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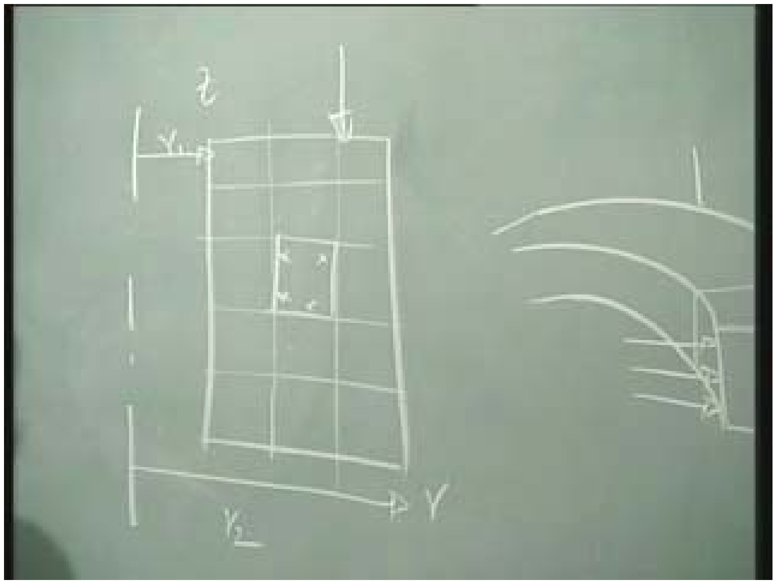

Suppose there is a load that is acting at that point. Then, actually this load acts throughout the ring. Is that clear? Throughout the ring. When it acts throughout the ring which passes through those point, that point there, this point, it is important to realize whether the software asks for total load or load per unit length or how does it handle that load? This usually varies from package to package. That is an important input that you have to give or else you will make a mistake. If it gives total load and it asks for unit length or something like that and normalize it, at 2 pi we will eliminate it, then it would be a problem. He would multiply by 2 pi, then that would be a problem. That is very important as to what it asks.

The other thing that you have to be very careful in axi-symmetric element is the r z coordinate system. Usually the x is r and the y is z. The softwares usually define this, the softwares usually define this as to which is r and which is z. You cannot go against it, then the problem is an entirely different problem.

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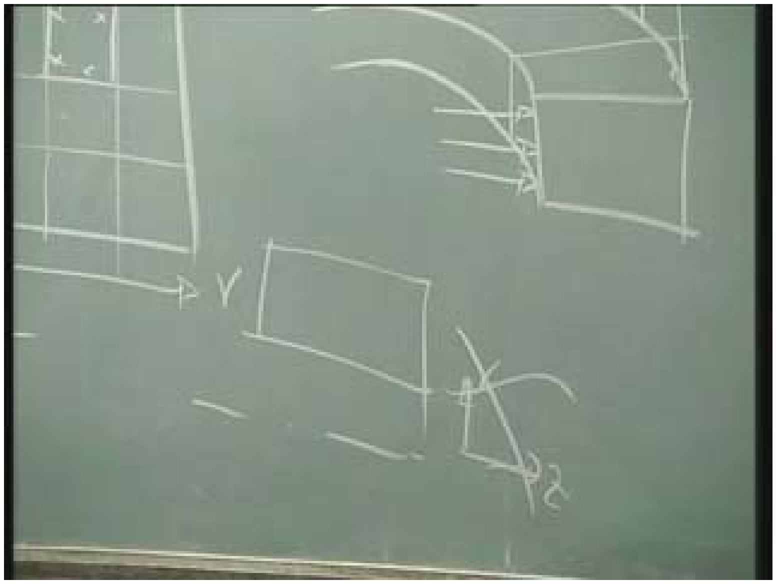

For example, this particular problem you cannot do it by making the ring like this and saying that I will take this as r and this as z, that is not allowed in most softwares. They have a fixed way of defining x and y. Is it clear and one more thing that comes out of this formulation is that since the pressure, actually the pressure is now rotated, the force is rotated, this pressure that is acting is rotated about 360 degrees. The ring is in equilibrium, automatically. Physically if you see, the ring is in equilibrium because of the force that acts.

What does it mean? It means that for an axi-symmetric element you need not specify a boundary condition in r, in r direction, r degree of freedom you need not specify a boundary condition. Why, because I am rotating this. On the other side also, as the ring comes to the other side, I will have a force that will be acting here as well, completely, radially force will be acting throughout the ring.

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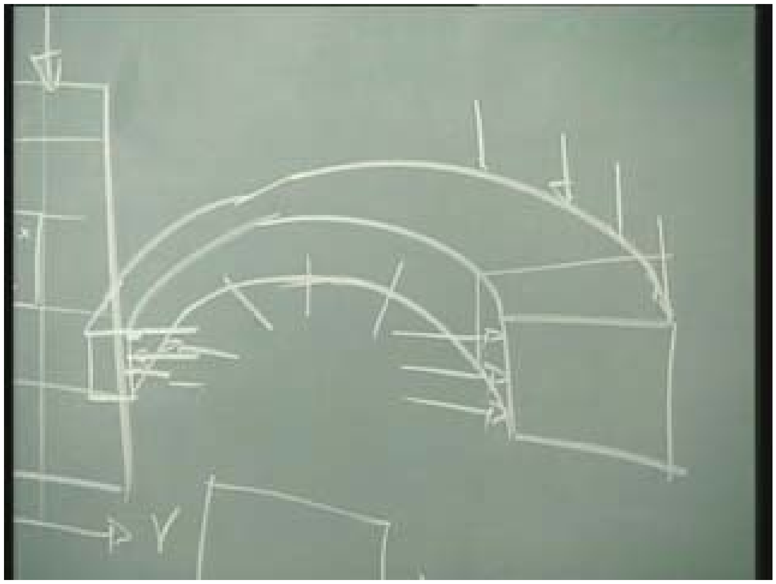

Hence that ring is under automatic equilibrium. You need not specify a boundary condition in the radial direction. In other words, suppose I have a problem where I have this particular tube, a hollow cylinder with internal pressures defined.

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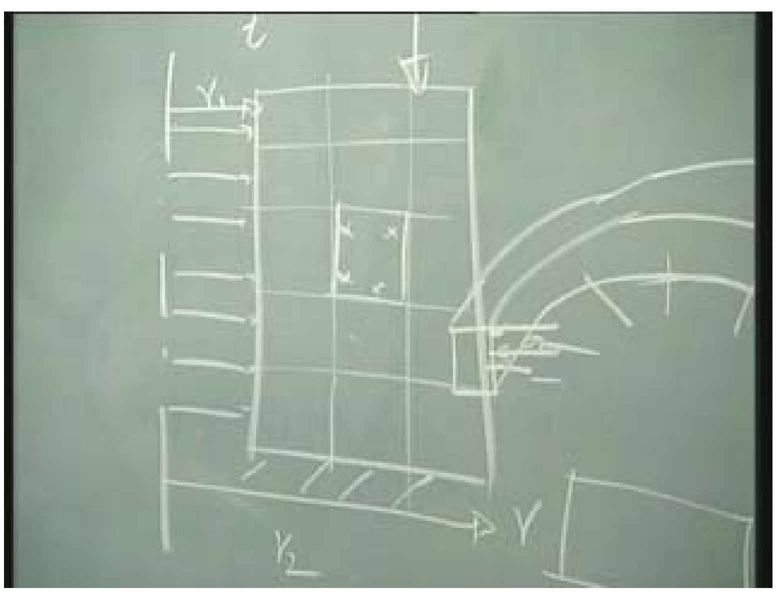

No doubt, I have to specify a boundary condition in the z direction, in the z direction, but I need not specify a boundary condition in the r direction, because the pressure when I integrated, would automatically equilibrate one side and the other side accurately. The formulation will take care of the rigid body motion in the r direction. After all why do you want to specify a boundary condition? We want to avoid a rigid body motion. If you the specify one f and another f, then you do not avoid rigid body motion. In this case, you definitely avoid rigid body motion in the r direction. It cannot just expand as it likes. The movement in the r direction is not equivalent to a rigid body motion. That should be very, very clear or in other words, when I say that this fellow moves like that if it had been just a plane strain problem or a plane stress problem, if I move from here, this position to another position, this position to another position in the x direction, then it is equivalent to a rigid body motion. Clear?

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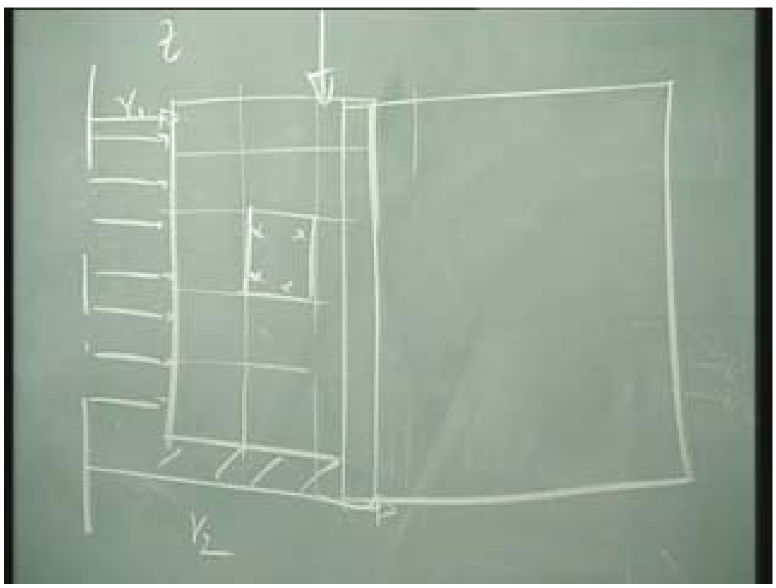

On the other hand, if I change this from here to say here, suppose I say I change this location from here to another location here; suppose I change, this whole block I move it like this, can I move it as a rigid body? No, because I am now increasing the radius. As I increase the radius, obviously, there is going to be strains. So, it is no more a rigid body motion. Hence there is no boundary condition that is required in the r direction and the pressures are also equilibrated on either side. On the other hand, there is a small warning. This is a very practical thing that when you apply this boundary condition, you need not worry about it. On the other hand, suppose I have a solid cylinder, instead of a hollow cylinder; if I have a solid cylinder instead of a hollow cylinder, I have a situation like this.

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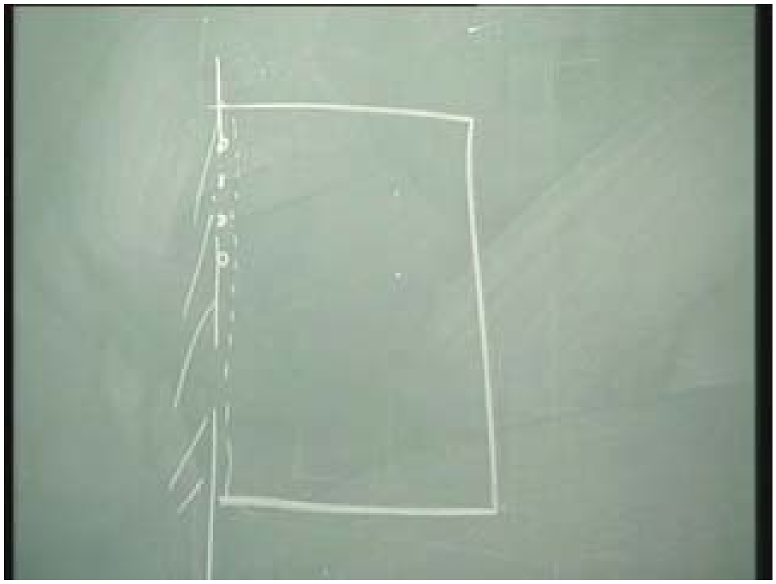

We can argue in a similar fashion and say that the center, this part need not be fixed, is that clear, the radius, the radius part in the radial direction, of course in the r direction. But, usually it is recommended that you fix the nodes at r is equal to zero in the r direction that is r is equal to zero line, that axi-symmetric line is the r is equal to zero line and it is important that you fix this r is equal to zero line, because usually what happens is that the computers have a fixed number of digits of accuracy and beyond which the computer is not very accurate; fixed number of digits. The computer is not accurate. This depends upon so many things.

Mathematically it depends upon what is called as the ….. conditioning of K matrix or condition number and so on; let us not worry about that. But nevertheless, just suffice it to understand that we are now looking at finite digits accuracy of any computer and hence though we are saying that at r is equal to zero, we need not fix it, if you look at the results, especially when you magnify the result, you would find that these nodes which are sitting here would have moved a bit and creating as if there is a hole there. Actually it is not a physical motion. There is nothing that has happened there, but nevertheless, because of the computer accuracy some sort of a hole seems to have been created at the center. If this happens, especially if you are working in a company, you will be hard pressed to explain to your boss why a hole has been created. He is immediately going to ask, oh, the material has failed at the center, hole is created. Then he is going to ask why there should be a hole at the centre that is created like this and so on, because this has happened to me before and then it is very difficult to explain it to people. Just to have a nice result, one of the reasons, you fix this nodes at r is equal to zero. Is this clear?

Please note that the failures are not, are not, shown as a hole in finite element analysis. This I told …….. within quotes who does not understand what is happening. The problem is that the failure has to be interpreted by you. Failure is not shown as a crack or as a displacement as two pieces and so on. It is only the stress which is going to come into picture for a linear case, which you have to interpret according to the failure criterion. Hence the finite element analysis will not show this kind of holes that are created. Nevertheless, in other words compatibility becomes very important. What are compatibility equations or compatibility conditions? Compatibility means that you do not create a hole. Though theoretically there are elements, in the last class I will talk about that, incompatible elements, but they are what I would call some crimes that we commit.

Some of the crimes, in fact there is an author called Strang. He called them as sundry variational crimes. We commit some crimes. If a mathematician looks at it, he would say that it is a crime but as engineers sometimes we make an approximation and make things work. There are some approximations that have been done. So, because of that we enter into what is called as incompatibility, incompatibility elements or incompatibility modes. But on the whole many of the elements are compatible or they are not there to produce a hole. Then there is a mistake. We are treating this as a continuum; we are treating this body as a continuum and in a continuum formulation, compatibility becomes very important, physically.

Physical compatibility becomes important or in other words in a continuum body we do not create a hole at the center. That is why, the whole science of elasticity or continuum mechanics is based on very strong foundations of topology, in fact, what we call as calculus and manifolds and there are a lot of things that go into what you have asked now. But nevertheless, at this point of time, it is important to understand that the results that we are going to get are results which are compatible. We are not going to create any hole anywhere in the body. If at all there is a failure or a fracture, it is up to you to interpret it from the results. Is that clear? That is a warning that I just wanted to give. These are some of the points that I wanted to make as far as the axi-symmetric element is concerned and we will now get back to our contact model that we had started in the last class. Is there any question on what we did in the last class regarding the temperature problem? So, if there are no questions on what we did, we will go ahead and look at the contact problem.

We had already defined or we had started our discussion on contact. We said that there is a special element called contact element. See, previously we used to have what is called as gap element, but nowadays this gap element is being replaced by what are called as contact elements, contact elements, in a two dimensional case.

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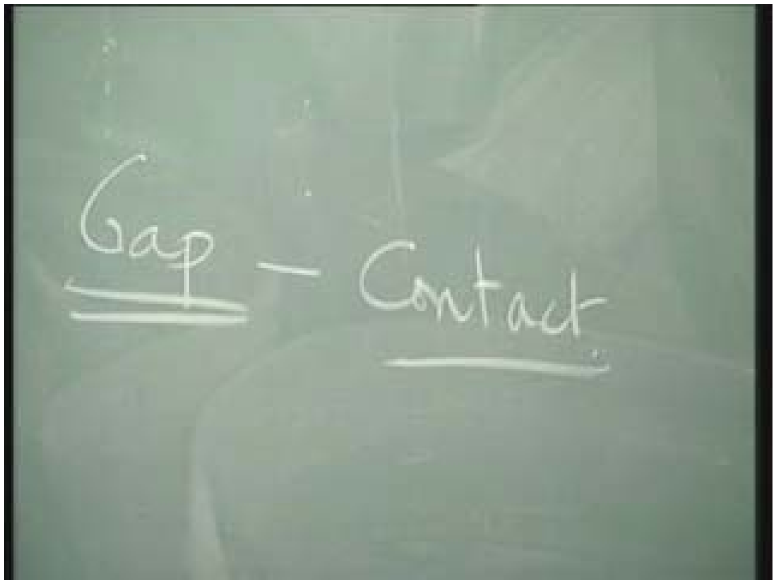

We will extend this to 3D in a minute. In two dimensional case, this contact element is defined by three nodes, is defined by three nodes.

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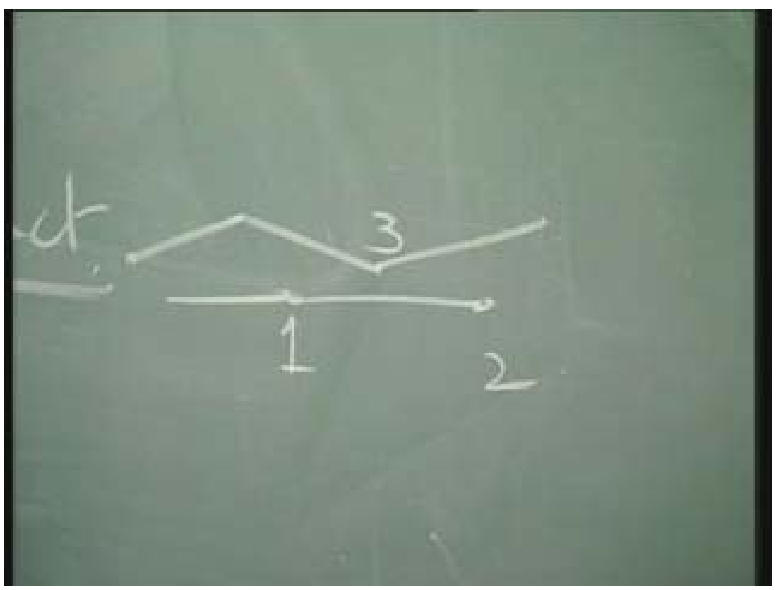

These nodes are two, what are called as master nodes say 1 and 2 and a slave node say 3. Now, how do we define this? What is the picture I have drawn here? It is nothing but a part of discretization of say a block sitting on a, say a rigid surface.

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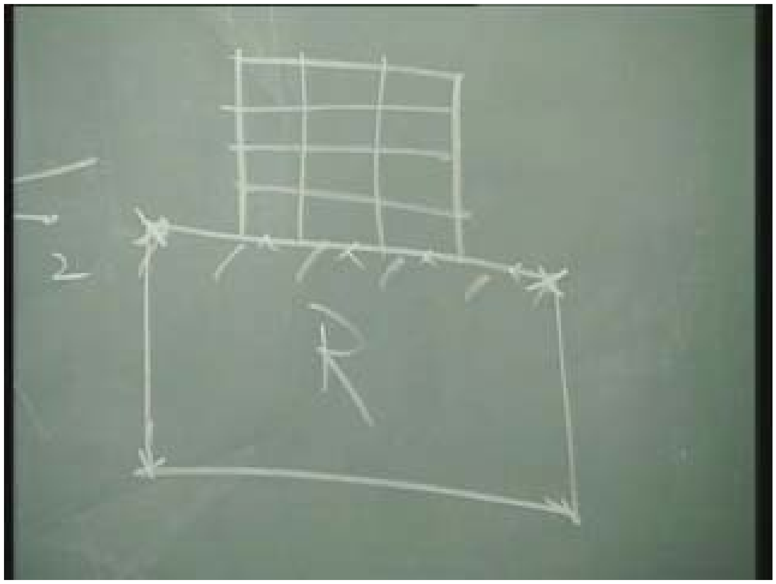

What I am doing is I am discretizing it, discretizing this block into number of elements and I am also putting down what are called as nodes, master nodes on this surface on which it is sitting, a rigid surface. For example, I can put down nodes like this. If you are confused, let us say that I have another big block, another big block say for example, I need not discretize it. I just put four nodes here. Let us say that this block is much more rigid when compared to this particular small or the ground is much more rigid when compared to the block that I am going to put or I have put.

I define as master nodes, the nodes belonging to the rigid body. Say for example, I have 4 nodes. Forget about these nodes for a minute. Suppose I have 4 nodes, then this node and this node are the rigid body's nodes or in other words master nodes. So, master node defines a surface, defines a surface, in this case a line, because it is 2D; defines a surface or a line into which a slave node cannot penetrate, into which a slave node cannot penetrate. What are the slave nodes?

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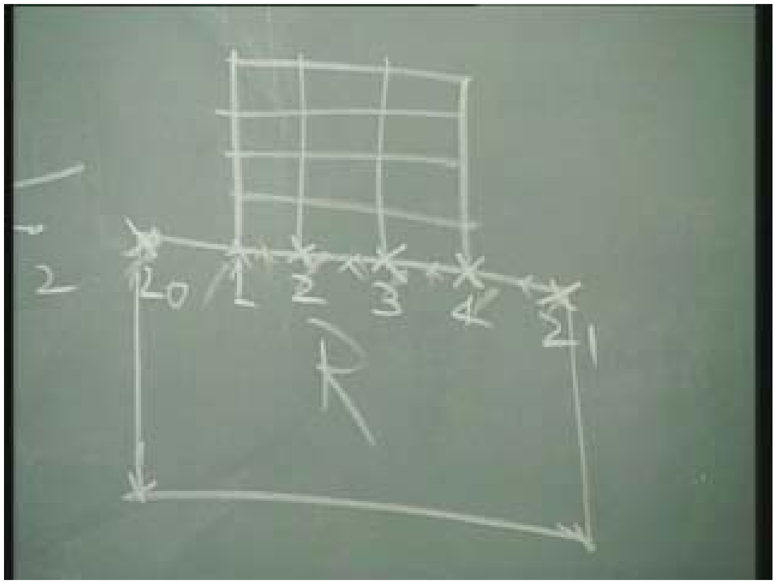

All these nodes sitting here are the slave nodes, all the nodes there, sitting there are the slave nodes. Let me number them, say for example, let me say that this is 1 2 3 4. Let me say that this node is say 20 and 21. Say, just for illustration purposes, let us say that the master nodes on either side is, say 20, 21. Sometimes people call this 20, 21 as a master surface or rigid surface. The contact in this case is between a deformable body and a rigid body. Is that clear? It is between a deformable body and a rigid body. My major aim in writing an algorithm is to see to it that these 1 2 3 4 nodes which belong to the deformable body, which I called as slave nodes should not penetrate the master surface, which is defined by 20 and 21.

So, number 1, when you use any contact algorithm you will be asked to define a master and a slave. Is that clear? What are these contact elements in this case? Contact elements will be formed between the slave nodes, note this carefully, slave nodes and the rigid surface. How many contact elements will be there? There will be 4 contact elements or in other words stiffness of these contact elements will be formed with 1 20 21, 2 20 21, 3 20 21 and 4 20 21. In other words, note this carefully that the contact element is a node to surface contact element; it is a node to surface contact element and not a surface-to-surface contact element.

Suppose you are working with a package like Ansys, say. The package will ask you to pick up two surfaces, target surface. One he will call as master surface and another is what he will call as slave surface. When you pick up, actually you will be defining a surface. In most packages, you will be defining a surface. Afterwards if you go and look at the input file, for example in Ansys, though you would have defined it as two surfaces, you will see that he would have split it up and he would have written some say, in this case, in this case what he would have done in say a package like Ansys, is that he would have defined 1 2 3 4 this surface as one of the surface and 20 21 as another surface. You would have defined only two surfaces.

You may think that it is some surface to surface contact algorithm and that the algorithm will not allow this surface to penetrate into this surface. No; it is not like that. All these algorithms are node to surface contact, node to surface contact.

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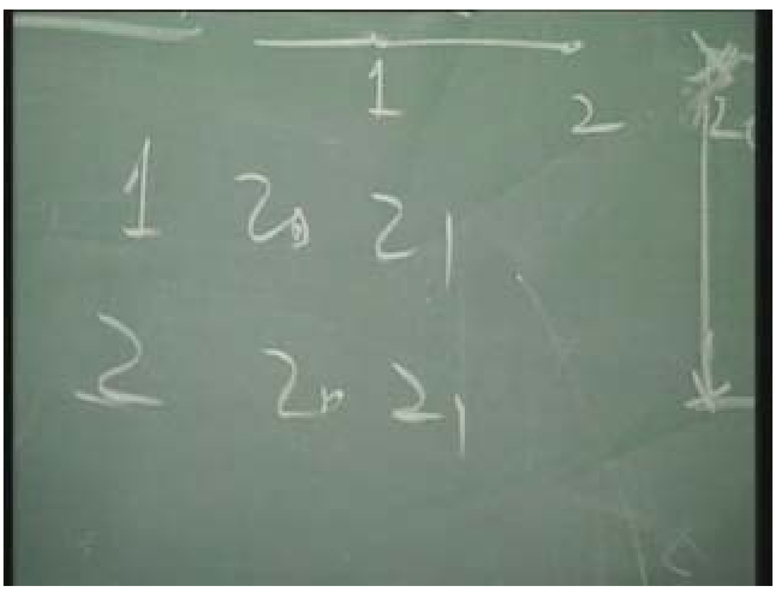

In fact if you had seen inside, go and see the data file, you would see that he would have defined 1 20 21, 2 20 21 and so on. Is that clear? That is the first point that they are node to surface algorithms. The problem in contact algorithm is that the algorithm's stiffness matrix is written in such a fashion that the slave nodes are not allowed to penetrate. But, unfortunately the master nodes have no such restrictions. Master nodes are allowed to penetrate. Is that clear? Before we go to that point, how does or how does this whole system of contact element operate? It is very simple actually, in the sense that, I am not going to the mathematical derivations, because the theory that is required is quite high and hence I am not going into the mathematical derivations of the stiffness matrix. That is not required also, unless you are going to work in contact.

The only thing I want you to know is that, the size of the stiffness matrix you can very easily understand, that the d's, what are d's? Understand that they are the degrees of freedom. The degrees of freedom that are associated with a slave node is 6 in number.

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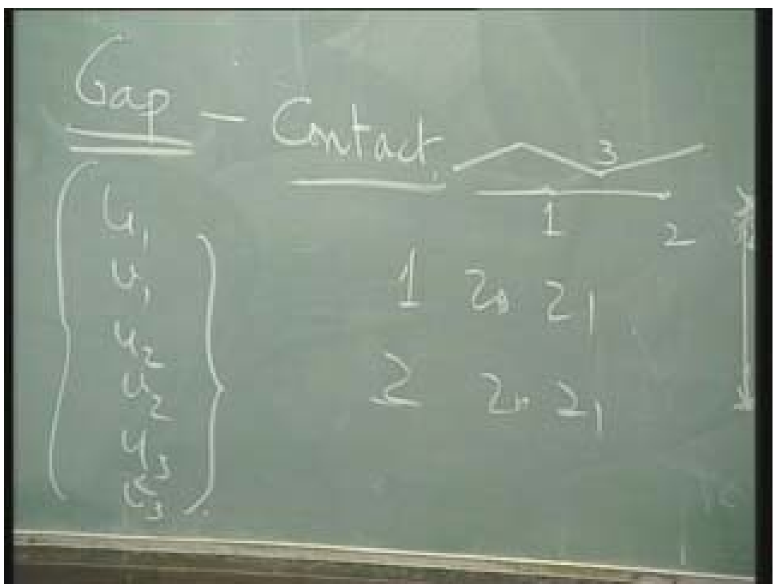

Suppose, I say 1 2 and 3, then u_1v_1 u_2v_2 and u_3v_3 and hence K matrix is now correspondingly, what? 6 by 6 and hence there is a force also. Correspondingly there are 6 forces. In other words, in other words, contact element can also take into account friction. There is friction which can also be taken into account contact element and that is the reason why I have displacement in the x direction as well, so, friction. There is a normal stiffness K_n as well as shear stiffness K_s. Normal stiffness comes from penalty; both of them come from penalty and the shear also take into account friction and so on. Point number 2 is that contact elements are capable of taking into account friction as well. Is that clear?

Please note that when I define the contact element, I need to, I need to make the algorithm recognize which is the segment in which it is operating. It is very important to realize that the algorithm inside a software would search and find out which is the segment of the master element or the rigid body that it is operating. What is it? Let me explain that more deeply.

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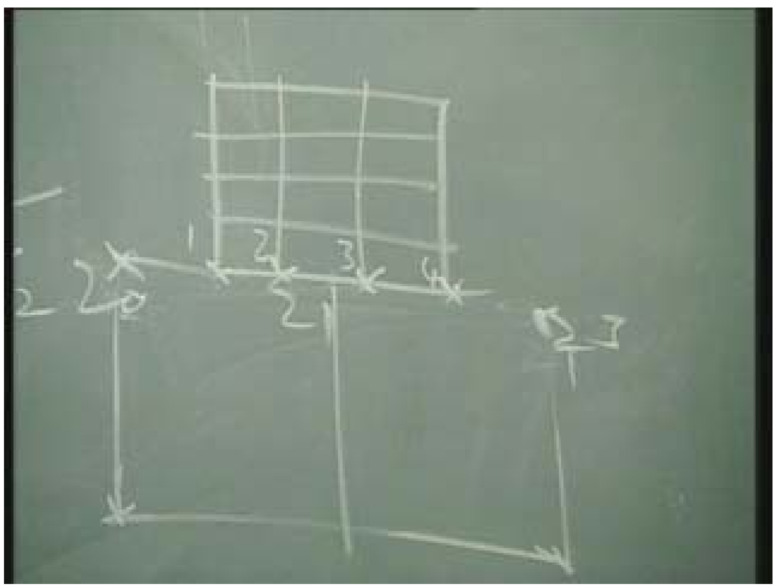

Due to some reasons, instead of saying 20 and 21, I can say that, I can divide the master into say two elements, say I say 20 21 and say 23. One of the jobs of the algorithm in any of the softwares that you may use is to find out whether this 1 say 2 3 4, these guys sit or come under the influence of which of the master segments? There are now two master segments. Remember that 20 21, 21 23. There is what is called as, in software they call this as contact searching algorithm, contact searching algorithm. The contact searching algorithm will find out whether one sits in 20 21 or in 21 23, so that the contact element is formed between 1 20 and 21; 3 is formed between 3 21 and 23. Note that carefully, 3 will be formed between 3 21 and 23 and so on. That is nice, you know.

You may think that is very simple. But, suppose this block has started moving due to application of your load.

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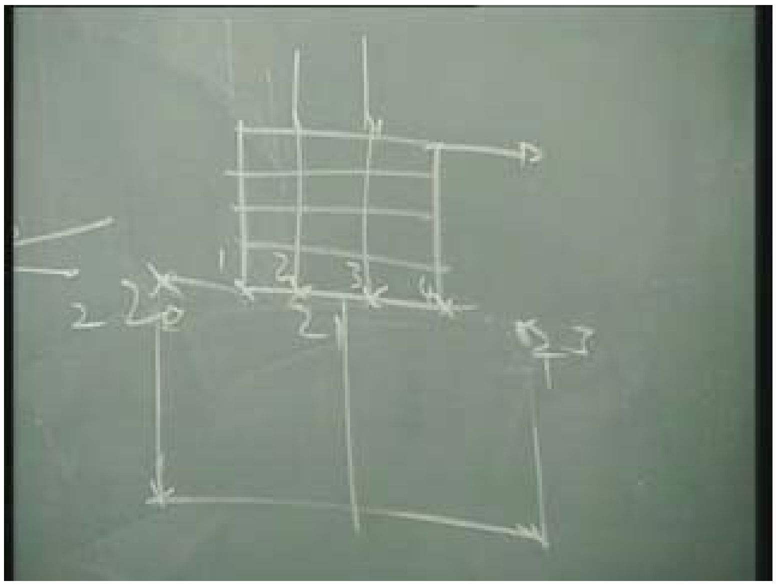

Suppose you can move the block. There is absolutely no problem with friction, because the stiffness matrix is such that the singularity will be removed and you can move the block. There will be a contact force, there will be a friction force. But if the force is such that, there will be some normal forces here of course and then suppose you are moving this block. It is possible the This is possible only if you have a friction or contact algorithm. Now what will happen? That 2 chap, this fellow who was under the influence of 20 21 may move to the other segment, 21 23.

Initially though you define, that is why you define it as a complete surface. You have to be very careful to see to it that you define all the surfaces, for which there is likelihood for the master segment to come into contact later during loading. Is it clear? The next step is for you to find out whether the segment, which segment it falls and so on. Now what happens?

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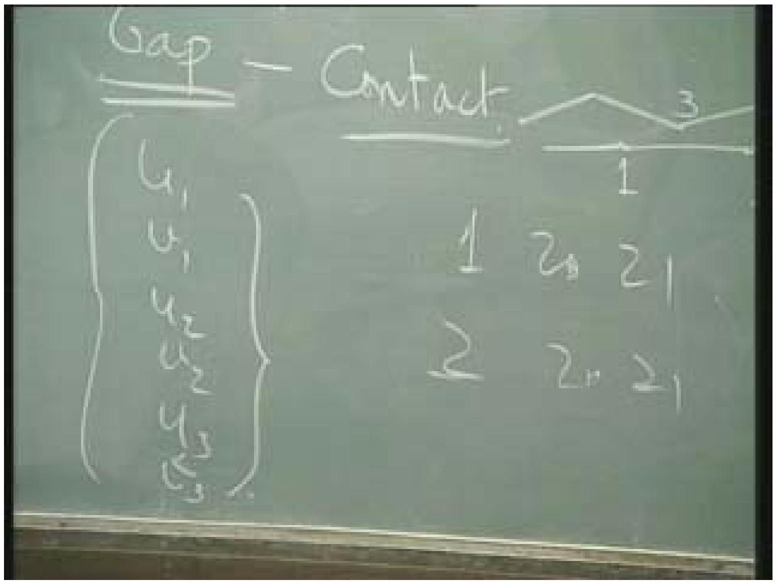

Now, this particular list u_1 u_2 u_3, initially for example for 2, it would have been u_{20} v_{20} u_{21} v_{21} and u_2 v_2; it would have been like that. But when this 2 moves to the segment 21 23, this would become now u_{21} u_{23}, sorry v_{23} and so on; u_{21} v_{21} u_{23} v_{23} and again u_2 v_2. Is it clear? The corresponding stiffness, where he assembles would also be different.

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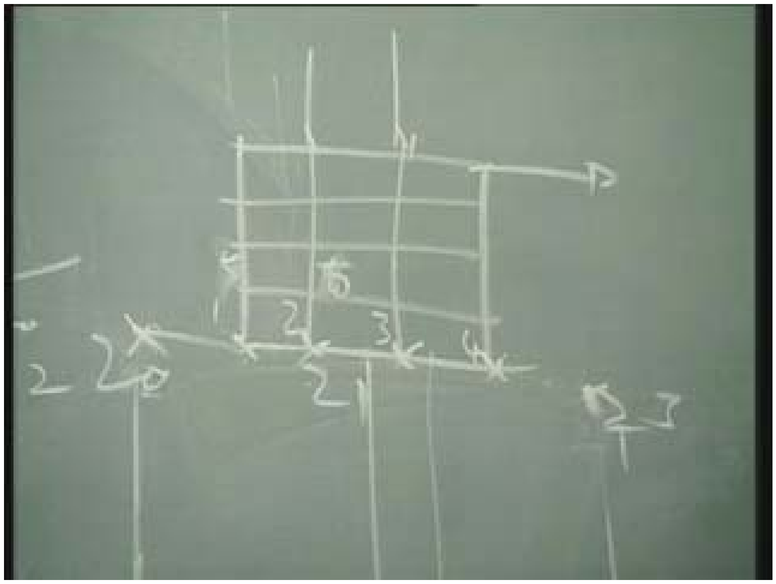

This will not happen, please note this would not have happened in our earlier finite element formulation, because if this had been say 1 2 3 sorry 4, 5 6 or sorry yeah, 5 6 and I defined an element 1 2 6 5, I define it as 1 2 6 5, that would be 1 2 6 5. When I assemble it, I will not have any problem. Here, note that the master and the slave, slave may remain the same or the master segments would change. What is the practical lesson behind it? Practical lesson is that you exhaust as a master segment, for a particular node all the segments that are possible in its history, as you load it as a master segment. In other words, again explaining it practically what it means is that if I just give 20 21 for the node 2 and forget about 21 23 and if it starts moving, when this fellow goes here into this segment, the fellow will go down straight away, because you would not have given 21 23 as a master segment. These are niceties, which we have look at the software which you define or which you are using in order to do that. Yeah, any question?

Fantastic; so, when the slave node and the master node coincide there are bound to be certain small problems, but when we write the code we see to it that there is some sort of a small tolerance that we give, to put it either in this segment or that segment. But when you write the code this is a problem. Many times, many times this would be, this would be a problem. There are a lot of places where a node would penetrate. What you say for example, it is a very good problem. It happens in many softwares also. The software people who write the software should take care of it. Let us say that, where is my duster? Yeah; let me take the same problem. Let us say that I am doing an axi-symmetric problem.

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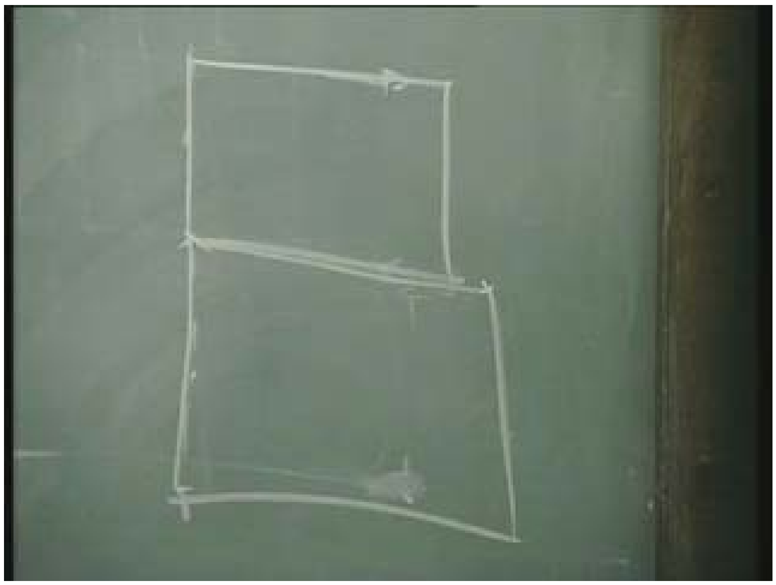

This is the axis of symmetry, that is a axis of symmetry. This you say, what does it mean? It means, it is a ring. The ring is now sitting on a block. This is a master node, that is a master node many times and there is a node, corresponding node, sitting in the axi-symmetric part of it. If both the nodes have the same coordinates, have the same coordinates, this node, this slave node will start penetrating inside. Many times this would happen and hence people who write softwares will adjust this. This is only a beginning. There are lots of issues here in contact. We will again look at contact in the next class and also certain nonlinear algorithms.