Introduction to Finite Element Method
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Lecture - 25

Let us now look at some heat transfer applications of finite element analysis.

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As I told you in the last class, thermal problems or heat transfer problems, …. lot of applications for manufacturing. I am going to concentrate only on manufacturing, not actually the thermal problems which are say dealt with in other areas of thermal engineering, for example, say heat exchangers and so on. Though the concept, which I am going to teach you is exactly the same as what you are going to apply or what you can apply for all these problems; there is no doubt about it, there is no questions about it. But nevertheless, after I teach this and when we go to case studies, I am going to give only problems from manufacturing. As I told you in the last class itself, the thermal problems application to manufacturing is very important and is the major step in welding problems, welding casting and so on, welding, casting and so on.

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The thermal problems can be broadly classified into what we call as steady state and transient problems. I hope all of you know what a steady statement problem is and what a transient problem is. Steady state problem is one where the temperatures do not change with time. Transient problems are ones where the temperature is a function of time. Is that clear? I hope all of you have studied this to certain extent in your heat transfer courses. Probably, some of you would have looked at the governing partial differential equations quite closely. May be, a few of you may know the type of governing partial differential equations for a steady state problem and a transient problem.

What are the types of partial differential equations? Can someone tell what the types of partial differential equations are? No, no, no; types of partial differential, say for example, it can be classified into elliptic, parabolic and yeah, hyperbolic. This is the type of partial differential equations under which you can classify any PDE and that the steady state problems are governed by what type of equations? Do you remember? Elliptic; very good. You know that steady state heat transfer equations are governed by elliptic partial differential equations and transient heat transfer equations are controlled by what are called as parabolic type of partial differential equations. This is just to review what you know and that this is what we are going to study.

In the first lecture, in today's lecture, hope we will be able to complete it or else it may spill over tomorrow or next lecture. In this lecture, we are going to look at the fornulations for steady state and transient phenomena and this we will follow it up with an example. Is that clear? We will follow it up with an example. Before we go to the finite element part, I have summarized here the equations, the governing equations, which are important for the analysis of heat transfer. Look at the first equation.

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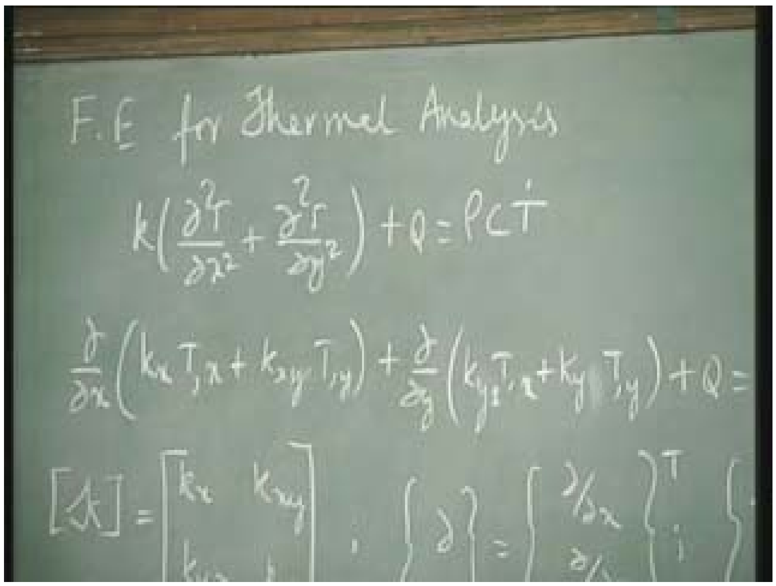

Are you all familiar with this equation? This is the heat transfer equation, which all of you would have seen. Any doubt? What is k? Thermal conductivity, density and specific heat. This is what we call as the governing differential equations, differential equation and this is very similar to, can you pick up a similarity between this and what we have done to our equilibrium equations that we have studied for stress analysis and so on? This is a general equation, sorry, or rather this is specific equation and this equation can be generalized a bit.

What do I mean by generalization? What do I mean by generalization?

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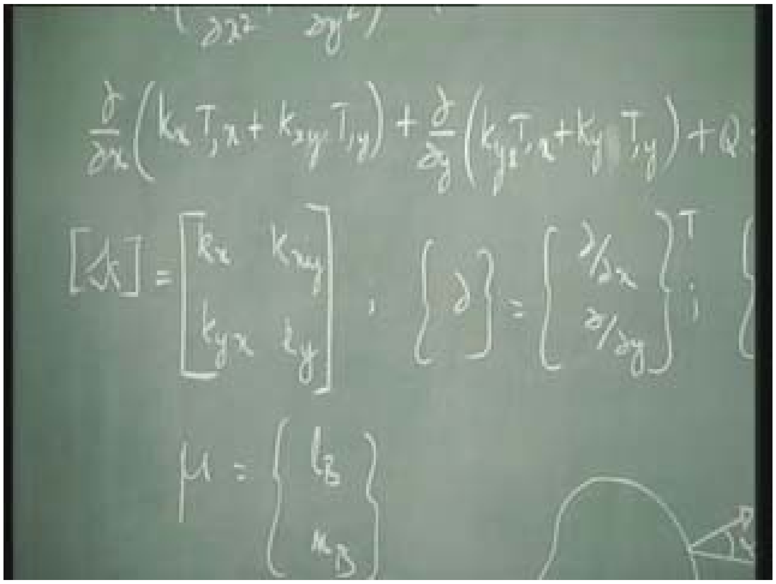

k here for example has been assumed to be the same or the material property k has been assumed to be isotropic. It is the same in all directions. You can in fact say that k may not be a constant. k may vary from one direction to the other. k can be looked at like a tensor with k_x k_{xy} k_{yx} and k_y and you can also look at k to have what are called as principle directions, k_1 and k_2 and then in fact in any material, we can have principle directions of k, k_1 and k_2 in which case, this matrix which I have written here for k where it is k_x k_{xy} k_{yx} k_y can be written as a how do I write? If it is principle directions, diagonal matrix; correct. So k_1 and k_2 zero and zero here.

It is very important to understand this. Why, because, why do you think it is important to understand the principle directions and this kind of k_x k_{xy} and so on? Why do you think it is important? I wanted to give an answer, but I thought that it is better that I know whether you have understood this. Because, principle directions of k_1 and k_2 is fixed with the material or the component. x y z axis's or axis's chosen by you.

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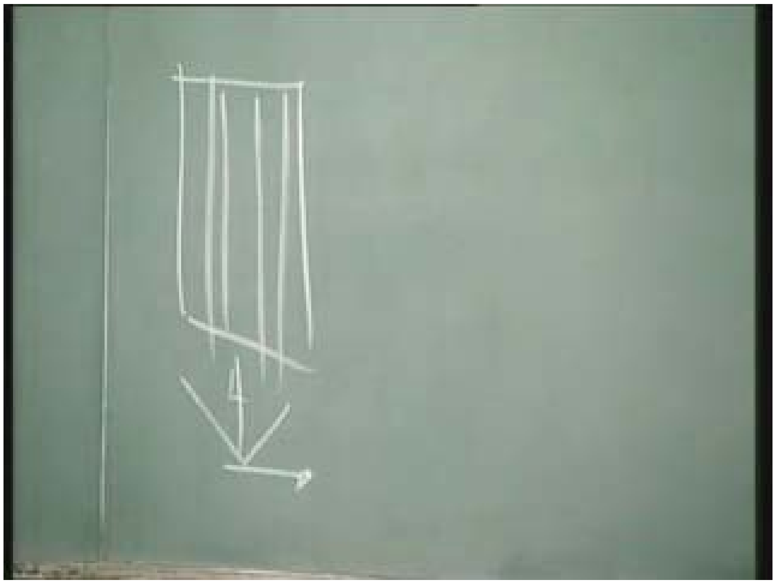

Say for example, I am going to do a problem, where some fiber is running like this and that the k values in this direction and in this directions are say, different and say they are the principle directions. Say, one direction is 1 and the other direction, is say 2. Suppose I say like this and then these two are fixed 1 and 2 directions are fixed. But due to some reason, it is not necessary due to some reason, I may shift my x and y axis. In other words, it is not necessary that I should have my x and y axis's to coincide with 1 and 2 due to some reason, you know. It may be I have just put a simple figure, where x and y can be chosen like this, but the picture may be more complicated than this. Then, I may choose, I may choose x and y to be say in this direction, at different directions. Then, it is important that the thermal conductivities are now expressed in terms of the x and the y coordinate system which I have chosen. Is that clear and hence we write like this. But, if you are doing normal metal, you know like in the sense that, say steel, most of the times we are not worried about the anisotropy of these metals, in which case how do I write this thermal conductivity matrix? This matrix, how do I write this?

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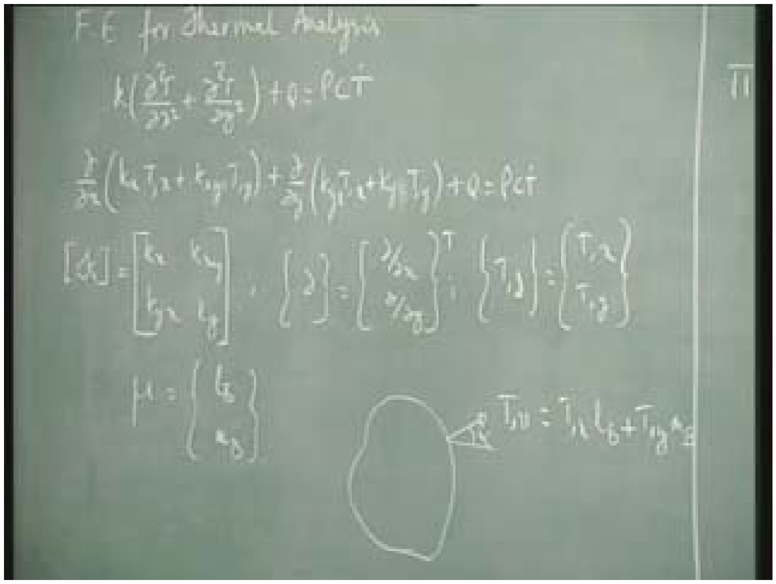

k k zero zero that is it. k into 1 1. This is just to make this equation to assume nuch more generalized form. So, in a very generalized form, this equation can be written in this fashion. By the way, what is q? Heat generate, what is it? I mean what is it, what is the unit? Now, what is it. Is it rate of heat generation or? I would like you to go and have a look. I am not going to give you the answer. All sorts of answers I am getting, so, have a look at that; rate of heat generation, internal heat, yeah, rate of internal heat generation. Have a look at that before you come to the next class, because you should be clear. These are all very fundamental things. You should be clear as to what are the terms that go into the picture. In a very general case, I can write the equation in this fashion. Since I have committed myself to the task that I will do manufacturing problems, there are other aspects as far as this equation is concerned, which enters into the picture.

What are the other equations or other aspects that enter into the picture? What do you think are the other aspects? Because, I have committed myself to look at say welding problem, what do you think are the other issues? We are now looking only at thermal problems. Expansion is important. When does an expansion become important? Because, when we look at a thermal problem and then have this as an input for a mechanics problem to look at deformation, then my coefficient of thermal expansion becomes important. But there are other things as well you know. Just think about it a minute. Like, we are looking at manufacturing problem. What is so very great about manufacturing, say welding problem or a casting problem?

Let us just look at these two problems. What is so very different from other things? Yes, that is it, very good. The temperatures are very high. It can vary quite a bit and hence k rho C_p all these things may depend upon temperature. This will make the problem highly non-linear. The k may not be a constant, will not be a constant. Why may, will not be a constant? Rho, C_p, everything would change with temperature. It is important that you realize this fact that k, whatever thermal or properties that we are talking about may not be a function or may not be a constant, but may be a function of temperature and that has to be known.

See, many of the failures of doing sinulation of manufacturing process comes about because of two things. One is not understanding first of all, that these values are not a constant. That would make a huge difference. It can vary two times, three times and so on. Number two is to get proper property values. That is also important. That is also important that you get a proper property values. We have defined what we called as the governing differential equation; defined what is called as the governing differential equation. What does this governing differential equation? Let us forget about this for a minute. Governing differential equation require, what is it that we should have along with it? Boundary conditions. What are the types of boundary conditions that you can have? What are the types of boundary conditions you can have? Let us look at this picture.

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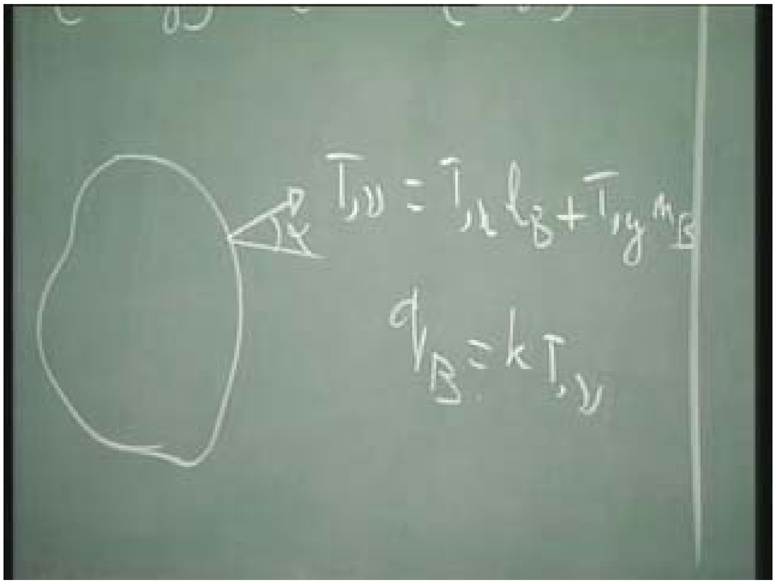

Let us now restrict ourselves to a plane problem. We will extend the other things later. Let us just look at this as a plane problem just for illustrating the boundary conditions per say. Now, what are the types of boundary conditions that can exist, that can exist in a problem? Heat flux and temperature; so, a part of the boundary, in a part of the boundary there can be heat flux that can be given. Let us call this as, say q_B. Let us say that this is equal to q_B. How do we write q_B, say the heat flux, may be say in this direction? How do you write q_B, the heat flux term? How do you write it? k into dow T by say dow n or dow nu. Keeping the heat flux to be positive, when the heat flux gets into the material, we can write this as say k into T comma nu.

We will just see that how to modify it for more practical manufacturing problems. How does What shape does it take usually? One is that you can look at it like this or you can specify heat flux directly. Let us just come to that in a minute, but T comma nu can be written as T comma x l_B. What is l_B? What I am trying to do is to just chain rolling. Dow T by dow nu is equal to dow T by dow x into dow x by dow nu plus dow T by dow y into dow y by dow nu. Let me call dow x by dow nu as l_B and dow T, sorry, dow y by dow nu as m_B and let me call this as mu, l_B and m_B. These are things which you already know.

Is that the only thing? Let us, I mean, let us first theoretically put down the boundary conditions and let us look at the boundary conditions more closely in a minute. Is that the only way? Is that only way to specify the boundary condition? Convection; let me capture this in a much more theoretical perspective. The boundary conditions are classified into what are called as essential boundary conditions and non-essential boundary conditions; essential and non-essential boundary conditions.

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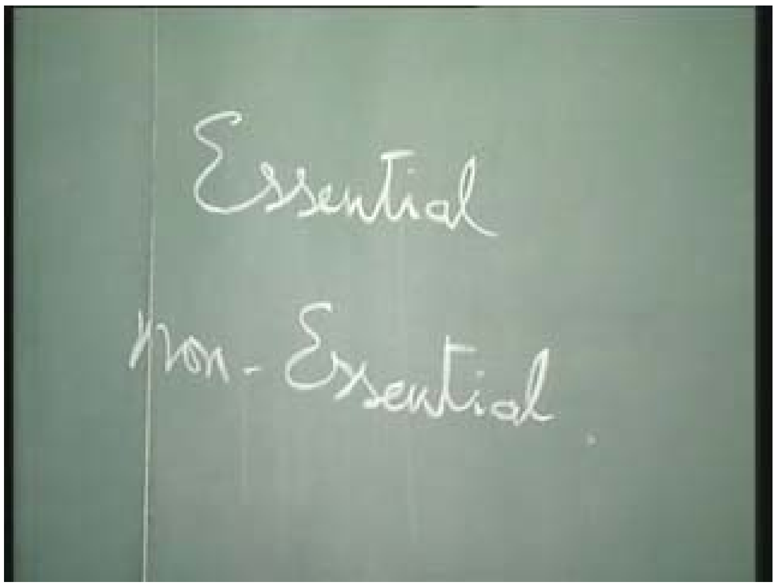

Let me ask you a small question. Let us not worry about what is essential and what is non-essential? We will come to that in a minute, you know. Before we proceed there, let me keep you thinking of it. Let us assume, let me assume or let me tell you that I have a body and that I am specifying the q_B by this formula throughout the surface. I am specifying it like this. Can I solve this problem by whatever means, by say finite element, because we are looking at finite element? Can I solve this problem directly and can you pick an analogy between your answer and what you already know? Do you understand? See, you already said that the boundary conditions can be temperature or heat flux. Heat flux, say for example, I am giving it in terms of a gradient like this. k into dow T by dow nu that is the heat flux term.

Suppose I say that this body, I am taking a body and that I am giving, I am giving the boundary condition to be only consisting of q_B or this. Is it possible for you to solve the problem? If yes why, if no, why not? No; very good. So, it is not possible to solve the problem only by specifying this. Why? You are bringing in a different effect. Let me explain this and come back to this question. Your no is right, but your explanations are not right. Let me come back to this. Let me go back to this steady state and transient. I thought you know it, but anyway, I will just review this.

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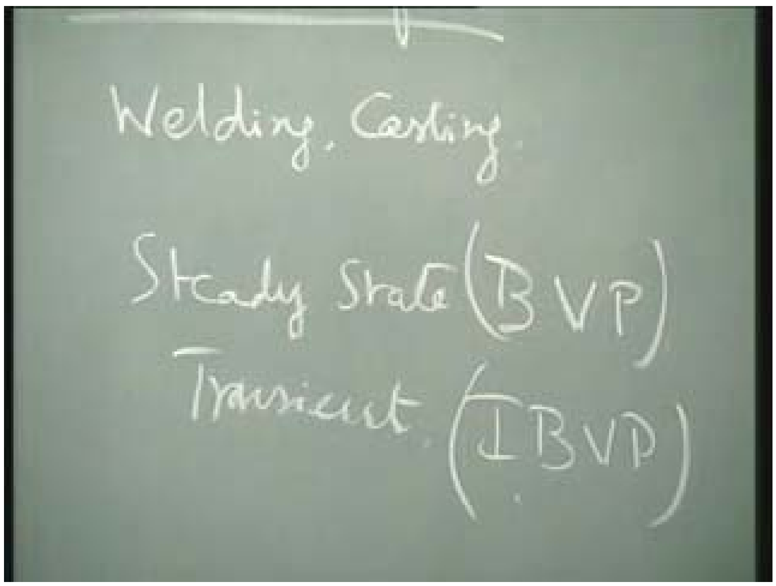

Because of your answer, I thought it is important that see, there are two things. The steady state heat transfer, as I told you before, is governed by elliptic partial differential equations. They are called the as boundary value problems, BVP. Many people call this as BVP, boundary value problems. In other words, what this requires is the boundary values, the values of say, temperature or flux or whatever it is, that is what we are discussing now, that has to be given at the boundary and these problems are called as boundary value problems. Do they require an initial condition? No; so, they do not require an initial condition because we are looking at steady state.

We have forgotten about transient. Transient has happened, we are in the steady state condition. Many problems are of importance in this region, steady state condition and hence these problems do not require initial conditions. They only require boundary values and hence these problems are called as boundary value problem. Is that clear? On the other hand, look at the transient problems. On the other hand, look at the transient problems. The transient problems are called as initial boundary value problems. Initial and boundary values, both of them have to be given, so, I would say initial boundary value problems. Is that clear?

These are the things that have to be given; initial value, initial condition at time T is equal to zero and for the steady state problem, I have to give only the boundary conditions. Here also, I have to give the boundary conditions. Of course, I have to give the boundary condition, but apart from the boundary condition I also have to give the initial condition. I will come back to this, because I think you have to catch this practically. May be, you have solved heat transfer problems, but more, see, my problem is that many students are very good in heat transfer, but many students look at heat transfer problems as a mathematics problem; as an initial value problem, as an initial boundary value problem, as a mathematics problem. There is lot more physics in it, especially when I come to manufacturing, there is lot more physical things that are happening and it is very important that you bridge this gap between what you know from heat transfer to what is happening in an actual manufacturing scene, you know. What is the analogy? It is very, very simple though. But, people make this mistake; I would like to explain that. But, I hope now you are clear that these are the two things. One is boundary value problem, initial boundary value problem, initial plus boundary value problem.

Let us come back to this. Now, let us just have a look at this. I think the clue is here.

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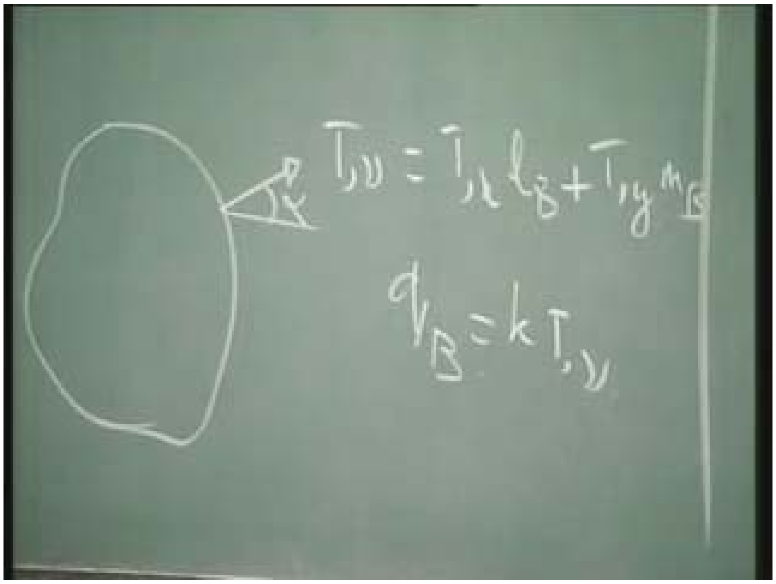

Why do you think, yes, what you say is right. I cannot solve a problem straight away by looking at this alone; absolutely right and you know the answer, because we have done this before. In another circumstances we have said that it may not be possible to solve it. Clear, anyone with a clue? No; yeah, will not be sufficient, but where are you make that is what I said, I agree with that answer. Yeah, I am saying, see okay fine; let me put it, let me give you another example.

Just to, just to, I said that or someone said beautifully that temperatures can be specified. Suppose I take the same body. I specify only the temperatures; I specify only the temperatures around, right now, I am specifying only temperature. Only one boundary condition I am specifying. Now, do you think that it can be solved or not, because what he said, answer he gave was that since you have given only one boundary condition, I cannot solve this problem. Now, I am saying that as you people told that, the boundary conditions can be classified into temperatures and flux, now I am saying fine, I do not want flux, I want only temperature. Now, can you answer whether I can solve this problem or not? You can solve it? No, yes; see, this is very important for an engineer to realize how you can solve this problem; they are the very, very fundamental.

If I give temperature, yes, yes majority say yes, but then why you solve it when I give temperature? Then you beat his answer, because now I have given only one boundary condition. No, no, no; I am not worried about the type of problem. I am only worried you know whether I am okay giving this boundary condition or the other boundary condition. That is all. Afterwards let us look at how to convert the physical situation into a boundary condition, you know. That we will see it later, but at this point of time, I just wanted to know, why? The answer is actually very simple. Let me say, let me give you a clue, because I want, I mean, this is not given in text books you know; this comes out of understanding a feel, you know, for this.

Let us look at this. I am looking at, say I am looking at a very physical answer. Let us look at this. No, this consists of, let me no, no, no, wait, wait, wait, wait; so, let me give you a clue. I think you are off tangent, but it is, I am happy that you are all thinking. But let me give you a clue. Let us look at this. What is this? This is a slope, so, I am specifying a slope. But what is that I want ultimately in this body? I want the temperature distribution, a unique temperature distribution. When I specify only q_B, what is that I am giving? I am giving, in other words, I am specifying a slope. Now, do you catch it? Not yet? Yes, that is it. So, it is similar to constant of integration.

I can give a slope here, 40 50 60 70, some value say, 70, whatever be the units. I can make the body. I will not get a unique temperature because, I have something like constant, exactly like that you know constant of integration. The body can be at temperature ranges, say for example, from 200 to 300, still have the same slope and still have the same q_B. The body can be from 500 to 700 or 650, still have the same slope and so on. In other words, I am not fixing the temperature, uniqueness of temperature is not specified. Why I do not get a unique solution by specifying q_B alone? Is that clear? Is that clear? So, the temperature part becomes important. The temperature part takes in what is called as the essential boundary condition, the heat flux part, heat flux part gives you the non- essential boundary condition.

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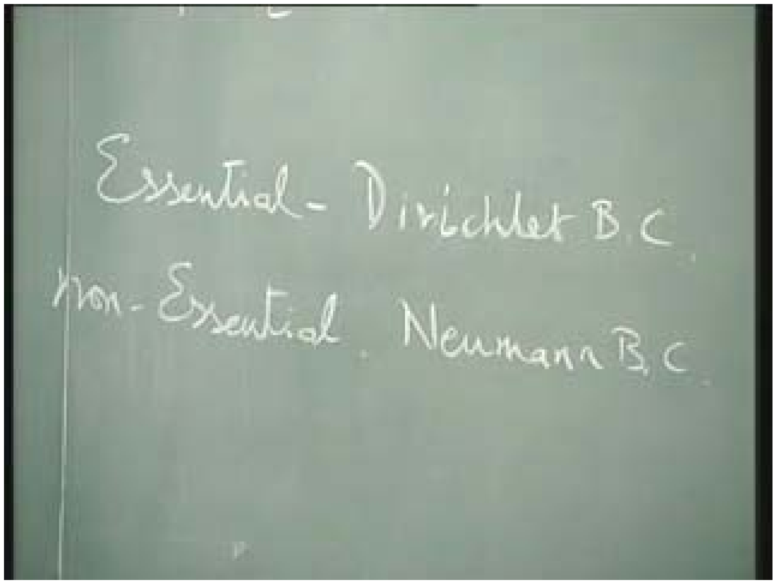

People call this essential boundary condition as Dirichlet boundary condition, I think Dirichlet boundary condition and the other one as Neumann boundary condition. Of course, because you are uniquely specifying, so, the question is can you solve it only if I give temperature? Of course, you can specify the temperature alone and you can solve it because, you can get a unique value. I am looking at it only physically. I am not going into the parabolic partial differential equation and so on, that aspect but I am looking at it more physically. So, if you specify temperature you can solve it.

I am going to confuse you a bit more, because this is important for us to later put boundary conditions. Suppose I am doing a problem where say welding, I am very interested in welding. I have done some work in welding; it is a very interesting topic. I will take a sheet; say for example, I take a sheet.

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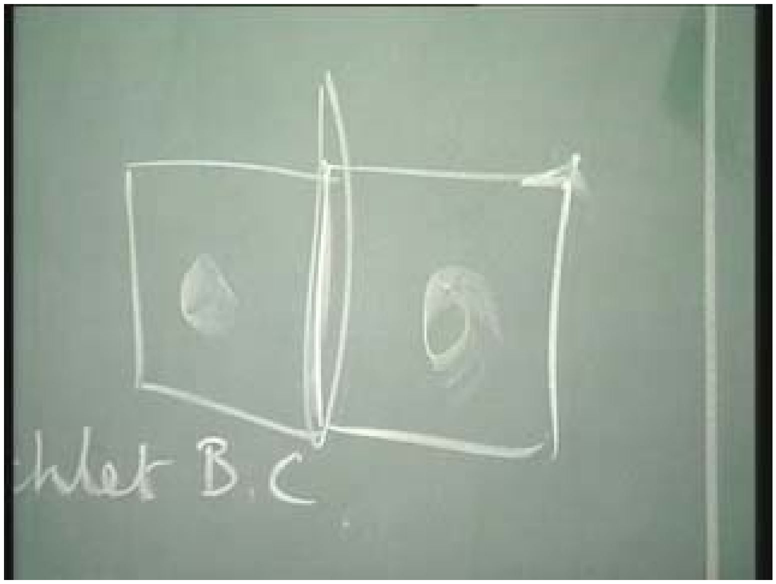

I am going to show you this example later and now this sheet I will weld it with another sheet. These two sheets are joined together and welded. I am welding it at this place. It is very clear looking at the problem that I am going to give heat inputs in this region. That is fine, q_B is given. Now, what happens in these regions? If you physically look at the problem, tell me what happens? There will be convection, heat transfer. So, convection that means heat flux is going to go out. There is going to be convection heat transfer. I am going to put down here a boundary condition of the form, how do I put it down? How do I say for example instead of this I am going to say that q due to convection is equal to, let me write it here so that it is clear.

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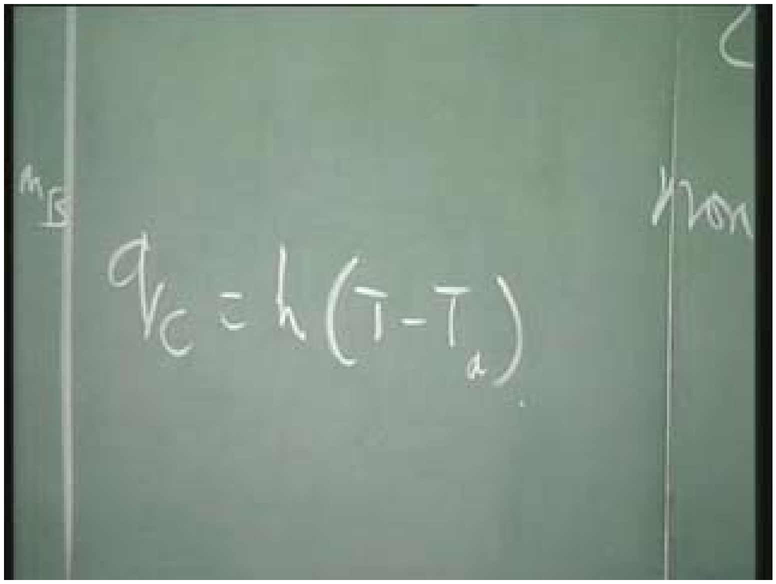

q due to convection is equal to h into T minus T infinity or T atmosphere or whatever you call it. What is it I have done? Instead of q_B in these areas, I have now replaced this by a convection condition. Now, can I solve this problem? Please note this carefully. You can see this sheet; you know, you would have seen it 100 times, just take two sheets and weld it. I have told you that there is an essential boundary condition called temperature, because I want to fix it. I want to say that look, this is going to be the unique value, so, I need temperatures. I have told you this and if I give you only the heat flux term, I have already told you that look, you are giving a slope as correctly told by one student that the integration, it becomes a headache or the constant of integration becomes a headache and so unique value cannot be obtained; all those things are very correct.

The question is I am asking you is look at this sheet and physically you see it. Where do you put now temperature boundary condition here and how am I going to solve this problem at all? Do you understand? Let us forget about the element type used and all those things. As you correctly said that there is going to be heat transfer from this region. If that is the case, how am I going to put temperatures here? If I am not going to put, one minute, let me complete. If I am not going to put temperature here, how am I going to give the boundary condition to solve this problem or can I solve this problem at all is the question? Is it clear? Yes; so, just look at it and tell me. Surrounding temperature is what I have. Can I put on the surface, the surrounding temperature?

I cannot put, obviously. If you go and touch the surrounding temperature say 35 degrees; in Chennai it may be 40 degrees or whatever it is, always hot. So, you cannot, you cannot say 40 degree, I am used to it, I will go and touch this place. It may not be possible. One minute, one minute; just a second, let me finish it. So, you cannot put as boundary condition on the surface, the atmospheric temperature, obviously; that is totally wrong. Then, I will get the same temperature throughout the bar, because there is heat transfer that is taking place from the body to the atmosphere. Yeah, what is your answer?

No, no, no. That is correct; no heat transfer, we will come to that afterwards. There is symmetry and I am going to take half the problem. What you say is absolutely right. We will come to that later. I will explain that in a minute, but here surface, I mean, surface is exposed to atmosphere. So, my question is this. Do you understand the question? My question is I am contradicting myself. I said essential boundary condition is temperature, then only I will be able solve the problem uniquely. You all agreed with me and I said that non-essential boundary condition called Neumann boundary condition is where I give q_B and I said you cannot solve a problem giving this alone, you all agreed. Now, I have posed you a problem like this.

Now I am asking you, if you have agreed with me on these two counts. Tell me how do I solve this problem? How do you find out, because, that is a good thing? Now you are saying that use this equation to find out surface temperature.

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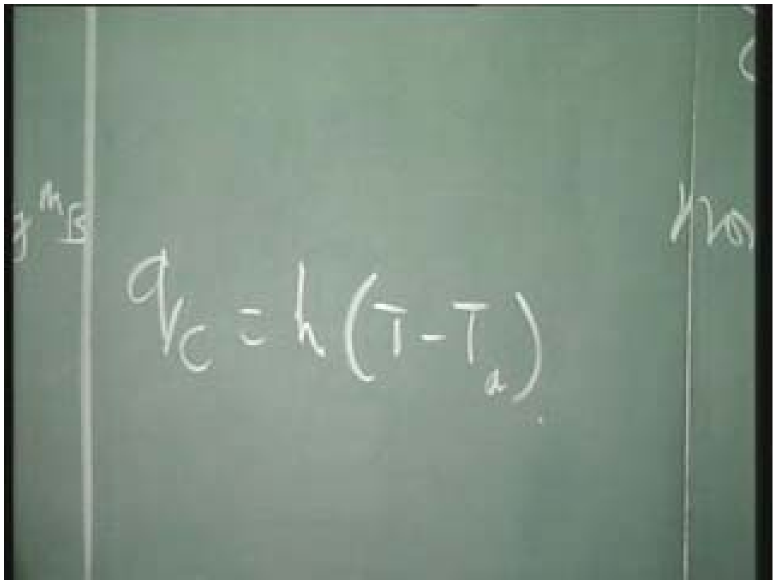

How do I find out surface temperature by using this equation? See, the equation can go only from right to left. It is not very easy to make it go from left to right. Knowing q_C, you can get T. But, how do you know? q_C is what I can get knowing T. If at all I know outside, see if at all I know T, my problem is solved. What am I trying to do? I am trying to find out the temperature distribution in this body. I cannot go from left to right; I can go only from right to left. So, that is not valid. Yeah, no, no. See, look at what I want? I want to know number one, can I solve this problem given this? If so, how do I beat, how do I beat my earlier problem of specifying only non- essential boundary conditions?

I am asking you, that is the question. What is the boundary condition I should give, can I solve it? That is exactly what I am trying to ask you. See, how do I give, do you understand everything? Everything is put, is there anyone who is confused about this? Is this a steady state problem? That is a good thing, you know. Is this a steady state problem? Is this a steady state problem? Why do you say it is a steady state problem? Someone said yes. It is obviously not a steady state problem, because how am I going to weld this? I am going to, I am going to change or I am going to charge this fellow, the arc to move along this direction.

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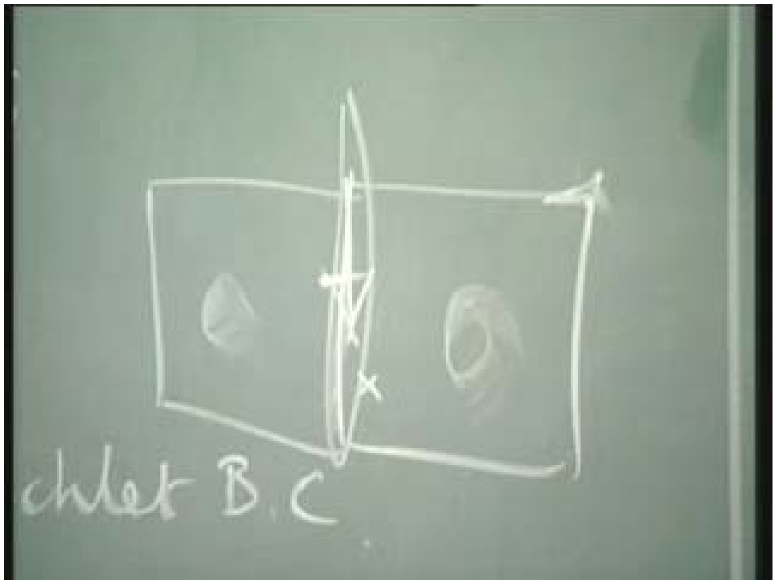

Obviously, when I am, when I am sitting here and say monitoring the temperature, initially I am happy, I am not hot and as this weld pool moves near me, towards me, what is going to happen? My temperature is going to increase. So, it is a question of temperature being a function of time. So, obviously this is not a steady state problem, it is a transient problem. Yeah, initial condition is required, I agree. You know initial condition is, let us not skirt the issue. You know, you have not still answered my previous question.

Yes, that is correct. So, let me rephrase your answer. That is very good. In other words, what it means is that there is a big difference between giving this and giving q_B. The big difference here is that I am fixing the slope with respect to T atmosphere.

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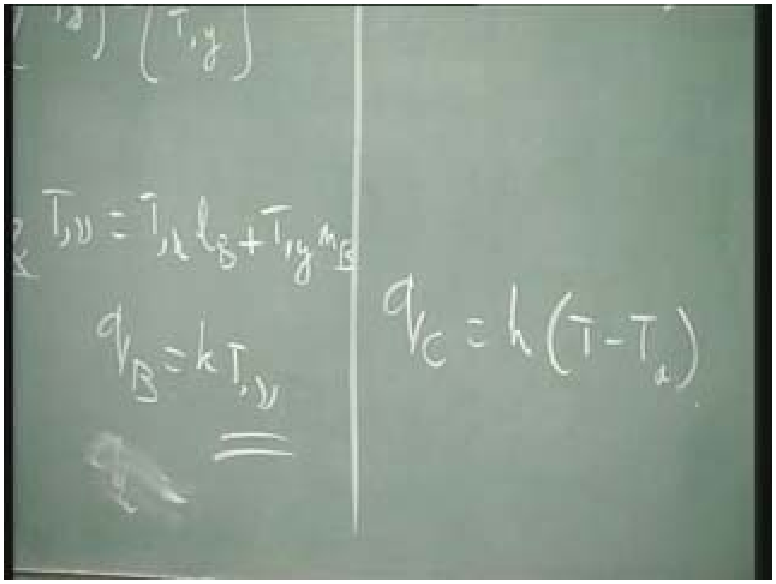

So, the uniqueness is obviously satisfied automatically, because I am fixing the slope or in other words, something like giving that constant, that integration constant. I am fixing the slope. When I say the slope is 300, one side is obviously 40 degrees, period. I am fixing the slope, so, you have to be careful in saying that if I give heat flux, q_B, I cannot solve it, but at the same time, I can remove singularity. That is the word, when I specify heat transfer coefficient or convection boundary condition. What is this like? This is similar to your force and displacement boundary conditions, which you have given in your earlier classes, where I told you that if I give force, I will not be able to solve the problem, force alone. I have to have displacement or else I will not have uniqueness of result. On the other hand, if I specify this kind of q_C for this problem, then I will be able to solve the problem without much issue. Is that clear, any question? Now is it clear? No, not clear?

Let me recapitulate. See, this is important for doing all practical problems. Let us recapitulate. I said, you tell me whether you have understood this point. I will just tell one point after the other. I said giving q_B alone it cannot be solved, understood one. I said that this is basically because there is a slope that is maintained or dow T by dow nu is given or dow T by dow n, however you call it, that is what is given. Then, that is not unique, because when I integrate this T plus a constant will come. So, when I specify just the slope say 300 or 400, slope is 400; it can be between, say, 500 and 100 or it can be between 600 and 200 and so on. So, it is not unique. So, by specifying slope alone, I cannot solve the problem. Agreed? That is what I said. I said if you give temperature this problem does not exist. Agreed? Fine.

But I said that in practical problems, as a boundary condition, I may have to give this convection as a boundary condition. Convection is something like heat coming out. Obviously you know it, but I asked the question whether I can solve this problem. Then, I said that yes, you can solve it or one of you answered it very well that it is possible to solve it, basically because, now it is not that I am specifying slope, but I am specifying a slope with respect to T_a. Say, when I said 400 and 500 and 100, 400 is the slope and it can be 500 100, 600 200 and so on. Here what I am trying to say is, I am not only specifying 500, but I am also specifying the lower limit 100, so, obviously that higher limit is fixed that is T.

In other words, this fellow here pegs down the result, physically. I want you to understand physically; there are other things to it. Physically, you can say that this fellow here pegs down the result. Is it clear now? So, these are the boundary conditions that you give. It is very important you recognize what is a boundary condition you give and one of our friends here said that what about initial condition? Obviously, this problem you have to specify what the initial condition is, as well. What can you take as an initial condition?

Ambient temperature; unless you do some preheating or you do something else, we can assume that the sheet has what we call as the ambient temperature. Let us just go ahead a bit to understand this problem. How am I going to give this heat flux? The torch that is going to move due to arc welding, how am I going to give this heat flux? This is the q input. This can be directly given. I need not have to go to k and so on. This is the heat input due to arc. It can directly be given as q_B and there are distributions to it. Say for example, you can say that the heat or the energy that you give in order to weld it, can be distributed say in a normal sense with respect to distance.

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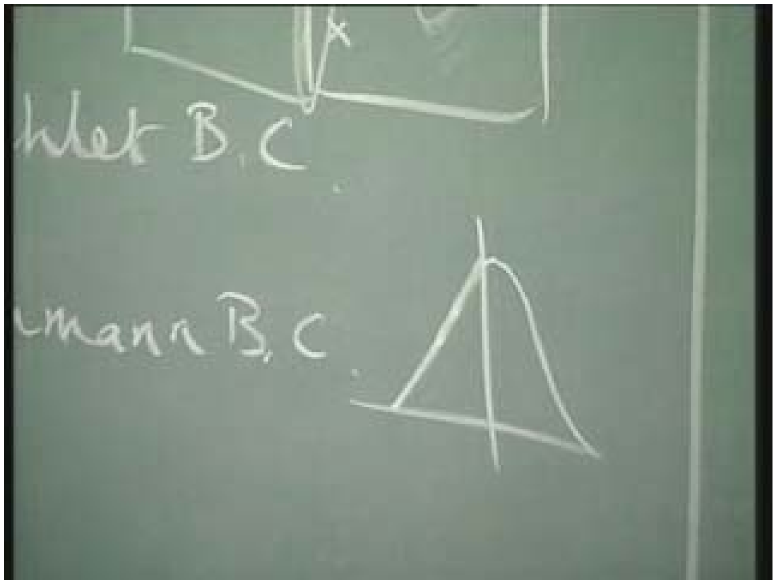

Say for example, you can say that the heat or the energy that you give, in order to weld it can be distributed say in a normal sense with respect to distance. We will talk about this again, later in the course or may be after we go through some theory. Right now, it is important to understand that it is possible to give this. We can give this as an input. The next issue that someone said is symmetry. In other words, what he means to say is why do you want to solve this problem taking both the sheets and that if you take only one sheet, that is a very good answer, if I take only one sheet that suffice it to analyze the problem.

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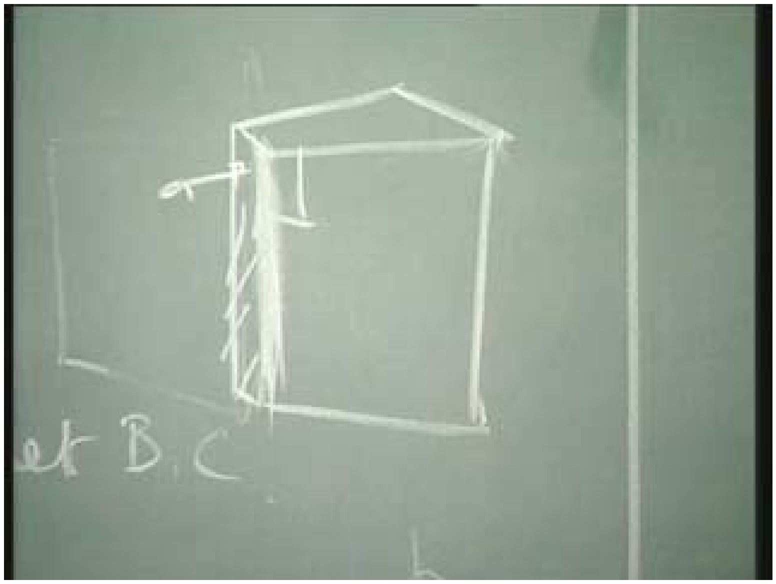

But, the only condition that I have to put here, in order to solve this problem is that this side, from this side there is no heat transfer or this side can be considered as insulator, because the other side it is like symmetry boundary condition. Other side will have the same temperature distribution, so, the slope there at that point will be zero and so there will not be any heat transfer coefficients. Yes, heat input is given in this nodes perpendicular to it. We will talk about the elements later, but let me put it like this. Let us say, let me put the third dimension. Now, heat input is given say in this region, at the top face, say xy face and symmetry boundary condition is applied in the other face, which is perpendicular to it, that phase. Is that clear now? That we are not giving. Clear? That is the practical things that we have to achieve. Knowing this or actually our job is very simple. Next step is very simple.

What is it that I have to do? For the governing differential equation, I have to write down a functional; I have to write down a functional. So, a functional, let us not go into the details of how to derive this functional. It can be very well nicely derived. But let us just say that this is the, have a look at that. This is the functional that we are going to put.

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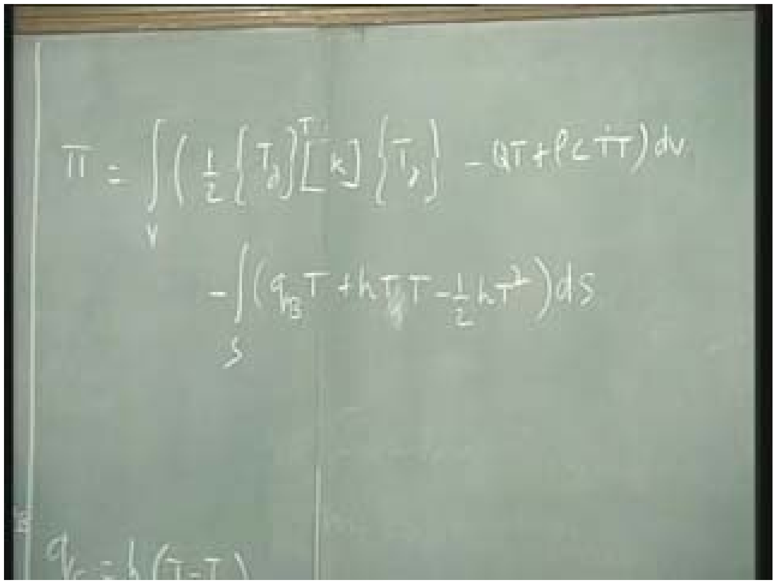

It consists of all the terms that you want and T_f is the final, actually this T_f is T_a or so on, all the types of boundary conditions in sense that both q_C and q_B are there. T_j sorry T_{delta} here is dow T by dow x and dow T by dow y. That is what I have written here, dow T by dow x and dow T by dow y. Del operator is dow by dow x and dow by dow y. We are only taking, looking at 2D, so that dv can be converted to dS, this to l. So, once we know functional, our job is quite easy. It is not very difficult. All of you by now are familiar as to how to convert this into a finite element form. What is more important is to understand what is the governing differential equation, what is the boundary conditions, how you give in a practical problem boundary conditions and so on, because once I come to that level and write down the governing functional, the problem becomes quite straight forward.

I want to convert this into a finite element form. So, what is the next step I am going to do? What is the next step I am going to do? Wait, wait, wait; you are jumping, you know. Let us go step by step. First thing is I have to write down d in terms of the nodal coordination. What is the field quantity or variable here?

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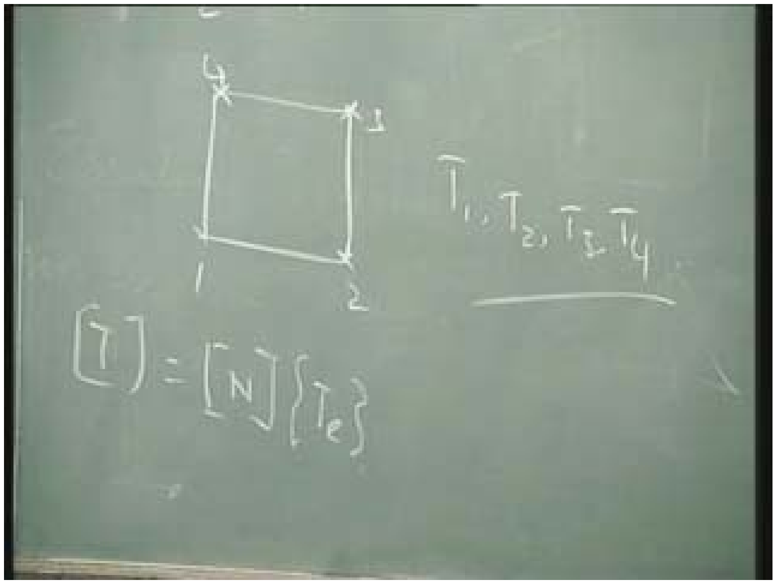

What is the variable? In those cases you had u v w and so on. What is the variable in this case? Temperature; so, the interpolation function or the shape function should be able to interpolate the temperatures. If I say that, here, T_1 T_2 T_3 T_4 are the unknown values or values to be obtained where 1 2 3 4 are the temperatures of these four nodes. So, we can write down T as N T in the same fashion as we did u v and w. Is that clear? T N, let me call this as T_e; T_1 T_2 T_3 and T_4. What else do we need in this? What else do we need? Look at this; look at the functional, look at the functional and look at this. So, what else do we need? Those are the things.

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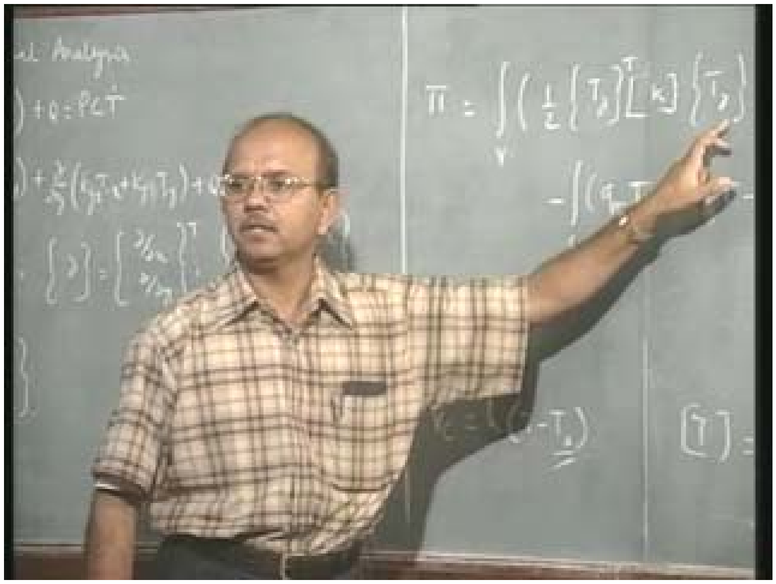

What is my, what is my strategy? Let us first understand my strategy. I will do that in the next class, but let me understand the strategy. What I am going to do is, I am going to expand or extend or write temperature in terms of nodal temperature, gradients in terms of nodal temperature, substitute it into my functional equation, then differentiate it with respect to T in order to minimize the functional, period. It is very, very straight forward or in other words extending, this can be written as just to, no, no, no. What did we call?

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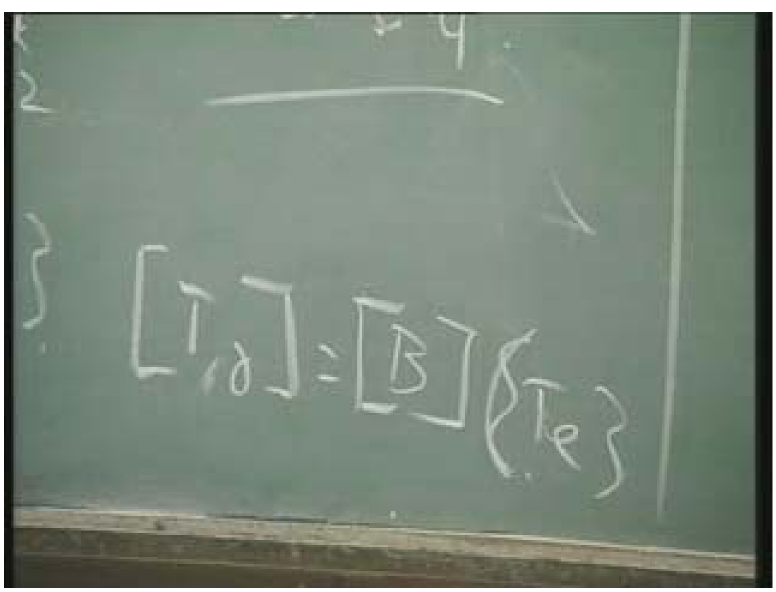

B; correct. That is it. So, this is similar to B into T_e, very good. So, we will develop this in the next class. That is exactly how things work. Substitute it and we will derive it. We will continue this, now that physically you have understood these things, we will continue with the derivation in the next class.